Higher Symmetries and the Neutron-Proton Magnetic-Moment Ratio*

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A quark model of $SU(4)$ is developed in which the quarks possess charge $\frac{2}{3}$ and $-\frac{1}{3}$ and baryon number $N=\frac{1}{3}$. The new particles predicted are characterized by a quantum number $W\neq 0$ (superstrangeness); they possess integer charge but fractional hypercharge $\frac{1}{3}$ or $\frac{2}{3}$, and are called hyperquarks. A spin extension of $SU(4)$ is formulated and leads to the study of the group $SU(8)$ and the subalgebra $SU(4)\otimes SU(2)$. The baryons and isobars are grouped in the representation **120** and the mesons in the adjoint representation **63**. It is found that the F/D ratio in $SU(8)$ is uniquely specified by the scheme. The ratio of the magnetic moments of the neutron and proton is uniquely determined by assuming that the magnetic-moment operator transforms as the adjoint representation of $SU(8)$, and by specifying the extended Gell-Mann–Nishijima relation for the quarks. The value $-\frac{2}{3}$ is found for the neutron-proton magnetic-moment ratio in agreement with the result found in $SU(6)$. The selection rules forbidding processes like $\phi\to\rho\pi$ and $\pi+N\to\pi+N+\phi$ are obtained from the conservation of the four quark spins. These results strongly indicate that the physical predictions of a symmetry scheme like $SU(6)$ are not unique and that there exists a hierarchy of symmetries all possessing equally good (or bad) physical predictions, but with new quantum numbers associated with superstrange particles.

I. INTRODUCTION

THERE have been two main reasons put forward for studying a higher symmetry scheme of strong interactions. The first was to circumvent the fractional charge of the quarks by introducing a new additive quantum number.[1] The second was to explain the following facts which go beyond the $SU(3)$ scheme:

1. The origin of the ω-ϕ mixing.
2. The decay of $Y^*\to\Sigma\pi$ appears to be forbidden.
3. The decay $\phi\to\rho\pi$ is forbidden.
4. The reactions $\phi\to n\pi$ and $\pi+N\to\pi+N+\phi$ are known to occur at a reduced rate compared with the production of the ω.
5. The relation between the masses of the vector and pseudoscalar octet $K^{*2}-\rho^2=K^2-\pi^2$.

The second selection rule is definitely in disagreement with $SU(3)$. If the mixture of the ϕ and ω is prescribed, then the selection rules (3) and (4) are accounted for in terms of an apparently accidental cancellation of $SU(3)$ amplitudes. Any proposed higher symmetry must accommodate these selection rules and at the same time preserve the selection rules and properties of $SU(3)$. A higher symmetry will incorporate new quantum numbers that predict new selection rules which could disagree with experiment.[2,3] The introduction of the concept of associated production based on the strangeness number selection rule provides a clue to the nature of the higher symmetry. The pion and the nucleon were classified as particles having a zero eigenvalue of strangeness, and no reactions involving these particles were forbidden. Similarly, if the new additive quantum number, which we shall denote by W and call "superstrangeness," has a zero eigenvalue for the known particles, then no known reactions involving these particles are forbidden. Lipkin[4] has pointed out that superstrange particles should be produced as strongly as ordinary particles albeit at a reduced rate due to their higher mass values. This could indicate that the higher symmetry scheme is badly broken[5] although the situation is not yet clear. It is possible that there exists a heirarchy of symmetries going up to the highest energies.[6] The elementary-particle physics world up to about 800 MeV would appear to us as an $SU(2)$ scheme. Up to, say, 30 BeV the world would appear approximately $SU(3)$ symmetric; with the discovery of new resonances at higher energies we should perhaps be induced to consider a higher symmetry, e.g., $SU(4)$—and so on. This leads us to the question: "Does there exist a unique internal symmetry group of elementary particles of reasonably small dimensions?" To answer this in terms of particle classifications is begging the question, since these classifications belong to the specific energy regions available for study with current accelerators. An attempt to answer the question at a more fundamental level is exemplified by the bootstrap program in which the symmetry may be derived from the dynamical scheme.[7–9] Such attempts suffer at present from the difficulty of being based on special models.

It is our purpose to investigate to what extent the successful experimental predictions of $SU(6)$ can be

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[1] L. Van Hove, Royal Society Discussion Meeting on Symmetries of Subnuclear Particles, 1964 (to be published).
[2] A. Pais, Phys. Rev. **110**, 574 (1958).
[3] A. Pais, Phys. Rev. Letters **12**, 632 (1964).
[4] H. J. Lipkin, Argonne National Laboratory, Report 1964 (unpublished).
[5] M. Parkinson, Phys. Rev. Letters **13**, 588 (1964).
[6] D. H. Neville, Phys. Rev. Letters **13**, 118 (1964).
[7] R. F. Dashen, S. C. Frautschi, and D. H. Sharp, Phys. Rev. Letters **13**, 777 (1964).
[8] R. E. Cutkosky, Phys. Rev. **131**, 1888 (1963).
[9] E. C. G. Sudarshan, Phys. Letters **9**, 286 (1964).

repeated in a higher symmetry scheme. The $SU(4)$ model provides the simplest extension of $SU(3)$ as a strong interaction symmetry scheme and has been studied with different points of view by various authors.[10–16] The symplectic group $SP(6)$ has also been studied as an alternative scheme,[17] and recently Van Hove[18] has made a nonrelativistic spin extension of $SP(6)$ and $SO(6)$ to $SO(12)$ and $SP(12)$, respectively, for the case of basic particles possessing integer charge.

The recent successes[19–22] of the extension of Wigner's supermultiplet theory of the nucleus to the elementary particles provides the motivation for deriving here a nonrelativistic spin extension of an $SU(4)$ quark model of strong interactions. The group used in the theory is $SU(8)$ which has a subalgebra $SU(4)\otimes SU(2)$. The group $SU(8)$ also has as subalgebras $SU(6)$ and Wigner's $[SU(4)]_W$.

The particles with $W\neq 0$ have integer charge but fractional hypercharge ($\frac{1}{3}$ or $\frac{2}{3}$) and we call these particles "hyperquarks." These particles are produced by associated production and occur as triplets and sextets.

The basic result obtained in the scheme is the famous ratio of $-\frac{2}{3}$ for the ratio of the neutron and proton magnetic moments.[21,22] This strongly suggests that internal symmetry schemes like $SU(6)$ are not unique and that there exists a hierarchy of symmetries possessing equally good physical predictions which are *uniquely* determined in the spin–unitary-spin scheme as soon as the quantum numbers of the quarks are specified and suitable transformation properties are assumed.

In Sec. II, we study a quark scheme of $SU(4)$ in which the charge operator is not a generator of the group. The quarks consist of a triplet $t_i=(t_1,t_2,t_3)$ and a scalar s. They possess charges $-\frac{1}{3}$ or $\frac{2}{3}$ and baryon number $\frac{1}{3}$. The mesons are formed from a quark-antiquark system and the baryons from the direct, threefold product of quarks, as in $SU(3)$ and $SU(6)$. We do not concern ourselves here with the problem of whether or not the quarks have physical reality. In Sec. III, we take up the problem of the nonrelativistic spin extension of $SU(4)$ which leads us to study $SU(8)$. The particle assignments are discussed in Sec. IV. It is pointed out that the F/D ratio is determined uniquely by the scheme, as is the case in $SU(6)$. In Sec. V, the ratio of the magnetic moments of the neutron and proton is calculated in terms of the vector model and the ratio $-\frac{2}{3}$ is obtained in good agreement with the experimental value -0.684. This calculation is unique once the transformation property of the magnetic-moment operator is prescribed and the Gell-Mann–Nishijima relation for the quarks is determined. In Sec. VI, it is pointed out that the quark spin conservation in $SU(8)$ leads to selection rules which suppress processes such as $\phi\rightarrow\rho\pi$ and $\pi+N\rightarrow\pi+N+\phi$. This implies invariance properties in subalgebras of $SU(8)$. Section VII ends the paper with a few concluding remarks.

II. $SU(4)$ QUARK MODEL OF STRONG INTERACTIONS

Let us examine the basic properties of the $SU(4)$ scheme of strong interactions based on quarks. One starts with a four-component spinor

$$\phi=\begin{pmatrix} t_1\\ t_2\\ t_3\\ s\end{pmatrix} \tag{1}$$

which transforms according to the four-dimensional representation of $SU(4)$. The representation **4** has the $SU(3)$ decomposition

$$\mathbf{4}=\mathbf{1}+\mathbf{3}. \tag{2}$$

$SU(4)$ is the covering group corresponding to the rank-three simple Lie algebra A_3. The three conserved quantum numbers I_3, Y, and X are related to the charge Q by the extended Gell-Mann–Nishijima relation[12]

$$Q=I_3+\tfrac{1}{2}Y+aX+bN\,, \tag{3}$$

where we have included the baryon number N and a, b are certain numerical coefficients that depend on the specific model considered. The superstrangeness number is defined by $W=aX+bN$.

Other authors have restricted themselves to basic particles possessing integral charge. In the following, we shall study a new $SU(4)$ model in which the basic triplet and singlet form "quarks"[23,24] with nonintegral charge and baryon number. We choose

$$W=\tfrac{1}{3}X-\tfrac{1}{4}N \tag{4}$$

and adopt the quantum numbers associated with the quarks in Table I. The operator Q is not a generator of the group $SU(4)$.

The outer Kronecker product of **4** and **4*** gives

$$\mathbf{4}\otimes\mathbf{4}^*=\mathbf{1}+\mathbf{15} \tag{5}$$

[10] P. Tarjanne and V. L. Teplitz, Phys. Rev. Letters **11**, 447 (1963).
[11] Z. Maki, Progr. Theoret. Phys. (Kyoto) **31**, 331 (1964).
[12] D. Amati, H. Bacry, J. Nuyts, and J. Prentki, Phys. Letters **11**, 190 (1964).
[13] B. J. Bjørken and S. L. Glashow, Phys. Letters **11**, 255 (1964).
[14] I. S. Gerstein and M. L. Whippman, Phys. Rev. **136**, B829 (1964).
[15] I. S. Gerstein and M. L. Whippman, Phys. Rev. **137**, B1522 (1965).
[16] Y. Hara, Phys. Rev. **134**, B701 (1964).
[17] H. Bacry, J. Nuyts, and L. Van Hove, Phys. Letters **9**, 279 (1964).
[18] L. Van Hove, CERN report, 1965 (unpublished).
[19] F. Gursey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964).
[20] A. Pais, Phys. Rev. Letters **13**, 175 (1964).
[21] B. Sakita, Phys. Rev. Letters **13**, 643 (1964).
[22] M. A. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13**, 514 (1964).
[23] M. Gell-Mann, Phys. Letters **8**, 214 (1964).
[24] G. Zweig, CERN report, 1964 (unpublished).

TABLE I. The quantum numbers I_3, Y, X, N, and Q for the quarks in $SU(4)$.

	I_3	Y	X	N	Q
t_1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{3}$
t_2	$-\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{3}$	$-\frac{1}{3}$
t_3	0	$-\frac{2}{3}$	$\frac{1}{4}$	$\frac{1}{3}$	$-\frac{1}{3}$
s	0	0	$-\frac{3}{4}$	$\frac{1}{3}$	$-\frac{1}{3}$

and the mesons are assigned to the adjoint representation **15**. This representation has the $SU(3)$ content

$$\mathbf{15}=\mathbf{1}+\mathbf{3}+\mathbf{3}^*+\mathbf{8}. \tag{6}$$

The mass relations for the mesons have been calculated by several authors.[12,13,15] An obvious generalization of the procedure in $SU(3)$ is to assume that the mass operator transforms like a member of the adjoint representation. The resulting mass relation is well satisfied by the *inverse* squares of the masses as suggested by the model of Coleman and Schnitzer.[25] It is not well satisfied by the squares of the masses.[12,15] If the vector mesons and pseudoscalar mesons are assigned to the adjoint representation but the mass operator is assumed to transform as the octet part of the representation **20″**, then the mass relations are well satisfied by the squares of the masses.[15] Gerstein and Whippman[15] make the additional requirement that the model should explain the anomalously small decay width of $\phi\rightarrow\rho\pi$. They found that if they place the vector mesons in **15**, the pseudoscalar mesons in **20″**, and assume that the mass operator transforms as the octet part of **20′**, then the mass relations are well satisfied by the squares of the masses and the decay width $\Gamma(\phi\rightarrow\rho\pi)=0.3-0.6$ MeV.

Thus $SU(4)$ predicts the correct mixing angle between ω and ϕ. The vector octet and singlet appear in the same representation **15** of $SU(4)$ and their mixing is prescribed. The octet and singlet have superstrangeness $W=0$, whereas the meson hyperquarks predicted within the multiplet have $W\neq0$ and fractional hypercharge. The ninth pseudoscalar meson is identified with the X^0 meson ($\eta 2\pi$) at 960 MeV with $J=0^-$.

The baryons in the scheme are formed from the product

$$4\otimes4\otimes4=4+2(20')+20. \tag{7}$$

The $J=\frac{1}{2}^+$ baryons are assigned to the 20-dimensional representation **20′**.[12] The octet in **20′** has $X=\frac{3}{4}$ and therefore has $W=0$. The $J=\frac{3}{2}^+$ isobars are placed in **20** which contains a decuplet with $W=0$. The representations **20′** and **20** have the $SU(3)$ contents

$$\mathbf{20}'=\mathbf{3}+\mathbf{3}^*+\mathbf{6}+\mathbf{8} \tag{8}$$

and

$$\mathbf{20}=\mathbf{1}+\mathbf{3}+\mathbf{6}+\mathbf{10}. \tag{9}$$

Mass relations for the baryons have been obtained[12] including the familiar Gell-Mann–Okubo mass relation when the superstrangeness number W is conserved. The representation **20** is described by a one-row Young diagram and satisfies a linear-spacing law including the usual decuplet mass relation.

In view of the number of parameters involved in the mass formulas, no definite predictions can be made at the moment about the masses of the baryon hyperquarks. By a suitable choice of parameters these particles can be made to lie well above the known strange baryons.

In view of the fact that the singlet s quark possesses $Y=0$ and the triplet of quarks t_1, t_2, and t_3 have $Y=\frac{1}{3}$, $Y=\frac{1}{3}$, and $Y=-\frac{2}{3}$, respectively, the superstrange particles with $W\neq0$ have fractional hypercharge but do have integer charge. The hyperquarks appearing in triplets and sextets can only be produced by associated production, and it is not known whether W is conserved weakly or semistrongly. It is possible that the hyperquarks do not satisfy the $\Delta Y=1$ rule of weak interactions.

III. THE REPRESENTATIONS OF $SU(8)$ AND QUARK SPIN CONSERVATION

The representations of $SU(8)$ can be characterized by seven nonnegative integers $(\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5\lambda_6\lambda_7)$ where the λ_i's are functions of the seven Casimir operators. Table II shows some of the lower dimensional representations of $SU(8)$ and their $SU(4)\otimes SU(2)$ content. The lowest nontrivial representation [1] has eight dimensions and represents a fundamental $SU(8)$ octet with ordinary spin $\frac{1}{2}$. Its $SU(4)\otimes SU(2)$ content is **(4,2)**, while the conjugate representation $[1^7]$ describes the anti-octet with the content **(4*,2)**.

The seven diagonal traceless generators of $SU(8)$ which describe conserved quantum numbers can be chosen as the three $SU(4)$ generators I_3, Y, and X and the third component of the four quark spins denoted by $J_{t_1}{}^3$, $J_{t_2}{}^3$, $J_{t_3}{}^3$, and $J_s{}^3$. These constitute the correct number of commuting generators required by the $SU(8)$ group, which is a group of rank seven. Thus $SU(8)$ implies $SU(4)$ invariance of the interactions and the conservation of the four quark spins.

TABLE II. Some representations in $SU(8)$ and their $SU(4)\otimes SU(2)$ decompositions.

Labeling	Dimensions	Unitary spin and spin multiplicities (n,m)
[1]	8	(4,2)
$[1^2]$	28	(10,1)+(6,3)
[2]	36	(6,1)+(10,3)
$[1^3]$	56	(20′,2)+(4*,4)
$[21^6]$	63	(15,1)+(15,3)+(1,3)
$[1^4]$	70	(20″,1)+(1,5)+(15,3)
[3]	120	(20′,2)+(20,4)
[21]	168	(4,2)+(20′,2)+(20′,4)+(20,2)

[25] S. Coleman and H. J. Schnitzer, Phys. Rev. **134**, B863 (1964).

IV. PARTICLE ASSIGNMENTS IN $SU(8)$

We assume that spin and unitary-spin independence of the interactions leads to the introduction of $SU(8)$ supermultiplets which contain states with different spin and different F spin. The group $SU(8)$ contains $SU(4)\otimes[SU(2)]_q$ as a subalgebra. $[SU(2)]_q$ is the unitary subalgebra of the Lorentz group that leaves invariant the momentum four-vector q. $SU(8)$ also contains $SU(6)$ and Wigner's supermultiplet scheme $[SU(4)]_W$ as subalgebras.

Let us first consider the mesons. We find that

$$\mathbf{8}\otimes\mathbf{8}^*=\mathbf{1}+\mathbf{63}. \qquad (10)$$

The subduction $SU(8)\supset SU(4)\otimes SU(2)$ gives the result

$$\mathbf{63}=(\mathbf{1},\mathbf{3})+(\mathbf{15},\mathbf{3})+(\mathbf{15},\mathbf{1}). \qquad (11)$$

Thus the adjoint representation of $SU(8)$ contains the $J=0^-$ and $J=1^-$ mesons in the representation **15** of $SU(4)$ together with a $J=1^-$ singlet. A similar calculation for the representation **70** gives

$$\mathbf{70}=(\mathbf{1},\mathbf{5})+(\mathbf{15},\mathbf{3})+(\mathbf{20}'',\mathbf{1}), \qquad (12)$$

so that **70** could accomodate the $J=0^-$ mesons in **20″** and the $J=1^-$ mesons in **15**.[15]

In the following, we shall assign the $J=0^-$ mesons to the adjoint representation **15** and the $J=1^-$ mesons to a **15** as well.

Let us now reduce the direct product

$$\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8}=\mathbf{56}+\mathbf{120}+2(\mathbf{168}). \qquad (13)$$

The $SU(4)\otimes SU(2)$ subduction of **120** gives

$$\mathbf{120}=(\mathbf{20}',\mathbf{2})+(\mathbf{20},\mathbf{4}) \qquad (14)$$

and the $J=\frac{1}{2}^+$ baryons in **20′** and the $J=\frac{3}{2}^+$ isobars in **20** are grouped as a 120-dimensional representation.

An important aspect of the theory is the $\bar{B}BM$ coupling. In the direct product

$$\mathbf{120}\otimes\mathbf{120}^*=\mathbf{1}+\mathbf{63}+\mathbf{1232}+\mathbf{13104} \qquad (15)$$

the adjoint representation **63** occurs only once and the coupling is therefore *unique*. This observation is made clear if we consider the direct product in $SU(4)$:

$$\mathbf{20}'^*\otimes\mathbf{20}'=\mathbf{1}+2(\mathbf{15})+\mathbf{20}''^*+\mathbf{45}'+\mathbf{45}^*+\mathbf{84}+\mathbf{175}. \qquad (16)$$

In this case, in analogy with the octets in $SU(3)$, the adjoint representation **15** occurs twice and there is both an F and D coupling. As is the case in $SU(6)$, the F/D ratio is determined by $SU(8)$.

We expect that the $J=\frac{3}{2}^+$ resonances in (20,4) will decay into baryons and mesons and we check that **120** is contained in the direct product **63**⊗**120**. We find that

$$\mathbf{63}\otimes\mathbf{120}=\mathbf{120}+\mathbf{168}+\mathbf{2520}+\mathbf{4752}. \qquad (17)$$

However, we find that this is not so for the direct product

$$\mathbf{70}\otimes\mathbf{120}=\mathbf{1800}+\mathbf{6600}. \qquad (18)$$

It is possible to discuss two-meson states by using the direct product

$$\mathbf{63}\otimes\mathbf{63}=\mathbf{1}+2(\mathbf{63})+\mathbf{720}+\mathbf{945}+\mathbf{945}^*+\mathbf{1232}. \qquad (19)$$

It is worth noting that

$$\mathbf{70}\otimes\mathbf{70}=\mathbf{1}+\mathbf{63}+\mathbf{720}+\mathbf{2352}+\mathbf{1764}. \qquad (20)$$

The representation **168** could be considered as a possible candidate for baryon + meson resonances. Its $SU(4)\otimes SU(2)$ subduction is

$$\mathbf{168}=(\mathbf{4},\mathbf{2})+(\mathbf{20}',\mathbf{2})+(\mathbf{20}',\mathbf{4})+(\mathbf{20},\mathbf{2}). \qquad (21)$$

V. RATIO OF NEUTRON AND PROTON MAGNETIC MOMENTS

If we assume that the strong interaction is $SU(8)$ invariant this implies that the eight states of the quarks rotate into one another. $SU(8)$ invariance imposes $SU(4)$ invariance and the conservation of all four quark spins. This suggests that the quark spins define algebraic subgroups of $SU(8)$.

One can use the quark spins J_{t_1}, J_{t_2}, J_{t_3}, and J_s in the following linear combination

$$S^\alpha=\tfrac{1}{3}(2J_{t_1}{}^\alpha-J_{t_2}{}^\alpha-J_{t_3}{}^\alpha-J_s{}^\alpha), \quad (\alpha=1,2,3). \qquad (22)$$

The operator S^α is a vector in ordinary spin space and transforms like the electric-charge operator Q under $SU(4)$. The operator (22) is not a generator of the group.

We shall assume that the electromagnetic interaction transforms according to the 63-dimensional adjoint representation plus a singlet part under $SU(8)$ transformations. However, the magnetic moment part of this belongs *entirely* to the **63** multiplet and the coupling of the **63** to the **120** to obtain another **120** is unique. The ratio of the singlet to the member of **63** must be specified in the theory, and we see that it is uniquely determined by the extended Gell-Mann–Nishijima relation for the quarks by assigning to these particles a charge $-\frac{1}{3}$ or $\frac{2}{3}$ and $N=\frac{1}{3}$.

Thus the fundamental assumptions are (a) the Gell-Mann–Nishijima formula for the quarks and (b) that the magnetic-moment operator transforms under $SU(4)$ like the charge operator Q and (c) that the baryons are assigned to the totally symmetric **120** multiplet. With these assumptions the ratio of the magnetic moments is uniquely determined.

The ratio of the neutron and proton magnetic moments can be calculated by obtaining the ratio of the expectation values of S^α between neutron and proton states. We shall employ the vector model and calculate the magnetic moments due to the constituent parts (quarks) of the physical proton and neutron. In order to do this we must consider the coupled spins of the quarks. We observe that the proton is made up of two t_1 quarks and a t_2 quark with total spin $\frac{1}{2}$. Owing to the symmetry of the two t_1 quarks in $SU(4)$ they couple as identical particles to give spin 1. The physical proton then consists of $J_{t_1}=1$, $J_{t_2}=\frac{1}{2}$, $J_{t_3}=0$, and $J_s=0$. The result $J_{t_3}=J_s=0$ follows from the fact that the t_3 and s quarks are absent. We choose the physical nucleons to be in a state with the third component of the total spin $J^3=\frac{1}{2}$. We find that

$$\langle p|J_{t_1}{}^3|p\rangle=\tfrac{2}{3}, \quad \langle n|J_{t_1}{}^3|n\rangle=-\tfrac{1}{6}, \tag{23}$$

$$\langle p|J_{t_2}{}^3|p\rangle=-\tfrac{1}{6}, \quad \langle n|J_{t_2}{}^3|n\rangle=\tfrac{2}{3}, \tag{24}$$

and

$$\langle p|S^3|p\rangle=\tfrac{1}{2}, \quad \langle n|S^3|n\rangle=-\tfrac{1}{3}. \tag{25}$$

This gives the celebrated result $\mu(n)/\mu(p)=-\frac{2}{3}$ for the ratio of the neutron and proton magnetic moments.

The ratio $-\frac{2}{3}$ has also been obtained in an appropriate reformulation of Wigner's supermultiplet theory, referred to as the $[SU(4)]_W$ theory.[26–30] In this scheme the charge operator also has the form

$$Q=\text{singlet}+\text{adjoint representation}$$

under $[SU(4)]_W$ transformations. However, the choice $\frac{1}{2}(\frac{1}{3}+\tau_3)$ is fixed by the Gell-Mann–Nishijima relation.[28]

The result $\mu(n)/\mu(p)=-\frac{2}{3}$, obtained in this $SU(8)$ scheme, is perhaps expected to appear from a threefold quark model of the type we have adopted, because $SU(6)$ and Wigner's $[SU(4)]_W$ are subalgebras of the group and both these schemes give the ratio $-\frac{2}{3}$. But this result already indicates that schemes of the $SU(6)$ type are nonunique in their physical predictions.

VI. SELECTION RULES

In $SU(6)$, selection rules can be obtained by assuming that strong interactions are invariant under $SU(6)$ and therefore the quark spins are individually conserved.[31,32] The conservation of total spin implies that the reactions $\rho\to 2\pi$ and $N^*\to N+\pi$ are forbidden.[31] The assumption of quark spin conservation leads to good selection rules for the ϕ and ω.[32,33]

Selection rules can also be derived in $SU(8)$ by assuming invariance of the strong interactions under $SU(8)$. The conservation of total spin also has the consequence that $\rho\to 2\pi$, etc., are forbidden. The conservation of the t_3-quark spin leads to the correct selection rules for ϕ and ω. This is easily seen by means of the following. The nucleons, pions, and ρ possess $J_{t_3}=J_s=0$. The Λ, Σ and K, $\bar{K}$ all have $J_{t_3}=\frac{1}{2}$ and $J_s=0$. The ϕ has $J_{t_3}=1$ and $J_s=0$, while the ω has $J_{t_3}=J_s=0$. The processes $\omega\to 3\pi$, $\pi+N\to\omega+N$, $\phi\to K+\bar{K}$, and $K+N\to\phi+\Lambda$ are permitted by t_3-quark spin conservation. The forbidden processes are $\phi\to 3\pi$, $\phi\to\rho\pi$, and $\pi+N\to\phi+N+\pi$.

Thus all the selection rules obtained in $SU(6)$ follow immediately from the quark model of $SU(8)$.

VII. CONCLUDING REMARKS

Our conclusion is that the key experimental prediction of the ratio $-\frac{2}{3}$ for the magnetic-moment ratio of the neutron and proton can be obtained equally well from a quark model of $SU(8)$ provided the extended Gell-Mann–Nishijima relation is determined beforehand. The selection rules obtained in $SU(6)$ follow equally well from $SU(8)$. The new particles with $W\neq 0$ possess fractional hypercharge and therefore suggest the existence of higher resonances with interesting properties.

From the point of view of agreement with experiment a symmetry scheme like $SU(6)$ appears to afford no reason for preference above higher symmetry schemes except that it *currently* provides the most economical classification of particles. Whether this reason for considering $SU(6)$ will prevail with time can only be decided by larger accelerators.

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[26] Y. C. Leung and A. O. Barut, Phys. Letters **15**, 359 (1965).
[27] S. Pakvasa, Purdue University, 1965 (unpublished report).
[28] A. J. Macfarlane, L. O'Raifeartaigh, and E. C. G. Sudarshan, Phys. Rev. Letters **14**, 755 (1965).
[29] Ching-Hung Woo and A. J. Dragt, Phys. Rev. **139**, B945 (1965).
[30] L. C. Biedenharn, J. Nuyts, and N. Straumann, Phys. Letters **16**, 92 (1965).
[31] H. J. Lipkin, *Proceedings of the Second Coral Gables Conference on Symmetry Principles at High Energies* (W. H. Freeman and Company, San Francisco, 1965), p. 202.
[32] H. J. Lipkin, Phys. Rev. Letters **13**, 590 (1964).
[33] F. Gursey, L. A. Radicati, and A. Pais, Phys. Rev. Letters **13**, 299 (1964).