

mechanisms and their relative strengths, we must conclude that the angular-momentum state of K^- absorption in liquid helium is still unknown.

As has been pointed out in Ref. 16, a measurement of the fraction of stopping mesons which yield x rays from transitions in the low-lying n orbits would help a great deal in resolving this problem. Assuming that the P -state capture rates are large compared to the P to S radiation rates as calculated, the value of this fraction would at least provide a measure of the minimum amount of P -state absorption.²⁹

²⁹ Note added in proof. Recent results, obtained after this paper was submitted, on x-ray emission from K^- absorption in helium [G. R. Bureson, D. Cohen, R. C. Lamb, D. N. Michael,

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R. A. Schluter, and T. O. White, Jr., Phys. Rev. Letters **15**, 70 (1965)] have indicated that a major fraction of the stopping K^- mesons reach the low-lying P states, consistent with the measured cascade time reported here.

Dynamical Basis of the Sum Rule $2\Sigma_{-}^{-} = \Lambda_{-}^{0} + \sqrt{3}\Sigma_{0}^{+}$. II

BENJAMIN W. LEE*

*The Institute for Advanced Study, Princeton, New Jersey
and*

Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania†

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The assumptions that (1) the weak processes $B \rightarrow B'$, $M \rightarrow M'$ ($\Delta S = \pm 1$) are mediated by an interaction that transforms like λ_6 , (2) the nonleptonic decay amplitudes of hyperons are dominated by pole terms, and (3) the mass-splitting pattern of the baryon octet can be approximated by $m = m_0 + (\Delta m)Y$ imply that the effective Lagrangian describing the parity-conserving hyperon decays transforms like λ_7 under $SU(3)$. The sum rule $2P(\Sigma_{-}^{-}) = P(\Lambda_{-}^{0}) + \sqrt{3}P(\Sigma_{0}^{+})$ follows immediately from this observation.

I. INTRODUCTION

IN a previous paper of the same title¹ it was shown that the parity-conserving amplitudes for nonleptonic hyperon decays satisfy the sum rule²

$$2P(\Sigma_{-}^{-}) = P(\Lambda_{-}^{0}) + \sqrt{3}P(\Sigma_{0}^{+}). \quad (1)$$

The assumptions made in deriving Eq. (1) are:

(i) The nonleptonic weak interaction is CP conserving and has the octet transformation property. In particular, it transforms like $\lambda_6 = \lambda_6^T$.³

(ii) The nonleptonic decay processes are dominated by the mechanism represented by baryon and meson poles (the model of Feldman, Matthews, and Salam,⁴ hereafter referred to as the FMS model).

(iii) The mass-splitting pattern among baryons is approximated by a simplified Gell-Mann-Okubo formula: $m = m_0 + (\Delta m)Y$, or $m_{\Lambda} = m_{\Sigma}$, $m_{\Sigma^{-}} - m_{\Lambda} = m_{\Lambda} - m_N$.

The purpose of this note is to show that under the assumptions (i)-(iii), the *effective* nonleptonic hyperon weak interaction (effective in the sense that the effect of mass splitting is taken into account) transforms like $\lambda_7 = -\lambda_7^T$. It follows immediately then, from the work of Coleman and Glashow,^{5,6} that the parity-conserving amplitudes obey the sum rule (1), since the circumstance at hand corresponds to what Coleman and Glashow⁵ called the abnormal octet dominance.

II. ANALYSIS

An effective Lagrangian which generates the pole-dominance model of FMS is

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}' \quad (2)$$

$$\begin{aligned} \mathcal{L}' = & (\Delta m)\bar{B}_i(F_8^{ij} + \alpha D_8^{ij})B_j + f\bar{B}_i(F_6^{ij} + \beta D_6^{ij})B_j \\ & + g\bar{B}_i(D_8^{ij} + \gamma F_8^{ij})\gamma_5 B_j M^k \\ & + (\Delta\mu^2)M_i D_8^{ij} M_j + f' M D_6^{ij} M_j, \quad (3) \end{aligned}$$

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† Permanent address.

¹ B. W. Lee and A. R. Swift, Phys. Rev. **136**, B228 (1964). See also J. D. Bjorken and B. Helleston, Phys. Letters **12**, 141 (1964).

² B. W. Lee, Phys. Rev. Letters **12**, 83 (1964); H. Sugawara, Progr. Theoret. Phys. (Kyoto) **31**, 213 (1964); B. Sakita, Phys. Rev. Letters **12**, 379 (1964); S. P. Rosen, *ibid.* **12**, 408 (1964).

³ M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964).

⁴ G. Feldman, P. T. Matthews, and A. Salam, Phys. Rev. **121**, 302 (1961); H. Sugawara, Nuovo Cimento **31**, 635 (1964).

⁵ S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).

⁶ S. Coleman, S. L. Glashow, and B. W. Lee, Ann. Phys. (N. Y.) **30**, 348 (1964).

where \mathcal{L}_0 is the free Lagrangian for the baryon octet B_i (spin $\frac{1}{2}$) and the meson octet M_i (pseudoscalar), and is $SU(3)$ invariant. For the definition of the coupling matrices $F_k^{ij} = -if_{kij}$, $D_k^{ij} = d_{kij}$, see Gell-Mann.⁷ f and f' measure the strength of the weak transitions $B \rightarrow B'$ ($\Delta S = \pm 1$) and $M \rightarrow M'$ ($\Delta S = \pm 1$); g measures the strength of the strong $\bar{B}BM$ coupling. We assume $\Delta m \gg f$ and $(\Delta\mu^2) \gg f'$.

The last term can always be transformed away to first order in f' by a unitary transformation that conserves charge and CP . We perform the transformation⁶:

$$\begin{aligned} M_i &\rightarrow \mathfrak{M}'_i = \left[1 + i \frac{2}{\sqrt{3}} \frac{f'}{(\Delta\mu^2)} F_7 \right]^{ii} M_j, \\ B_i &\rightarrow \mathfrak{B}'_i = \left[1 + i \frac{2}{\sqrt{3}} \frac{f'}{(\Delta\mu^2)} F_7 \right]^{ii} B_j. \end{aligned} \quad (4)$$

In terms of the new variables \mathfrak{M}'_i and \mathfrak{B}'_i , \mathcal{L}_0 remains unchanged, while Eq. (3) holds without the last term with a redefinition of f and β . If

$$f/(\Delta m) = \beta/\alpha = f'/(\Delta\mu^2) \quad (5)$$

as in a pure tadpole model,⁵ then both the second and the last terms get transformed away. Let us assume therefore that at least one of the equalities in (5) is not true. We shall henceforth drop the term proportional to f' in our analysis.

We shall further assume that $\alpha = 0$ in (3). This is for the sake of simplicity. The effect of nonvanishing α will be discussed later. We perform a unitary transformation on B_i :

$$B_j \rightarrow \mathfrak{B}_j = \left[1 + i \frac{2}{\sqrt{3}} \frac{f}{(\Delta m)} (F_7 + \beta D_7) \right]^{ij} B_j. \quad (6)$$

This transformation is *not* an $SU(3)$ transformation, but can be embedded in $SU(8)$. This transformation conserves charge and the property under charge conjugation.

Proof: The fact that a transformation of the form $(1 + i\theta F_7)$ conserves both charge and charge-conjugation property is well known.⁸ Consider the transformation $(1 + i\theta D_7)$. To show that $[Q, D_7] = 0$, we note $[F_i, D_j] = if_{ijk} D_k$ and $Q = F_3 + (3)^{-1/2} F_8$. Under charge

⁷ M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

⁸ See for instance Ref. 6 and N. Cabibbo, Phys. Rev. Letters **12**, 62 (1964).

conjugation

$$\begin{aligned} C: \mathfrak{B}_i &\rightarrow \mathfrak{B}_i^c = \sum_j [1 + i\theta D_7]^{ij} \epsilon_j \bar{B}_j \\ &= \epsilon_i \sum_j \bar{B}_j (1 - i\theta D_7)^{ji} \\ &= \epsilon_i \bar{\mathfrak{B}}_i, \end{aligned}$$

where $\epsilon_i = -1$ for $i = 2, 5, 7$ and $+1$ otherwise. We have used the identities⁹ $D_k^{ij} = D_k^{ji}$, $D_7^{ij} \epsilon_j = -\epsilon_i D_7^{ji}$ (not summed). Q.E.D.

To lowest order in f , \mathcal{L}' in (3) is now transformed into

$$\begin{aligned} \mathcal{L}' &\rightarrow (\Delta m) \bar{\mathfrak{B}}_i F_8^{ij} \mathfrak{B}_j + (\Delta\mu^2) M_i D_8^{ij} M_j \\ &\quad + g \bar{\mathfrak{B}}_i (D_k^{ij} + \gamma F_k^{ij}) \gamma_5 \mathfrak{B}_j M_k \\ &\quad + i \frac{2}{\sqrt{3}} \frac{gf}{\Delta m} \bar{\mathfrak{B}}_i [F_7 + \beta D_7, D_k + \gamma F_k]^{ij} \gamma_5 \mathfrak{B}_j M_k \quad (7) \end{aligned}$$

which shows, to lowest order in f , that the nonleptonic weak interaction of hyperons transform effectively like λ_7 . The last term in (7) can be cast in a more familiar form by writing the $\bar{B}BM$ coupling in Eq. (3) in the form

$$g (\text{Tr} \bar{b} \gamma_5 \{m, b\} + \gamma \text{Tr} \bar{b} \gamma_5 [m, b]), \quad (8)$$

where $b = \lambda_i B^i$, $m = \lambda_i M^i$ are the 3×3 representations of the octets of baryons and mesons, and noting that the transformation (7) is equivalent to

$$b \rightarrow \mathfrak{b} = b + i \frac{2}{\sqrt{3}} \frac{f}{\Delta m} ([\lambda_7, b] + \beta \{\lambda_7, b\}). \quad (9)$$

The change in (8) due to the transformation (8) is

$$\begin{aligned} &i \frac{2}{\sqrt{3}} \frac{gf}{\Delta m} [(1 + \beta)(1 + \gamma) \text{Tr}(\bar{b} \gamma_5 [\lambda_7, m] b) \\ &\quad - (1 - \beta)(1 - \gamma) \text{Tr}(\bar{b} \gamma_5 \bar{b} [\lambda_7, m])] \quad (10) \end{aligned}$$

which is equivalent to the last term of Eq. (7). The sum rule (1) holds independently of the choice of β and γ .

When $\alpha \neq 0$ in Eq. (3), the part of \mathcal{L}' quadratic in baryon fields can in principle be diagonalized by a $SU(8)$ transformation, but the *effective* nonleptonic weak interaction transforms no longer like a pure octet.

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⁹ The secondary identity follows from $D_k^{ij} = d_{kij}$, $\epsilon_{k\ell i} \epsilon_j d_{kij} = d_{kij}$.