Baryon Resonance Production by Neutrinos and the Relativistic Generalizations of $SU(6)$*

C. H. ALBRIGHT AND L. S. LIU
Department of Physics, Northwestern University, Evanston, Illinois

(Received 6 August 1965)

An extension of our phenomenological study of baryon resonance production by neutrinos is carried out to include the relativistic generalizations of $SU(6)$. The semileptonic weak-interaction Lagrangian invariant under the group $SL(6,c)$ is constructed for the **364**-to-**364** supermultiplet transitions. Two special cases are considered for this interaction Lagrangian: One invokes invariance under the larger group $M(12)$, while the other includes a simple $M(12)$ symmetry-breaking term. Detailed comparisons with the present experimental information on weak N^* production and with the $SU(6)$ theory are made for each of these relativistic symmetry schemes.

IN a recent paper,[1] we have presented a phenomenological study of N^* production by neutrinos in terms of three models. Of particular interest was the one incorporating the $SU(6)$ work of Bég and Pais on semileptonic interactions[2] in the no-recoil approximation. The cross-section predictions for this model were found to be in good agreement with the published experimental information.[3] A more recent analysis[4] by the CERN group indicates, however, that the experimental inelastic cross section was underestimated, so that now the $SU(6)$ predictions appear to be too low.

In any event, it is of interest to extend our phenomenological study to include the predictions of relativistic $SU(6)$. As has been emphasized many times in the literature,[5] the relativistic completion of $SU(6)$ is not unique; however, the simplest one has been proposed independently by several groups[6] and is known variously as $M(12)$, $\tilde{U}(12)$, $SU(12)_{\mathcal{L}}$, and $SV(12)$. Partly because the simple $M(12)$ symmetry is intrinsically broken by the free Lagrangian and the equations of motion, the predictions of $M(12)$ have not met with overwhelming success.[7] On the other hand, since the theory appears to fare better in dealing with vertex functions than with scattering amplitudes, there is hope that its predictions for the N-N^* semileptonic transition will not be entirely meaningless.

Two points are of special interest. First, the $M(12)$ theory will provide more complete information on the set of octet-decuplet form factors evaluated at zero momentum transfer than could be obtained from the nonrelativistic $SU(6)$ theory and, at the same time, will reveal any deviations from those determined in the no-recoil limit. And second, since the vector form factors involve the proton-magnetic-moment prediction of $M(12)$ which is known to be too large, it is of interest to introduce some $M(12)$ symmetry-breaking interactions.

To facilitate this study, in Sec. I we shall discuss the semileptonic weak-interaction Lagrangian in the more general relativistic scheme[8] known as $SL(6,c)$, then restrict our study to the predictions of $M(12)$ in Sec. II, and finally consider $M(12)$ symmetry-breaking interactions in Sec. III. A brief summary of our work is presented in Sec. IV.

I. INVARIANCE OF L_W UNDER $SL(6,c)$

In our previous account of N^* production by neutrinos,[9] the octet-decuplet transition vertex was written in terms of eight form factors. It was shown there that the $SU(6)$ theory of semileptonic interactions according to Bég and Pais[2] together with the conserved-vector-current (CVC) hypothesis lead to predictions for three of these form factors at zero momentum transfer. No clear-cut knowledge can be obtained for the remaining five form factors, however, because of their velocity-induced nature. In contrast, the recent development on the relativistic generalizations[6] of $SU(6)$ enables one to obtain normalization predictions for all eight form factors.

Since the relativistic completion procedure for $SU(6)$ is not unique, various relativistic predictions are to be expected for the form factors. Here we shall follow the relativistic formulation of Sakita and Wali[8] based upon the group $SL(6,c)$ to construct an effective semileptonic interaction. We begin with a brief review of their work.

The group $SL(6,c)$ of 6×6 complex matrices with unit determinant contains $SL(2,c)\otimes SU(3)$, where $SL(2,c)$ can be identified as the covering group of the

* Supported in part by the National Science Foundation.
[1] C. H. Albright and L. S. Liu, Phys. Rev. **140**, B748 (1965). We subsequently refer to this as I.
[2] M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 51 (1965).
[3] M. M. Block *et al.*, Phys. Letters **12**, 281 (1964).
[4] CERN Internal Report, NPA/Int. 65-11.
[5] See, for example, M. A. B. Bég and A. Pais, Phys. Rev. **137**, B1514 (1965); **138**, B692 (1965).
[6] K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters **14**, 48 (1965); A. Salam, R. Delbourgo, and T. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965); M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 267 (1965); B. Sakita and K. C. Wali, *ibid.* **14**, 404 (1965); P. Roman and J. J. Aghassi, Phys. Letters **14**, 68 (1965); W. Rühl, *ibid.* **14**, 334 (1965).
[7] M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 509, 577(E) (1965); J. M. Cornwall, P. G. O. Freund, and K. T. Mahanthappa, *ibid.* **14**, 515 (1965); R. Blankenbecler, M. L. Goldberger, K. Johnson, and S. B. Treiman, Phys. Rev. Letters **14**, 518 (1965).
[8] B. Sakita and K. C. Wali, Phys. Rev. Letters **14**, 404 (1965) and Phys. Rev. **139**, B1355 (1965).
[9] C. H. Albright and L. S. Liu, Phys. Rev. Letters **13**, 673 (1964); **14**, 324, 532(E) (1965), and Ref. 1.

homogeneous Lorentz group and $SU(3)$ as the internal symmetry group. Hence a 12-component representation ψ_A of $SL(6,c)$ can be decomposed into representations of $SL(2,c)\otimes SU(3)$ by assigning a pair of indices $i\alpha$ to A, where i is the Dirac spinor index and α refers to the unitary spin index. The inner products $\bar{\psi}\psi$ and $\bar{\psi}\gamma_5\psi$ are both invariant under the $SL(6,c)$ transformation group.

The fields associated with the elementary particles are assumed to transform like the products of ψ's and $\bar{\psi}$'s. Thus the meson field is represented by a second-rank mixed tensor, $\Phi_A{}^B=\Phi_{i\alpha}{}^{j\beta}$, with 144 components and the baryon field by a totally symmetric third-rank tensor, $\Psi_{ABC}=\Psi_{i\alpha,j\beta,k\gamma}$, with 364 components. These field tensors, Φ and Ψ, are then required to satisfy the Duffin-Kemmer and Bargmann-Wigner free field equations, respectively, so as to lead to a particle-supermultiplet structure in agreement with that of the $SU(6)$ symmetry scheme. The meson field, which represents a nonet of vector mesons $V_{\mu,\alpha}{}^\beta$ of mass m_T and a nonet of pseudoscalar mesons $P_\alpha{}^\beta$ of mass m, assumes the following form:

$$\Phi_{i\alpha}{}^{j\beta}=(\gamma_\mu)_i{}^j V_{\mu,\alpha}{}^\beta-\frac{1}{2(m_T)_\alpha{}^\beta}(\sigma_{\mu\nu})_i{}^j F_{\mu\nu,\alpha}{}^\beta+(\gamma_5)_i{}^j P_\alpha{}^\beta-(\gamma_\mu\gamma_5)_i{}^j\partial_\mu(P/m)_\alpha{}^\beta, \quad (1.1)$$

where

$$F_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu. \quad (1.2)$$

The baryon field can be expressed as a sum of a decuplet field D and an octet field B,

$$\Psi_{i\alpha,j\beta,k\gamma}=D_{ijk,\alpha\beta\gamma}+B_{ijk,\alpha\beta\gamma}, \quad (1.3)$$

where D and B are spelled out in Ref. 8.

We now turn our attention to a discussion of the semileptonic weak interactions.[10] With the conventional assumption of current-current coupling and a point $V-A$ interaction for the leptons, the effective weak-interaction Lagrangian is given by

$$L_W=-i\left(\frac{G_V}{\sqrt{2}}\mathcal{J}_\mu{}^V+\frac{G_A}{\sqrt{2}}\mathcal{J}_\mu{}^A\right)_\beta^\alpha L_{\mu,\alpha}{}^\beta. \quad (1.4)$$

Here $\mathcal{J}_\mu{}^V$ and $\mathcal{J}_\mu{}^A$ are the vector and axial-vector currents of the hadrons, and L_μ is the leptonic current expressed in terms of the Cabibbo angle

$$L_\mu=\begin{bmatrix} 0 & l_\mu\cos\theta & l_\mu\sin\theta \\ l_\mu{}^\dagger\cos\theta & 0 & 0 \\ l_\mu{}^\dagger\sin\theta & 0 & 0\end{bmatrix}, \quad (1.5)$$

with $-il_\mu=\bar{\psi}_l\gamma_\mu(1+\gamma_5)\psi_\nu$. To construct L_W in the $SL(6,c)$ scheme for the **364** to **364** transitions, we identify $\Phi_{W,A}{}^B$ with the point leptonic current[11] according to Eq. (1.1) as follows:

$$\Phi_{W,i\alpha}{}^{j\beta}=(\gamma_\mu)_i{}^j L_{\mu,\alpha}{}^\beta-\frac{1}{2(m_T)_\alpha{}^\beta}(\sigma_{\mu\nu})_i{}^j(\partial_\mu L_\nu-\partial_\nu L_\mu)_\alpha{}^\beta+(G_A'/G_V')(\gamma_\mu\gamma_5)_i{}^j L_{\mu,\alpha}{}^\beta. \quad (1.6)$$

The constants G_A' and G_V' are inserted in order to yield the proper normalization for the elastic process $\nu_l+n\to p+l^-$ and will be specified in the next section.

The most general L_W which is invariant under $SL(6,c)$ is a linear combination of the following twelve forms:

$$\bar{\Psi}(\Phi_W\otimes 1\otimes 1)\Psi, \quad (1.7a)$$
$$\bar{\Psi}(\gamma_5\Phi_W\otimes 1\otimes 1)\Psi, \quad (1.7b)$$
$$\bar{\Psi}(\Phi_W\gamma_5\otimes 1\otimes 1)\Psi, \quad (1.7c)$$
$$\bar{\Psi}(\Phi_W\otimes\gamma_5\otimes 1)\Psi, \quad (1.7d)$$
$$\bar{\Psi}(\gamma_5\Phi_W\gamma_5\otimes 1\otimes 1)\Psi, \quad (1.7e)$$
$$\bar{\Psi}(\gamma_5\Phi_W\otimes\gamma_5\otimes 1)\Psi, \quad (1.7f)$$
$$\bar{\Psi}(\Phi_W\gamma_5\otimes\gamma_5\otimes 1)\Psi, \quad (1.7g)$$
$$\bar{\Psi}(\Phi_W\otimes\gamma_5\otimes\gamma_5)\Psi, \quad (1.7h)$$
$$\bar{\Psi}(\gamma_5\Phi_W\gamma_5\otimes\gamma_5\otimes 1)\Psi, \quad (1.7i)$$
$$\bar{\Psi}(\gamma_5\Phi_W\otimes\gamma_5\otimes\gamma_5)\Psi, \quad (1.7j)$$
$$\bar{\Psi}(\Phi_W\gamma_5\otimes\gamma_5\otimes\gamma_5)\Psi, \quad (1.7k)$$
$$\bar{\Psi}(\gamma_5\Phi_W\gamma_5\otimes\gamma_5\otimes\gamma_5)\Psi, \quad (1.7l)$$

where Φ_W is given by Eq. (1.6) and Ψ by Eq. (1.3). Contraction of all indices is implied by the direct products appearing above. If one inserts the expansions for Ψ and Φ_W into Eqs. (1.7), it becomes clear that L_W can be expressed in the form of Eq. (1.4) or more simply as

$$L_W=-i\frac{G_V'}{\sqrt{2}}J_{\mu,\beta}{}^\alpha L_{\mu,\alpha}{}^\beta, \quad (1.8)$$

where, for our purposes, the total current J_μ is a sum of four currents:

$$J_\mu=J_\mu(\bar{D}D)+J_\mu(\bar{D}B)+J_\mu(\bar{B}D)+J_\mu(\bar{B}B). \quad (1.9)$$

We have elaborated all the possible $SL(6,c)$ interaction forms above in order to emphasize the great generality of couplings which this symmetry scheme admits. Obviously, it is much too general to have any desirable predictive power. In the next two sections, we restrict this generality considerably.

II. INVARIANCE OF L_W UNDER $M(12)$

Here we limit ourselves to a weak-interaction Lagrangian which is invariant under the larger group,

[10] An alternative approach to the theory of weak semileptonic interactions based on consideration of the group $SU(6)$ as a little group is given by W. Rühl, Phys. Letters **15**, 99 (1965). $M(12)$ symmetry in weak interactions was also considered briefly by K. Kawarabayashi and R. White, Phys. Rev. Letters **14**, 527 (1965).

[11] The factor $(1+\gamma_5)$ is considered to be associated with the neutrino field in the standard fashion.

$M(12)$. This is the group of 12×12 complex matrices, M, which satisfy the condition

$$M^\dagger[\gamma_4\otimes 1]M=\gamma_4\otimes 1\,, \qquad (2.1)$$

where the unit matrix is a 3×3 matrix in $SU(3)$ space. In terms of the 12-component representation ψ_A of $SL(6,c)$, the inner product $\bar\psi\psi$ remains invariant under the group $M(12)$ while the inner product $\bar\psi\gamma_5\psi$ does not. Since the fields Φ_W and Ψ of Sec. I are assumed to transform like products of the fundamental tensors ψ_A, the only term in L_W which remains invariant under $M(12)$ is (1.7a). Hence we are able to write simply

$$L_W=-i\frac{G_V'}{\sqrt{2}}\bar\Psi^{ADC}\Phi_{W,A}{}^{B}\Psi_{BDC}\,. \qquad (2.2)$$

With the appropriate expansions for Φ_W and Ψ, we can relate Eqs. (1.4) and (2.2) according to

$$\mathcal{J}_\mu{}^V L_\mu=(G_V'/G_V)\left[J_\mu{}^V L_\mu - J_{\mu\nu}{}^T\frac{F_{W,\mu\nu}}{m_T}\right] \qquad (2.3)$$

and

$$\mathcal{J}_\mu{}^A L_\mu=-(G_A'/G_A)J_\mu{}^A L_\mu\,, \qquad (2.4)$$

where contraction of the $SU(3)$ tensor indices is implied with L_μ specified in Eq. (1.5) and the weak antisymmetric tensor defined by

$$F_{W,\mu\nu}=\partial_\mu L_\nu-\partial_\nu L_\mu\,. \qquad (2.5)$$

The baryon-baryon vector, tensor, and axial-vector currents have been given explicitly by Sakita and Wali[8] and are summarized below for convenience.

For the octet-octet transition,

$$J_\mu{}^V(\bar BB)=\frac{1}{6M_1M_2}[\bar\psi\{q^2\gamma_\mu+(M_1+M_2)i\sigma_{\mu\rho}q_\rho-(M_2-M_1)iq_\mu\}\psi]_D$$
$$+\frac{1}{18M_1M_2}[\bar\psi\{(6M_1M_2H-q^2)\gamma_\mu-(M_1+M_2)i\sigma_{\mu\rho}q_\rho+(M_2-M_1)iq_\mu\}\psi]_F\,, \qquad (2.6a)$$

$$J_{\mu\nu}{}^T(\bar BB)=\frac{1}{6M_1M_2}[\bar\psi\{M_1M_2H\sigma_{\mu\nu}+(p_{2\mu}p_{1\nu}-p_{2\nu}p_{1\mu})\}\psi]_D+\frac{1}{18M_1M_2}[\bar\psi\{2M_1M_2H\sigma_{\mu\nu}-(p_{2\mu}p_{1\nu}-p_{2\nu}p_{1\mu})\}\psi]_F\,, \qquad (2.6b)$$

$$J_\mu{}^A(\bar BB)=\frac{1}{3}H[\bar\psi\gamma_5\gamma_\mu\psi]_D+\frac{2}{9}H[\bar\psi\gamma_5\gamma_\mu\psi]_F\,. \qquad (2.6c)$$

The D-type and F-type currents are defined as follows:

$$\begin{aligned}[\bar\psi O\psi L]_D&=\mathrm{Tr}(\bar\psi OL\psi)+\mathrm{Tr}(\bar\psi O\psi L)\,,\\ [\bar\psi O\psi L]_F&=\mathrm{Tr}(\bar\psi OL\psi)-\mathrm{Tr}(\bar\psi O\psi L)\,,\end{aligned} \qquad (2.7)$$

where O is any Dirac matrix and ψ here represents one of the baryon octet fields. We have omitted the S-type current terms in the above, since they all involve the trace of L_μ which is zero in the absence of neutral leptonic currents.

For the octet-decuplet transition,

$$J_\mu{}^V(\bar DB)=\tfrac{2}{3}\bar\psi_\lambda\left[H\delta_{\lambda\mu}-i\frac{p_{1\lambda}}{M_1}\gamma_\mu+\frac{p_{1\lambda}p_{2\mu}}{M_1M_2}\right]\gamma_5\psi\,, \qquad (2.8a)$$

$$J_{\mu\nu}{}^T(\bar DB)=\tfrac{1}{3}\bar\psi_\lambda\left[H(\delta_{\lambda\nu}\gamma_\mu-\delta_{\lambda\mu}\gamma_\nu)+\frac{p_{1\lambda}}{M_1M_2}(\gamma_\mu p_{2\nu}-\gamma_\nu p_{2\mu})\right]\gamma_5\psi\,, \qquad (2.8b)$$

$$J_\mu{}^A(\bar DB)=\tfrac{2}{3}\bar\psi_\lambda\left[H\delta_{\lambda\mu}+\frac{p_{1\lambda}p_{2\mu}}{M_1M_2}\right]\psi\,. \qquad (2.8c)$$

The kinematical factors are defined as follows:

$$H=\frac{(M_1+M_2)^2+q^2}{2M_1M_2}\,, \qquad (2.9)$$

where M_1 and M_2 refer to the initial and final baryon masses in the semileptonic process $\nu_l+B_1\to B_2+l^-$, with four momenta denoted by $k_1+p_1\to p_2+k_2$ and $q=p_2-p_1=k_1-k_2$, respectively. It can easily be checked that the vector currents $J_\mu{}^V(\bar BB)$, $J_\mu{}^V(\bar DB)$, and $\mathcal{J}_\mu{}^V$ are individually conserved.

We now proceed to determine the constants G_V' and G_A' appearing in Eqs. (2.3) and (2.4) by considering the elastic process, $\nu_l+n\to p+l^-$. Use of Eqs. (2.3), (2.4), (2.5), (1.4), and (1.5) leads to

$$L_W(n\to p)=\frac{G_V'}{\sqrt{2}}(\cos\theta)\tfrac{1}{9}\bar\psi_p\left[(5H-4)\gamma_\mu+i\left(\frac{2}{M}+\frac{5H}{m_\rho}\right)\sigma_{\mu\nu}q_\nu-i\frac{q^2}{M^2m_\rho}(p_1+p_2)_\mu\right]\psi_n\bar\psi_l\gamma_\mu$$
$$\times(1+\gamma_5)\psi_\nu+\frac{G_A'}{\sqrt{2}}(\cos\theta)\frac{5}{9}H\bar\psi_p\gamma_\mu\gamma_5\psi_n\bar\psi_l\gamma_\mu\times(1+\gamma_5)\psi_\nu\,. \qquad (2.10)$$

At zero momentum transfer, we are able to identify

$$G_V'=\tfrac{3}{2}G_V\,,\quad G_A'=(9/10)G_A\,. \qquad (2.11)$$

Hence the effective semileptonic interaction Lagrangian which is invariant under $M(12)$ can be written in terms of the baryon-baryon currents of Sakita and Wali as

$$L_W=-\tfrac{3}{2}i\left[\frac{G_V}{\sqrt{2}}J_\mu{}^{V,\mathrm{Total}}-\frac{3}{5}\frac{G_A}{\sqrt{2}}J_\mu{}^A\right]_\beta^\alpha L_{\mu,\alpha}{}^\beta\,, \tag{2.12}$$

where

$$J_\mu{}^{V,\mathrm{Total}}L_\mu=J_\mu{}^VL_\mu-J_{\mu\nu}{}^T\frac{F_{W,\mu\nu}}{m_T}\,. \tag{2.13}$$

The factor 3/5 appearing above in Eq. (2.12) for the relative weighting of the V and A terms is characteristic of the corresponding $SU(6)$ relations and has been commented on previously.[12]

We now turn to the inelastic reaction of interest

$$\nu_\mu+n\rightarrow N^{*+}+\mu^-\,. \tag{2.14}$$

For this process, the effective-interaction Lagrangian derived from Eq. (2.12) is given by

$$\begin{aligned}L_W(n\rightarrow N^{*+})=&\frac{G_V}{\sqrt{2}}(\cos\theta)\frac{1}{\sqrt{3}}\left(1+\frac{M_1+M_2}{m_\rho}\right)\bar{\psi}_{N^{*+},\lambda}\\&\times\left[H\delta_{\lambda\mu}-i\frac{p_{1\lambda}}{M_1}\gamma_\mu+\frac{p_{1\lambda}p_{2\mu}}{M_1M_2}\right]\gamma_5\psi_n\bar{\psi}_l\gamma_\mu(1+\gamma_5)\psi_\nu\\&-\frac{G_A}{\sqrt{2}}(\cos\theta)\frac{\sqrt{3}}{5}\bar{\psi}_{N^{*+},\lambda}\left[H\delta_{\lambda\mu}+\frac{p_{1\lambda}p_{2\mu}}{M_1M_2}\right]\psi_n\bar{\psi}_l\gamma_\mu\\&\times(1+\gamma_5)\psi_\nu\,.\end{aligned} \tag{2.15}$$

When this form is compared with the matrix element given in Eqs. (2.8) and (2.9) of I, one obtains for the eight form factors at zero momentum transfer:

$$\begin{aligned}&F_1{}^A(0)=-\frac{G_A}{G_V}\frac{\sqrt{3}}{5}\frac{(M_1+M_2)^2}{2M_1M_2}\,,\\&F_2{}^A(0)=0\,,\\&F_3{}^A(0)=-F_4{}^A(0)=F_1{}^A(0)\,,\\&F_1{}^V(0)=\frac{1}{\sqrt{3}}\left(1+\frac{M_1+M_2}{m_\rho}\right)\frac{(M_1+M_2)^2}{2M_1M_2}\,,\\&F_2{}^V(0)=-\frac{2M_2}{M_1+M_2}F_1{}^V(0)\,,\\&F_3{}^V(0)=-F_4{}^V(0)=F_1{}^V(0)\,,\end{aligned} \tag{2.16}$$

where M_1, M_2, and m_ρ denote the masses of the neutron, N^* isobar, and ρ meson, respectively. Note that the multiplicative factor,

$$\mu^*=1+\frac{M_1+M_2}{m_\rho}\,, \tag{2.17}$$

appearing in the vector form factors is the analog of the proton-magnetic-moment prediction[6] of $M(12)$ in units of the nuclear magneton:

$$\mu_p=1+2M_N/m_\rho\,. \tag{2.18}$$

The form-factor normalizations derived from nonrelativistic $SU(6)$ and $M(12)$ are summarized in Table I as models (3) and (4), respectively. For the three form factors ($F_1{}^A$, $F_1{}^V$, and $F_2{}^V$) for which a comparison can be made, it is clear that the direct axial-vector form factor remains essentially unmodified while the two vector form factors are substantially increased over their $SU(6)$ values. This may be understood as follows: In the nonrelativistic $SU(6)$ theory, the term for which $F_1{}^A$ represents the leading contribution is independent of v/c, so neglect of baryon recoil has no influence on the determination of $F_1{}^A(0)$. On the other hand, the vector interactions are magnetic in origin and are proportional to v/c. As such, the *exact* $SU(6)$ symmetry scheme gives no information about the vector form factors, while $M(12)$ theory permits their determination in terms of the derived "magnetic moment," μ^*. In the work of Bég and Pais,[2] the authors have broken the exact $SU(6)$ symmetry scheme by including first-order effects in v/c in terms of the empirical proton and neutron magnetic moments. The enhancement of the vector form factors thus arises from the fact that the derived value of μ_p (or μ^*) is considerably larger than its observed value. However, if one identifies μ^* with the observed proton moment and uses the relation $\mu_p/\mu_n=-1.5$ in the nonrelativistic formulas of I, it is clear that the results for $F_1{}^A(0)$, $F_1{}^V(0)$, and $F_2{}^V(0)$ derived from $M(12)$ and $SU(6)$ are compatible. In other words, the no-recoil limit is a good approximation in the sense stated above.

Concerning the remaining five form factors, nonrelativistic $SU(6)$ has nothing to say while the $M(12)$ theory permits the following observations. Wheras the static theory leads one to believe that $F_2{}^A(0)$ is small,[13] we now see that it is predicted to be identically zero.

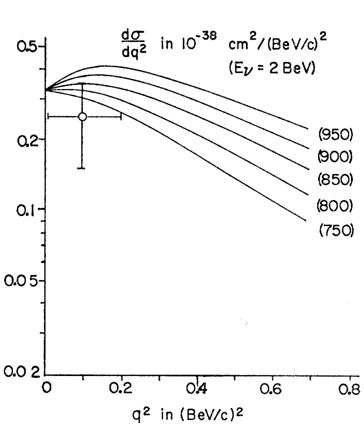

FIG. 1. Invariant differential cross section for the inelastic process, $\nu_\mu+n\rightarrow N^{*+}+\mu^-$, in the $M(12)$ theory. The number in parenthesis for each curve is equal to $\sqrt{b}$ in mega-electron volts. The experimental information comes from the CERN heavy-liquid bubble-chamber group of Ref. 3.

[12] R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters 13, 678 (1964) and Ref. 2.

[13] J. S. Bell and S. M. Berman, Nuovo Cimento 25, 404 (1962).

TABLE I. Form-factor parameters for the models investigated here and in Ref. 1. The $F_i^{V,A}$ refer to the values at zero momentum transfer, while the values quoted for $\sqrt{b}$ are characteristic of the experimental information.

Model	Form-factor parameters F_1^A	F_2^A	F_3^A	F_4^A	F_1^V	F_2^V	F_3^V	F_4^V	$\sqrt{b}$[a] (MeV)	$\sqrt{b}$[b] (MeV)
(1) Pure F_1^A	1.0	0	0	0	0	0	0	0	1220	1700
(2) N^* photoproduction+CVC+F_1^A	−0.87	0	0	0	5.6	−5.6	0	0	670	860
(3) $SU(6)$+CVC	−0.83	...	...	...	3.75	−3.75	...	...	820	1030
(4) $M(12)$	−0.85	0	−0.85	0.85	4.55	−5.18	4.55	−4.55	730	920
(5) Broken $M(12)$, $\xi=0.72$	−0.85	0	−0.85	0.85	3.62	−4.11	3.62	−3.62	800	1030
(6) Broken $M(12)$, $\xi=0.5$	−0.85	0	−0.85	0.85	2.86	−3.25	2.86	−2.86	850	1100

[a] These values of $\sqrt{b}$ pertain to the results of Ref. 3.
[b] The same for Ref. 4.

The values of $F_3^A(0)$ and $F_3^V(0)$ are equal to those of $F_1^A(0)$ and $F_1^V(0)$, respectively; however, the velocity-induced nature of F_3^A and F_3^V is responsible for somewhat smaller contributions to the cross sections.[1] The terms in the matrix element involving F_4^A and F_4^V are proportional to the lepton mass and are completely negligible.

The differential and total cross section results are presented in Figs. 1 and 2 for the $M(12)$ symmetry scheme with Hofstadter-type form factors[14] and selected values of b:

$$F_i^{V,A}(-q^2)=\frac{F_i^{V,A}(0)}{(1+q^2/b)^2}. \qquad (2.19)$$

The value of $d\sigma/dq^2$ at zero momentum transfer is slightly increased from its $SU(6)$ value as a result of the positive F_1^A, F_3^A interference term, cf. Eq. (3.8) of I. Both the published and the new experimental numbers from the CERN bubble-chamber group are presented in Fig. 2. Whereas a value of $b=(730\text{ MeV})^2$ is characteristic of the older results, a larger value of $(920\text{ MeV})^2$ is more compatible with the new CERN analysis. In making this comparison, we have assumed as in I that all the true one-pion events with $M_{N\pi}^{*2}\lesssim 2$ $(\text{BeV})^2$ go through the N^* channel. This assumption is attractive, though it tends to overestimate somewhat the N^* production cross section to which the theoretical predictions are compared[15]; hence the characteristic values quoted for b are expected to be slightly large.

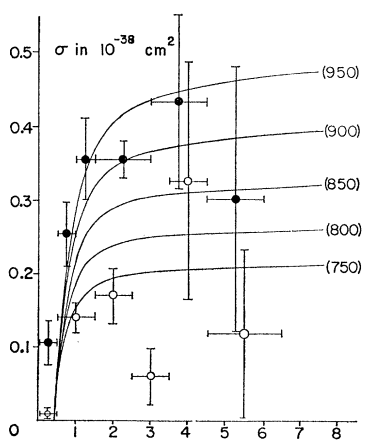

FIG. 2. Total cross section corresponding to Fig. 1. The experimental results of Refs. 3 and 4 are represented by open and filled circles, respectively.

For the Y_1^* (1385) production process, $\bar{\nu}_\mu+p\to Y_1^{*0}+\mu^+$, the $M(12)$ form-factor contributions can also be derived from Eq. (2.12). One finds that they are correspondingly altered from their $SU(6)$ values, but that the ratio of Y_1^* to N^* production as given in Fig. 14 of I remains essentially unchanged.

III. BREAKING OF $M(12)$ SYMMETRY FOR L_W

The enhancement of the vector form factors encountered in the $M(12)$ theory leads to larger cross sections for N^* production by neutrinos than those obtained from the nonrelativistic $SU(6)$ theory for a given value of the parameter b. It is interesting to note that the new results are in better agreement with those derived from the N^* photoproduction analysis of Gourdin and Salin[16] and the CVC theory [cf. model (2) of Table I].

This better agreement notwithstanding, one should recall that the vector form-factor enhancement results from the appearance of the derived proton moment (more correctly μ^*). Since the latter is known to be too large, it is natural to consider some $M(12)$ symmetry-breaking terms in the weak-interaction Lagrangian L_W. $M(12)$ symmetry-breaking interactions have already been considered by several authors. Among those of interest here, we cite the work of Bég and Pais[17] and also that of Oehme.[18] In the first-named work, the symmetry is broken by a ξ-admixture parameter, while the latter work involves a symmetry-breaking spurion. If only one spurion is inserted for the vertices in ques-

[14] As determined from our study in I, this type structure appears to give the most reasonable fit to the experimental information. We have used this phenomenological q^2 dependence, since that obtained from the $M(12)$ theory in Eq. (2.15) is valid only in the low-q^2 limit. For interesting speculations on the general q^2 dependence, see A. Pais, (to be published).

[15] See Refs. 3 and 4. The experimental uncertainties are too large at the present time to make any corrections for this overestimate meaningful.
[16] M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 193, 309 (1963). See also the recent work by Ph. Salin (to be published).
[17] M. A. B. Bég and A. Pais, see Ref. 7.
[18] R. Oehme, Phys. Letters **15**, 284 (1965); Phys. Rev. Letters **14**, 664, 866 (1965).

tion, the results of Oehme and of Bég and Pais are equivalent. In the framework of Bég and Pais, the proton magnetic moment is given by

$$\mu_p = 1 + \xi(2M_N/m_p)\,, \tag{3.1}$$

in place of Eq. (2.18), where ξ may be set equal to 0.72 to yield the observed value.[19]

The simplest manner in which to break the $M(12)$ symmetry of L_W and to incorporate this ξ parameter into the theory is achieved by considering in addition to (1.7a) also (1.7e) for L_W. In other words, we select only one term from the remaining eleven forms of (1.7) which are invariant under $SL(6,c)$. In place of Eq. (2.2) we now write

$$L_W = -i\frac{G_V'}{\sqrt{2}}\bar{\Psi}^{ADC}[\alpha\Phi_W - \beta\gamma_5\Phi_W\gamma_5]_A{}^B\Psi_{BDC}\,, \tag{3.2}$$

where $G_V' = 1.5\, G_V$ as given in (2.11). For the other two parameters, α and β, we identify

$$\alpha+\beta = 1\,, \quad \alpha-\beta = \xi\,. \tag{3.3}$$

These two relations follow from the fact that Eq. (3.2) can be cast into the form

$$L_W = -\tfrac{3}{2}i(\alpha+\beta)\left\{\frac{G_V}{\sqrt{2}}\left[J_\mu{}^V L_\mu - \frac{\alpha-\beta}{\alpha+\beta}J_{\mu\nu}{}^T\frac{F_{W,\mu\nu}}{m_T}\right] - \frac{3}{5}\frac{G_A}{\sqrt{2}}J_\mu{}^A L_\mu\right\}\,, \tag{3.4}$$

in place of Eqs. (2.12) and (2.13). Only the tensor term in L_W is modified with the sole effect that the mass tensor m_T is replaced by m_T/ξ. As such, μ^* of (2.17) now becomes

$$\mu^* = 1 + \xi\frac{M_1+M_2}{m_p}\,, \tag{3.5}$$

with a similar substitution for the four vector form factors of Eq. (2.16).

In Fig. 3 we have presented the total cross section predictions of $SU(6)$, $M(12)$, and broken $M(12)$ with $\xi = 0.72$ and 0.5 for the process, $\nu_\mu + n \to N^{*+} + \mu^-$, with $b = (850\text{ MeV})^2$. The dashed lines refer to the corresponding antineutrino process, $\bar{\nu}_\mu + p \to N^{*0} + \mu^+$, and have been plotted to exhibit the large $V-A$ interference effect. The enhancement of the $M(12)$-predicted cross section is quite apparent. For the broken $M(12)$ theory, this enhancement is considerably reduced. With $\xi = 0.72$, the deviation from the $SU(6)$ result is essentially a reflection of the new $F_3{}^A$ and $F_3{}^V$ contributions which are relatively small. A value of $\xi = 0.5$, which is suggested if one uses central mass values for the supermultiplets, reduces the enhancement of the cross section even more.

[19] If one uses the central masses of the supermultiplets in place of M_N and m_p, the value for ξ is approximately 0.5, see Ref. 17.

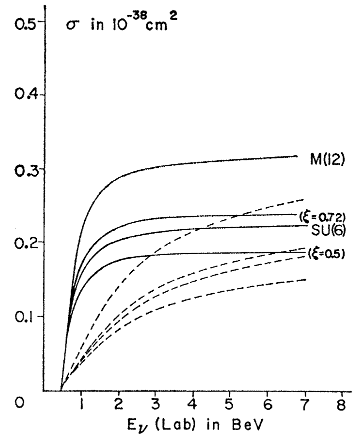

FIG. 3. Total cross section for $\nu_\mu + n \to N^{*+} + \mu^-$ in the $SU(6)$, $M(12)$, and broken $M(12)$ theories with $b = (850\text{ MeV})^2$. The dashed curves refer to the antineutrino process, $\bar{\nu}_\mu + p \to N^{*0} + \mu^+$, with the same sequence of labels.

IV. SUMMARY

The form-factor parameters of Eq. (2.19) have been summarized in Table I for all the models of weak N^* production considered in I and in this paper. The values of $\sqrt{b}$ quoted are characteristic of the experimental information presented in Refs. 3 and 4. Owing to the experimental uncertainties in the cross-section results both for single-pion production and for direct N^* production, it does not seem possible to single out a preferred model at this time.

The experimental uncertainties aside, the following remarks deserve attention. It was first pointed out by Bég, Lee, and Pais[20] that the $SU(6)$ prediction for the p-N^{*+} transition moment,

$$\langle N^{*+}|\mu|p\rangle = \frac{1}{\sqrt{6}}\left(F_1{}^V - \frac{2M_1}{M_1+M_2}F_2{}^V\right)\,, \tag{4.1}$$

is about 1.5 times too small compared to that derived from photopion production near the 3-3 resonance. For the $M(12)$ theory, this transition-moment prediction is still too small but now only by a factor of 1.2. On the other hand, the broken $M(12)$ theory with $\xi = 0.72$ yields the same prediction as $SU(6)$. In our formulation of the $M(12)$ symmetry-breaking interaction for L_W, ξ must be regarded as a universal admixture parameter. Hence it must be taken less than unity if Eq. (3.1) is to be associated with the total proton magnetic moment. The disagreement between the photopion prediction and the broken $M(12)$ prediction for the p-N^{*+} transition moment then follows.

From the group-theoretical point of view, the broken $M(12)$ theory with $\xi = 0.72$ is probably the most attractive. On the other hand, we have seen that its predictions are at odds with those derived from the photoproduction analysis for equivalent values of the cutoff parameter b and, also more recently, with the

[20] M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters 13, 514 (1964).

calculation of Salin[16] on single-pion production if one assumes N^*-channel dominance. When more reliable experimental information becomes available and the invariant differential cross section can also be determined, the correctness of the broken $M(12)$ model will be subject to a more severe test. We wish to emphasize that the type of $M(12)$ symmetry breaking introduced in Sec. III is severely limited by the single parameter ξ, which is related to the proton magnetic moment. It may well be the case that one is forced to admit some of the additional symmetry-breaking terms elaborated in Sec. I.

ACKNOWLEDGMENT

We are grateful to Professor K. C. Wali for several helpful discussions during the early stages of this work.

PHYSICAL REVIEW VOLUME 140, NUMBER 6B 20 DECEMBER 1965

Experimental Tests of Broken $\tilde{U}(12)$ and Approximate $U(6)\times U(6)$

H. Harari
Israel Atomic Energy Commission, Soreq Nuclear Research Center, Yavne, Israel

AND

H. J. Lipkin*
International Center for Theoretical Physics, Trieste, Italy

(Received 24 June 1965; revised manuscript received 30 July 1965)

Broken $\tilde{U}(12)$ and the Dashen–Gell-Mann nonchiral $U(6)\times U(6)$ symmetry are shown to lead to equivalent predictions for a large number of processes. Predictions based on invariance under certain subgroups of $\tilde{U}(12)$ are shown to be valid in $\tilde{U}(12)$ calculations which include symmetry breaking to all orders by momentum spurions. New predictions from the W-spin collinear group include $M1$ dominance for N^* photoproduction and electroproduction and justify the Stodolsky-Sakurai assumption of $M1$ dominance in vector-meson-exchange peripheral reactions.

SYMMETRY breaking has recently been introduced into calculations based on the $\tilde{U}(12)$ theory[1] in order to treat disagreements with experiment and difficulties in principle which follow from the assumption of strict $\tilde{U}(12)$ invariance. Although the use of momentum spurions (kinetons) and simple derivative couplings[2] lead to some useful results, the number of independent amplitudes and free parameters appearing in these treatments greatly reduces the predictive power of the symmetry scheme. We should like to point out that predictions from certain subgroups of $\tilde{U}(12)$ remain valid for appropriately chosen sets of processes *to any order in such symmetry-breaking terms.* The chain of subgroups obtained in this way is just the chain proposed by Dashen and Gell-Mann[3] in their nonchiral $U(6)\times U(6)$ approximate symmetry. Predictions based on these subgroups are therefore valid both in broken $\tilde{U}(12)$ to all orders in kinetons and simple derivative couplings and in the DGM theory. We also give some examples of new predictions which follow from the W-spin collinear subgroup.[4]

The kinetic spurion has the form $\gamma^\mu p_\mu$, where the gamma matrices can be considered as acting individually on each "quark component" in a meson or baryon. If all four γ matrices are present and break the symmetry, $\tilde{U}(12)$ is reduced to ordinary $SU(3)$ and no new predictions are obtained. However, if any component of the momentum is zero *for all particles* in the process considered in a particular Lorentz frame, the corresponding γ matrix does not appear in any symmetry-breaking term, and a nontrivial subgroup of $\tilde{U}(12)$ remains unperturbed to all orders in the symmetry breaking. Consider the following cases:

(1) *Zero-dimensional processes.* If all particles in a particular state are at rest, $p_x=p_y=p_z=0$ and the $\tilde{U}(12)$ symmetry is broken only by $\gamma^0 p_0$ spurions. The subgroup of $\tilde{U}(12)$ which commutes with γ^0 remains a

* Permanent address: Weizmann Institute of Science, Rehovoth, Israel.

[1] A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965); M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 269 (1965); B. Sakita and K. C. Wali, *ibid.* **14**, 404 (1965).

[2] The use of "irregular" derivative couplings and momentum spurions has been independently proposed and used by many investigators. Some examples are R. Oehme, Phys. Rev. Letters **14**, 664 (1965) and **14**, 866 (1965); J. M. Charap and P. T. Matthews, Phys. Letters **16**, 95 (1965); K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters **14**, 264 (1965); R. Ferrari and M. Konuma, *ibid.* **14**, 378 (1965). See also Footnotes 5 and 11 below. Note that once spurions are introduced, there is no reason to expect their effects to be small, and to expect results to be valid if only low-order contributions are included in the calculations.

[3] R. F. Dashen and M. Gell-Mann, Phys. Letters **17**, 142 (1965). This work is referred to subsequently in this paper as DGM.

[4] H. J. Lipkin and S. Meshkov, Phys. Rev. Letters **14**, 670 (1965).