Pomeranchuk Repulsion and Resonance Narrowing*

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The contribution of the P (Pomeranchuk) and P' trajectories to the generalized two-particle (low-energy) potential is shown to be repulsive and effectively of long range. A rough expression for the P potential is given in terms of the high-energy total cross section and associated diffraction peak. It is argued that Pomeranchuk repulsion represents the many-particle channels that dominate high energies and that have an important narrowing effect on resonance widths even though these channels are closed in the low-energy resonance region.

I. INTRODUCTION

In a recent paper there were discussed certain consequences of employing Regge poles rather than fixed J poles as the source of two-particle generalized potentials.[1] An important omission in that paper was an estimate of potentials arising from trajectories for which the first physical J value fails to have an associated pole of the S matrix. Two well-established trajectories are of this type, the so-called P (Pomeranchuk) and P' trajectories, where the first associated particles have $J=2$,[2] whereas the first physical angular momentum value is $J=0$. The purpose of the present paper is to show that the $J=0$ components of the P and P' Regge potentials are repulsive and effectively of long range. They may constitute the major bootstrap component, so far overlooked, that tends to make resonances narrow.

Bootstrap calculations of low-baryon-number particles on the basis of fixed-spin potentials have always yielded larger widths than experimentally observed.[3] It is well known from the dynamics of particles with large baryon number (classical nuclear physics) that the proliferation of many-body channels, open at high energies, systematically narrows the widths of low-energy resonances for which these channels are closed. No estimates have heretofore been given of this effect for particles of low baryon number, but the Reggeized strip approximation[4] includes the high-energy inelastic effect and therefore should manifest the narrowing tendency.

In the new form of strip approximation the generalized two-body potential is represented as a sum over contributions from the leading Regge trajectories of crossed reactions. Reference 1 shows that when the leading physical J value on the trajectory has an associated physical particle, one may associate the potential in the conventional manner with "exchange" of this particle, although there is an important form factor which reduces the strength and extends the range—relative to a fixed-spin (elementary) particle potential. A small part of the P and P' potentials may be associated in such a sense with exchange of the $J=2$ $f(1250)$ and $f'(1525)$ particles, but the major component belongs to $J=0$—where no particles exist. We suggest that physically this latter component represents the aforementioned dynamical effect of many-particle channels, closed inside the strip where the potential is to be employed, but open above the strip boundary.

Why is such an identification plausible? First of all, the P and P' trajectories account for most of the total cross section in the high-energy region where multiple production dominates.[5] Second, as we shall see, the $J=0$ component of the P and P' potentials is always repulsive and of a range—corresponding to the forward peaks of high-energy diffraction scattering—that is relatively long. When such a long-range repulsion is added to a shorter range attraction from "ordinary" particle exchange, one has the dynamical situation favorable to narrow resonances.[6]

II. THE $J=0$ COMPONENT OF THE POMERANCHUK POTENTIAL

In Ref. 1 it was explained that inside the s strip one may make a Legendre polynomial expansion in z_t of the s-reaction potential associated with the ith Regge pole communicating with the t reaction. Since the Pomeranchuk trajectory is of even signature, we have

$$V_P^s(t,s)=\sum_{J\ \mathrm{even}}(2J+1)V_J^P(t)P_J(z_t)\,, \quad \text{(II.1)}$$

where

$$z_t(s,t)=[s+q_a^2(t)+q_b^2(t)]/2q_a(t)q_b(t)\,, \quad \text{(II.2)}$$

$$q_a^2(t)=t/4-m_a^2\,,\quad q_b^2(t)=t/4-m_b^2\,, \quad \text{(II.3)}$$

if the s reaction connects channels with particle masses

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[1] G. F. Chew, University of California. Lawrence Radiation Laboratory Report No. UCRL-16101, 1965 [Progr. Theoret. Phys. (Kyoto) Suppl., to be published].

[2] Presumed to be the $f(1250)$ and the $f'(1525)$, the latter reported by Barnes *et al.*, Phys. Rev. Letters **14**, 82 (1965).

[3] See, for example, J. R. Fulco, G. L. Shaw, and D. Y. Wong, Phys. Rev. **137**, B1242 (1965).

[4] G. F. Chew, Phys. Rev. **129**, 2363 (1963); G. F. Chew and C. E. Jones, *ibid.* **135**, B208 (1964).

[5] See R. J. N. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965), for a recent review of all high-energy πN and KN experiments and for additional references.

[6] One may say that the system becomes "trapped" inside the repulsive barrier and takes a long time to find its way out. A long lifetime means a narrow width.

m_a and m_b. It should suffice for our qualitative discussion here, as it did in Ref. 1, to employ the Khuri-Jones formula for $V_J{}^P(t)$:

$$V_J{}^P(t)=\beta_P[q_a(t)q_b(t)]^{\alpha_P(t)}\{\gamma_P(t)/[J-\alpha_P(t)]\}\times e^{-[J-\alpha_P(t)]\xi_1(t)}, \quad \text{(II.4)}$$

where β_P is a crossing matrix element (always positive for the Pomeranchuk pole), $\gamma_P(t)$ is the reduced residue (also positive near $t=0$), and $\alpha_P(t)$ is the Pomeranchuk trajectory. The function $\xi_1(t)$ is given by

$$\xi_1(t)=\ln\{z_1(t)+[z_1{}^2(t)-1]^{1/2}\}, \quad \text{(II.5)}$$

where $z_1(t)=z(s_1,t)$, s_1 being the strip width, that is, the lowest energy at which the imaginary part of the full amplitude can be approximated by the imaginary part of the potential. It appears experimentally that $s_1\gtrsim 4$ GeV^2.

The qualitative discussion of Ref. 1 may be applied to $V_{J=2}{}^P(t)$, associating this force component with $f(1250)$ exchange, although the damping here with respect to elementary-particle exchange is severe. Our rough estimate would give a reduction at $t=0$ by a factor $\approx e^{-2}$,[7] almost one order of magnitude, so the $J=2$ component of the Pomeranchuk potential is relatively minor, although attractive (positive). The $J=0$ component, on the other hand, is, for $|t|\ll s_1$,

$$V_{J=0}{}^P(t)=-\beta_P\alpha_P{}^{-1}(t)\gamma_P(t)s_1{}^{\alpha_P(t)}, \quad \text{(II.6)}$$

strongly repulsive. The result for P' is similar. One may usefully compare (II.6) to the high-energy limit of the imaginary part of the amplitude—which is the same as the high-energy limit of the imaginary part of the Pomeranchuk potential:

$$\text{Im}V_P{}^s(t,s) \underset{s\to\infty}{\longrightarrow} \beta_P \times\left[\frac{\sqrt{\pi}}{2}(2\alpha_P(t)+1)\frac{\Gamma(\alpha_P(t)+\frac{1}{2})}{\Gamma(\alpha_P(t)+1)}\right]\gamma_P(t)s^{\alpha_P(t)}. \quad \text{(II.7)}$$

Observe that for s not enormously larger than s_1 the t dependence of the two forms is similar. Thus the "shape" of the Pomeranchuk potential is essentially that of the high-energy diffraction peak. Using the optical theorem,

$$\sigma^{\text{tot}}(s) \underset{s\gg s_1}{\approx} (16\pi/s)\,\text{Im}V_P{}^s(t=0,s), \quad \text{(II.8)}$$

together with the fact that $\alpha_P(0)=1$, we may establish the normalization to be

$$V_{J=0}{}^P(t=0)\approx-(s_1/24\pi^2)\sigma^{\text{tot}}(\infty). \quad \text{(II.9)}$$

Had we used the Chew-Jones expression for the Regge formula[1,4] rather than the Khuri-Jones expression, we should have found in (II.9) a coefficient $-s_1/16\pi^2$, corresponding to a slightly different significance for the parameter s_1. Since actual dynamical calculations are more likely to be based on the Chew-Jones expression, we shall use this latter normalization in what follows. (The arguments to be made here are only qualitative, so a factor of $\frac{3}{2}$ is of no consequence.)

III. AN APPARENT CONTRADICTION

Estimating the t discontinuity (or imaginary part) of (II.6), one finds it negative in the region between the 2π threshold and the mass squared of $f(1250)$.[8] Since the t discontinuity of any t-reaction partial-wave elastic amplitude must be positive, a doubt arises about the correctness of (II.6).

In fact, Chew and Teplitz[9] proposed a technique for evaluation of the potential which precludes a negative result for the potential carrying the vacuum quantum numbers. The reasoning of these authors, however, depended on the neglect of double spectral functions throughout the "corner" regions where both variables (s and t) are inside their respective strips. This is equivalent to assuming that inside the t strip the entire t discontinuity is contained in the potential for the s reaction.

Such is, of course, not strictly the case, and if one asks where (in t) the discontinuity of (II.6) becomes large, one sees that it is in the region where $\text{Im}\alpha_P(t)$ is large, that is, the upper portion of the t strip above the mass squared of $f(1250)$. In view of the relatively narrow width of the f we can be sure that $\text{Im}\alpha_P(t)$ remains small for $t\leqslant m_f{}^2$.[10] Now, in the upper portion of the t strip (inside the s strip) there may be substantial components of the Mandelstam double spectral function arising from iteration of lower t components in the potential. This double spectral function contributes to the total t discontinuity but is excluded (by definition) from the potential. Were the double spectral function sufficiently large it could produce the required positive sign for the complete t discontinuity, even though the potential (II.6) may be negative.

Towards the lower edge of the t strip the potential must dominate the t discontinuity, so (II.6) cannot there be a good approximation to the complete (vacuum-like) potential. Here the procedure recommended by Chew and Teplitz seems appropriate in order to include the effect of secondary trajectories and "background."

Notice that our conjected mechanism for removing the contradiction between (II.6) and the positive-definiteness requirement, through the double spectral function, implies the inadequacy of approximating the left-hand discontinuities in an N/D calculation by the

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[7] This result is confirmed by numerical calculations of Collins and Teplitz based on the Chew-Jones potential (private communication).

[8] The essential point is that $\text{Im}\alpha_P(t)$ is positive.

[9] G. F. Chew and V. L. Teplitz, Phys. Rev. **137**, B139 (1965).

[10] For $t=m_f{}^2$, $\text{Im}\alpha_P=\Gamma_f m_f(d\,\text{Re}\alpha_P/dt)_{t=m_f{}^2}$, and the trajectory slope appears to be less than 1 GeV^{-2}. Thus for $\Gamma_f\approx 100$ MeV, $\text{Im}\alpha_P\lesssim 0.1$ at $t=m_f{}^2$.

discontinuities of the potential. This is perhaps not surprising if one recalls that this latter approximation has been especially deficient in handling strongly repulsive forces.[11]

A lesser paradox is the circumstance that the "range" of the Pomeranchuk potential (II.6), as measured by its logarithmic derivative at $t=0$, is longer than would be given by a dispersion-relation estimate based on the region of $t(>m_f^2)$ where the imaginary part becomes large. For pion-pion scattering, as an example, the inverse logarithmic derivative with respect to t of the diffraction amplitude (II.7) at $t=0$ and $s\approx s_1$ is ≈ 0.5 GeV^2,[12] while $m_f^2=1.6$ GeV^2. The explanation here is that the imaginary part of $V_{J=0}{}^P(t)$ oscillates when the imaginary part of $\alpha_P(t)$ becomes large, leading to cancellations in the dispersion integral, so the dependence on t near $t=0$ may be steeper than given by the elementary estimate, which tacitly assumes an absence of cancellations. This circumstance means that Pomeranchuk repulsion even while behaving dynamically like a long-range force, does not correspond to a "nearby" left-hand singularity in partial-wave amplitudes. It is a superposition of distant singularities on *both* right (outside the strip) and left, in which the oscillatory character of the discontinuity is an essential feature. To represent such an effect in N/D models by a few phenomenological poles on the left is probably hopeless.

IV. ESTIMATE OF THE IMPORTANCE OF POMERANCHUK REPULSION

Let us now examine for a much studied example, the $I=1$ $\pi\pi$ channel, the relative importance of the potentials associated with the P and ρ trajectories, the latter being the only one usually considered for this system.

In our previous paper[1] we have roughly estimated the ρ potential as

$$V_{\pi\pi,I=1}{}^\rho(t)=3(1+s/2q_t^2)V_1{}^\rho(t)\,, \qquad \text{(IV.1)}$$

where, for $|t|\ll s_1$,

$$V_1{}^\rho(t)\approx q_t^2[(4\Gamma_\rho/m_\rho)/(m_\rho^2-t)]e^{-2\alpha_\rho'(m_\rho^2-t)}, \qquad \text{(IV.2)}$$

the effective crossing matrix element here being equal to $\frac{1}{2}$.[13] The potential is attractive, to be compared to our estimate above of the repulsive Pomeranchuk potential:

$$V_{\pi\pi,I=1}{}^P(t)\approx -\frac{s_1}{8\pi^2}\sigma_{\pi\pi}{}^{\text{tot}}(\infty)\frac{\operatorname{Im}A_{\pi\pi}(s_1,t)}{\operatorname{Im}A_{\pi\pi}(s_1,0)}\,. \qquad \text{(IV.3)}$$

Although the detailed shape of the high-energy $\pi\pi$ forward diffraction amplitude is not known, it should suffice here to represent it by a simple exponential of the above-mentioned width 0.5 GeV^2. The value of $\sigma^{\text{tot}}(\infty)$ is taken as 10 mb,[12] leading to

$$V_{\pi\pi,I=1}{}^P(t)\approx -0.3s_1e^{2t}, \qquad \text{(IV.4)}$$

where s_1 and t are to be evaluated in units of GeV^2. For the ρ potential, using a width $\Gamma_\rho=110$ MeV, a mass $m_\rho=0.77$ GeV, and a trajectory slope $\alpha_\rho'=0.5\ m_\rho^{-2}$, we have

$$V_{\pi\pi,I=1}{}^\rho(t)\approx 1.1(t/4-m_\pi^2+s/2)e^{1.7t}/(1-1.7t)\,. \qquad \text{(IV.5)}$$

Comparing (IV.4) and (IV.5), one should notice two points: (a) The t dependence of the two potentials is not very different, but the ρ potential has a major component increasing linearly with s, while the Pomeranchuk potential is independent of s. (b) In the lower half of the strip, where $s<s_1/2$, the Pomeranchuk repulsion is entirely comparable in magnitude to the ρ attraction.

The s-increasing aspect of the ρ potential means that in N/D dynamics this component, acting like a very short-range attractive force, tends to dominate the denominator function and thus to control the existence and location of resonances in the amplitude. On the other hand, the width of a resonance (resonances are expected to occur in the lower half of the strip) is proportional to the numerator function at the resonance energy—which is sensitive to the value of the potential in this low-energy region (the "long-range force"). Thus a drastic reduction of the potential in the resonance region should lead to an important resonance narrowing effect.

It has already been remarked that with such a strong repulsion one may not employ the N/D device of replacing left-hand partial-wave cuts by the cuts of the potential. It will be necessary to perform at least a few steps of the Mandelstam iteration in order to achieve a believable dynamical result. The results of such calculations will, one hopes, be reported at a later time.

V. CONCLUSION

The presence of Pomeranchuk repulsion in all two-particle channels may explain why resonance widths have so uniformly been overestimated in non-Reggeized bootstrap calculations. At the same time, certain aspects of the qualitative estimates heretofore given of the attractive forces essential to forming bound states and resonances are not invalidated by Reggeization. There remains a correlation with the concept of particle exchange, and the sign (attraction or repulsion) generally survives. We can understand in this way the success of crude bootstrap arguments that use crossing matrices

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[11] J. G. Bjorken and A. Goldberg, Nuovo Cimento **16**, 539 (1960).
[12] G. F. Chew and V. L. Teplitz, Phys. Rev. **136**, B1154 (1964).
[13] In the $\pi\pi$ problem, *both* crossed reactions contain the poles in question, so the total potential treated in this section is twice that from the t reaction above.

and almost nothing more. The estimates given in this paper and in Ref. 1 indicate, however, that to achieve even semiquantitative accuracy in the dynamics it will be necessary to employ Regge potentials together with the Mandelstam iteration or the equivalent thereto.

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Measurement of the K^+ Branching Ratio into the τ Mode

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A determination of the K^+ decay branching ratio into the mode $K^+ \rightarrow \pi^+\pi^+\pi^-$ is obtained in a H_2 bubble chamber with a beam of stopping K^+. With a total τ^+ count of 2186, the branching ratio obtained is $(5.71\pm0.15)\%$. The estimated world average is $(5.54\pm0.075)\%$.

INTRODUCTION

THE precise determination of the τ branching ratio is very important, since this information is used in most $K^\pm$ experiments to establish the $K^\pm$ flux. Many values have been obtained in the past years, not always compatible within the errors. In Table I we have collected the most significant ones.

We present here a new determination of the τ branching ratio obtained in the 81 cm Saclay-CERN hydrogen bubble chamber exposed to a beam of stopping K^+. The statistical accuracy of our determination is comparable with those of Refs. 1 and 2. However, the use of the H_2 chamber allows, in our opinion, a more certain reduction of the background.

EXPERIMENTAL DETAILS

The entire analysis was carried out at the scan table with visual separation between decays into the τ mode and all other K^+ decays. Two kinds of scans have been done. In the first scan (scan A) all the tracks entering

TABLE I. Published values of the τ branching ratio.

References	Technique	No. of τ	Branching ratio into τ mode
G stack coll.[a]	emulsion (cosmic rays)	30	$(8.5\ \pm1.6)\%$
Ritson *et al.*[b]	emulsion	58	$(7.6\ \pm1)\%$
Brussard *et al.*[c]	emulsion	30	$(7.1\ \pm1)\%$
Hoang *et al.*[d]	emulsion	9	$(5.2\ \pm2)\%$
Birge *et al.*[e]	emulsion	171	$(5.6\ \pm0.4)\%$
Alexander *et al.* (see Ref. 6)	emulsion	226	$(6.8\ \pm0.4)\%$
Taylor *et al.*[f]	emulsion	263	$(5.2\ \pm0.3)\%$
Roe *et al.*[g]	xenon bubble chamber	359	$(5.7\ \pm0.3)\%$
Bøggild *et al.* (see Ref. 7)	emulsion	98	$(7.7\ \pm0.8)\%$
Shaklee *et al.* (see Ref. 1)	xenon bubble chamber	540	$(5.1\ \pm0.2)\%$
Callahan *et al.* (see Ref. 2)	Freon bubble chamber	2332	$(5.54\pm0.12)\%$
Present experiment: scan A		504	$(5.65\pm0.26)\%$
Present experiment: scan B	hydrogen bubble chamber	1682	$(5.74\pm0.18)\%$
Present experiment: total			$(5.71\pm0.15)\%$
Weighted mean			$(5.54\pm0.075)\%$

[a] J. H. Davies *et al.*, Nuovo Cimento 2, 1063 (1955).
[b] D. M. Ritson *et al.*, Phys. Rev. 101, 1085 (1956).
[c] J. Crussard *et al.*, Nuovo Cimento 3, 731 (1956); 4, 1195 (1956).
[d] T. F. Hoang, M. F. Kaplon, and G. Yekutieli, Phys. Rev. 102, 1185 (1956).
[e] R. W. Birge *et al.*, Nuovo Cimento 4, 834 (1956).
[f] S. Taylor, *et al.*, Phys. Rev. 114, 359 (1959).
[g] B. P. Roe *et al.*, Phys. Rev. Letters 7, 346 (1961).

[1] F. S. Shaklee, G. L. Jensen, B. P. Roe, and D. Sinclair, Phys. Rev. 136, B1423 (1964).
[2] A. Callahan, R. March, and R. Stark, Phys. Rev. 136, B1463 (1964).