# Moderation and Cascade Times of Negative Pions and Negative Kaons in Liquid Helium*

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We have measured separately the mean moderation times $T_\pi^{\rm He}(v_A)$ and $T_K^{\rm He}(v_A)$, spent by $\pi^-$ and $K^-$, respectively, in atomic orbits before nuclear capture in the Northwestern helium bubble chamber. It was necessary to resolve the velocities of $\pi^-$ (or $K^-$) down to $v\approx 0.02c$. The resolution was obtained in the pion case by measuring the $\mu^-$ range from $\pi^-$-$\mu^-$-$e^-$ decays and in the kaon case by the kinematic analysis of the $\tau$-decay mode. We obtained the following results:

$$T_\pi^{\rm He}(0.02c)=(3.19\pm0.23)\times10^{-10}\ {\rm sec},$$
$$T_K^{\rm He}(0.02c)=(2.4\pm0.4)\times10^{-10}\ {\rm sec}.$$

These measured times are two orders of magnitude longer than that predicted by Day assuming dominant Stark effect and thus $S$-state capture. A recent calculation by Russell reduces the rate of Stark effect yet predicts cascade times still five times shorter than our experimental values. Thus the angular-momentum states of $K^-$ capture in liquid helium, whose determination was the motivation for the present experiments, are still unknown without a detailed understanding of the de-excitation mechanism of the $K$-mesonic atom.

## 1. INTRODUCTION

THIS report describes the results of a completed measurement of the cascade times of negative pions and kaons in liquid helium. There have been three previous reports on this subject.[1–3] The quantity which we call the cascade time in liquid helium $T^{\rm He}(v_A)$ is the time taken by the meson in going from an initial velocity $v_A$ to nuclear absorption. The method used to determine $T_\pi^{\rm He}(v_A)$ was first employed by Fields *et al.* in hydrogen[4] and subsequently by others in hydrogen[5,6] and deuterium.[7] The measurement of $T_K^{\rm He}(v_A)$ reported here is done by a new technique involving the use of the tau-decay mode of the $K^-$.[8] We present here a complete analysis of the data and an attempt to interpret the results in terms of the latest theoretical work done in this field.

*Note added in proof.* Since this paper was submitted, two experimental groups,[8a,8b] using techniques first suggested in Ref. 3, have published the results of measurements of the moderation time of $K^-$ mesons in liquid hydrogen.

These measurements were motivated by the possibility of ascertaining the predominant angular-momentum state ($l$) of nuclear absorption of $K^-$ mesons in liquid helium. This knowledge is important in the analysis of the spin of the ground state of the $_\Lambda H^4$ ($_\Lambda He^4$)[9] and the spin and parity of the 1385-MeV $Y_1^*$.[10,11] Both of these analyses relied in part upon the assumption that negative kaons are predominantly absorbed by the helium nucleus from $l=0$ states.

In principle, the rates of absorption from each $l$ state could be found by a detailed consideration of the important de-excitation processes as revealed by the experimental results for the cascade times. For hydrogen the calculations of Day, Snow, and Sucher,[12] Russell and Shaw,[13] and Leon and Bethe[14] indicated that the pion should be absorbed very rapidly from a high $nS$ state ($n\sim4$) as a result of Stark oscillations arising from collisions of the mesonic atom with neighboring molecules. Leon and Bethe predict $3.5\times10^{-12}$ sec for the cascade time $T_\pi^{\rm H_2}(0.01c)$. The experimental value[5] $T_\pi^{\rm H_2}(0.01c)=(2.5\pm0.6)\times10^{-12}$ sec lends strong support to the Stark-effect calculation and therefore to the argument for high-$nS$ state absorption of mesons in hydrogen.

The first calculation of the de-excitation rates for mesons stopping in liquid helium was done by Day and Snow.[15,16] They pointed out that this problem in helium

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is quite different from the hydrogen case and quite a bit more complicated. The de-excitation processes expected for the positive charged mesonic helium ion (after the ejection of both orbital electrons) are not the same as those of the neutral mesonic hydrogen atom. From a detailed calculation of the competing processes, Day[16] concluded that $K^-$ stopping in liquid helium should also be captured in times $\lesssim 10^{-12}$ sec from high $nS$ states.

A recalculation of the atomic capture and cascade processes of both $K^-$ and $\pi^-$ mesons in liquid helium has recently been done by Russell.[17] He obtains rates for the same de-excitation and absorption processes which are substantially different from those of Day. Consequently, the cascade times predicted by Russell are about two orders of magnitude longer than that obtained by Day and raise the possibility that there might be a significant amount of $P$-state absorption at lower principal quantum numbers.

In 1961, the helium-bubble chamber[18] was exposed to a stopping $K^-$ beam at the Lawrence Radiation Laboratory Bevatron. In 1963, this same chamber was exposed to a stopping $\pi^-$ beam at the University of Chicago Cyclotron. From these two exposures we were able to obtain the data necessary to determine the cascade times of both $\pi^-$ and $K^-$ mesons in liquid helium.

The description of the experimental details and results are contained in Secs. 2, 3, and 4. A comparison of the theoretical work of Day, Russell, and others, with the experimental results, is done in Sec. 5.

## 2. EXPERIMENTAL CONSIDERATIONS

The cascade time, $T(v_A)$, which we have defined as the average time a meson spends in going from velocity $v_A$ to nuclear absorption, can be obtained as follows. If we assume that upon being captured into atomic orbit, there are $r$ channels open to a meson in its cascade to the nucleus, we may define the average cascade time as

$$T=(\sum_{k=1}^{r} N_k t_k)/(\sum_{k=1}^{r} N_k), \tag{1}$$

where $t_k$ is the time spent by a meson in channel $k$ ($t=0$ corresponds to the instant at which the meson has velocity $v_A$), and $N_k$ is the number of mesons cascading through channel $k$ at $t=0$. If we assume $t_k \ll \tau$ for all $k$, where $\tau$ is the meson mean lifetime, we get

$$N_k t_k = n_k \tau, \tag{2}$$

where $n_k$ is the number of decays observed in the $k$th channel. Thus, using (2), the cascade time defined in (1) becomes

$$T=\tau(\sum n_k/\sum N_k)$$

or

$$T=\tau(N_d/N_s),$$

where

$$N_s=\sum_{k=1}^{r} N_k$$

is the total number of mesons slowing down to velocity $v=v_A$ and

$$N_d=\sum_{k=1}^{r} n_k$$

is the total number of decays observed with meson velocities $v\leq v_A$. We emphasize that this relation is only true if there are no channels open where the time to nuclear absorption is comparable with the decay lifetime of the meson. This seems like a valid assumption when we consider that the longest calculated cascade times[16] (associated with channels in which radiative processes occur) are about two orders of magnitude shorter than the decay lifetimes of the mesons.[19]

A measurement of $T(v_A)$ is most meaningful for $v_A$ as small as possible because we are primarily interested in the time a meson spends in atomic orbits. In particular, we would like to be able to resolve meson velocities down to values which are comparable with the velocity of the orbital atomic electrons. This corresponds to $v_A \sim Z\alpha c$, where $Z$ is the effective atomic number of the moderator and $\alpha$ is the fine structure constant. Calculations[20] have shown that the time taken by the meson to go from $v=Z\alpha c$ to atomic capture is much shorter than the cascade time measured here. Therefore, for helium we would like to have good resolution down to $v_A \lesssim 0.02c$. Radius of curvature measurements of the meson's momentum extrapolated to the decay vertex through the range-momentum relationships are not sufficiently accurate because they have large uncertainties below meson momenta of about 100 MeV/$c$ due to multiple scattering. Therefore, to obtain an accurate determination of $N_d$ we must rely upon the meson's decay kinematics.

The method used to determine $N_d$ for pions employs the unique range of the $\mu^-$ from the two-body decay of pions at rest, $\pi^- \rightarrow \mu^- + \nu$. This range is 1.03 cm in liquid helium of density $\rho=0.141$ g/cm$^3$. The procedure consists of looking for muons decaying into the backward hemisphere, i.e., those decays where the laboratory opening angle ($\theta_{\pi\mu}$) between the extension of the pion track and the direction of the muon is greater than 90°. (A correction for solid angle must then be included.) The backward decays are chosen for two reasons. First, examination of Fig. 1 shows that the kinematics are single valued and most sensitive to the pion velocity in

[17] J. E. Russell (to be published).

[18] M. M. Block, W. M. Fairbank, E. M. Harth, T. Kikuchi, C. Meltzer, and J. Leitner, in *Proceedings of the International Conference on High-Energy Accelerators and Instrumentation, Geneva, 1959* (CERN Scientific Information Services, Geneva, Switzerland, 1959), p. 461.

[19] We used $\tau_\pi=(2.551\pm0.026)\times10^{-8}$ sec and $\tau_K=(1.229\pm0.008)\times10^{-8}$ sec from A. Rosenfeld *et al.*, Rev. Mod. Phys. **36**, 977 (1964).

[20] E. Fermi and E. Teller, Phys. Rev. **72**, 399 (1947).

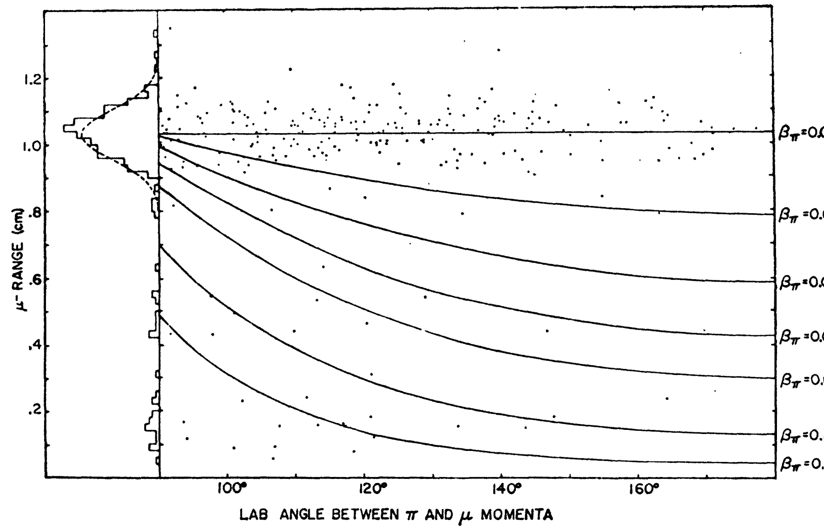


FIG. 1. Illustrated on a graph of the kinematics of pion decay is a scatter plot of the range of the $\mu^-$ versus the angle between the laboratory $\pi$ and $\mu$ momenta ($\theta_{\pi\mu}$) for $\theta_{\pi\mu}>90°$. The range distribution histogram of these events plotted along the ordinate is compared to that of 483 $\pi^+$ decays at rest presented as a Gaussian normalized to the $\pi^-$ sample.

this region. Second, it solves the problem of eliminating the only source of background which cannot be separated from $\pi$-$\mu$-$e$'s by ionization considerations alone—namely, a $\mu^-$ in the beam that Coulomb scatters through a small angle and subsequently decays. For the low contamination of stopping muons in the beam, estimated at about 1 muon per 5 pions, the probability of finding a muon scattering through an angle greater than 90° is negligible. By this procedure we are able to measure pion velocities down to $v_\pi \sim 0.02c$.

The measurement of $N_d$ for $K$ mesons is more difficult. Because of the large rest mass of the $K$ meson, the two-body decay modes for the $K$'s, i.e., $K^- \rightarrow \mu^- + \nu$, $K^- \rightarrow \pi^- + \pi^0$, result in energetic charged secondaries which invariably leave the chamber. This eliminates the possibility of obtaining accurate momentum resolution from range-energy relations. Then too, the momenta of these decay particles is in a region ($\sim$200 MeV/$c$) of considerable background from the products of $K^-$ interactions (i.e., $K^- + He^4 \rightarrow \pi^-$ + invisible $\Lambda^0$ + invisible $He^3$). However, the relatively rare (5.7%) $\tau$ decay mode ($K^- \rightarrow \pi^- + \pi^- + \pi^+$) provides an excellent means of obtaining the desired momentum resolution and is free of background. All of the $\tau$-decay products are charged and hence measurable. In general this gives us four constraints on the decay, or three constraints if we ignore the measured $K$ momentum. Even a simple coplanarity check on the decay pions determines if a given event is a possible decay "at rest." Using the kinematical optimization program HEGUTS we obtained a $K^-$ momentum resolution of about 8 MeV/$c$ for $p_K > 10$ MeV/$c$ and 6 MeV/$c$ for decays at rest. Thus we are able to measure the velocity of the $K$ to better than $v_K \sim 0.02c$.

### 3. ANALYSIS OF THE PION FILM

Approximately four pions were observed to come to rest in each frame. The pictures were scanned for two types of events: $\pi$-$\mu$-$e$'s and pion interactions producing either a 0-pronged or a 1-pronged star. The scanning criteria imposed were:

(1) The point of pion decay or interaction must lie within a fixed fiducial volume ($\sim 15 \times 10 \times 8$ cm$^3$) whose limits are at least 1 cm from each wall.

(2) The muon must make an angle of no less than 60° with respect to the parent pion in the projection of at least one view.

The first condition was imposed to ensure that the decay muon would lie entirely within the chamber, that the decay electron from the muon would be easily identifiable (since the $\pi$-$\mu$ decay configuration is not kinematically overdetermined), and that the curvature of the beam track would be unambiguous (to distinguish pion interactions from background proton scattering). The second condition was made liberal enough to guarantee that we include all backward-decaying muons.

Despite the careful selection of fiducial volume, we found that it was difficult to distinguish pion interactions with a prong which leaves the chamber after a short distance ($\sim$1 cm) from the case where a beam pion scatters and leaves in this distance. In order to correct for this, but to avoid further reduction of the fiducial volume, we counted only those pion interactions which gave either no prong or a prong which stops in the chamber, and then corrected the latter for geometric detection efficiency. This amounted to a 14% correction. We obtained 34 600 pion interactions corrected for a measured scanning efficiency of 95%.[21] In this same sample there were 409 pion decays satisfying the criteria

[21] The pion beam enters the chamber at about 70 MeV/$c$ and the probability of interaction in the momentum interval $mZ\alpha < p < 70$ MeV/$c$ is negligible compared to the probability for the pion coming to rest in the liquid helium. For the $K$ beam we have empirically determined the fraction of $K^-$ interacting before coming to rest (10%) and this amount is subtracted from the total number of $K$ interactions observed.

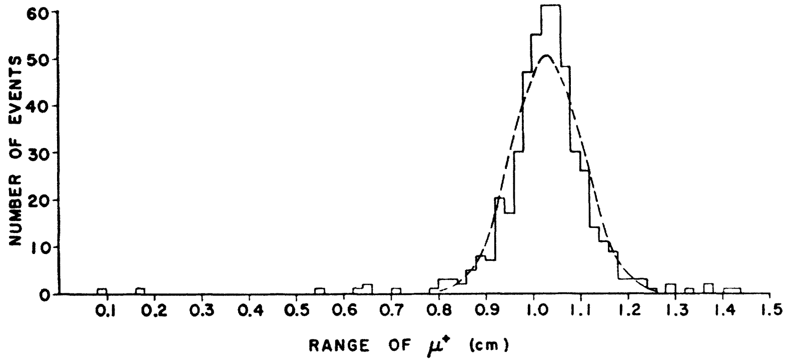


FIG. 2. Muon range distribution from $\pi^+$ decays. The broken curve is a Gaussian fitted to those events in the muon range interval between 0.70 and 1.30 cm.

listed above, all of which were hand measured. Of these, 241 had $\theta_{\pi\mu}\geq 90°$ and are presented as points in Fig. 1. The measured scanning efficiency for pion decays is 96% for muon ranges $\geq 0.7$ cm and 94% for shorter ranges.

To determine the effect that range straggling would have on the distribution of muon ranges, we ran a separate exposure of the chamber to a stopping $\pi^+$ beam. From measurements in this film we obtained the range distribution of $\mu^+$'s from 483 $\pi^+$ decays at rest. A histogram of this distribution is presented in Fig. 2. The dashed curve is a Gaussian fitted to those events in the interval 0.70 cm$\leq R_\mu \leq$1.30 cm. Along the $R_\mu$ axis of Fig. 1, we have drawn this same Gaussian normalized to the number ($n=211$) of $\pi^-$ decays in the same interval. The solid histogram in Fig. 1 is the experimental distribution for all 241 $\pi^-$ decays. The average measurement uncertainties in this region were about 3° for $\theta_{\pi\mu}$ and 0.033 cm for $R_\mu$. Therefore, the observed width of the $\mu^+$ Gaussian (full width $\sim$0.2 cm) is almost entirely due to range straggling. The shape, peak, and width of the histogram are in good agreement with the Gaussian. On the basis of this agreement, and since we expect only about three events in flight with $R_\mu\geq 0.7$ cm and $\theta_{\pi\mu}\geq 90°$, we can infer that the events in the histogram largely correspond to decays "at rest."

From ordinary stopping power theory we have calculated the number of expected decays in any $\Delta\beta$ interval. Table I gives the expected background for $\theta_{\pi\mu}\geq 90°$, and the corresponding observed values for various $\Delta\beta$ intervals. Above $\beta_\pi=0.06$ (at which point the contribution from decays at rest due to range straggling is negligible), the agreement between the predicted and observed values is good. The last column of Table I gives the time spent by the pion in these $\Delta\beta$ intervals as evaluated from the observed events.

TABLE I. Energy-loss predictions and transit times for various pion-velocity intervals.

| $\Delta\beta_\pi$ | Expected | Observed | Time of transit ($10^{-11}$ sec) |
|---|---|---|---|
| 0.27–0.16 | 14.4 | 11 | 8.4±2.4 |
| 0.16–0.12 | 8.3 | 9 | 2.9±0.9 |
| 0.12–0.08 | 7.0 | 7 | 1.7±0.6 |
| 0.08–0.06 | 2.3 | 3 | 0.6±0.3 |
| 0.06–0.02 | 2.3 | ··· | 0.4[a] |
| 0.06–nuclear capture | ··· | 211 | 32.3±2.1[b] |

[a] Evaluated using expected number of events.
[b] Evaluated using observed number of events and estimate of number expected (2.5) between $\beta=0.06$ and atomic capture.

As a final check on the consistency of the data, we calculated the cascade times obtained using various values of the minimum $\theta_{\pi\mu}$ in addition to 90°. The cascade time was found to be essentially independent of $(\theta_{\pi\mu})_{\min}$. The best value for this time is

$$T_\pi^{\text{He}}(0.02c)=(3.19\pm 0.23)\times 10^{-10}\text{ sec.}$$

This value is consistent with the earlier preliminary result[1] based on about half of the present data, as well as with the result obtained by Fetkovich and Pewitt[2] on the basis of a similar study involving 11 pion decays. It should be emphasized that the measured helium cascade time is about two orders of magnitude greater than the cascade time in liquid hydrogen.[4–6]

## 4. ANALYSIS OF THE KAON FILM

The film was scanned in three views for all interactions (excluding elastic scattering) and all decays. The measured scanning efficiency for finding $\tau$'s from two scans was 99%. Tau-decay candidates from 30 400 $K$'s stopping in the fiducial volume[21] were measured on a Hydel digital measuring machine and sent through a reconstruction program written for the Northwestern helium bubble chamber.

Because the $\tau$ decay has such an unusual configuration, all background from $K^-$-$He^4$ interactions could be eliminated either on the scanning table or at the measuring stage by imposing the following condition:

(1) A $\tau$-decay candidate is rejected if the length of either of the negative pions is less than 1.0 cm and there is no visible recoil at the vertex where the pion stops.

This condition is necessary because it is difficult to distinguish the legitimate $\tau$ decay where one of the negative pions stops within 1 cm of the decay vertex (thereby eliminating observation of the negative curva-

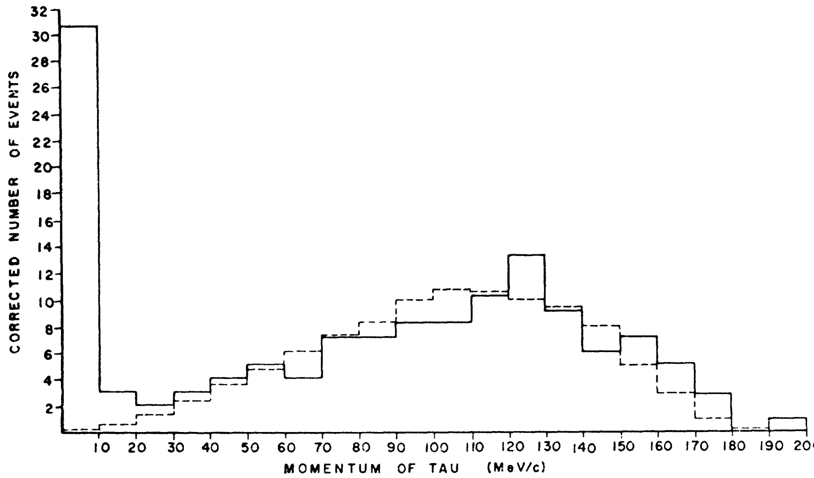


FIG. 3. The solid histogram is the kinematically optimized momentum distribution for all $\tau$ decays whose full potential range lies within the fiducial region. The dashed histogram is the prediction from ordinary energy-loss theory corrected for the escape probability.

ture) and yields a 0-prong star, from the case:

$$K^- + He^4 \rightarrow \Sigma^\pm + \pi^\mp + H^3$$
$$\searrow \pi^\pm + n,$$

where the $\Sigma$ decays before making a bubble and the triton looks like a short stopping pion. From the experimental triton momentum distribution in reaction (1) [10] it is found that a negligible fraction of the events have triton momenta corresponding to ranges greater than 1.0 cm. Hence, condition (1) effectively eliminates this source of background and all 510 events passing this condition were fitted satisfactorily to the $\tau$-decay hypothesis by HEGUTS. Using the distribution[22] for pion momenta from $\tau$ decays and the experimentally determined fraction of at rest $\pi^-$ interactions in helium yielding a 0-prong star,[1] we find that imposing condition (1) necessitates a 1.3% correction to our data.

Pion momenta obtained from curvature measurements gave average uncertainties of about 12%. For the 1 pion in ~6 which stopped in the chamber, the momentum uncertainty, arising primarily from range straggling, was about 2%. The $K^-$ curvature measurements for vertex momenta ($p_K$) below about 150 MeV/$c$ had such large uncertainties due to multiple scattering they were not used. Typical uncertainties in dip and projected angles were about one degree.

For each $\tau$ decay it was required from the optimized momentum and direction that

(2) the $K^-$ would have come to rest within the chosen fiducial volume had it continued along its path instead of decaying.

Those failing this criterion were eliminated from the sample.[23] In the case of 15 high-momentum decays ($p_K > 150$ MeV/$c$) occurring near the boundaries of the fiducial volume, in which none of the pions were long enough ($l_\pi \geq 3$ cm) for a momentum measurement, the directly measured $K^-$ momentum was sufficiently accurate to show that they would not have come to rest within the fiducial volume. In the remaining 136 events satisfying condition (2), all but six had at least two usable momentum measurements and consequently had at least two constraints on the kinematic solution.

We found it was necessary to treat the HEGUTS solutions for low-momentum decays with considerable caution. HEGUTS would not obtain a satisfactory fit for many of those cases at first. This difficulty arises from the extreme directional sensitivity of the three-pion resultant vector momentum as the magnitude of this vector approaches zero at low $\tau$-decay velocity. It was often necessary to measure the same event several times before a good fit was obtained. Further, all events satisfying $p_K \leq 50$ MeV/$c$ were sent through one ad-

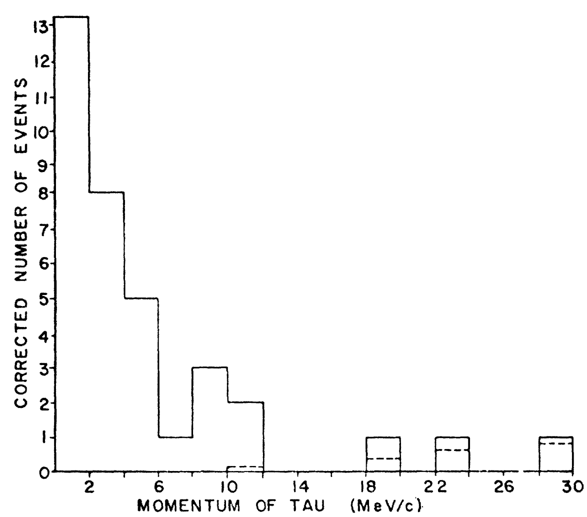


FIG. 4. The same distributions as in Fig. 3 on an expanded scale up to 30 MeV/$c$.

[22] M. Ferro-Luzzi, D. H. Miller, J. J. Murray, A. H. Rosenfeld, and R. D. Tripp, Nuovo Cimento 22, 1087 (1961).

[23] Note that this consideration was not important in the pion-decay case, since the requirement of a backward decaying muon restricted the pion momentum to a maximum of 40 MeV/$c$ corresponding to a residual pion range of only 1.4 cm.

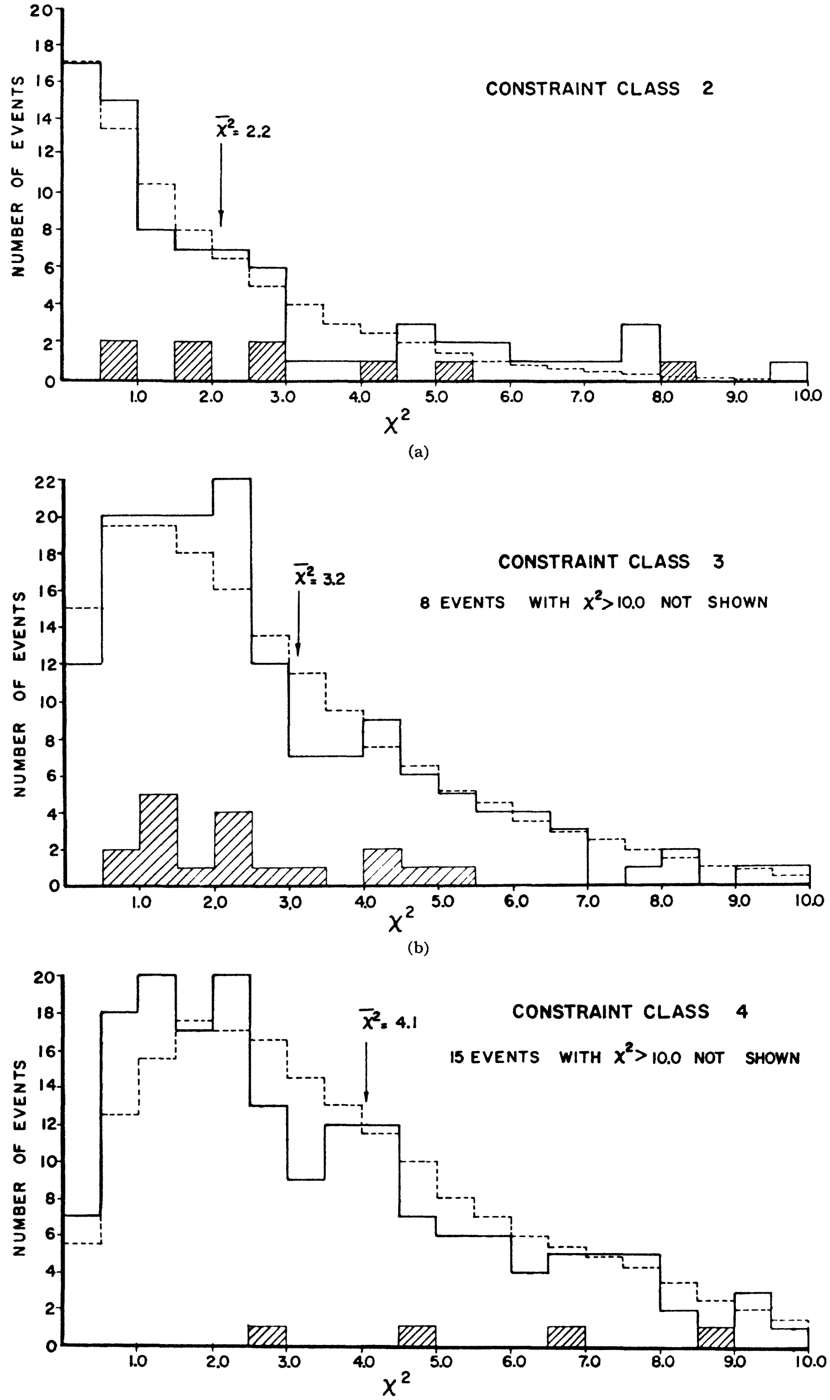


FIG. 5. The solid histograms are the $\chi^2$ distributions for all fitted $\tau$-decay events (a) with two constraints, (b) with three constraints, (c) with four constraints. The broken histograms are the theoretical predictions for a pure sample. The hatched areas represent those events whose fitted momenta were below 20 MeV/$c$.

ditional step to determine how consistent the event was with a decay at rest. Under the assumption that $p_K=0$ a coplanarity optimization for the three decay pions, with a subsequent determination of energy and momentum conservation in the optimized plane, was done with the use of the computer program LIBRARY. This was a very sensitive indicator of whether an event could be included as a possible "at rest" candidate. All events which HEGUTS had placed in the region $p_K \geq 30$ MeV/$c$ were rejected by LIBRARY because either the coplanarity $\chi^2$ or the energy-momentum $\chi^2$ was greater than 10.0.

The decay momentum distribution is drawn as the solid histogram in Fig. 3 in intervals of 10 MeV/$c$. Also drawn in as the dashed histogram are the corresponding predictions from energy-loss theory. The large peak rising well above energy-loss predictions at low momenta corresponds to those events decaying from atomic orbits. In Fig. 4 we have replotted the $p_K$ distribution up to 30 MeV/$c$ in smaller intervals. Above 10 MeV/$c$, the estimated background integrated over 5 MeV/$c$ intervals is indicated in those bins where at least one event appears.

Distributions of the final HEGUTS $\chi^2$ values in each constraint class are presented in Fig. 5, including all events satisfying condition (1) whose decay vertex lies in the fiducial region. The average values of these distributions and the expected[24] shapes for a pure sample are also indicated. The cross-hatched areas correspond to the events in the final sample with $p_K \leq 20$ MeV/$c$.

The net area under the histogram of Fig. 4 and the result for the number of stopping $K$'s, $N_s = (3.04 \pm 0.13) \times 10^4$, are used to obtain the cascade time of $K$'s from $v \sim 0.02c$ to nuclear absorption

$$T_K{}^{\mathrm{He}}(0.02c) = (2.4 \pm 0.4) \times 10^{-10} \text{ sec.}$$

Thus, both pion and kaon times are two orders of magnitude longer than pion cascade times in hydrogen.

## 5. DISCUSSION AND CONCLUSIONS

The results obtained for the cascade time of pions

$$T_\pi{}^{\mathrm{He}}(0.02c) = (3.19 \pm 0.23) \times 10^{-10} \text{ sec}$$

and of $K$'s

$$T_K{}^{\mathrm{He}}(0.02c) = (2.4 \pm 0.4) \times 10^{-10} \text{ sec}$$

in liquid helium are considerably different from what was expected on the basis of the theoretical work done by Day and from the corresponding time measured for pions in liquid hydrogen and deuterium.

If Day's model described completely the fate of $K^-$ mesons being captured into atomic levels in liquid helium, one would expect the following picture. A thermal energy $K^-$ is captured by a helium atom via the ordinary Auger process in which one of the two orbital electrons is ejected. The release of the 24.56-eV ionization energy brings the meson to $n_K \sim 30$ and radius $r \sim 0.6\, a_0$ (where $n_K$ is the principal quantum number for the $K$-meson atomic orbit and $a_0$ is the radius of the first atomic orbit of electrons in hydrogen), where there is considerable overlap with the second orbital electron. A second Auger process takes place where energy conservation brings the meson to $n_K \sim 25$ and $r \sim 0.3\, a_0$ within a total time of $10^{-15}$ sec. The ionization of the second electron makes the system a highly excited $(K^- \alpha)^+$ ion. Day evaluates the strengths of the various interatomic de-excitation processes which the $K^-$ mesonic ion experiences. Among the processes considered are: (1) the external Auger effect, arising from the ionization of one of the electrons in the colliding atom and leading to mesonic transitions of the type $n,l \rightarrow n',l'$ where $\Delta n = n - n'$ is consistent with energy conservation and where $|l - l'| = 1$ is most likely, (2) $S$-state polarization capture in which a meson in the $nP$ state, by polarizing a neighboring helium atom, makes a virtual transition into the $nS$ state from which it is absorbed, and (3) the molecular-field Stark effect arising from the weak molecular electric field "felt" by the mesonic ion in proximity to a helium atom and causing oscillations between angular-momentum states of opposite parity. Other cascade processes considered are radiative transitions and direct nuclear capture for mesons in the $nS$ or $nP$ state. A tabulation of Day's results for kaons as well as a re-evaluation of these rates for pions is presented here in Table II. From Table II it is clear that for the large initial values of $n$ ($n \approx 25$ for $K^-$, $n \approx 13$ for $\pi^-$) in which the mesons find themselves, $S$-state Stark capture is dominant. These considerations lead to the conclusion that mesons are absorbed quickly from a high $nS$ state and that we should measure an average cascade time $\lesssim 10^{-12}$ sec. The experimental results imply that either the meson spends an anomalously long time between $\beta = 0.02$ and atomic capture, in direct contradiction to the results of Ref. 20, or something is wrong or incomplete in Day's theoretical calculation of the cascade time.

TABLE II. Summary of the theoretical predictions for the rates of various atomic processes of $K$-mesonic and $\pi$-mesonic helium ions using the theory of Day.

| Process | $K$ rate (sec$^{-1}$) | $\pi$ rate (sec$^{-1}$) |
|---|---|---|
| Radiation from $nP$ to all $S$: | | |
| $\Gamma_{\mathrm{rad}}(nP)$ | $7.1 \times 10^{13}/n^3$ | $2.2 \times 10^{13}/n^3$ |
| Nuclear capture: | | |
| $\Gamma_{\mathrm{cap}}(nS)$ | $2 \times 10^{19}/n^3$ | $\sim 5 \times 10^{17}/n^3$ |
| $\Gamma_{\mathrm{cap}}(nP)$ | $4 \times 10^{15}/n^3$ | $5 \times 10^{12}/n^3$ |
| Polarization capture: | | |
| $\Gamma_{\mathrm{pol}}(nP \rightarrow nS)$ | $2 \times 10^{11} n$ | $\sim 5 \times 10^{10} n$ |
| External Auger transitions: | | |
| $\Gamma_{\mathrm{Aug}}(n)$, $n = 5$ | $9 \times 10^5$ | $1.6 \times 10^9$ |
| $n = 6$ | $3.4 \times 10^7$ | $4.3 \times 10^{10}$ |
| $n = 7$ | $5.8 \times 10^8$ | $5.4 \times 10^{11}$ |
| $n = 8$ | $6.1 \times 10^9$ | $3.7 \times 10^{12}$ |
| $n = 9$ | $4.4 \times 10^{10}$ | $1.4 \times 10^{13}$ |
| $n = 10$ | $2.2 \times 10^{11}$ | $7.6 \times 10^{12}$ |
| $n = 20$ | $\sim 10^{12}$ | |
| Stark capture: | | |
| $\Gamma_{\mathrm{Stark}}(nS)$ | $2 \times 10^5 n^6$ | $8 \times 10^7 n^6$ |
| $\Gamma_{\mathrm{Stark}}(nP)$ | $2 \times 10^{15}/n^4$ | $2.5 \times 10^{12}/n^4$ |

If the Stark effect as calculated by Day were not present, a $K^-$ in low $n$ levels (undergoing radiative transitions only) would be captured from a $P$ state before reaching an $S$ state. This is not true for pions where the $P$ to $S$ radiation rate dominates $P$-state capture for all $n$. Therefore pions are predominantly absorbed from $l = 0$ states in helium regardless of whether Stark capture is present or not.

Since both kaon and pion cascade times are times comparable to those taken by mesons undergoing tran-

[24] H. Cramer, *The Elements of Probability Theory* (John Wiley & Sons, Inc., New York, 1955).

TABLE III. Average times to nuclear absorption via radiative transitions from states $n,l$.

| $n$ | $l$ | $t_\pi(n,l)$ (units of $10^{-12}$ sec) | $t_K(n,l)$ (units of $10^{-12}$ sec) |
|---|---|---|---|
| 3 | 1 | 1.009 | 0.0066 |
| 3 | 2 | 3.946 | 1.121 |
| | | $\bar{t}_\pi(3)=2.528$ | $\bar{t}_K(3)=0.625$ |
| 4 | 1 | 2.371 | 0.0157 |
| 4 | 2 | 8.745 | 2.617 |
| 4 | 3 | 21.02 | 6.368 |
| | | $\bar{t}_\pi(4)=12.37$ | $\bar{t}_K(4)=3.607$ |
| 5 | 1 | 4.606 | 0.0307 |
| 5 | 2 | 16.68 | 5.060 |
| 5 | 3 | 35.79 | 10.95 |
| 5 | 4 | 76.36 | 23.37 |
| | | $\bar{t}_\pi(5)=41.40$ | $\bar{t}_K(5)=12.50$ |
| 6 | 1 | 7.932 | 0.0530 |
| 6 | 2 | 28.45 | 8.682 |
| 6 | 3 | 59.04 | 18.11 |
| 6 | 5 | 108.5 | 33.31 |
| 6 | 5 | 219.5 | 67.36 |
| | | $\bar{t}_\pi(6)=110.3$ | $\bar{t}_K(6)=33.64$ |
| 7 | 1 | 12.57 | 0.0842 |
| 7 | 2 | 44.83 | 13.72 |
| 7 | 3 | 91.63 | 28.13 |
| 7 | 4 | 161.2 | 49.51 |
| 7 | 5 | 272.5 | 83.69 |
| 7 | 6 | 537.3 | 165.0 |
| | | $\bar{t}_\pi(7)=251.8$ | $\bar{t}_K(7)=77.09$ |
| 8 | 1 | 18.73 | 0.1257 |
| 8 | 2 | 66.57 | 20.40 |
| 8 | 3 | 134.9 | 41.44 |
| 8 | 4 | 232.9 | 71.54 |
| 8 | 5 | 373.3 | 114.7 |
| 8 | 6 | 599.5 | 184.2 |
| 8 | 7 | 1169 | 359.2 |
| | | $\bar{t}_\pi(8)=513.6$ | $\bar{t}_K(8)=157.5$ |
| 9 | 1 | 26.63 | 0.1790 |
| 9 | 2 | 94.42 | 28.96 |
| 9 | 3 | 190.4 | 58.49 |
| 9 | 4 | 325.2 | 99.93 |
| 9 | 5 | 509.4 | 156.5 |
| 9 | 6 | 769.8 | 236.6 |
| 9 | 7 | 1196 | 367.5 |
| 9 | 8 | 2326 | 714.6 |
| | | $\bar{t}_\pi(9)=961.7$ | $\bar{t}_K(9)=295.2$ |
| 10 | 1 | 36.50 | 0.2456 |
| 10 | 2 | 129.1 | 39.63 |
| 10 | 3 | 259.6 | 79.74 |
| 10 | 4 | 440.5 | 135.4 |
| 10 | 5 | 681.5 | 209.4 |
| 10 | 6 | 1003 | 308.2 |
| 10 | 7 | 1456 | 447.5 |
| 10 | 8 | 2211 | 679.5 |
| 10 | 9 | 4308 | 1324 |
| | | $\bar{t}_\pi(10)=1684$ | $\bar{t}_K(10)=517.0$ |

sitions in the region $n<10$, a possible explanation for the results could be found by deriving a mechanism which explains the suppression of the processes leading to high $n$ capture and brings the meson down to $n\lesssim 10$ quickly (in times $\leq 10^{-10}$ sec). This would require that the Stark capture process be much weaker than calculated by Day, while the external Auger is about as strong as calculated.

Using this phenomenological approach we assumed that the Stark effect was completely inoperative and that the de-excitation must go via Auger and radiative transitions only. This approach was first tried in Ref. 2. However, since the tables in Bethe and Salpeter[25] and Burhop[26] used in the calculation of Ref. 2 are incomplete and contain some errors and since the Auger rates we have calculated for pions are somewhat different from those of Ref. 2, we have redone this calculation. An IBM 709 computer was used to evaluate the radial dipole matrix elements for hydrogen

$$R_{nl}^{n'l'}=\int_0^\infty R_{nl}R_{n'l'}r^3dr$$

as given in Eqs. (63.2) and (63.3) of Ref. 25, for $l'=l-1$.

From these transition probabilities and the $P$-state capture rates of Table II we have calculated the average time $t(n,l)$ spent by both $\pi$ and $K$ mesons in going from state $n,l$ to nuclear capture in helium via radiative transitions. In this calculation it was assumed that $\Gamma_{\rm rad}(n,l\to n',\ l+1)$ is negligible compared with $\Gamma_{\rm rad}(n,l\to n',\ l-1)$ for all $n,l$, and $n'$. The times $t(n,l)$ along with the statistical average for each $n$

$$\bar{t}(n)=\frac{1}{n^2}\sum_{l=0}^{n-1}(2l+1)t(n,l)$$

are presented in Table III from $n=3$ up to $n=10$ for both $\pi$'s and $K$'s. From these times, we can see that the hypothesis that only Auger and radiative transitions are important fits the data for $K$'s very well, but only very roughly explains the pion results. In particular, the Auger transitions for the pion will bring it down to $n=6$ in times $\lesssim 10^{-11}$ sec, or down to $n=5$ in $0.3\times10^{-10}$ sec. The largest $t(6,l)$ is $t(6,5)=2.2\times10^{-10}$ sec and the largest $t(5,l)$ is $t(5,4)=0.8\times10^{-10}$ sec. Thus, the longest possible time spent by a pion going to nuclear capture through this mechanism is $2.2\times10^{-10}$ sec. We emphasize that this would be the cascade time were every pion to take the longest possible path through the radiative transitions. Assuming a statistical distribution in the level from which the first radiative transitions take place, we get essentially the same result as Fetkovich and Pewitt, $T_\pi=1\times10^{-10}$ sec. For any distribution less heavily weighted to high $l$, the result would be correspondingly shorter.

The work of Russell, mentioned earlier, provides a justification for the above phenomenological approach. Russell argues that during a collision time of $\sim2\times10^{-15}$ sec, the electric field giving rise to the Stark effect maintains a strictly statistical distribution among the $n^2$ equally probable states corresponding to different values

[25] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Academic Press Inc., New York, 1957), p. 248 ff. and Tables 13 and 15.

[26] E. H. S. Burhop, *The Auger Effect* (Cambridge University Press, New York, 1952), p. 168.

of the magnetic quantum number $m_l$. For a uniform electric field only the $n$ states with $m_l=0$ are coupled with the $nS$ state. Day claims that this selection rule breaks down because the positive ion in liquid helium leads to large clusters which give rise to weak electric fields with rapidly changing directions. Russell ignores this argument and consequently there is an additional factor of $1/n$ in his absorption rates. This factor, together with a higher estimate of the $S$-state level shift, and a different method of averaging over the ion-atom separation ($R$), at high $n$ reduces Russell's $S$-state absorption estimates below Day's by two orders of magnitude. The use of somewhat different wave functions in the external Auger calculation makes Russell's Auger rates larger than Day's at low $n$. The effect of the changes in these rates is to make Russell's predicted cascade time about five times shorter than the experimental results, as opposed to the previous discrepancy of about two orders of magnitude.

The importance of the remaining discrepancy is minimized by Russell. He states that the problem might be solved by considering the meson's initial atomic capture. If this capture occurs when the meson has a few electron volts kinetic energy (as opposed to essentially zero kinetic energy as assumed by Day), it will be captured into an orbit of somewhat greater principal quantum number than $n_K=30$, and, after ejection of the second electron, will result in a mesonic ion of $n_K>25$. For states somewhat higher than $n_K=25$, the meson must make a $|\Delta n|\geq 4$ transition to conserve energy in the external Auger process. Since these transitions are expected to be considerably slower for larger $\Delta n$, Russell concludes that this effect might account for the final discrepancy.

In another discussion of this problem, Condo,[27] trying a different approach, presents a qualitative argument attempting to explain the discrepancy by considering a possible channel for de-excitation in which a small percentage of the mesons are "trapped" for very long times. In his picture this trapping occurs after the initial atomic capture ($n_\pi\sim 16$, $n_K\sim 30$) for those mesons in the most circular obtits ($l\sim n$). Condo followed the history of a pion being captured into the state $n=16$, $l=15$. In this ordinary Auger capture, the pion ejects one of the two orbital electrons of the helium atom and in so doing falls to essentially the same radius as the electron replaced.[28] Due to the large overlap of the meson's wave function at this point with that of the second orbital electron, one would ordinarily expect another rapid Auger process. This would eject the second electron and result in the formation of a mesonic ion whose subsequent motion gives rise to all of the interatomic processes (Stark, external Auger, polarization capture) that Day and Russell evaluated. However, in order to eject this second electron the pion must give up 25 eV, and in so doing fall to $n=13$, making a $\Delta n=3$ transition. For the pion under consideration this would mean that $\Delta l=3$. According to Condo, quadrupole and higher-pole Auger transitions are exceedingly weak and are not expected to be able to compete with radiation. Since the mesonic atom is neutral as long as the electron is present, none of the ionic interactions are available to de-excite the meson and Condo claims it must de-excite by radiative transitions alone. These transitions are on the order of $10^{-8}$ sec each at $n=16$, and it is expected that essentially all mesons so trapped will decay before reaching a level where a $\Delta n=1$ transition releases at least 25 eV. This trapped state will also exist for those pions initially captured into $n=16$, $l=14$.

A similar argument applies to $K^-$ capture. Here we consider those mesons initially captured into the states $n=30$, $l=26$ through 29. Since the $K$ must undergo a $\Delta n=5$ transition to eject the electron, for these four states the Auger effect is suppressed and mesons in these states will decay before they are absorbed. Therefore, if this picture is correct, after atomic capture the mesons are split into two populations. The majority are captured into low and intermediate orbital-angular-momentum states and form a mesonic ion. These are absorbed relatively quickly either from an $S$ state as a result of Stark oscillations arising from interactions with neighboring helium atoms (process 3), or in a $P$ state at low $n$. A maximum of $\frac{1}{2}\%$ of these mesons would decay before nuclear absorption. The others, in a much smaller group, are trapped in high angular-momentum states, never get to form an ion, and all decay in orbit. This latter group would account for most of the contribution to $N_d$ measured in this experiment.

To put this analysis on a more quantitative basis would require a calculation of the higher multipole Auger transitions, a calculation of the population of the initial $n,l$ levels of mesonic helium atoms, and a demonstration that for mesons captured initially in circular states there exist no other means of de-excitation via interatomic interactions. Indeed, as Russell has pointed out, it is likely that the rearrangement of angular momenta occurring during collisions depopulates the circular states at Auger rates which are not much different from the original results. It might be expected that Condo's mechanism would be more applicable in a gas than in liquid helium where quasimolecular $\Delta l$ transitions are likely at rates $\sim 10^{10}$ sec$^{-1}$ as evaluated by Russell. However, it is possible that the time of these additional transitions provides an alternative explanation for the remaining discrepancy between theory and experiment.

It appears that although Russell's calculations go a long way toward increasing our confidence in what major processes are involved in the de-excitation, a reliable quantitative estimate of the cascade times of mesons in liquid helium has yet to be made. Since the rate of capture of $K$ mesons from the different angular-momentum states cannot be determined without a detailed understanting of the important de-excitation

[27] G. T. Condo, Phys. Letters **9**, 65 (1964).
[28] G. A. Baker, Jr., Phys. Rev. **117**, 1130 (1960).

mechanisms and their relative strengths, we must conclude that the angular-momentum state of $K^-$ absorption in liquid helium is still unknown.

As has been pointed out in Ref. 16, a measurement of the fraction of stopping mesons which yield x rays from transitions in the low-lying $n$ orbits would help a great deal in resolving this problem. Assuming that the $P$-state capture rates are large compared to the $P$ to $S$ radiation rates as calculated, the value of this fraction would at least provide a measure of the minimum amount of $P$-state absorption.[29]


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[29] *Note added in proof.* Recent results, obtained after this paper was submitted, on x-ray emission from $K^-$ absorption in helium [G. R. Burleson, D. Cohen, R. C. Lamb, D. N. Michael, R. A. Schluter, and T. O. White, Jr., Phys. Rev. Letters **15**, 70 (1965)] have indicated that a major fraction of the stopping $K^-$ mesons reach the low-lying $P$ states, consistent with the measured cascade time reported here.

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# Dynamical Basis of the Sum Rule $2\Xi_-^- = \Lambda_-^0 + \sqrt{3}\Sigma_0^+$. II


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The assumptions that (1) the weak processes $B \to B'$, $M \to M'$ ($\Delta S = \pm 1$) are mediated by an interaction that transforms like $\lambda_6$, (2) the nonleptonic decay amplitudes of hyperons are dominated by pole terms, and (3) the mass-splitting pattern of the baryon octet can be approximated by $m = m_0 + (\Delta m)Y$ imply that the effective Lagrangian describing the parity-conserving hyperon decays transforms like $\lambda_7$ under $SU(3)$. The sum rule $2P(\Xi_-^-) = P(\Lambda_-^0) + \sqrt{3}P(\Sigma_0^+)$ follows immediately from this observation.


## I. INTRODUCTION

In a previous paper of the same title[1] it was shown that the parity-conserving amplitudes for nonleptonic hyperon decays satisfy the sum rule[2]

$$2P(\Xi_-^-) = P(\Lambda_-^0) + \sqrt{3}P(\Sigma_0^+). \quad (1)$$

The assumptions made in deriving Eq. (1) are:

(i) The nonleptonic weak interaction is $CP$ conserving and has the octet transformation property. In particular, it transforms like $\lambda_6 = \lambda_6^T$.[3]

(ii) The nonleptonic decay processes are dominated by the mechanism represented by baryon and meson poles (the model of Feldman, Matthews, and Salam,[4] hereafter referred to as the FMS model).

(iii) The mass-splitting pattern among baryons is approximated by a simplified Gell-Mann–Okubo formula: $m = m_0 + (\Delta m)Y$, or $m_\Lambda = m_\Sigma$, $m_\Xi - m_\Lambda = m_\Lambda - m_N$.

The purpose of this note is to show that under the assumptions (i)–(iii), the *effective* nonleptonic hyperon weak interaction (effective in the sense that the effect of mass splitting is taken into account) transforms like $\lambda_7 = -\lambda_7^T$. It follows immediately then, from the work of Coleman and Glashow,[5,6] that the parity-conserving amplitudes obey the sum rule (1), since the circumstance at hand corresponds to what Coleman and Glashow[5] called the abnormal octet dominance.

## II. ANALYSIS

An effective Lagrangian which generates the pole-dominance model of FMS is

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}' \quad (2)$$

$$\mathcal{L}' = (\Delta m)\bar{B}_i(F_8^{ij} + \alpha D_8^{ij})B_j + f\bar{B}_i(F_6^{ij} + \beta D_6^{ij})B_j + g\bar{B}_i(D_k^{ij} + \gamma F_k^{ij})\gamma_5 B_j M^k + (\Delta\mu^2)M_i D_8^{ij} M_j + f' M D_6^{ij} M_j, \quad (3)$$

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[1] B. W. Lee and A. R. Swift, Phys. Rev. **136**, B228 (1964). See also J. D. Bjorken and B. Helleson, Phys. Letters **12**, 141 (1964).

[2] B. W. Lee, Phys. Rev. Letters **12**, 83 (1964); H. Sugawara, Progr. Theoret. Phys. (Kyoto) **31**, 213 (1964); B. Sakita, Phys. Rev. Letters **12**, 379 (1964); S. P. Rosen, *ibid.* **12**, 408 (1964).

[3] M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964).

[4] G. Feldman, P. T. Matthews, and A. Salam, Phys. Rev. **121**, 302 (1961); H. Sugawara, Nuovo Cimento **31**, 635 (1964).

[5] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).

[6] S. Coleman, S. L. Glashow, and B. W. Lee, Ann. Phys. (N. Y.) **30**, 348 (1964).