Mass Formulas and Baryon States in $SU(6)$ Symmetry*

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The mass formulas of baryon states in $SU(6)$ symmetry are examined in detail to exhibit (a) the deep structural similarity between the 56- and 70-dimensional representations and (b) the existence of a hierarchy of mass-breaking terms in these formulas which suggest strongly the dominance of the contribution due to the **35** representation. Implications of this form resemblance for the existence of a possible higher symmetry, in particular relativistic generalizations of $SU(6)$, are briefly discussed. Finally a critical analysis is made of the basis for a $(70)^-$ representation of baryon states.

1. INTRODUCTION

THE evidence that the strong interactions are approximately invariant under an $SU(3)$ transformation is based in a very important part on the remarkably accurate "mass formulas." Predictions of the existence of the η meson and the Ω^- before their experimental discovery, on the basis of these formulas, play a critical role in increasing our confidence in the existence of a higher unitary symmetry. Similarly, we hope that predictions of yet-to-be-discovered particles on the basis of mass formulas[1–3] of $SU(6)$ symmetry[4] will be of relevance to an understanding of possible approximate spin-unitary spin independence in particle physics.

In the derivation of $SU(3)$ mass formulas, one presumes, among other assumptions, that the symmetry-violating interactions are small. On the other hand, since the mass differences among, say, K, π, and η which are members of the same multiplet are not small compared to their actual masses, the violation of $SU(3)$ symmetry is apparently not weak.[5] The important and perhaps paramount question to ask concerning $SU(3)$ symmetry is whether basic triplets[6] or quarks[7] exist with a higher mass scale and strength of interaction [invariant under $SU(3)$], such that the mass formulas involving the known particles can be usefully described in terms of a small perturbation on the fundamental entities. The mass formulas of $SU(6)$ symmetry are obtained analogously and there is here the same need for justification in terms of more basic fields.[8] In addition, we are aware of the special problems associated with the interpretation of $SU(6)$ theory and Lorentz invariance. Several relativistic extensions[9,10] of $SU(6)$ symmetry have been proposed by many authors, though at the present moment, problems associated with unitarity as well as experimental implications of some of these theoreies[11] are not fully understood. An appropriate question to ask here is whether the generalizations will introduce important modifications of the usual $SU(6)$ mass formulas.

It is evident that the compositions of the mass equations themselves are likely to yield important clues concerning the basic problems posed above. In the present paper we shall examine three questions in this context. They are: (i) The deep structural similarity between the 56- and 70-dimensional representations of $SU(6)$ is very strongly evident when analyzed in terms of known empirical data. Implications of this form resemblance for the possible existence of higher symmetries, in particular relativistic generalizations of $SU(6)$, are naturally of great interest.[12] (ii) The simplicity of $SU(3)$ mass formulas is that the symmetry-breaking mass operator can be ascribed to very simple

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† John S. Guggenheim Fellow.
[1] A. Pais, Phys. Rev. Letters **13**, 415 (1964).
[2] M. A. B. Bég and V. Singh, Phys. Rev. Letters **13**, 418 (1964); **13**, 509 (1964); see also T. K. Kuo and T. Yao, *ibid* **13**, 415 (1964).
[3] I. P. Gyuk and S. F. Tuan, Phys. Rev. Letters **14**, 121 (1965).
[4] F. Gürsey and L. Radicati, Phys. Rev. Letters **13**, 173 (1964); B. Sakita, Phys. Rev. **136**, B1756 (1964).
[5] This has given rise to the interesting remark [S. Coleman, S. L. Glashow, and D. J. Kleitman, Phys. Rev. **135**, B779 (1964)] that the mass formula is better established than any of its derivations.
[6] T. D. Lee, Nuovo Cimento **35**, 933 (1965); F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. **135**, 467 (1964).
[7] M. Gell-Mann, Phys. Letters **3**, 214 (1964). The simplicity of a model based on a single triplet (quark) is appealing; on the other hand the requirement of fractional charges for the quarks is difficult to visualize for leptons which have no strong interactions.
[8] A. Pais and M. A. B. Bég [Phys. Rev. **137**, B1514 (1965)] have pointed out that in the context of the magnetic-moment results, the only acceptable sextet is the straight extension of the quark model (Ref. 7) to $SU(6)$.
[9] M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 267 (1965); R. Delbourgo, A. Salam, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965); B. Sakita and K. C. Wali, Phys. Rev. Letters **14**, 404 (1965).
[10] R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters **13**, 678 (1964); K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, *ibid.* **13**, 698 (1964); **14**, 48 (1965); S. Okubo and R. E. Marshak, *ibid.* **13**, 818 (1964).
[11] M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 509 (1965); R. Blankenbecler *et al.*, *ibid.* **14**, 518 (1965).
[12] This has also been noted by K. Bardakci *et al.*, Phys. Letters **15**, 79 (1965).

transformation properties; namely, the operator is assumed to transform as the $T=Y=0$ member (consistent with conservation of isospin and hypercharge in strong interaction) of an octet. In this framework, one might infer by analogy that mass formulas for $SU(6)$ symmetry can be adequately written in terms of a mass-breaking operator $M_{35}^{(8)}$ of the **35** representation alone. The actual presence, in general, of contributions from both **189** and **405** terms[2] to obtain the observed mass splittings in baryon and meson multiplets is thus a less well-understood (and aesthetically less satisfying) feature of $SU(6)$ mass formulas. We shall show here that there exists a hierarchy of mass-breaking terms in these formulas nevertheless, which suggests strongly the dominance of the contribution due to the **35** representation. In a loose sense therefore, the contributions for higher representations can be regarded as 'higher order perturbations' with the leading and essential mass-breaking term coming from $M_{35}^{(8)}$. As a corollary to this study, we are able to determine those aspects of $SU(6)$ mass formulas which are in actuality and to a good approximation to be ascribed to $SU(3)$ properties only. (iii) A critical analysis is made of the basis for a $(70)^-$ representation of baryon states.

In Secs. 2 and 3, the mass formulas for the **56** and **70** are written in the most convenient form appropriate to the present study. Section 4 examines the comparison between the **56** and **70** of $SU(6)$ symmetry based on the numerical estimates detailed in Sec. 3; the emphasis here is on the relevance of the phenomenological information here obtained to higher symmetry schemes like $W(6)$ and $\tilde{U}(12)$. In Sec. 5, we discuss the question of a hierarchy of mass-breaking terms and approximate mass formulas on the basis of the general mass operator of Bég and Singh[2]

$$\begin{aligned}M=a+bC_2^{(3)}+cJ(J+1)+dY\\+e[2S(S+1)-C_2^{(4)}+\tfrac{1}{4}Y^2]\\+f[N(N+1)-S(S+1)]\\+g[I(I+1)-\tfrac{1}{4}Y^2].\end{aligned}\quad(1)$$

Section 6 is devoted to a critical appraisal of the status of the $(70)^-$ representation proposed earlier,[1–3] and in Sec. 7 we conclude with some summary comments on the over-all situation.

We have mentioned[11] that attempts to reconcile the essentially nonrelativistic $SU(6)$ and relativity have not met with total success. Coleman[13] has pointed out that even if $SU(6)$ symmetry is regarded as valid only in the static limit, there remains the conceptual problem of understanding baryons and mesons (described by apparently successful mass formulas) as bound states of fundamental triplets. The triplets[6,7] are expected to have enormous mass ($\sim$10 BeV) and the very large binding energies for the known particles thus represent an ultrarelativistic situation, not a nonrelativistic one.

[13] S. Coleman, Phys. Rev. **138**, B1262 (1965).

Indeed Lee[14] has offered the elegant suggestion that many (perhaps all) of the successes of the nonrelativistic $SU(6)$ can be explained in terms of the algebra of currents and the highly convergent nature of dispersion-theoretic form factors, assuming $SU(3)$ symmetry but *without assuming that strong interactions are $SU(6)$ invariant.*

Independent of the details of whether a true symmetry is the underlying basis for $SU(6)$, the actuality that nature takes advantage of groupings into **35**, **56**, and perhaps **70** multiplets, together with their subpartitions into appropriate isospin and J-spin members, is of substantial significance[15] in itself. The situation reminds one of the analogous case of degeneracy between the $2S$ and $2P$ states of the hydrogen atom where no deeper invariance principle is claimed for the result. As emphasized by Yang,[16] empirical regularities[17] are always worthy of attention and can sometimes give suggestive leads as to the underlying principle that causes such remarkable regularity. In this context, our analysis here on structural similarity and the existence of a hierarchy of mass-breaking terms inherent in the apparently successful $SU(6)$ mass formulas is likely to afford useful building blocks and windows for the correct future theory.

2. MASS FORMULAS FOR THE 56

For purposes of comparison, we shall put down the well-known mass relations for the **56** baryon representation. However, it is convenient to introduce a new set of variables:

$$\begin{aligned}\alpha_1&=\Xi-\Sigma, & \beta_1&=\Xi^*-Y_1^*,\\ \alpha_2&=\Sigma-N, & \beta_1&=Y_1^*-N^*,\\ \alpha_3&=\Lambda-\Sigma, & \beta_3&=\Omega^--\Xi^*.\end{aligned}\quad(\mathrm{A})$$

The α's and β's are mass differences within the two $SU(3)$ representations contained in the **56** octet and decuplet. These mass differences are completely adequate to express all the mass relations and, in fact, will allow us to display them in a particularly simple form. After all, the mass operator is really a mass-splitting operator, producing the individual particle masses of a multiplet from a degenerate average mass.[18]

Using the definitions (A), we obtain for the mass

[14] B. W. Lee, Phys. Rev. Letters **14**, 676 (1965).

[15] We wish to thank Professor T. D. Lee for a useful discussion.

[16] C. N. Yang, Proceedings of the Argonne Users' Group, 1963 (unpublished); R. J. Oakes and C. N Yang, Phys. Rev. Letters **11**, 174 (1963).

[17] Observations of empirical relations among strongly interacting particles have been known for some time; see for instance S. F. Tuan, Nuovo Cimento **23**, 448 (1962). A recent summary has been given by R. M. Sternheimer, Phys. Rev. **136**, B1364 (1964).

[18] We use the connotation *average mass*, as that mass obtained by averaging masses of members of a given $SU(3)$ multiplet, say, taking into account weighting factors due to the differing isospins of the constituents. This is to be contrasted with the central mass of an $SU(3)$ multiplet as defined by Pais (Ref. 1).

relations[2]

$$3\alpha_3 = 2(\alpha_1 - \alpha_2), \tag{2}$$

$$\beta_1 = \beta_2, \tag{3}$$

$$\beta_1 = \beta_3, \tag{4}$$

$$\beta_1 = \alpha_1. \tag{5}$$

Equation (2) and Eqs. (3) and (4) are, respectively, the Gell-Mann–Okubo mass formula for an octet, and equal spacing for the decuplet. The $SU(3)$ formulas are seen to remain unchanged—as they must since there is no spin degeneracy between octet and decuplet multiplets of the **56**. Equation (5) is specifically $SU(6)$ and relates the octet splittings to the decuplet splittings. We note that there is no relation between the mass centers of the multiplets involved. Hence each $SU(3)$ representation could undergo a scale transformation as a whole in mass, without violating the formula. Only the *mass differences* within an $SU(3)$ representation enter.

Looking at experimental results,[19] we have (in MeV):

$$\begin{aligned} \alpha_1 &= 122, & \beta_1 &= 147, \\ \alpha_2 &= 253, & \beta_2 &= 146, \\ \alpha_3 &= -77, & \beta_3 &= 146. \end{aligned} \tag{A'}$$

Thus Eq. (2) is satisfied to about 30 MeV, Eqs. (3) and (4) almost exactly, and Eq. (5) to 25 MeV. That is an accuracy of 1 to 3% of the masses involved[20]—even if only of order 10 to 20% of the mass differences themselves. As might be expected, the accuracy for Eq. (5), which is specifically due to $SU(6)$, is less good than that for the other relations. For future reference, we remark here that 'increasing' the value of α_1 would have the combined effect of not only improving the $SU(6)$ relation (5), but also enhance the accuracy for the Gell-Mann–Okubo octet formula (2) as well. Taken in conjunction with the known very-well-obeyed equal spacing for the decuplet, our heuristic expectations would lead us to conclude that $\beta_1 = 147$ MeV is the more 'significant' entity.

3. MASS FORMULAS FOR THE 70

It is evidently desirable to express the mass formulas for the **70** representation[2,3] in a form which will exhibit to a maximal degree its possible structural similarity to the **56**. We shall therefore introduce the following mass differences for the **70** with content $70 = (1,2) + (8,2) + (10,2) + (8,4)$:

$$\begin{aligned} \alpha_1 &= \tilde{\Xi} - \tilde{\Sigma}, & \beta_1 &= \tilde{\Xi}^* - \tilde{Y}_1^*, & \gamma_1 &= \Xi_\gamma - \Sigma_\gamma, \\ \alpha_2 &= \tilde{\Sigma} - \tilde{N}, & \beta_2 &= \tilde{Y}_1^* - \tilde{N}^*, & \gamma_2 &= \Sigma_\gamma - N_\gamma, \\ \alpha_3 &= \tilde{\Lambda} - \tilde{\Sigma}, & \beta_3 &= \tilde{\Omega}^- - \tilde{\Xi}^*, & \gamma_3 &= \Lambda_\gamma - \Sigma_\gamma, \\ \Sigma_- &= \tilde{Y}_1^* - \tilde{\Sigma}, & \Lambda_- &= \tilde{\Lambda} - \tilde{\Lambda}', & \Xi_- &= \tilde{\Xi}^* - \tilde{\Xi}. \end{aligned} \tag{B}$$

α, β, and γ refer to mass differences within the $SU(3)$ representations (8,2), (10,2), and (8,4), respectively. Λ_- and Σ_- are essentially the splittings between spin-degenerate $SU(3)$ structures.

With these definitions, the mass formulas[2,3] for the **70** assume the following form (again seven relations):

$$3\gamma_3 = 2(\gamma_1 - \gamma_2), \tag{6}$$

$$2\beta_1 = \beta_3 + \beta_2, \tag{7}$$

$$[\Sigma_- + (\beta_3 - \beta_2)]^2 = (\Sigma_- + \gamma_3)(\Sigma_- - \gamma_3), \tag{8}$$

$$\beta_1 = \gamma_1 - \gamma_3, \tag{9}$$

$$3(\Lambda_- - \Sigma_-) + 2[2(\alpha_1 - \alpha_2) - 3\alpha_3] - 4(\beta_3 - \beta_2) = 0, \tag{10}$$

$$[\Sigma_- + (\beta_3 - \beta_2)]^2 = \Lambda_-^2 - [(2/3)(\beta_3 - \beta_2) + (4/3)(\alpha_1 - \alpha_2) + \gamma_3]^2, \tag{11}$$

$$\alpha_1 = \beta_1. \tag{12}$$

Equation (6) is the Gell-Mann–Okubo formula for the γ octet, corresponding to (2) for the **56**. Equation (7) is the direct sum of (3) and (4). We see that for the **70**, in place of equal spacing, we have generally an 'average-spacing law'—β_1 is the mean of β_2 and β_3. The magnitude of departure from equal spacing is given by quadratic equation (8). For small γ_3, Eq. (8) yields

$$(\beta_3 - \beta_2) \approx -\gamma_3^2/2\Sigma_-.$$

Hence for small $\gamma_3^2/2\Sigma_-$, Eqs. (7) and (8) reduce to the form (3) and (4) for the usual equal-spacing law. Equation (9) is analogous to Eq. (5), except for the presence of γ_3. Equation (10) represents the sum of three terms which taken together vanish exactly; however the vanishing of the individual terms will correspond to linear relationships involving a Gell-Mann–Okubo formula for the (8,2) (the so-called η octet),[3] a partial equal-spacing law for the (10,2), and a formula according to which $\Lambda_- = \Sigma_-$. Equation (11) actually gives the amount of mixing between these above relationships. For $\gamma_3^2/2\Sigma_-$ and $\alpha_3^2/2\Sigma_-$ small, the linear formulas hold themselves separately. Equation (12) is again, finally, a typical $SU(6)$ formula [like (5) for the **56**] relating the (8,2) octet and the (10,2) decuplet to each other.

One should note that the complexity of Eqs. (10) and (11), for example, are not really due to $SU(6)$ specifically. Rather, they arise from the *mixing* between $SU(3)$ multiplets. In an $SU(3)$ theory, as emphasized by Coleman, Glashow, and Kleitman,[5] mixing between

[19] A. H. Rosenfield *et al.*, Rev. Mod. Phys. **36**, 977 (1964). We take mass differences between uncharged particles following the convention adopted by S. Okubo, J. Phys. Soc. Japan **19**, 1507 (1964). Experimental uncertainties are neglected.

[20] Note that the accuracy is doubled, if we write Eq. (2), say, in the conventional octet form $(N+\Xi)/2 = (3\Lambda+\Sigma)/4$.

8$\oplus$**10** will give the following two mass relations for these multiplets:

$$[2(\alpha_1-\alpha_2)-3\alpha_3]+(\beta_1-\beta_3)=0\,, \tag{13}$$

$$[\Sigma_- -(2\beta_2-\beta_1-\beta_3)](2\beta_2-\beta_1-\beta_3) =[\Xi_- -(\beta_1+\beta_2-2\beta_3)](\beta_1+\beta_2-2\beta_3)\,. \tag{14}$$

Again (14) determines the mixing between linear equations $2(\alpha_1-\alpha_2)-3\alpha_3=0$ and $\beta_1-\beta_3=0$, constituents of Eq. (13) which would hold independently for the two respective representations **8** and **10** in the absence of mixing. The similarity of Eqs. (13) and (14) to Eqs. (10) and (11) is self-evident, though in the **70** we are dealing with **1**$\oplus$**8**$\oplus$**10**. $SU(6)$ actually greatly simplifies the situation since it yields explicitly the equality of all the off-diagonal mass matrix elements and of the coefficients in the three representations.[21]

We now examine the numerical values of the mass differences for the **70**, using solution (a) of Table I. Solution (a), a complete set of $SU(6)$ states, is derived with input $\Lambda'=1405$ MeV, $(\tilde{N},\tilde{\Lambda})$ at their threshold values (1483,1663) and the new γ octet $(N_\gamma,\Sigma_\gamma,\Xi_\gamma)=(1512,1660,1817)$; the slight deviation from the corresponding solution (a+) of Gyuk and Tuan[3] is due to the small change in the best experimental value for Ξ_γ as reported recently.[22] The mass differences (in MeV) are

$$\begin{array}{lll}\gamma_1=157\,, & \alpha_1=\ \ 151\,, & \beta_1=151\,,\\ \gamma_2=148\,, & \alpha_2=\ \ 205\,, & \beta_2=151\,,\\ \gamma_3=\ \ \ 6\,, & \alpha_3=-30\,, & \beta_3=151\,,\\ \Lambda_-=258\,, & \Sigma_-=\ \ 246\,, & \Xi_-=246\,.\end{array} \tag{B$'$}$$

TABLE I. Possible solutions of the **70**$^-$ mass formulas with appropriate input. The solutions here correspond to the (+) type solutions of Ref. 3. We have discarded here the (−) solutions discussed previously since these do not converge properly towards mass degeneracy (when certain input masses approach each other) nor towards $SU(3)$ formulas.

Input: $\Lambda'=1405$; $(N_\gamma,\Sigma_\gamma,\Xi_\gamma)=(1512,1660,1817)$; $(\tilde{N},\tilde{\Lambda})=(1488,1663)$
(a)
$\Lambda_\gamma=1666$, $(\tilde{\Sigma},\tilde{\Xi})=(1693,1844)$
$(\tilde{N}^*,\tilde{Y}_1^*,\tilde{\Xi}^*,\tilde{\Omega}^-)=(1788,1939,2090,2241)$
Input: $\Lambda'=1405$; $(N_\gamma,\Sigma_\gamma,\Xi_\gamma)=(1512,1660,1817)$; $(\tilde{N},\tilde{\Lambda})=(1455,1660)$
(b)
$\Lambda_\gamma=1666$, $(\tilde{\Sigma},\tilde{\Xi})=(1706,1857)$
$(\tilde{N}^*,\tilde{Y}_1^*,\tilde{\Xi}^*,\tilde{\Omega}^-)=(1768,1919,2070,2221)$
Input: $\Lambda'=1405$; $(N_\gamma,\Sigma_\gamma,\Xi_\gamma)=(1512,1660,1817)$; $(\tilde{N},\tilde{\Lambda})=(1488,1688)$
(c)
$\Lambda_\gamma=1666$, $(\tilde{\Sigma},\tilde{\Xi})=(1735,1886)$
$(\tilde{N}^*,\tilde{Y}_1^*,\tilde{\Xi}^*,\tilde{\Omega}^-)=(1833,1984,2135,2286)$

We see that indeed $\gamma_3^2/2\Sigma_-$ and $\alpha_3^2/2\Sigma_-$ are quite small. Consequently the decuplet (10,2) is almost exactly equal spaced and the Gell-Mann–Okubo octet formula holds quite well for the (8,2) η octet. We can therefore replace Eqs. (7) and (8) and Eqs. (10) and (11) by the corresponding linear equations, incurring an error of less than 1% of the baryon masses. This is better than we can generally expect from an $SU(6)$ formula in any case—as pointed out in the previous section with respect to the **56**$^+$. The new linear relations for the **70**$^-$ are

$$3\gamma_3=2(\gamma_1-\gamma_2)\,, \tag{15}$$

$$\beta_1=\beta_2\,, \tag{16}$$

$$\beta_1=\beta_3\,, \tag{17}$$

$$\beta_1=\gamma_1-\gamma_3\,, \tag{18}$$

$$3\alpha_3=2(\alpha_1-\alpha_2)\,, \tag{19}$$

$$\Sigma_-=\Lambda_-\,, \tag{20}$$

$$\alpha_1=\beta_1\,. \tag{21}$$

Thus, finally, pure $SU(3)$ formulas emerge for the γ octet, the decuplet, and the η octet, Eqs. (15), (16), (17), and (19). As with the case of the **56** [cf. Eq. (5)], mass differences within a given multiplet are related to some combination of splittings of another—Eqs. (18) and (21). Equation (20) is a mass-difference equality involving $Y=0$ members of the three $SU(3)$ multiplets. We must emphasize however that the set of Eqs. (15) to (21) will hold only if the baryon states $\tilde{\Lambda}$ and $\tilde{N}$ are indeed in the vicinity of their respective thresholds; otherwise the exact set (6) to (12) must be used.

There is one important way though in which the quadratic nature of the mass equations does enter. For certain values of the variables, complex solutions occur. As an example, we have, for γ_3 small,

$$\Lambda_-^2>\Sigma_-^2\,, \tag{22}$$

for real solutions. This has the effect of enforcing upper and lower bounds for some of the variables. Thus, for $\tilde{\Lambda}$ between 1660 ± 100 MeV, we must restrict $\tilde{N}$ to the interval of energy

$$\Lambda'-\alpha_1<\tilde{N}<\tilde{\Lambda}-\alpha_1\,. \tag{23}$$

In Fig. 1, we have plotted the $SU(6)$ solution and the corresponding linear $SU(3)$ solution as functions of α_3 and α_2, assigning fixed values for $\alpha_1(=151$ MeV) and $\tilde{\Lambda}(1660$ MeV) consistent with (B$'$). Note the very good agreement between the $SU(6)$ solution and a pure

[21] This is evident explicitly from Eqs. (5), (6), and (7) of Ref. 3. Here $(1/4)\{3[(\tilde{Y}_1^*+\tilde{\Sigma})-(\Lambda'+\tilde{\Lambda})]-4(\tilde{N}^*-\tilde{N})\}$ is the common off-diagonal mass matrix element for the three pairs of mixed states $(\Lambda',\tilde{\Lambda})$, $(\tilde{\Sigma},\tilde{Y}_1^*)$, and $(\tilde{\Xi},\tilde{\Xi}^*)$ from $(1,2)\oplus(8,2)\oplus(10,2)$; likewise the coefficients $a, b, c, \ldots, f$ are the same for the three representations.

[22] G. Smith *et al.*, Phys. Rev. Letters **14**, 25 (1965).

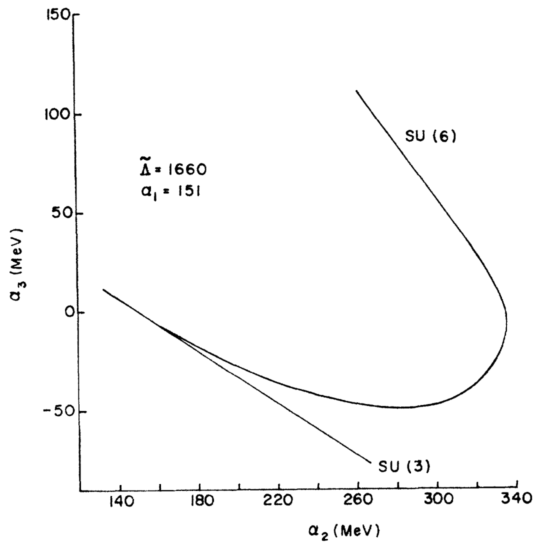

FIG. 1. Plot of α_3 versus α_2 in MeV for both $SU(6)$ and $SU(3)$ solutions; $\tilde{\Lambda}$ and α_1 have been assigned values of 1660 and 151 MeV, respectively, in this graph.

$SU(3)$ solution of the Gell-Mann–Okubo octet formula, for α_3 small (and negative) and $\alpha_2 \lesssim 220$ MeV. These ranges cover the numerical values detailed in (B′).

4. COMPARISON OF 56^+ AND 70^- REPRESENTATIONS

Examining the sets of Eqs. (2) to (5) and (15) to (21), we see that there is a complete formal analogy between the decuplet and the baryon octet in the **56**, and the (10,2) decuplet with either the γ octet (8,4) or the η octet (8,2) of the **70**. The singlet (1,2) member merely comes in via $\Sigma_- \cong \Lambda_-$. We have the scheme,

56	70	
(8,2)	(8,2)	(8,4)
(10,4)	(10,2)	(10,2).

Each octet-decuplet pair is thus related in exactly the same way [except for the presence of γ_3 in Eq. (18) of **70**]. This is the underlying reason why solution (a) of the baryon states for the **70** is so remarkably consistent with sum rules involving octet-decuplet relations within the **70** derived on the basis of Pais' dynamical model,[3] *without assuming state mixing.*[23] What is not clear at the present moment is why nature should so express herself dynamically such that the premise,[1] that $SU(6) \rightarrow$ factorized $\{SU(3)\times SU(2)_J\} \rightarrow$ broken $SU(3)$ be additive in the first- and second-stage breakdowns, is so well respected by the rigorously derived mass formulas[2] as well.[24] We must emphasize in this connection, that the derived regularities of the $(70)^-$ mass formulas of Bég and Singh[2,3] represent almost as far as pure group theory can tell us; the subtle interplay between these and dynamics is a subject which deserves further study.[24a]

Let us next look at the mass differences themselves in detail. We note at once that β_1 has numerically the same value in both **56** and **70**. In fact numerically we have $\alpha_1=\beta_1=\beta_2=\beta_3$ for the **56** and $\gamma_1=\gamma_2=\beta_1=\beta_2=\beta_3=\alpha_1$ for the **70**, all reasonably well satisfied. Like the similarity between the octet and decuplet in $SU(3)$, this suggests that we equate coefficients of the mass operator [Eq. (1)] for the **56** and **70**.[12] Three equations result:

$$\beta_1{}^+=(\beta_1+\gamma_3)^-, \tag{24}$$

$$\alpha_3{}^+=[2\alpha_3-(\Lambda_--\Sigma_-)-4(\beta_1-\beta_3)]^-, \tag{25}$$

$$(Y_1{}^*-\Sigma)^+=[(\tilde{Y}_1{}^*+\Sigma_\gamma-2\tilde{\Sigma})-3(\beta_1-\beta_3)+(1/2)\gamma_3]^-, \tag{26}$$

where the superscripts $(\pm)$ refer to mass differences defined in the 56^+ and 70^-, respectively. Numerically the left-hand sides are 147, -77, and 190 MeV, while for the right-hand sides we have 157, -72, and 216 MeV, respectively. We can thus say that the assumption of equal coefficients $b, c, \ldots, f$ of Eq. (1) for the two representations is borne out very well.

The physical content of Eqs. (24)–(26) become more transparent if we use the approximate linear set of Eqs. (15) to (21) instead and take $\gamma_3=0$. We obtain

$$\beta_1{}^+=\beta_1{}^- \quad \text{or} \quad \alpha_1{}^+=\alpha_1{}^-, \tag{27}$$

$$\alpha_3{}^+=2\alpha_3{}^-, \tag{28}$$

$$(Y_1{}^*-\Sigma)^+=(\tilde{Y}_1{}^*+\Sigma_\gamma-2\tilde{\Sigma})^-. \tag{29}$$

Equation (27) is the persistent equal spacing we have come to recognize; Eq. (29) relates the octet-decuplet splitting in 56^+ to the splitting between $SU(3)$ representations of the 70^-. The result that the η octet is now brought into close relationship with the baryon octet, by $\alpha_1{}^+=\alpha_1{}^-$ and $\alpha_3{}^+=2\alpha_3{}^-$, is very pleasing since it tends to confirm the conjectured genetic relationship[3] between the two octets.

We have of course obtained only three relationships linking the two $SU(6)$ representations. This can be readily understood when we recognize that the 56^+ [when analyzed in terms of mass operator Eq. (1)] is completely described by three parameters: α_1 the

[23] Note the use of the Glashow-Rosenfeld γ octet [S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963)] will involve using $\gamma_3=\Lambda_\gamma-\Sigma_\gamma=1520-1660=-140$ MeV, hence γ_3 is no longer small. This implies that Eqs. (15) to (21) will no longer be good approximations to the complete **70** mass formulas (6) to (12).

[24] This mystery is accentuated, when we recognize that the dynamical model breaks the J-spin degeneracy at the first stage, whereas the Bég-Singh mass formulas (as discussed in Sec. 5) show a first-"order" mass breaking due to $35^{(8)}$ which does not remove J-spin degeneracy. See, however, Sec. 7.

[24a] *Note added in proof.* See, however, Sec. 7 where it is shown that Pais' dynamical model (Ref. 1) is equivalent to the first-order mass equations (40) of Bég and Singh when analyzed in terms of **35** dominance.

'basic' splitting, α_3 the second-order departure[25] from equal spacing, and $Y_1^*-\Sigma$ which fixes the octet and the decuplet relative to each other.

In terms of the several versions of relativistic generalizations[10,11] of $SU(6)$, the structural similarity here noted between the **70**$^-$ and the **56**$^+$ can be explained in terms of the $W(6)=U(6)\times U(6)$ type theories.[10] As emphasized by Bardakci *et al.*,[12] an N-type $W(6)$ invariance can unite **56**$^+$ and **70**$^-$ in a single representation $(21,6)+(6,21)$ with $L=0$ of $W(6)$. According to Bardakci *et al.*,[12] $W(6)$ makes the prediction that the mass-splitting parameters [b, c, d, e, f, of Eq. (1)] of the $SU(6)$ mass formulas for the **56** and **70** are equal—leading to the sum rules (24) to (26).

Interpretation of this form affinity between **70**$^-$ and **56**$^+$ is less clear-cut when evaluated in terms of relativistic theories like the $SU(12)_\mathcal{L}$ theory of Bég and Pais[9] or the $\tilde{U}(12)$ theory of Delbourgo, Salam, and Strathdee.[9,26] The $SU(12)_\mathcal{L}$ unites **56**$^+$ and **70**$^+$ in the representation **364**, the **70**$^-$ and a **20**$^-$ can be accommodated by the representation **220** of the higher relativistic symmetry. While the joining of the latter $SU(6)$ representations in one supersymmetry representation **220** is attractive, in that one can understand qualitatively why phenomonological solutions for the **70**$^-$ are of comparable mass to those suggested by Gyuk and Tuan[3] for the **20**$^-$, it nevertheless raises the question of what possible connections **56**$^+$ and **70**$^-$ can have—if they belong to *different* $SU(12)_\mathcal{L}$ representations. In a subsequent section, Sec. 6, we shall examine briefly the empirical status of a possible **70**$^+$ $SU(6)$ representation.

A consistent assignment of parities for the $\tilde{U}(12)$ multiplets gives again a somewhat different picture. According to Salam *et al.*,[26] the assignments are

$$220^+,\ 364^+,\ 572^+,\ 5720^-,\ \cdots \text{ for baryons.}$$

These basic multiplets correspond, respectively, to the **20**$^+$, **56**$^+$, **70**$^+$, and **70**$^-$ for baryons in the $SU(6)$ language. There is again here the problem of understanding the structural similarity noted for the **56**$^+$ and **70**$^-$ when they belong to different $\tilde{U}(12)$ multiplets **364** and **5720**, respectively.[26a]

It has been pointed out to us by Sudarshan,[27] however, that $\tilde{U}(12)$ without the constraint of the Bargmann-Wigner equation allows enough freedom to have both the 56 and the 70 in **364**. Parities can then be defined consistently to obtain a **56**$^+$ and a **70**$^-$ in a very natural way.

[25] This terminology will become clear in terms of the discussion on hierarchy of interactions in Sec. 5.

[26] A. Salam, J. Strathdee, J. M. Charap, and P. T. Matthews, Phys. Letters **15**, 184, (1965).

[26a] *Note added in proof.* Harari *et al.* (to be published), have shown that the **70**$^-$ of $SU(6)$ cannot be accommodated by the **5720** as suggested (Ref. 26), because this representation does not appear in the product **143**$\times$**364**. However, this **70**$^-$ can belong to the **35100**.

[27] E. C. G. Sudarshan (private communication).

5. HIERARCHY OF MASS-BREAKING TERMS AND APPROXIMATE MASS FORMULAS

We shall now use the phenomenological solution of the **70**$^-$ representation, discussed in the previous section, for a detailed investigation of the mass operator itself. Special emphasis shall be paid to the *relative* importance of various contributions to the mass operator.

The complete mass operator of Eq. (1) contains contributions from $35^{(8)}$, $189^{(1)}$, $189^{(8)}$, $405^{(1)}$, and $405^{(8)}$. As emphasized recently by Harari and Lipkin,[28] the octet contribution from **189**, $189^{(8)}$, must vanish in order to obtain agreement with the observed meson masses belonging to the **35** representation. This requirement, which imposes the constraint $g=f$ on Eq. (1), will have no effect on the mass formulas for the baryon **56** since **189** does not contribute to **56**$^*\times$**56** $=$ **1**$+$**35**$+$**405**$+$**2695**. The contribution of **189**$^{(8)}$ is a relevant question for baryon states of the **70**, since the breakdown of **70**$^*\times$**70**$=$**1**$+$**35**$+$**35**$+$**189**$+$**280**$+$**280*** $+$**405**$+$**3675** contains **189** and **405**, among others. If we include $189^{(8)}$ in the **70**, only six sum rules would result. We note, though, that Eqs. (6) and (10) still hold. Equations (7) and (12), however, collapse into one equation:

$$2(\beta_3+\beta_2-2\beta_1)+3(\alpha_1-\beta_1)=0. \tag{30}$$

As long as a Gell-Mann–Okubo type $(8,2)^-$ is borne out reasonably well, we will indeed have $\alpha_1\cong\beta_1$, so that (7) and (12) hold separately—thus justifying the exclusion of $189^{(8)}$.

The remaining coefficients of Eq. (1) can then be expressed in terms of the masses:

$$\begin{aligned}
b&=\tfrac{1}{6}\Sigma_-+\tfrac{1}{6}(\beta_3-\beta_2)\approx\tfrac{1}{6}\Sigma_-,\\
c&=\tfrac{1}{3}(\Sigma_\gamma-\tilde{\Sigma})+\tfrac{1}{3}(\beta_3-\beta_2)+\tfrac{1}{6}\gamma_3\approx\tfrac{1}{3}(\Sigma_\gamma-\tilde{\Sigma}),\\
d&=-\gamma_1+2\gamma_3\approx-\alpha_1,\\
e&=-\tfrac{1}{4}[\alpha_3+\gamma_3-\tfrac{1}{2}(\Lambda_--\Sigma_-)+(\beta_3-\beta_2)]\approx-\tfrac{1}{4}\alpha_3,\\
f&=2e+\tfrac{1}{2}\gamma_3\approx-\tfrac{1}{2}\alpha_3.
\end{aligned} \tag{31}$$

This in turn enables us to evaluate the coefficients of the contributing representations which are[2]

$$\begin{aligned}
M_{35}{}^{(8)}&=a_1+b_1Y+c_1[2S(S+1)-C_2{}^{(4)}+\tfrac{1}{4}Y^2],\\
M_{189}{}^{(1)}&=a_2+b_2[2J(J+1)-C_2{}^{(3)}],\\
M_{405}{}^{(1)}&=a_4+b_4[2J(J+1)+C_2{}^{(3)}],\\
M_{405}{}^{(8)}&=a_5+b_5\{[2J(J+1)+C_2{}^{(3)}]\\
&\quad+(21/8)[2S(S+1)-C_2{}^{(4)}+\tfrac{1}{4}Y^2]\}\\
&\quad+3[2I(I+1)-\tfrac{1}{2}Y^2+2N(N+1)-2S(S+1)].
\end{aligned} \tag{32}$$

The numerical values for the coefficients which are obtained by utilizing the masses of the **70**$^-$ [solution (a)

[28] H. Harari and H. J. Lipkin, Phys. Rev. Letters **14**, 570 (1965).

of Table I] are (in MeV)

$$\begin{aligned} a&=1472, & a&=1472,\\ b&=43, & b_1&=-145,\\ c&=-10, & c_1&=-19,\\ d&=-145, & b_2&=-24,\\ e&=7, & b_4&=16,\\ f&=17, & b_5&=3. \end{aligned} \tag{33}$$

We see that by far the most significant contribution to mass splitting comes from $35^{(8)}$—in fact it is roughly 10% of the constant mass term ($a=1472$ MeV). The contributions due to **189** and **405** are about one order of magnitude smaller. It is tempting to infer, therefore, that the leading symmetry-breaking term is indeed the **35**; reasonable 'first-order' mass formulas should be possible on this basis alone. Some care must be exercised in the application of this rule, however, since here particles with different spins or isospins but the same hypercharge would be mass degenerate (e.g., $\tilde{N}=\tilde{N}^*$). Because of the relative importance of the term whose coefficient is b_2, this would lead to unsatisfactory formulas. To obviate this difficulty, we shall work (as we have done so far) only with mass differences—here the linear contributions from terms with $J(J+1)$ will tend to cancel. All undesirable mass formulas are thus excluded methodically and a consistent set of mass relations results. These are then the *first-order* mass formulas with pure $35^{(8)}$ mass splitting, contributions from **189** and **405** are then 'higher order' corrections to the basic symmetry-breaking interaction.

The existence of a hierarchy in the representations important for symmetry breaking is actually very plausible. We observe that **35** is precisely the representation containing the better established pseudoscalar and vector mesons. The contributions from **189** and **405** are much smaller, since these representations come from **35**×**35**(=**1**+**35**+**35**+**189**+**280**+**280***+**405**); there will course also be a residual second-order contribution from **35**×**35** to the first-order **35**. It is plausible to assume that the contributions of **35**×**35**×**35** would be even smaller, and are in fact negligible. Assuming that such an ansatz (which as we have seen is quite reasonable in terms of known features of empirical solutions to the **70**$^-$) is in fact a general property of $SU(6)$ theory, it will be instructive to obtain mass formulas for the different $SU(6)$ representations on the basis of **35** dominance.[29] For the mesons of **35** we obtain

$$3\alpha_3=2(\alpha_1-\alpha_2), \tag{34}$$

$$\gamma_1=\alpha_1, \tag{35}$$

$$\gamma_3=0, \tag{36}$$

$$\varphi_-=\gamma_1-\gamma_2. \tag{37}$$

[29] Mass formulas for **35** dominance have been given by T. K. Kuo and T. Yao, Phys. Rev. Letters **13**, 415 (1964); however, in the case of the **70**, these are inconsistent with formulas obtained from the full treatment. They can be rearranged into the form we have given.

Here α_1, α_2, α_3 and γ_1, γ_2, γ_3 refer to pseudoscalar and vector mesons, respectively, with

$$\begin{aligned} &\alpha_1=K-\pi, \quad \gamma_1=K^*-\rho,\\ &\alpha_2=\pi-\bar{K}, \quad \gamma_2=\rho-\bar{K}^*, \quad \varphi_-=\varphi-\omega,\\ &\alpha_3=\eta-\pi, \quad \gamma_3=\omega-\rho, \end{aligned} \tag{C}$$

and meson label = (meson mass)^2. Of course we have the restriction

$$\alpha_2=-\alpha_1, \quad \gamma_2=-\gamma_1. \tag{38}$$

In the full treatment of Bég and Singh[2] which includes higher order terms like **189** and **405**, Eq. (34) remains valid but Eqs. (35), (36), and (37) unite into a single quadratic equation [Bég and Singh, Phys. Rev. Letters **13**, 418 (1964), Eq. (30)]. This quadratic equation would be an identity if the linear equations were true individually. On the other hand, equations such as (35) and (37) *do emerge* from dynamical calculations on the mesons.[30] This seems sensible, since it is presumably dynamics which ensures **35** dominance in the first place.

For the **56**$^+$, the first-order symmetry breaking due to $35^{(8)}$ just gives universal equal spacing:

$$\alpha_1=\alpha_2=\beta_1=\beta_2=\beta_3, \quad \alpha_3=0. \tag{39}$$

In Sec. 2 we have already discussed the experimental situation for the **56**. An examination of Eq. (A′) will show that $\alpha_3=0$ is rather unsatisfactory. However, empirically α_3 is only about, say one-half of β_1, hence qualitatively Eq. (39) is in the right direction. On the other hand we must remember that the Gell-Mann–Okubo octet mass formula obtained on the basis of the full treatment does allow in general for solutions with $\alpha_3>\alpha_1$ (as for instance in the meson **35** case). The two facts taken together suggest the following heuristic rule: namely, the baryons of **56** (unlike the somewhat analogous meson situation) obey a Gell-Mann–Okubo octet formula with the Λ-Σ($=\alpha_3$) mass difference rather smaller than the Σ-N and Ξ-Σ mass differences (α_2 and α_1, respectively) consistent with trends indicated by the lowest order mass equations (39). It is interesting to note that **35** dominance already yields a Gell-Mann–Okubo octet formula for the mesons, while it yields more or less equal spacing for the baryons of **56** and **70**. This is in accord with the fact that $\alpha_1=-\alpha_2$ for the mesons while $\alpha_1\approx\alpha_2$ for the baryons.

Finally, for the **70**, the first-order mass relations with **35** dominance are

$$\begin{aligned} &\gamma_2=\gamma_1-\tfrac{3}{2}\gamma_3,\\ &\beta_1=\gamma_1-\gamma_3, \quad \beta_2=\beta_1+\tfrac{1}{2}\gamma_3, \quad \beta_3=\beta_1-\tfrac{1}{2}\gamma_3,\\ &\alpha_1=\beta_1, \quad \alpha_2=\alpha_1-\tfrac{1}{2}\alpha_3, \quad \alpha_3=\gamma_3. \end{aligned} \tag{40}$$

Comparing this set with the linear equations of (15)–(21) we see that three of the relations contained in

[30] See, for example, H. Pietschmann, Phys. Rev. **139**, B446 (1965).

(40) are exact. The others are identical to the results of the full treatment only for $\gamma_3=\alpha_3=0$. Hence first-order **35** dominance for the **70** representation implies that γ_3 and α_3 should be small, as indeed they are with our choice of phenomenological solutions [cf. Eq. (B′) of Sec. 3].

6. STATUS OF $(70)^-$ REPRESENTATION IN $SU(6)$ THEORY

Let us consider next the general question of classification of baryon states (with baryon number 1) based on the assumption that the (spin, F-spin) multiplets need not be strongly recoupled to l, the orbital angular momentum of the meson-baryon system. As first emphasized by Pais,[1] if we assign the better known mesons and vector mesons to the **35**-dimensional representation of $SU(6)$ and the stable baryons and decuplet states to the **56**, then, to the extent we can ignore spin-orbit coupling,

$$(\mathbf{35}\times\mathbf{56})^{\pm}=\mathbf{1134}^{\pm}+\mathbf{700}^{\pm}+\mathbf{70}^{\pm}+\mathbf{56}^{\pm}, \qquad (41)$$

where $(\pm)$ denote the parity. The right-hand side represents possible representations which can be filled by baryon states and baryon reasonances. For $l=1$ (P-wave meson-baryon interaction), the $(+)$ sign is appropriate. It is thus gratifying to see that $\mathbf{56}^+$ appears on the right-hand side of (41) as it should since a (10,4) decuplet decays into (baryon and meson) where energetically possible. The possible existence of a $\mathbf{70}^+$ representation is a very interesting question since several relativistic extensions of $SU(6)$, notably the $SU(12)_{\mathcal{L}}$ scheme of Bég and Pais[9] and the $\tilde{U}(12)$ of Salam *et al.*[26] can accommodate such a multiplet; we shall examine later on in this section the experimental situation with respect to the $\mathbf{70}^+$. However, a P-wave bootstrap of meson-baryon systems in the static limit[31] for $SU(6)$ symmetry yields crossing matrices which are repulsive in $\mathbf{70}^+$ and $\mathbf{1134}^+$ and mildly attractive in $\mathbf{700}^+$. On this basis the existence of a second $(+)$ parity multiplet is not required and we have just the basic $\mathbf{56}^+$ where the crossing matrices indicate strong attraction. For the $(-)$ parity multiplets involving lowest orbital states ($l=0$), the majority of the particles predicted are S-wave baryon 'resonances'[32] and hence, because of the lack of centrifugal barrier containment, conventional bootstrap counter arguments are less severe here. In other words, the $\mathbf{70}^-$ with $l=0$ that we proposed earlier[3] is the lesser evil of all possible **70**'s ($l=1,2,3\cdots$). Actually Capps[33] has shown recently that a bootstrap philosophy with exchange of the 35-fold meson multiplet μ and the singlet meson $[X^0(959)]$ can lead to S-wave bound states of the type μB (where B is the baryon $\mathbf{56}^+$ multiplet) for the $\mathbf{70}^-$ and $\mathbf{56}^-$. We are aware also that in the static limit of $SU(6)$ symmetry, the problem of coupling between the baryon resonance and the constituent meson and baryon (like $N^*\pi N$, etc.) does not in general arise for S waves as it does for P waves etc., where some form of relativistic completion (boosting)[34] through spin-orbit coupling is needed to obtain the required coupling.

For the $(-)$ parity of Eq. (41) and S-wave meson-baryon interactions, the $\mathbf{56}^-$ is a possible candidate for occupancy of baryon resonant states. The spin-unitary spin content has no mixing problem and is just $\mathbf{56}^-=(8,2)^-+(10,4)^-$. If we choose to assign the $Y_1^*(1660)$ and $\Xi^*(1817)$[22] as $(\frac{3}{2}^-)$ states belonging to the $(10,4)^-$ sector, these determine an equal spacing of 157 MeV. The remaining states are then an $\Omega^-(1974)$ with spin-parity $(\frac{3}{2}^-)$ and a $T=\frac{3}{2}$, $J=\frac{3}{2}^-$ $N^*(1503)$. This latter state looks rather improbable since this energy region of pion-nucleon scattering has been fairly well explored experimentally; there is evidence on the contrary for a $T=\frac{1}{2}$ resonant state—the $N^*(1512)$ with probable spin-parity $(\frac{3}{2}^-)$.[35] So momentarily at least, the $(56)^-$ is not a strong candidate for occupancy. Let us then consider the $(700)^-$ and $(1134)^-$ representations. Pais[1] has emphasized that an $SU(6)$ classification of baryon states is probably only satisfactory up to $J\leq\frac{5}{2}$ with an energy range of the order of 2 BeV, since symmetry breakdown will be substantial for large spin-orbit coupling effects. Both the (700) and (1134) representations will in all likelihood include many $J=\frac{5}{2}$ states at a fairly substantial energy for which classification will be ambiguous if possible at all. Simplicity of a physical theory alone will dictate that the lower representations (where possible) should have first priority for occupancy. Note the attempt to construct basic triplets[6] or quark models[7] to use up the representations 3 and 3* (smaller representations than the **8** of the eightfold way) in order to give expression to these lower representations in $SU(3)$.

We turn now to the $(70)^-$ as perhaps the most reasonable contender. Both Feynman, Gell-Mann, and Zweig[10] and Bardakci *et al.*[12] have pointed out that in the framework of the $U(6)\times U(6)$ type theory, first-order perturbation in the masses suggests that the $\mathbf{70}^-$ is likely to bear a relative mass ratio of order unity in relation to the $\mathbf{56}^+$. This will bring the expected particle states of the $(70)^-$ in the rough energy range of 1 to 2 BeV.

The spin-unitary spin content of the **70**, as emphasized earlier, is

$$(70)^-=(1,2)^-+(8,2)^-+(10,2)^-+(8,4)^-. \qquad (42)$$

Identification of $(1,2)^-$ with $Y_0^*(1405)$ appears to be in good agreement with the recent analyses of low-

[31] J. G. Belinfante and R. E. Cutkosky, Phys. Rev. Letters **14**, 33 (1965); R. H. Capps, *ibid* **14**, 31 (1965).

[32] The (8,4) γ octet of $(\frac{3}{2}^-)$ baryon states in the $\mathbf{70}^-$ can be regarded as dynamically composed of S-wave interactions of bound systems of vector meson-baryon [e.g., $N^*(1512)\leftrightarrow(\rho N)_S$] composite. However such features as the experimentally allowed D-wave $\pi+N$ decay of $N^*(1512)$ will have to arise from the completed $SU(6)$ (cf. Ref. 34 below).

[33] R. H. Capps, Phys. Rev. Letters **14**, 842 (1965).

[34] M. A. B. Bég and A. Pais, Phys. Rev. **138**, B692 (1965).

[35] See for instance P. Auvil and C. Lovelace, Nuovo Cimento **33**, 473 (1964); M. Olsson and G. Yodh, University of Maryland Technical Report No. 358 (unpublished).

energy K^--p data.[36] There is also empirical data[37] for copious decay of $Y_1^*(1660) \rightarrow Y_0^*(1405)+\pi$ [or in our notation $\Sigma_\gamma(1660) \rightarrow \Lambda'(1405)+\pi$]. This is in qualitative agreement with expectations from P-wave decay of a $Y_1^*(1660)$ with $J^P=\frac{3}{2}^-$ into $Y_0^*(1405)$ with $J^P=\frac{1}{2}^-$ and pion; the small amount of phase space available for this process will make it difficult to understand the large branching ratio into this channel with D-wave decay and $Y_1^*(1660)$ spin-parity $(\frac{3}{2}^+)$. In addition Glashow and Rosenfeld[38] have emphasized that the existence of such a decay in $SU(3)$ will require that if $\Lambda'(1405)$ is a unitary singlet then $\Sigma_\gamma(1660)$ must be assigned to an octet. Taken together, we feel the assignment of $Y_1^*(1660)$ to the $J^P=(\frac{3}{2}^-)$ γ octet (8,4) is probably not unreasonable. The evidence that $N_\gamma(1512)$ is $(\frac{3}{2}^-)$ is good[35] while the more recent experimental data[22] are quite consistent with a $(\frac{3}{2}^-)$ assignment for $\Xi_\gamma(1817)$. The Gell-Mann–Okubo octet formula for the $(8,4)^-$ members, Eq. (6), then yields a Λ_γ at energy 1666 MeV. It is important to note here that the quadratic mass formulas of the **70**$^-$ [Eqs. (8) and (11)] do impose fairly stringent constraints on possible assignments for the γ-octet members as input. To take an example, use of $N_\gamma(1512)$, $\Xi_\gamma(1817)$, and $\Lambda_\gamma(1520)$ [corresponding[39] to the $J^P=(\frac{3}{2}^-)Y_0^*(1520)$] as input assignment will give complex solutions for the **70**.

The experimental status of $\tilde{N}$ and $\tilde{\Lambda}$ members of a possible η octet $(8,2)^-$ has been reviewed in Ref. 3. It is possible to understand these baryon states[40] as virtual or bound states of the appropriate η-baryon systems. The quadratic conditions of the **70** mass formulas again impose restrictions on the choice of $\tilde{N}$ and $\tilde{\Lambda}$ as input. In fact, Eq. (23) with $\tilde{\Lambda}$ at 1660 MeV, say, will require that $\tilde{N}$ lie in the range (1250, 1510) MeV for *real solutions*. In Table I, we have listed three typical solutions (a), (b), and (c) with $\Lambda'(1405)$, $N_\gamma(1512)$, $\Sigma_\gamma(1660)$, and $\Xi_\gamma(1817)$ plus suitable values for $\tilde{N}$ and $\tilde{\Lambda}$ (close to their respective η+baryon thresholds) as input data to the $(70)^-$ mass equations. These solutions correspond to the (+) solutions of Ref. 3. The (−) solutions there can be discarded since they do not converge properly towards mass degeneracy (when certain input masses approach each other) nor towards $SU(3)$ formulas in the sense of Sec. 3.

It is evidently desirable, in the first instance, to have information about the remaining members of the η octet at the earliest opportunity. The search for Ξ can proceed by analyzing three-body final-state reactions like

$$K^-+p \rightarrow K^+ + \tilde{\Xi} \rightarrow K^+ + \eta^0 + \Xi^- . \qquad (43)$$

Since the reaction threshold is about 2.36 BeV the K^- beam with c.m. energy in the range 2.4 to 2.8 BeV is most suitable for this purpose. Bubble-chamber investigation of the $\tilde{\Sigma}$ from a two-body reaction is likely to be complex because of the presence of two neutrals $(\pi^0\gamma)$ in final-state

$$K^-+p \rightarrow \eta^0+\Sigma^0 \rightarrow \pi^+\pi^-\pi^0\Lambda^0\gamma .$$

On the other hand, should $\tilde{\Sigma}$ turn out to be a bound-state resonance of $\eta+\Sigma$,[40] or if it should decay copiously into coupled channels, direct study of the two-body reaction $K^-+p \rightarrow \pi^0+\Lambda^0$ in the energy range of 1690 to 1740 MeV (c.m.) suggested by solutions (a) and (b) of Table I, should be appropriate. Otherwise the indirect three-body approach should be feasible, viz.,

$$K^-+p \begin{cases} \nearrow (\eta^0+\Sigma^+)+\pi^- \rightarrow \pi^+\pi^-\pi^0\Sigma^+\pi^- \\ \searrow (\eta^0+\Sigma^-)+\pi^+ \rightarrow \pi^+\pi^-\pi^0\Sigma^-\pi^+ ; \end{cases} \qquad (44a)$$

$$\pi^-+p \rightarrow K^+ + \Sigma^- + \eta^0 \rightarrow K^+\Sigma^-\pi^+\pi^-\pi^0 ; \qquad (44b)$$

$$K^-+d \rightarrow (\eta^0+\Sigma^-)+p \rightarrow \pi^+\pi^-\pi^0\Sigma^- p . \qquad (44c)$$

These involve only one final-state neutral π^0. Equations (44a) and (44b) have thresholds in c.m. energy of order 1.9 and 2.25 BeV, respectively; the analysis should be relatively straightforward if at the appropriate energy, backgrounds due to, say, ρ production do not become overwhelmingly dominant. It has been conjectured that η production does not necessarily test the strangeness of the incoming meson[40]; Eq. (44c) will supply an interesting test of this since some of the earliest evidence for $\tilde{N}$ (η-N interaction near threshold) was obtained from the analogous process $\pi^+ + d \rightarrow \eta^0 + p + p$ by Pauli *et al.*[41]

We have assigned the $\tilde{N}$ and $\tilde{\Lambda}$ associated, respectively, with the η-N and η-Λ S-wave interactions near threshold as input into the **70**$^-$. It is evidently desirable to have information on the $SU(6)$ allowed decays for members of this multiplet. Dyson and Xuong[42] have pointed out that the $\eta+N$ decay mode of $\tilde{N}$ is forbidden in strict $SU(6)$ symmetry for the **70**; no such problem arises for the remaining members of the η octet. This can be seen in the following manner.[43] In $SU(6)$ symmetry the mesons transform like the tensor φ_ν for the 35-fold meson multiplet $\mu(\kappa,\nu=1, \cdots, 6)$; the baryons transform like the symmetric tensor $B_{\alpha\beta\gamma}(\alpha,\beta,\gamma=1, \cdots, 6)$ for the **56**$^+$. The tensor for the **70** with mixed symmetry

[36] J. K. Kim, Phys. Rev. Letters **14**, 29 (1965); G. S. Abrams and B. Sechi-Zorn, Phys. Rev. **139**, B454 (1965).
[37] P. Eberhard *et al.*, Phys. Rev. Letters **14**, 466 (1965).
[38] S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963).
[39] M. Ferro-Luzzi, M. B. Watson, and R. D. Tripp, Phys. Rev. **131**, 2248 (1963).
[40] S. F. Tuan, Phys. Rev. **139**, B1393 (1965).
[41] E. Pauli *et al.*, in *Proceedings of the Sienna International Conference on Elementary Particles*, edited by G. Bernadini and G. P. Puppi (Societa Italiana di Fisica, Bologna, Italy, 1963), Vol. I, p. 92.
[42] F. J. Dyson and N. Xuong (private communication).
[43] The argument here is due to S. Pakvasa. We thank Dr. R. Socolow for a pertinent communication as well.

is constructed from the above as

$$B_{\alpha\beta\delta}\varphi_\gamma{}^\delta - B_{\alpha\gamma\delta}\varphi_\beta{}^\delta - B_{\beta\gamma\delta}\varphi_\alpha{}^\delta. \qquad (45)$$

Identification by charge, hypercharge, isospin, and spin along the lines suggested by Bég and Singh,[44] will show that η transforms as $\phi_3{}^3+\phi_6{}^6-\frac{1}{3}(\phi_\mu{}^\mu)$, whereas N transforms like $B_{\alpha\beta\gamma}$ with $\alpha\beta\gamma\neq 3$ or 6—and no coupling is present with $\tilde{N}$ according to Eq. (45). It is readily seen that no such problem arises for $\eta+\Lambda$, $\eta+\Sigma$, and $\eta+\Xi$, nor for the π^-+p decay of $\tilde{N}$. These results can also be seen by studying the reduction chain of Bég and Singh[2]:

$$SU(6)\supset U(1)\otimes SU(2)\otimes SU(4)$$
$$\supset U(1)\otimes SU(2)\otimes SU(2)\otimes SU(2);$$

the $\eta+N$ decay breaks $SU(4)$ in this reduction.

The absence of $\eta+N$ coupling to $\tilde{N}$ in strict $SU(6)$ symmetry is not necessarily a serious drawback to the assignment of $\tilde{N}$ to the **70**$^-$. We have long recognized the important effects due to broken symmetry. To take an example, Gupta and Singh[45] have pointed out that the inclusion of broken $SU(3)$ effects is very important for understanding the abnormally small branching ratio for $Y_1{}^*(1385)$ decay into $\pi+\Sigma$—in the limit of exact $SU(3)$ symmetry this branching ratio has been variously estimated[46] as high as 20% (in sharp disagreement with experiment). For $SU(6)$, there are two ways of breaking the symmetry: (a) introduction of spin-orbit coupling and (b) the $SU(3)$ chain is a broken symmetry in, say, Pais' dynamical model,[1] i.e., $SU(6)\rightarrow\{SU(3)\times SU(2)\}\rightarrow$ *broken* $SU(3)$. Point (a) is irrelevant for the η octet since we deal with S-wave effects. Point (b) is not irrelevant since the $\eta+N$ decay can go via broken $SU(3)$. In fact Gupta and Singh[47] have discussed width relationships for $B^*(8)\rightarrow B(8)+\mu(8)$ (appropriate to the η-octet) in which the $SU(3)$ symmetry breaking is achieved through an added term having the transformation properties of the $I=0$, $Y=0$ component of a unitary octet. We have

$$2X(\tilde{\Sigma},N\bar{K})+2X(\tilde{\Sigma},\Sigma\pi)+2X(\tilde{N},\Sigma K)$$
$$=2X(\tilde{\Sigma},\Sigma\eta)+X(\tilde{N},N\pi)+X(\tilde{N},N\eta), \quad (46)$$

where X is closely related to the appropriate couplings. In principle, Eq. (46) shows how broken $SU(3)$ may enter to restore $\tilde{N}\rightarrow N+\eta$ coupling. Practically, a test of Eq. (46) will not be easy since one of the coupling $X(\tilde{N},\Sigma K)$ involve a 'bound' state for the η octet; quantitative evaluation will require dispersion-theoretic technique.[48] Again it may be possible to discuss the general question of sum rules for broken $SU(6)$ without reference to a specific dynamical model such as those recently proposed by Chan and Sarker.[49]

We conclude this section with some brief comments about the possible existence of a **70**$^+$. The large P_{11} phase shift in pion-nucleon scattering between 1400 and 1480 MeV has often been attributed to a resonance N^* with $T=\frac{1}{2}$ and $J=\frac{1}{2}^+$ as first emphasized by Roper,[50] though the enhancement observed need not necessarily be identified with a resonant state in the conventional sense.[51] Such a 'state' can be assigned to the $(8,2)^+$ sector of a possible **70**$^+$. A rich source of probe for possible $T=1$ partners to this state, is to study the reaction

$$K^-+p\rightarrow(\Sigma_-{}^+\pi_+{}^-\pi_+{}^-)\pi^\pm. \qquad (47)$$

Independent of detailed considerations, the Dalitz plot for the three-body system $(\Sigma_-{}^+\pi_+{}^-\pi_+{}^-)$ in the physical region where the resonance bands due to $Y_0{}^*(1405)$ cross, is likely to show some enhancement.[52] For a $\pi+Y_0{}^*(1405)$ interpretation and the Q value available (dominant S-wave $\pi+Y_0{}^*(1405)$ interaction), this effect will manifest itself most prominently for $(\Sigma_-{}^+\pi_+{}^-\pi_+{}^-)$ in $T=1$, $J=\frac{1}{2}^+$ at an energy neighborhood of 1600 to 1660 MeV. In summary, we see that while there are some indications for (+) parity baryon states other than the basic **56**$^+$, the evidence is rather weak compared with the status of a **70**$^-$. This bears out in part bootstrap notions[31,33] concerning these multiplets.

7. CONCLUDING REMARKS

In this paper we have exhibited the rather striking structural similarity between the **56** and the **70** dimensional representations. Use of input empirical information consistent with the existence of the η and γ octets conjectured earlier[3] is shown to give important simplification and regularity to the **70**$^-$ multiplet. Comparison of the numerical results for the **56**$^+$ and **70**$^-$ suggests strongly that there exists a hierarchy of mass-breaking terms with the dominant contribution coming from the lowest order term $M_{35}{}^{(8)}$. We are able to determine those aspects of $SU(6)$ mass formulas which are in reality, and to a good approximation, to be ascribed to $SU(3)$ properties only. A critical examination is made of the theoretical and experimental basis for the existence of a **70**$^-$ representation in $SU(6)$ symmetry, as well as the implication of such a multiplet for relativistic extensions of $SU(6)$ theory. It is shown that the **70**$^-$ is the most natural multiplet for occupancy after the **35**$^-$ and **56**$^+$ whose members are already known.

The importance of the existence of a hierarchy of mass-breaking terms in the group-theoretic derivation

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[44] See in particular M. A. B. Bég and V. Singh, Phys. Rev. Letters **13**, 418 (1964).
[45] V. Gupta and V. Singh, Phys. Rev. **135**, B1444 (1964).
[46] S. L. Glashow and J. J. Sakurai, Nuovo Cimento **25**, 337 (1962); *ibid* **26**, 622 (1962).
[47] V. Gupta and V. Singh, Phys. Rev. **136**, B782 (1964).
[48] See, for instance, J. Franklin and S. F. Tuan, Nuovo Cimento **20**, 1024 (1961).
[49] C. H. Chan and A. Q. Sarker, Phys. Rev. **139**, B626 (1965).
[50] L. D. Roper, Phys. Rev. Letters **12**, 340 (1964).
[51] R. H. Dalitz and R. G. Moorhouse, Phys. Letters **14**, 159 (1965).
[52] S. F. Tuan, Phys. Rev. **125**, 1761 (1962). Such types of enhancement in a Lee-model-type calculation have been discussed by A. Pagnamenta (private communication).

of $SU(6)$ mass formulas, is underlined when comparison is made with Pais' dynamical model.[1] Pais assumes that $SU(6)\to$ broken $SU(3)$ is *additive* in the first- and second-stage breakdowns with coefficients that depend on the (five) Casimir operators C_i of $SU(6)$ only. This latter assumption is the most natural way of introducing *additivity*, since the coefficients $a(C_i)$ and $b(C_i)$ of the Pais mass formula[1]

$$M=M_0+a(C_i)Y+b(C_i)[I(I+1)-\tfrac{1}{4}Y^2-\tfrac{1}{3}F^2] \quad (48)$$

are effectively constants for $SU(3)$ contents of an $SU(6)$ multiplet [M_0 is the central mass of an $SU(3)$ multiplet]. For a discussion of the **70** representation, introduction of the γ octet with $\Sigma_\gamma\approx\Lambda_\gamma$ (i.e., $\gamma_3\approx 0$) will require $b(C_i)\approx 0$ for Eq. (48). This in turn implies that for the remaining unitary spin contents of the **70**, the $(10,2)^-$ must be equally spaced, the $(8,2)^-$ is also 'equally' spaced with $\tilde{\Sigma}=\tilde{\Lambda}$. These results are contained in the statement of Eq. (40) about first-order mass relations with **35** dominance in the framework of the complete treatment. Contributions to state mixing and mass splitting of $\tilde{\Sigma}$-$\tilde{\Lambda}$ [both small for solution (a) of Table I] are in the large part second-order effects in the hierarchy noted from the group-theory mass formulas.

We have used the formalism of Bég and Singh[2] as a basis of discussion. This is completely adequate for treatment of the **35** meson multiplet as well as the **56** and **70** baryon states if we agree to ignore mass-splitting contributions for representations >1000. The baryon mass formulas for the **20** representation[1,3] can show the influence of mass-breaking terms other than the 35, 189, and 405 splittings considered by Bég and Singh. Breakdown of $\mathbf{20}^*\times\mathbf{20}=\mathbf{1}+\mathbf{35}+\mathbf{175}+\mathbf{189}$ contains an extra term **175** mass-breaking contribution and deserves further study.

The η octet of $(8,2)^-$ states associated with η-baryon S-wave threshold interactions, has been discussed here in terms of its relevance to $SU(6)$ symmetry. It is to be noted however that such an octet can play a role in alternative higher symmetry schemes. In particular it satisfies (trivially) the mass formula, $N-\tilde{N}=\Sigma-\tilde{\Sigma}=\Xi-\tilde{\Xi}$, proposed by Marshak, Mukunda, and Okubo[53] in connection with the higher symmetry group W_3 with parity interchange.

Note added in proof: There are some bubble-chamber data [Phys. Letters **17**, 166 (1965)] at CERN where they see a $Y_1^*(1942)$. This is in remarkable agreement with the position predicted for $\tilde{Y}_1^*(1939)$ from solution (a) of the present paper. On the other hand, solution (a) predicts the corresponding $(8,2)^-$ member at 1693 MeV. This raises the question, should $\tilde{\Sigma}(1693)$ turn out to be a virtual state, whether it will manifest itself strongly at the $\eta^0+\Sigma^-$ threshold from an experimental study of reaction (44b) (cf. Sec. 6). The appropriate threshold of $\eta^0+\Sigma^-$ is at 1745 MeV—some 53 MeV away. It is also of interest to note that a recent zero effective range study of the reaction $\pi^-+p\to\eta^0+n$ by Dobson [University of Hawaii Report HEPG-5-65 (unpublished)], based on the S_{11} pion-nucleon phase shift analysis of Cence [University Hawaii HEPG-3-65 (unpublished)], predicts a virtual η-N state with a mass of 1465 ± 20 MeV. This is in good accord with our assignment of the $\tilde{N}$ member, as given in solutions (a) and (b) of Table I.

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[53] R. E. Marshak, N. Mukunda, and S. Okubo, Phys. Rev. **137**, B698 (1965).