New Sum Rule for Meson-Baryon Total Cross Sections at High Energy*

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(Received 7 July 1965)

Assuming that the dominant underlying mechanism of high-energy meson-nucleon elastic scattering is the exchange of SU_3 octet meson states, a new sum rule relating meson-nucleon total cross sections is derived:

$$\sigma_t(K^-p)-\sigma_t(K^+p)=\sigma_t(K^-n)-\sigma_t(K^+n)+\sigma_t(\pi^-p)-\sigma_t(\pi^+p).$$

Comparison of the sum rule with experiment indicates substantial agreement from 10 to 18 BeV/c. The f/d ratio for the charge coupling of the vector-meson octet to the baryon octet is also determined from ratios of total cross-section differences.

ACCUMULATING evidence supports the assignment of the observed low-mass meson states to either octet or singlet (or nonet) representations of SU_3. The classification of the known mesons and meson resonances in a 0^- octet and singlet $[\pi,K,\eta,X^0]$, a 1^- nonet $[\rho,K^*,\varphi,\omega]$ and a 2^+ nonet[1] $[A_2,\hat{K}(1430),\hat{f}^0(1525),f^0]$ fairly well exhausts the meson mass spectrum, provided that certain other enhancements $[A_1,B,K^{**}(1175)]$ prove to be of kinematic origin.[2] In any case since these other enhancements have parity assignments $(-1)^{J+1}$, and thus are not coupled to a pseudoscalar-meson pair, they are not relevant to our subsequent analysis of pseudoscalar meson-nucleon scattering. The recently completed 2^+ nonet is presumably the physical manifestation of Pignotti's conjectured SU_3 octet and singlet of Regge poles $[R,Q,P',P]$ implied by bootstrap dynamics.[3]

The occurrence of only the 1 and 8 representations of SU_3 for the observed bosons suggests a picture of high-energy elastic amplitudes dominated by exchanges of unitary singlet and octet states in the crossed $1+\bar{1}\to 2+\bar{2}$ channel. In this article we derive a sum rule for meson-nucleon total cross sections at high energies based on that assumption. The Regge-pole hypothesis provides a natural framework for this picture but is by no means an essential part of this analysis.

If the dominant underlying mechanism of high-energy elastic scattering is the exchange of singlet and octet meson states of arbitrary number and spin, then the elastic meson-baryon scattering amplitudes (MB) may be written as

$$\begin{pmatrix}(K^-p)\\(K^+p)\\(K^-n)\\(K^+n)\\(\pi^-p)\\(\pi^+p)\end{pmatrix}=\begin{pmatrix}1&\frac{2}{3}&-2&0&0\\1&\frac{2}{3}&2&0&0\\1&-\frac{1}{3}&-1&-1&1\\1&-\frac{1}{3}&1&-1&-1\\1&-\frac{1}{3}&-1&1&-1\\1&-\frac{1}{3}&1&1&1\end{pmatrix}\times\begin{pmatrix}1\\8_{ss}\\8_{aa}\\8_{sa}\\8_{as}\end{pmatrix},\quad(1)$$

where the s, a subscripts label the symmetric and antisymmetric octet representations of the coupling to the exchanged mesons. Applying the optical theorem to these amplitudes, we find directly a sum rule relating the meson-nucleon total cross sections

$$\Delta_{Kp}=\Delta_{Kn}+\Delta_{\pi p},\quad(2)$$

where we have introduced the notation

$$\Delta_{MB}=\sigma_t(M^-B)-\sigma_t(M^+B).$$

Comparison of the sum rule with experiment[4] in Table I indicates substantial agreement from 10 to 18 BeV/c. The sum rule appears to be in quantitatively better agreement with the data than the Johnson-Treiman relations[5]:

$$\Delta_{Kp}=2\Delta_{\pi p}=2\Delta_{Kn}.\quad(3)$$

TABLE I. Comparisons of the sum rule $\Delta_{Kp}=\Delta_{Kn}+\Delta_{\pi p}$ and the Johnson-Treiman relations $\Delta_{Kp}=2\Delta_{\pi p}=2\Delta_{Kn}$ with experiment. (Data from Ref. 4).

$P_{\rm LAB}$ (BeV/c)	Total cross section differences (mb)			
	Δ_{Kp}	$\Delta_{\pi p}+\Delta_{Kn}$	$2\Delta_{\pi p}$	$2\Delta_{Kn}$
6	7.0±0.3	6.7±0.7	4.6±0.8	8.8±1.1
8	6.3±0.2	4.5±0.7	4.8±0.8	4.2±1.1
10	5.2±0.2	4.8±0.7	3.4±0.8	6.2±1.1
12	4.3±0.2	4.3±0.7	3.4±0.8	5.2±1.1
14	4.1±0.2	4.1±0.7	3.0±0.8	5.2±1.1
16	4.3±0.4	4.6±0.8	3.4±0.8	5.8±1.4
18	3.9±0.8	4.2±1.2	3.0±0.8	5.4±2.3

* Work supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation and in part by the U. S. Atomic Energy Commission under contract No. AT(11-1)-30 (COO-30-104).

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Contributions to the cross-section differences Δ_{MB} are entirely due to octets whose neutral members are odd under charge conjugation. Consequently the observed 2^+ meson octet is not relevant to the relations in Eqs. (2) and (3). If we make a further assumption that only the one vector-meson octet V is responsible for the Δ_{MB}, then the f/d ratio for the $VB\bar{B}$ charge coupling can be calculated from the experimental total cross sections (the VMM coupling is pure f-type, and the $VB\bar{B}$ magnetic coupling vanishes for forward scattering). The f/d ratio can be calculated from the following expressions:

$$\begin{aligned} f/d &= \Delta_{Kp}/(2\Delta_{\pi p}-\Delta_{Kp}) \\ &= \Delta_{Kp}/(\Delta_{Kp}-2\Delta_{Kn}) \\ &= (\Delta_{\pi p}+\Delta_{Kn})/(\Delta_{\pi p}-\Delta_{Kn}) \end{aligned} \tag{4}$$

which are equivalent according to Eq. (2). Pure f-type coupling yields the Johnson-Treiman relations as previously noted by Sawyer.[6] The determination of the f/d ratio from the data[4] through Eq. (4) is given in Table II. The results indicate a mean value somewhere between $f/d\approx-3$ and $f/d\approx-5$, showing an appreciable deviation from the universality prediction $d=0$.[7] The errors on the cross sections are sufficiently large to make a precise determination of f/d difficult.

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TABLE II. Determinations of the vector meson-nucleon charge coupling f/d ratio from experiment. (Data from Ref. 4.)

P_{LAB} (BeV/c)	f/d ratio for Vector Meson-Baryon charge coupling: $\frac{f}{d}=\frac{\Delta_{Kp}}{(2\Delta_{\pi p}-\Delta_{Kp})}$	$\frac{f}{d}=\frac{\Delta_{Kp}}{(\Delta_{Kp}-2\Delta_{Kn})}$	$\frac{f}{d}=\frac{(\Delta_{\pi p}+\Delta_{Kn})}{(\Delta_{\pi p}-\Delta_{Kn})}$
6	−2.9	−3.9	−3.2
8	−4.2	+3.0	15.0
10	−2.9	−5.2	−3.4
12	−4.8	−4.8	−4.8
14	−3.7	−3.7	−3.7
16	−4.8	−2.9	−3.8
18	−4.3	−2.6	−3.5

In any event we emphasize that the sum rule of Eq. (2) is dependent only on the general octet-dominance property of the $MM\to\bar{B}B$ channel and not upon these further detailed considerations.

We are grateful to Professor R. F. Sawyer for a discussion of this work. We are indebted to Professor C. Goebel for a helpful discussion and for a careful reading of the manuscript.

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PHYSICAL REVIEW VOLUME 140, NUMBER 5B 6 DECEMBER 1965

$SU(6)$ Predictions for s-Wave Baryon-Baryon Scattering*

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The predictions of the $SU(6)$-symmetry model for s-wave baryon-baryon scattering are derived and compared with some low-energy experimental data.

THE application of $SU(6)$ symmetry to meson-baryon scattering yielded a number of relations that can be compared with experiment.[1] Such comparisons are of interest in determining the validity of $SU(6)$ symmetry and the nonrelativistic limit of its relativistic extensions.[2] In this paper the $SU(6)$ predictions for s-wave baryon-baryon scattering are tabulated and compared with some low-energy experimental data.

* Work supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation and in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-30. (C00-30-101).

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[2] A list of references to much of the current literature on $SU(6)$ symmetry and its extensions is given by B. Sakita and K. C. Wali, Phys. Rev. Letters 14, 404 (1965).

For the s-wave baryon-baryon scattering process

$$B(4)+B(2)\to B(3)+B(1),$$

the $SU(6)$-invariant scattering operator S, which incorporates the generalized Pauli principle, can be written in terms of the 56-dimensional $SU(6)$ baryon wave function[3] as

$$\begin{aligned} S=A\{&\bar{\psi}^{ABC}(1)\psi_{ABC}(2)\bar{\psi}^{DEF}(3)\psi_{DEF}(4) \\ &-\bar{\psi}^{ABC}(1)\psi_{ABC}(4)\bar{\psi}^{DEF}(3)\psi_{DEF}(2)\} \\ &+(C/81)\{\bar{\psi}^{ABC}(1)\psi_{ABD}(2)\bar{\psi}^{EFD}(3)\psi_{EFC}(4) \\ &-\bar{\psi}^{ABC}(1)\psi_{ABD}(4)\bar{\psi}^{EFD}(3)\psi_{EFC}(2)\}. \end{aligned} \tag{1}$$

The result of Eq. (1) for BB reactions of experimental interest are listed in Table I. Only amplitudes which are isospin-independent are included. The quantities α and β of Table I are scalar products of Pauli spinors,

$$\begin{aligned} \alpha&=\chi^i(1)\chi_i(2)\chi^j(3)\chi_j(4)\,, \\ \beta&=\chi^i(1)\chi_i(4)\chi^j(3)\chi_j(2)\,. \end{aligned} \tag{2}$$

[3] B. Sakita, Phys. Rev. Letters 13, 643 (1964).