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Field Theory of Matter. IV

JULIAN SCHWINGER*
Harvard University, Cambridge, Massachusetts
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The relativistic dynamics of 0^- and 1^- mesons in the idealization of U_3 symmetry is derived from the hypothesis that a compact group of transformations on fundamental fields induces a predominantly local and linear transformation of the phenomenological fields that are associated with particles. The physical picture of phenomenological fields as highly localized functions of fundamental fields implies that the interaction term of the phenomenological Lagrange function can have symmetry properties, expressed by invariance under the compact transformation group, that have no significance for the remainder of the Lagrange function, which describes the propagation of the physical excitations. It is verified that the meson interaction term derived by considering fundamental fermion fields is invariant under the parity-conserving group $U_6\times U_6$. The implied connection between the $\rho\pi\pi$ and $\omega\rho\pi$ coupling constants is well satisfied. There is a brief discussion of the dynamics of fermion-particle triplets, from which it is shown that the invariance of the similarly derived interaction term implies the mass degeneracy of the singlet and octuplet of 1^- mesons, without relation to 0^- masses. The triplets are also used to illustrate the derivation of gauge- and relativistically invariant electromagnetic properties. The mass degeneracy of the nine 1^- mesons, and of nine 2^+ mesons, can be inferred from the commutation properties of bilinear combinations of the fundamental field.

INTRODUCTION

THE preceding paper of this series[1] describes a program for establishing contact between the fundamental fields $\psi_{\zeta a}(x)$ and $V_a{}^\mu(x)$, $V_a{}^{\mu\nu}(x)$, and the phenomenological fields that represent observed particles. This is accomplished by a technique of comparative kinematical transformation. Compact groups of kinematical transformations on the fundamental fields are exploited as a device for conveying dynamical information concerning the highly localized structure of the phenomenological fields. The hypothesis of completeness for stable and unstable particles permits a linear representation of the essentially localized transformations induced on the phenomenological fields. This implies a correspondence between group generators at the fundamental and the phenomenological levels. Each kind of generator is a quadratic function of the appropriate kind of field. The quadratic functions of the fundamental fields, as objects with various tensor transformation properties, are also represented linearly by the phenomenological fields of bosons with suitable spins and parities. Through the machinery of relativistic field theory, particularly the distinction and the relation between independent and dependent field components, these alternative phenomenological identifications of group generators serve to determine phenomenological field dynamics. In this paper the program will be illustrated by the dynamics of 0^- and 1^- bosons, in the idealization of U_3 symmetry. We shall also consider briefly the dynamics of spin-$\frac{1}{2}$ particle triplets. The extension to baryon interactions and the inclusion of unitary symmetry-breaking effects will be dealt with separately.

MESON FIELDS

The discussion of III directed particular attention to the group of parity-preserving transformations on the twelve-component field $\psi_{\zeta a}(x)$. This group has the structure $U_6\times U_6$. The corresponding generators can be displayed as

$$M_{AB}{}^{(\pm)}(x)=\psi_a{}^\dagger(x)\tfrac{1}{2}(1\pm\gamma^0)\frac{1}{2}\begin{pmatrix}1+\sigma_3 & \sigma_1+i\sigma_2\\ \sigma_1-i\sigma_2 & 1-\sigma_3\end{pmatrix}\psi_b(x),$$

where A, for example, is a sextuple-valued index that combines a and the double-valued spin label. The equal-time commutation relations obeyed by these combinations are

$$[M_{AB}{}^{(+)}(x),M_{CD}{}^{(-)}(x')]=0$$

and

$$[M_{AB}{}^{(\pm)}(x),M_{CD}{}^{(\pm)}(x')]=\delta(\mathbf{x}-\mathbf{x}')\{\delta_{BC}M_{AD}{}^{(\pm)}(x)-\delta_{AD}M_{CB}{}^{(\pm)}(x)\}.$$

We also note that

$$(M_{AB}{}^{(\pm)})^\dagger=M_{BA}{}^{(\pm)}.$$

The transformation induced on the odd-parity objects

$$M_{AB}=-\psi_a{}^\dagger\gamma_5\tfrac{1}{2}(1+\gamma^0)\frac{1}{2}\begin{pmatrix}1+\sigma_3 & \sigma_1+i\sigma_2\\ \sigma_1-i\sigma_2 & 1-\sigma_3\end{pmatrix}\psi_b$$

is represented by the equal-time commutation relations

$$[M_{AB}(x),M_{CD}{}^{(+)}(x')]=\delta(\mathbf{x}-\mathbf{x}')\delta_{BC}M_{AD}(x),$$

$$[M_{AB}(x),M_{CD}{}^{(-)}(x')]=\delta(\mathbf{x}-\mathbf{x}')(-)\delta_{AD}M_{CB}(x).$$

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[1] J. Schwinger, Phys. Rev. 135, B816 (1964); 136, B1821 (1964), which are referred to in the text as I and II, respectively. The third article is contained in the published account of the second *Coral Gables Conference on Symmetry Principles at High Energy* (W. H. Freeman and Company, Inc., San Francisco, 1964).

To these we add the commutation relations of

$$M_{AB}{}^{\dagger}=\psi_a{}^{\dagger}\tfrac{1}{2}(1+\gamma^0)\gamma_5\frac{1}{2}\begin{pmatrix}1+\sigma_3 & \sigma_1+i\sigma_2\\ \sigma_1-i\sigma_2 & 1-\sigma_3\end{pmatrix}\psi_b$$

$$=(M_{BA})^{\dagger},$$

namely,

$$[M_{AB}{}^{\dagger}(x),M_{CD}{}^{(+)}(x')]=\delta(\mathbf{x}-\mathbf{x}')(-)\delta_{AD}M_{CB}{}^{\dagger}(x),$$

$$[M_{AB}{}^{\dagger}(x),M_{CD}{}^{(-)}(x')]=\delta(\mathbf{x}-\mathbf{x}')\delta_{BC}M_{AD}{}^{\dagger}(x).$$

When completed by

$$[M_{AB}(x),M_{CD}{}^{\dagger}(x')]=\delta(\mathbf{x}-\mathbf{x}')\{\delta_{BC}M_{AD}{}^{(-)}(x)-\delta_{AD}M_{CB}{}^{(+)}(x)\},$$

we have also indicated the commutator structure of the group U_{12}.

The matrices that represent parity-preserving transformations, on the phenomenological fields of octuplet-plus-singlet families of 0^- and 1^- particles, commute with A^0. The latter is the antisymmetrical real matrix that specifies boson-field equal-time commutation relations. Accordingly, it is possible and convenient to diagonalize A^0. This is accomplished by introducing the non-Hermitian combinations

$$\phi^{\pm}=(\tfrac{1}{2}m_0)^{1/2}\phi\mp i(2m_0)^{-1/2}\phi^0$$

and

$$U_k{}^{\pm}=(2m_1)^{-1/2}U^0{}_k\pm i(\tfrac{1}{2}m_1)^{1/2}U_k,$$

which obey the commutation relations (all other commutators vanish)

$$[\phi_{ab}{}^{+}(x),\phi_{cd}{}^{-}(x')]=\delta(\mathbf{x}-\mathbf{x}')\delta_{bc}\delta_{ad},$$

$$[U_{kab}{}^{+}(x),U_{lcd}{}^{-}(x')]=\delta(\mathbf{x}-\mathbf{x}')\delta_{kl}\delta_{bc}\delta_{ad}.$$

The quantities m_0 and m_1 are mass scaling factors which could vary arbitrarily among the multiplet members, subject only to the symmetry requirements of the assumed dynamical level. Thus, with the assumption of U_3 symmetry, we might use a different mass factor for a singlet than for the members of an octuplet. We shall, in fact, employ a unique mass for all nine fields of a given spin so that m_0 and m_1 are just numbers. The necessity for this will be indicated later. It will also be discovered that $m_0=m_1$.

The 72 odd-parity field components comprised in the 36 non-Hermitian combinations $\phi_{ab}{}^{+}$, $U_{kab}{}^{+}$ can be arrayed in the matrix

$$M_{AB}=\frac{(\tfrac{1}{2}m)^{3/2}}{(-g)}\begin{pmatrix}\phi_{ab}{}^{+}+U_{3ab}{}^{+} & U_{1ab}{}^{+}+iU_{2ab}{}^{+}\\ U_{1ab}{}^{+}-iU_{2ab}{}^{+} & \phi_{ab}{}^{+}-U_{3ab}{}^{+}\end{pmatrix}$$

which is displayed more compactly as

$$M_{AB}=(\tfrac{1}{2}m)^{3/2}/(-g)(\phi_{ab}{}^{+}+\sigma_k{}^{T}U_{kab}{}^{+}).$$

The adjoint matrix is

$$M_{AB}{}^{\dagger}=(M_{BA})^{\dagger}=(\tfrac{1}{2}m)^{3/2}/(-g)(\phi_{ab}{}^{-}+\sigma_k{}^{T}U_{kab}{}^{-}).$$

Here, m is a mass parameter which can be identified conveniently with the common value of m_0 and m_1, while g is an arbitrary dimensionless constant. Note that the commutation properties of the phenomenological fields are given by

$$[M_{AB}(x),M_{CD}{}^{\dagger}(x')]=\delta(\mathbf{x}-\mathbf{x}')\delta_{BC}\delta_{AD}(m^3/4g^2).$$

The use of a common notation indicates the linear correspondence that is being established between the phenomenological fields and bilinear combinations of the fundamental ψ operators. This correspondence is expressed covariantly by

$$-g\bar{\psi}_a\gamma_5\psi_b\leftrightarrow m^{3/2}m_0{}^{1/2}\phi_{ab}$$

$$-g\bar{\psi}_k\gamma^{\mu}\psi_b\leftrightarrow m^{3/2}m_1U_{ab}{}^{\mu}$$

$$g\bar{\psi}_a i\gamma^{\mu}\gamma_5\psi_b\leftrightarrow m^{3/2}m_0{}^{-1/2}\phi_{ab}{}^{\mu}$$

$$g\bar{\psi}_a\sigma^{\mu\nu}\psi_b\leftrightarrow m^{3/2}m_1{}^{-1/2}U_{ab}{}^{\mu\nu},$$

where a distinction among the various mass constants is still retained.

The correspondence has another meaning. It presents the phenomenological fields as the basis for a particular representation of the group $U_6\times U_6$. The commutation relations between M_{AB} and $M_{CD}{}^{(\pm)}$ imply that M_{AB} transforms as the product of independent six-dimensional representations. This is symbolized by

$$M\sim 6_-{}^*\times 6_+,$$

and similarly

$$M^{\dagger}\sim 6_+{}^*\times 6_-,$$

while

$$M^{(+)}\sim 6_+{}^*\times 6_+,\quad M^{(-)}\sim 6_-{}^*\times 6_-.$$

Accordingly, the generators $M^{(\pm)}$ must be identified with suitable bilinear combinations of the phenomenological fields M and $M^{\dagger}$. The required combinations are simply

$$M_{AB}{}^{(+)}=(4g^2/m^3)(M^{\dagger}M)_{AB}$$

and

$$M_{AB}{}^{(-)}=-(4g^2/m^3)(MM^{\dagger})_{AB},$$

which possess all the commutation properties required of $U_6\times U_6$ generators.

The linear correspondence between phenomenological fields and bilinear ψ combinations also implies the following correspondence

$$M_{AB}{}^{(\pm)}\leftrightarrow-(m^{3/2}/4g)(\pm m^{1/2}S_{ab}+m_1{}^{1/2}U_{ab}{}^0-\sigma_k{}^{T}m_0{}^{-1/2}\phi_{kab}\mp\sigma_k{}^{T}m_1{}^{-1/2}U_{[k]ab}),$$

where

$$U_{[k]}=\tfrac{1}{2}\epsilon_{klm}U_{lm}$$

and

$$g\bar{\psi}_a\psi_b\leftrightarrow m^2S_{ab}.$$

The result is a local relation between the dependent and independent components of the fields associated with 0^- and 1^- particles. It is expressed by the matrix equations

$$-m_1^{1/2}U^0+\sigma_k{}^T m_0^{-1/2}\phi_k = (g/m^{3/2})[\phi^-+\sigma_k{}^T U_k{}^-, \phi^+ +\sigma_k{}^T U_k{}^+]$$

and

$$-m^{1/2}S+\sigma_k{}^T m_1^{-1/2}U_{[k]} = (g/m^{3/2})\{\phi^-+\sigma_k{}^T U_k{}^-, \phi^+ +\sigma_k{}^T U_k{}^+\}.$$

These are written out somewhat more explicitly as

$$-m^{3/2}m_1^{1/2}U^0=g([\phi^-,\phi^+]+[U_k{}^-,U_k{}^+]),$$

$$m^{3/2}m_0^{-1/2}\phi_k=g([\phi^-,U_k{}^+]+[U_k{}^-,\phi^+] -i\epsilon_{klm}\{U_l{}^-,U_m{}^+\}),$$

$$m^{3/2}m_1^{-1/2}U_{[k]}=g(\{\phi^-,U_k{}^+\}+\{U_k{}^-,\phi^+\} -i\epsilon_{klm}[U_l{}^-,U_m{}^+]),$$

and

$$-m^2S=g(\{\phi^-,\phi^+\}+\{U_k{}^-,U_k{}^+\}).$$

MESON DYNAMICS

The Lagrange function of octuplet plus singlet families of 0^- and 1^- particles is given by

$$\mathfrak{L}=\mathrm{tr}[-\phi^\mu\partial_\mu\phi+\tfrac{1}{2}\phi^\mu\phi_\mu-\tfrac{1}{2}\phi M_0{}^2\phi] +\mathrm{tr}[-\tfrac{1}{2}U^{\mu\nu}(\partial_\mu U_\nu-\partial_\nu U_\mu)+\tfrac{1}{4}U^{\mu\nu}U_{\mu\nu}-\tfrac{1}{2}U^\mu M_1{}^2 U_\mu] +\mathfrak{L}_{\mathrm{int}}(\phi,U),$$

where $M_0{}^2$ and $M_1{}^2$ only distinguish between singlet and octuplet of the corresponding species. The constraint equations are

$$M_1{}^2U^0=-(\partial\mathfrak{L}_{\mathrm{int}}/\partial U^0)-\partial_k U^{0k},$$

$$\phi_k=-(\partial\mathfrak{L}_{\mathrm{int}}/\partial\phi_k)+\partial_k\phi,$$

$$U_{[k]}=-(\partial\mathfrak{L}_{\mathrm{int}}/\partial U_{[k]})+(\nabla\times\mathbf{U})_k,$$

and we identify the local parts of these relations with those established in the previous section. The resulting tentative construction of $\mathfrak{L}_{\mathrm{int}}$ is indicated by

$$\mathfrak{L}_{\mathrm{int}}=\mathrm{tr}[-U^0(M_1{}^2/m^2)\text{`}m^2U^0\text{'}-\phi_k(1/m)\text{`}m\phi_k\text{'} -U_{[k]}(1/m)\text{`}mU_{[k]}\text{'}],$$

where the quantities in quotation marks are the quadratic functions of the independent field variables, and a partial anticipation of the relations $m_0=m_1=m$ has been introduced, merely for typographical simplification.

We now invoke the requirement of relativistic invariance in order to obtain connections among the various constants that have been employed. To illustrate this procedure let us isolate the terms in $\mathfrak{L}_{\mathrm{int}}$ that are linear in the components of ϕ^μ. These terms are

$$(g/m^{3/2})\ \mathrm{tr}[m_0^{1/2}\phi^k\{m_1^{1/2}U^l,m_1^{-1/2}\,{}^*U_{kl}\} +m_0^{-1/2}\phi^0\{m_1^{1/2}U^k,m_1^{1/2}\,{}^*U_{0k}\}],$$

where we have introduced the dual tensor

$${}^*U_{\mu\nu}=\tfrac{1}{2}\epsilon_{\mu\nu\lambda\kappa}U^{\lambda\kappa}.$$

Since the two contributions are parts of a single scalar they must possess the same coefficients for any combination of unitary components. The unitary singlets contribute to this interaction as well as the octuplets. It is now clear that all possible values for m_0 and m_1 must be identical, which we also adopt as the natural choice for m,

$$-m_0=m_1=m.$$

One should note that there is another term obtained in this way, which is not contained in the tentative form of $\mathfrak{L}_{\mathrm{int}}$. It is

$$-(g/m)\ \mathrm{tr}\phi^k\{U^0,{}^*U_{0k}\},$$

which is a product of three dependent field components. It is an example of a structure that is outside the simplified framework of local and linear transformations, but one which is demanded by the requirements of relativistic invariance.

Another such pair of terms, parts of a single relativistic scalar, is

$$-ig\ \mathrm{tr}[(M_1{}^2/m^2)U^0[\phi^0,\phi]+U_k[\phi^k,\phi]].$$

Evidently,

$$M_1(\text{octuplet})=m,$$

but no information is obtained in this way about M_1 (singlet), since $\mathrm{tr}[\phi^0,\phi]=0$. (See the last section, however.) There is no necessary relation between m and the masses of the 0^- particles.

The outcome of these considerations is the following scalar interaction term of the Lagrange function,[2]

$$\mathfrak{L}_{\mathrm{int}}=ig\ \mathrm{tr}(\phi^\mu[U_\mu,\phi]+\tfrac{1}{2}U^{\mu\nu}[U_\mu,U_\nu]) -(g/m)\ \mathrm{tr}(\phi\tfrac{1}{4}\{{}^*U_{\mu\nu},U^{\mu\nu}\}+\phi^\nu\{U^\mu,{}^*U_{\mu\nu}\}) -i(g/m^2)\ \mathrm{tr}(\tfrac{1}{2}U^{\mu\nu}[\phi_\mu,\phi_\nu]+\tfrac{1}{3}U^{\mu\nu}U_{\nu\lambda}U_\mu{}^\lambda).$$

It is particularly interesting that this structure can also be presented as

$$\mathfrak{L}_{\mathrm{int}}=-((2g)^4/m^5)\ \mathrm{Tr}[M^{(+)}M^\dagger M-M^{(-)}MM^\dagger -\tfrac{1}{3}(M^{(+)})^3+\tfrac{1}{3}(M^{(-)})^3],$$

where Tr refers to the sextuple-valued indices that label the fields $M^{(\pm)}$, M, and $M^\dagger$. The latter form makes it evident that $\mathfrak{L}_{\mathrm{int}}$ is invariant under the transformations of the group $U_6\times U_6$. Let it be emphasized that we have *derived* this property of the interaction term[3] from our fundamental dynamical assumptions concerning localizability and completeness. It is entirely comprehensible that $\mathfrak{L}_{\mathrm{int}}$ should possess this invariance as a kinematical expression of the highly localized dynamical relation between phenomenological and fundamental

[2] These results were reported at the second Coral Gables Conference on Symmetry Principles at High Energy, January, 1965.

[3] Some authors have postulated that the interaction term in a Lagrange function referring to fundamental fields (translated from the original Quark) is invariant under a $U_6\times U_6$ transformation group, which is also regarded as imbedded in a larger noncompact group. See, for example, K. Bardakci, J. Cornwall, P. Freund, and B. Lee, Phys. Rev. Letters **14**, 48 (1965). In our view, such hypotheses are irrelevant to the emergence of the parity-conserving $U_6\times U_6$ group at the phenomenological level.

fields, without such transformations having the slightest relevance to the remainder of the phenomenological Lagrange function, which characterizes the propagation of the physical excitations.[4]

There is one difficulty, at least, with this result. It concerns the manner in which U^μ appears, through the implication for electromagnetic interactions. Let us recall that the ψ contribution to the electric current vector is

$$j^\mu(x) = -e\bar{\psi}_1\gamma^\mu\psi_1(x) \to (e/g)m^2U_{11}{}^\mu(x).$$

The field equations deduced from the action principle by varying U^μ are

$$M_1{}^2U^\mu + \partial_\nu U^{\mu\nu} = ig[\phi,\phi^\mu] + ig[U_\nu,U^{\mu\nu}] - (g/m)\{{}^*U^{\mu\nu},\phi_\nu\},$$

and the consequence for the operator of total electrical charge is contained in

$$(M_1{}^2/g)\int(d\mathbf{x})U^0 = \int(d\mathbf{x})(i[\phi,\phi^0] + i[U_k,U^0{}_k] - (1/m)\{{}^*U^0{}_k,\phi_k\}).$$

The last term on the right-hand side is objectionable. In its absence, only octuplet terms appear and, on replacing M_1 with the constant m, we obtain just the total charge operator anticipated for 0^- and 1^- particles. It is true that the additional term vanishes in a first approximation, for which $\phi_k = \partial_k\phi$ and $\partial_k{}^*U^{0k} = 0$. But we should prefer that the required charge operator emerge precisely. This can be achieved by a redefinition of the fields ϕ_μ and $U_{\mu\nu}$ within the framework of cubic interaction terms, to which our discussion is limited. The required transformation is given by

$$\phi^\mu \to \phi^\mu - (g/m)\{{}^*U^{\mu\nu},U_\nu\},$$
$$U_{\mu\nu} \to U_{\mu\nu} - (g/m)\{{}^*U_{\mu\nu},\phi\} - (g/m){}^*(\partial_\mu\{U_\nu,\phi\} - \partial_\nu\{U_\mu,\phi\}),$$

and the new version of the interaction is

$$\mathcal{L}_{\text{int}} = ig\,\mathrm{tr}(\phi^\mu[U_\mu,\phi] + \tfrac{1}{2}U^{\mu\nu}[U_\mu,U_\nu]) - \tfrac{3}{2}(g/m)\,\mathrm{tr}(\phi^*U^{\mu\nu}U_{\mu\nu}) - i(g/m^2)\,\mathrm{tr}(\tfrac{1}{2}U^{\mu\nu}[\phi_\mu,\phi_\nu] + \tfrac{1}{3}U^{\mu\nu}U_{\nu\lambda}U_\mu{}^\lambda).$$

COMPARISON WITH EXPERIMENT

Tests of the prediction that $U\phi\phi$, ϕUU, and UUU couplings are all governed by a single coupling constant are available only for the first two, which are involved in various meson decays. In order to minimize the effects of symmetry-breaking interactions, which have not yet been included, we confine our attention to the particles π, ρ, and ω. We shall also identify m with the observed mass of the ρ particle. This is based on the treatment of interactions that break U_3 symmetry, discussed in II, which associates the perturbation with the third axis of the unitary space. The phenomenological analysis of $\rho\pi\pi$ and $\omega\rho\pi$ couplings given in I (which also explained the suppression of $\phi\rho\pi$ coupling) employed the interaction terms

$$\mathcal{L}_{U\phi^2} = \tfrac{1}{2}g_{U\phi^2}i\,\mathrm{tr}\{U^\mu[\phi_\mu,\phi]\}$$

and

$$\mathcal{L}_{\phi U^2} = \tfrac{1}{2}g_{\phi U^2}\,\mathrm{tr}\{\phi\,{}^*U^{\mu\nu}U_{\mu\nu}\},$$

where

$$(g_{U\phi^2})^2/4\pi \sim 5, \quad (m_\omega g_{\phi U^2})^2/4\pi \sim 6.$$

The effective coupling constant $g_{\phi U^2}$ is identified with $-3(g/m)$, $m = m_\rho$, while $g_{U\phi^2} = -3g$. The latter connection can be verified by removing the interaction term $U^{\mu\nu}[\phi_\mu,\phi_\nu]$, either with a field transformation in the manner indicated before, or by a perturbation approximation appropriate to the decay $\rho \to \pi + \pi$. Thus, the two independent evaluations of g agree closely,[2,5] and

$$g^2/4\pi \sim \tfrac{1}{2}.$$

FERMION TRIPLETS

The inability to obtain information about the mass M_1 (singlet) can be traced back to the correspondence between the field $U_{aa}{}^\mu$ and the current $-\bar{\psi}_a\gamma^\mu\psi_a$, which is the current of ψ field charge (as defined in II). The 0^- and 1^- mesons carry zero field charge. Accordingly, we now consider the simplest particle multiplet that possess nonzero field charge. This is the spin-$\frac{1}{2}$ unitary triplet field $\Psi_a(x)$, which is in correspondence with the fundamental Fermi field $\psi_a(x)$. The decomposition

$$\Psi_a = \tfrac{1}{2}(1+\gamma^0)\Psi_a + \tfrac{1}{2}(1-\gamma^0)\Psi_a$$

supplies the bases $\Psi_A{}^{(+)}$ and $\Psi_A{}^{(-)}$ for the representations 6_+ and 6_- of the kinematical group $U_6 \times U_6$. The phenomenological generators $M^{(\pm)}$ now acquire a fermion contribution which is given by

$$(M_{AB}{}^{(+)})_f = \Psi_A{}^{(+)\dagger}\Psi_B{}^{(+)}.$$
$$(M_{AB}{}^{(-)})_f = \Psi_A{}^{(-)\dagger}\Psi_B{}^{(-)}.$$

Through addition and substraction we obtain the equivalent unitary forms

$$(m^2/g)(-U^0 + (1/m)\sigma_k{}^T\phi_k)_f = \Psi^\dagger\Psi + (\Psi^\dagger\sigma_k\Psi)\sigma_k{}^T,$$
$$(m^2/g)(-S + (1/m)\sigma_k{}^TU_{[k]})_f = \Psi^\dagger\gamma^0\Psi + (\Psi^\dagger\gamma^0\sigma_k\Psi)\sigma_k{}^T,$$

[4] This distinction, which requires that one take seriously the possibility of spatio-temporal description at subparticle dimensions, is lost completely in a global or S-matrix particle description. The physical incorrectness of S-matrix theories built on relativistically generalized SU_6 symmetry is beginning to be recognized (see the 29 March 1965 issue of Physical Review Letters).

[5] Some other discussions of the relation between the $\rho\pi\pi$ and $\omega\rho\pi$ coupling constants have appeared very recently: K. Bardakci, J. Cornwall, P. Freund, and B. Lee, Phys. Rev. Letters 14, 264 (1965); B. Sakita and K. Wali, *ibid.* 14, 404 (1965); I. Gerstein, *ibid.* 14, 453 (1965). Also W. Rühl, CERN, February, 1965 (unpublished).

or

$$-m^2(U^0)_f=g\bar{\Psi}\gamma^0\Psi\,,$$
$$m(\phi_k)_f=g\bar{\Psi}i\gamma_k\gamma_5\Psi\,,$$

and

$$m(U_{kl})_f=g\bar{\Psi}\sigma_{kl}\Psi\,,$$
$$-m^2(S)_f=g\bar{\Psi}\Psi\,.$$

The part of $\mathcal{L}_{\rm int}$ that refers to the fermion triplets and the 0^-, 1^- boson fields is then obtained in the tentative form

$$(\mathcal{L}_{\rm int})_f=g\,\mathrm{tr}[U^0(M_1{}^2/m^2)\bar{\Psi}\gamma^0\Psi-(1/m)\phi^k\bar{\Psi}i\gamma_k\gamma_5\Psi-(1/2m)U^{kl}\bar{\Psi}\sigma_{kl}\Psi].$$

Now, these coupling terms can also be displayed as the $U_6\times U_6$ invariant structure

$$(\mathcal{L}_{\rm int})_f=-(2g/m)^2\,\mathrm{Tr}[M^{(+)}\Psi^{(+)\dagger}\Psi^{(+)}+M^{(-)}\Psi^{(-)\dagger}\Psi^{(-)}],$$

provided only that

$$M_1=m$$

is also valid for the unitary singlet combination of 1^- particles. Thus, it requires only a very weak extension of the invariance property of $\mathcal{L}_{\rm int}$ already established for mesons to reach the empirically valid conclusion that the octuplet and singlet of 1^- particles are mass degenerate in the dynamical idealization of U_3 symmetry. How different this dynamical argument is from the SU_6 assertion that the 0^- and the 1^- mesons constitute an initially mass degenerate multiplet, which is split by a spin-dependent perturbation.[6]

The relativistically invariant form of the triplet interaction term is

$$(\mathcal{L}_{\rm int})_f=g\,\mathrm{tr}[-U^\mu\bar{\Psi}\gamma_\mu\Psi-(1/m)\phi^\mu\bar{\Psi}i\gamma_\mu\gamma_5\Psi-(1/2m)U^{\mu\nu}\bar{\Psi}\sigma_{\mu\nu}\Psi].$$

The additional terms demanded by relativistic invariance are

$$g\,\mathrm{tr}[-U^k\bar{\Psi}\gamma_k\Psi+(1/m)\phi^0\bar{\Psi}i\gamma^0\gamma_5\Psi+(1/m)U^{0k}\bar{\Psi}i\gamma^0\gamma_k\Psi].$$

If one were to supplement these by the pseudoscalar interaction

$$-g\,\mathrm{tr}(\phi\bar{\Psi}\gamma_5\Psi)\,,$$

all the additional terms would be comprised in the $U_6\times U_6$ invariant contribution

$$(2g/m)^2\,\mathrm{Tr}[M\bar{\Psi}^{(+)}\gamma_5\Psi^{(-)}+M^\dagger\bar{\Psi}^{(-)}\gamma_5\Psi^{(+)}].$$

The fermion triplets illustrate most simply the derivation of electromagnetic properties from the coupling

$$\mathcal{L}_{\rm em}{}^A=j^\mu A_\mu\to(e/g)m^2U_{11}{}^\mu A_\mu\,.$$

We give first the general verification of the gauge-invariant nature of this description. The field transformation

$$U_{11}{}^\mu\to U_{11}{}^\mu+(e/g)A^\mu\,,$$
$$F^{\mu\nu}\to F^{\mu\nu}-(e/g)U_{11}{}^{\mu\nu}$$

converts the electromagnetic coupling into the explicitly gauge-invariant form

$$\mathcal{L}_{\rm em}{}^F=-(e/g)\tfrac{1}{2}U_{11}{}^{\mu\nu}F_{\mu\nu}$$

while introducing, through the substitution $gU_{11}{}^\mu\to gU_{11}{}^\mu+eA^\mu$, the gauge-covariant combination $\partial_\mu-ieqA_\mu$ involving the appropriate charge matrix q.

If we simplify the dynamics of the U field by retaining only the interaction with the Ψ field, the field equations become

$$m^2U^\mu+\partial_\nu U^{\mu\nu}=-g\bar{\Psi}\gamma^\mu\Psi\,,$$
$$U_{\mu\nu}-(\partial_\mu U_\nu-\partial_\nu U_\mu)=(g/m)\bar{\Psi}\sigma_{\mu\nu}\Psi\,,$$

and therefore

$$(m^2-\partial^2)(e/g)U_{11}{}^\mu=-e[\bar{\Psi}_1\gamma^\mu\Psi_1+(1/m)\partial_\nu\bar{\Psi}_1\sigma^{\mu\nu}\Psi_1].$$

This gives immediately a form-factor description of electromagnetic properties. In particular, the anomalous magnetic moment of a positively charged particle is obtained as two ρ meson magnetons[7] (in the approximation that ignores the indirect contribution of other particles).

Note added in proof. The relations between dependent and independent field variables contain nonlocal contributions, which are demanded by the structure of phenomenological relativistic field theory. These include the linear space-derivative terms illustrated by

$$\phi_k=-(\partial\mathcal{L}_{\rm int}/\partial\phi_k)+\partial_k\phi\,.$$

It is important to recognize that such terms are implicit in the commutation properties of bilinear combinations of the ψ field. This fact is the basis for another, and more fundamental proof that the nine 1^- mesons are mass degenerate in the U_3 dynamical idealization.

It has been known for some time[8] that the commutation properties of local bilinear combinations of Dirac fields, defined by a limiting procedure from initially distinct points, contain additional space-derivative terms which are overlooked in a formal calculation. (Such terms were not mentioned in the text precisely because they are nonlocal. A question on this point by Coleman served to elicit these additional remarks.) Consider, for example, the equal-time commutator

$$[\bar{\psi}_a(x)\gamma^0\psi_b(x),\bar{\psi}_c(x')\gamma_k\psi_d(x')]=\lim_{\epsilon\to0}[\bar{\psi}_a(x)\gamma^0\psi_b(x),\bar{\psi}_c(x'-\tfrac{1}{2}\epsilon)\gamma_k\psi(x'+\tfrac{1}{2}\epsilon)]$$

in which the limit $\epsilon\to0$ is performed symmetrically in

[6] F. Gürsey and L. Radicati, Phys. Rev. Letters **13**, 173 (1964).

[7] A related statement can be found in P. Freund, Institute for Advanced Study, March, 1965 (unpublished).

[8] J. Schwinger, Phys. Rev. Letters **3**, 296 (1959).

space. The result is

$$[\bar{\psi}_a(x)\gamma^0\psi_b(x),\bar{\psi}_c(x')\gamma_k\psi_d(x')] = \delta(\mathbf{x}-\mathbf{x}')\{\delta_{bc}\bar{\psi}_a(x)\gamma_k\psi_d(x)-\delta_{ad}\bar{\psi}_c(x)\gamma_k\psi_b(x)\} - \tfrac{1}{2}i\partial_k\delta(\mathbf{x}-\mathbf{x}')(\delta_{bc}K_{ad}+\delta_{ad}K_{cb}),$$

where

$$K_{ab}=\lim_{\epsilon\to 0}\tfrac{1}{3}i\langle\bar{\psi}_a(x)\boldsymbol{\epsilon}\cdot\boldsymbol{\gamma}\psi_b(x+\epsilon)\rangle$$

is assumed to be nonzero, in consequence of the singularity of the vacuum expectation value, and finite. In the idealization of U_3 dynamical symmetry, we have

$$K_{ab}=K\delta_{ab}$$

and the additional nonlocal term in this commutator becomes

$$-iK\delta_{ad}\delta_{bc}\partial_k\delta(\mathbf{x}-\mathbf{x}').$$

Similar commutators are

$$[\bar{\psi}_a(x)i\gamma_k\gamma_5\psi_b(x),\ \bar{\psi}_c(x')i\gamma^0\gamma_5\psi_d(x')] = \delta(\mathbf{x}-\mathbf{x}')\{\delta_{bc}\bar{\psi}_a(x)\gamma_k\psi_d(x)-\delta_{ad}\bar{\psi}_c(x)\gamma_k\psi_b(x)\} - iK\delta_{ad}\delta_{bc}\partial_k\delta(\mathbf{x}-\mathbf{x}')$$

and

$$[\bar{\psi}_a(x)\sigma_k\psi_b(x),\bar{\psi}_c(x')i\gamma^0\gamma_l\psi_d(x')] = -\delta(\mathbf{x}-\mathbf{x}')\epsilon_{klm}\{\delta_{bc}\bar{\psi}_a(x)\gamma_m\psi_d(x)-\delta_{ad}\bar{\psi}_c(x)\gamma_m\psi_b(x)\} + iK\delta_{ad}\delta_{bc}\epsilon_{klm}\partial_m\delta(\mathbf{x}-\mathbf{x}').$$

The transcription of these commutators into properties of phenomenological fields gives

$$m^2[U_{ab}{}^0(x),U_{kcd}(x')] = \delta(\mathbf{x}-\mathbf{x}')g\{\delta_{ad}U_{kcb}(x)-\delta_{bc}U_{kad}(x)\} - i(g^2/m^2)K\delta_{ad}\delta_{bc}\partial_k\delta(\mathbf{x}-\mathbf{x}')$$

and

$$[\phi_{kab}(x),\phi_{cd}{}^0(x')] = \delta(\mathbf{x}-\mathbf{x}')g\{\delta_{ad}U_{kcb}(x)-\delta_{bc}U_{kad}(x)\} - i(g^2/m^2)K\delta_{ad}\delta_{bc}\partial_k\delta(\mathbf{x}-\mathbf{x}'),$$

$$[U_{[k]ab}(x),U_{lcd}{}^0(x')] = -\delta(\mathbf{x}-\mathbf{x}')\epsilon_{klm}g\{\delta_{ab}U_{mcb}(x)-\delta_{bc}U_{mad}(x)\} + i(g^2/m^2)K\delta_{ad}\delta_{bc}\epsilon_{klm}\partial_m\delta(\mathbf{x}-\mathbf{x}').$$

These are statements about the dependence of U^0, ϕ_k, and $U_{[k]}$ on $U^0{}_k$, ϕ, and U_l, respectively. The appearance of the space derivative of the three-dimensional delta function implies that U^0, ϕ_k, and $U_{[k]}$ additively contain ∂_kU^{0k}, $\partial_k\phi$, and $(\nabla\times U)_k$, respectively. The unit coefficients that occur in the last two relations are reproduced by the value

$$K=m^2/g^2.$$

We conclude that $m^2U_{ab}{}^0(x)$ contains the additive linear term $-\partial_kU_{ab}{}^{0k}(x)$. The comparison with the Lagrangian relation

$$M_1{}^2U^0=-(\partial\mathcal{L}_{\text{int}}/\partial U^0)-\partial_kU^{0k}$$

shows that

$$M_1{}^2=m^2$$

is valid for all nine 1^- mesons, at the dynamical level of U_3 symmetry.

Note added in proof. These considerations can be extended to an octuplet plus singlet family of 2^+ mesons. A correspondence exists between the phenomenological tensor fields $h^{\mu\nu}{}_{ab}$, ${}^\lambda\Gamma^{\mu\nu}{}_{ab}$ and bilinear combinations of the ψ fields constructed with the aid of the Dirac matrices γ^μ, $\sigma^{\mu\nu}$, and single coordinate derivatives. In particular, the commutation properties of $h^{kl}{}_{ab}$ and ${}^0\Gamma^{mn}{}_{ab}-\frac{1}{2}\partial^mh^{0n}{}_{ab}$ can be compared with those of $i\bar{\psi}_a(\gamma^k\partial^l+\gamma^l\partial^k)\psi_b$ and $i\bar{\psi}_a\sigma^{0n}\partial^m\psi_b-\frac{1}{3}\delta^{mn}i\bar{\psi}_a\sigma^{0k}\partial_k\psi_b$, where the coordinate derivatives act antisymmetrically on both fields. These phenomenological-field commutation relations depend explicitly upon the particle masses. In the idealization of U_3 dynamical symmetry, the comparison implies the mass degeneracy of octuplet and singlet for the 2^+ mesons. This result is very gratifying in view of the experimental situation concerning the following particles: f(1.25 BeV, $T=0$), A_2(1.32 BeV, $T=1$), K^*(1.43 BeV, $T=\frac{1}{2}$, $Y=\pm1$), and f'(1.50 BeV, $T=0$), which seem to form just such a family of 2^+ mesons, in striking analogy with the well-known 1^- mesons [see three contributions on spin-2 mesons in Phys. Rev. Letters, 16 August 1965]. The dynamical couplings of 2^+ mesons should also have a universal character akin to those of 1^- mesons, since the former provide a basis for the representation of gravitational interactions analogous to the electromagnetic use of spin-1 mesons.

Finally, we must mention an important aspect of the comparative transformation program which was discussed in the lecture upon which III is based, but is not contained in the published version. The assumption that fundamental field transformations induce linear transformations on phenomenological fields forces one to restrict the compact group U_{12} to the parity-preserving subgroup $U_6\times U_6$. But parity-changing transformations can be represented if nonlinear transformations are admitted. This leads to a further unveiling of phenomenological dynamics, including quartic couplings among the mesons and quadratic meson couplings with baryons, which are also specified by the universal coupling constant g.