(1) A discrete state in the sum (2) with $p^2=M^2$, and $p_0>0$ leads to a term in $F(a)^2$ proportional to $\delta(a^2-M^2)$ and therefore to a term in $\Delta_F'(p)$ proportional to $1/(p^2-M^2+i\epsilon)$. Conversely, the existence of such a pole term in the propagator $\Delta_F'(p)$ implies the existence of a particle of mass M in the theory.

(2) A discrete state in (2) with $p^2=M^2$, $p_0<0$ leads to a term in $G(a^2)$ proportional to $\delta(a^2-M^2)$ and thence to a pole in $\Delta_F'(p)$ of the form $1/(p^2-M^2-i\epsilon)$. Conversely, the existence of such a pole in the propagator implies the existence of a discrete state in the theory with $p^2=M^2$, $p_0<0$.

(3) A discrete state in (2) with $p^2=-M^2$ leads to a term in $H(-a^2)$ proportional to $\delta(a^2-M^2)$ and therefore to a term in $\Delta_F'(p)$ proportional to $\delta(p^2+M^2)$. Conversely, the presence of such a delta-function term in the propagator implies the existence of a discrete state in the theory with $p^2=-M^2$.

It is thus seen that, while a pole in the propagator of the form $1/(p^2-M^2\pm i\epsilon)$ implies the existence of a discrete state with eigenvalue $p^2=M^2$, it is the presence of a term proportional to $\delta(p^2+M^2)$ rather than the presence of a pole term $1/(p^2+M^2\pm i\epsilon)$ that implies the existence of a discrete state with $p^2=-M^2$. What distinguishes the cases of positive and negative p^2 in this respect is the fact that the sign of the time is not an invariant for negative p^2. Whereas the term $-2\pi i \times\delta(p^2+a^2)$ in (6) can be written

$$-2\pi i\delta(p^2+a^2)=\frac{1}{p^2+a^2+i\epsilon}-\frac{1}{p^2+a^2-i\epsilon}, \qquad (7)$$

the two pole terms cannot be separated. On the other hand, the poles occurring at positive values of p^2 can be separated by means of the two separate functions $F(a^2)$ and $G(a^2)$.

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Predictions for $\pi^-+p\rightarrow\eta^0+n$ from Regge Poles and SU_3*

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It is assumed that the charge-exchange process $\pi^-+p\rightarrow\eta^0+n$ is dominated by the R Regge pole. A prediction for the cross section is then made, by taking Regge-pole parameters from a previous analysis of πN and KN scattering, and by invoking SU_3 symmetry.

THERE has recently been renewed interest in Regge-pole models for high-energy scattering, especially for processes in which the number of poles in the crossed channel is severely limited by selection rules.[1–4] One such process is $\pi^-+p\rightarrow\pi^0+n$ charge exchange, where of the known Regge poles only ρ can contribute; another is $K^-+p\rightarrow\bar{K}^0+n$, where only ρ and R contribute[5]; another is $\pi^-+p\rightarrow\eta^0+n$, where only R contributes. Explicit models have already been constructed for the first two processes, and the ρ couplings are found to obey the expected SU_3 symmetry.[4] The present note is to show that, by requiring SU_3 symmetry for the R couplings, we get a prediction for the third process, $\pi^-+p\rightarrow\eta^0+n$.

The models of Ref. 4 fit available πN data, using the P, P', and ρ Regge poles. KN and $\bar{K}N$ data are then fitted, with P, P', and ρ contributions restricted by the factorization principle, and with the ω and R poles added. The best solutions are found to obey SU_3 symmetry for the P and ρ couplings. The πN and $\bar{K}N$ charge-exchange data are particularly valuable in determining the ρ and R contributions, although the other data are also important in this.

Now, R is supposed to belong to an SU_3 octet. The coupling between this particular octet and the octet containing π, η, K, and $\bar{K}$ must be pure D type, to preserve charge-conjugation invariance.[6] Hence, at high energies, at which the η-π mass difference has negligible effect, the amplitude for $\pi^-+p\rightarrow\eta^0+n$ is essentially the same—apart from an extra factor $2/\sqrt{3}$—as the R contribution to $K^-+p\rightarrow\bar{K}^0+n$.

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[4] R. J. N. Phillips and W. Rarita, Phys. Rev. **140**, B200 (1965).
[5] R is the even-signature trajectory with $I=1$, $G=-1$, proposed by Pignotti. It is presumed to be associated with the A_2 meson.

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Consider therefore the R contributions to $K^- + p \to \bar{K}^0 + n$, as given in Ref. 4. There is a helicity-flip amplitude B and a nonflip amplitude A (following Ref. 7, where, however, the notation is B and A'). The differential cross section is

$$\frac{d\sigma}{dt}=\frac{1}{\pi s}\left(\frac{m_N}{4k}\right)^2\left\{\left(1-\frac{t}{4m_N^2}\right)|A|^2+\frac{t}{4m_N^2}\left(s-\frac{s+p^2}{1-t/4m_N^2}\right)|B|^2\right\}, \quad (1)$$

where s and t are the invariant squares of energy and momentum transfer, p is the kaon lab momentum, k is the c.m. momentum, and m_N is the nucleon mass. The R contributions to A and B are written[4]

$$A_R=-2C_0e^{C_1t}\alpha(2\alpha+1)\frac{1+\exp(-i\pi\alpha)}{\sin\pi\alpha}\left(\frac{E}{E_0}\right)^{\alpha}, \quad (2)$$

$$B_R=-2D_0e^{D_1t}\alpha\frac{1+\exp(-i\pi\alpha)}{\sin\pi\alpha}\left(\frac{E}{E_0}\right)^{\alpha-1}. \quad (3)$$

Here α is the R trajectory, $E=(p^2+m_K^2)^{1/2}$ is the total kaon lab energy, and E_0 is an arbitrary scale parameter, taken for convenience to be 1 GeV; C_0, C_1, D_0, and D_1 are coefficients which parametrize the residue functions. The trajectory α is given the form

$$\alpha(t)=-1+[1+\alpha(0)]^2/[1+\alpha(0)-\alpha'(0)t], \quad (4)$$

where $\alpha(0)$ and $\alpha'(0)$ are the value and slope at $t=0$. In Ref. 4, the various parameters for R, as well as other relevant Regge poles, are determined by least-squares fitting to πN, KN, and $\bar{K}N$ data.

Using these R Regge-pole parameters, a prediction for $\pi^- + p \to \eta^0 + n$ can immediately be made. Table I shows a set of parameters, representing a slightly modified[8] form of solution 1 of Ref. 4; Fig. 1 shows the predicted cross section at 10 GeV/c.

Let us consider how much uncertainty attaches to this prediction.

(*i*) *SU_3 symmetry.* If SU_3 symmetry fails for the R couplings by 10%, we may expect a 20% effect in the predicted cross section. This symmetry also enters indirectly, via the analysis of pole parameters. The numbers in Table I are a best fit if exact SU_3 symmetry is assumed for the $\rho\bar{K}K$ and $\rho\pi\pi$ couplings; if this relation is relaxed by 10%, the best fits predict cross sections that differ by less than 20% on the whole.

TABLE I. Parameters for the R Regge pole in K-N scattering.

$\alpha(0)$	$\alpha'(0)$ [(GeV/c)$^{-2}$]	C_0 (mb×GeV)	C_1 (GeV^{-2})	D_0 (mb)	D_1 (GeV^{-2})
0.32	0.80	3.1	0.4	−29	2.4

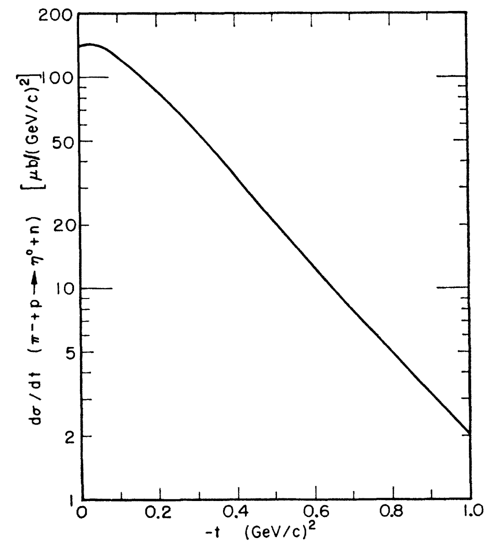

FIG. 1. Predicted $\pi^- + p \to \eta^0 + n$ cross section at 10 GeV/c.

(*ii*) *The helicity-flip amplitude.* The cross-section predictions are rather sensitive to this amplitude. If it increases by 20%, the dip at $t=0$ becomes more pronounced; if it decreases by 20% the dip vanishes. Our solution happens to lie in a rather sensitive intermediate region.

(*iii*) *Model dependence.* The four different models in Ref. 4 all give similar predictions for the R contribution, which suggests there is not much uncertainty in this respect.

(*iv*) *Energy-dependence.* The cross section $d\sigma/dt$ behaves like $E^{2\alpha-2}$; hence for our model it behaves like $E^{-1.4}$ at $t=0$ and like E^{-2} near $t=-0.5$ (GeV/c)^2. There is some uncertainty in α, and hence in the energy dependence, but this should scarcely affect our prediction at 10 GeV/c, which is effectively normalized to the nearby K^-p charge-exchange measurements at 9.5 GeV/c.

We understand that experimental data on $\pi^- + p \to \eta^0 + n$ have recently been taken and are being analyzed, by the MIT/Pisa and Saclay/Orsay groups.[9] The results should throw much light, both on the R-Regge-pole model and on the applicability of SU_3 symmetry in this context.

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[8] The analysis was modified by using final rather than preliminary K^-p charge-exchange data, and by requiring exact SU_3 symmetry for the ρ couplings.

[9] Janos Kirz, Lawrence Radiation Laboratory, R. K. Logan, Massachusetts Institute of Technology, and P. Sonderegger, Centre d'Etudes Nucléaires, Saclay (private communications).