Coupling Constants in Muon Capture*

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(Received 19 July 1965)

The measured capture rate of muons in C^{12} leading to the ground state of B^{12}, in combination with other data, is employed as a basis for a determination of the weak-interaction coupling constants associated with muon capture. The study of this transition has the advantages: (1) that the rate depends only weakly on the vector, induced pseudoscalar, and (possible) tensor coupling constants, but not at all on a possible scalar coupling constant; (2) that empirical information from inelastic electron scattering on C^{12} leading to the excitation of the 15.1-MeV level, the $M1$ lifetime of this level, and the $f\tau_{1/2}$ value for the beta decay of B^{12} allows one to determine the required nuclear matrix elements of major importance (as well as some of minor importance) in practically a model-independent way. The capture rate can thus be expressed in terms of the axial vector coupling constant and the weak-magnetism coupling constants. Assuming the validity of the conserved-vector-current hypothesis (CVC), and $5<C_p<28$, one then finds that $F_A^{\mu}/F_A^{\beta}=1.04_{-0.10}^{+0.07}$. On the other hand, if one assumes $F_A^{\mu}=F_A^{\beta}$, one obtains for the isotopic vector magnetic moment of the nucleon (at the appropriate momentum transfer) $\mu(\nu^2)=5.7_{-1.6}^{+1.1}$ nuclear magnetons, which is consistent with the CVC prediction of 4.60 nm and can be considered as evidence for weak magnetism in muon capture.

INTRODUCTION

An important goal of muon-capture experiments is to attempt to establish quantitatively the equality of the weak-interaction coupling constants for muon capture with the corresponding constants for electron capture (or, more generally, beta decay). The difficulties in securing the information desired from experimentally determined muon capture rates are of several kinds:

(1) The capture rate generally involves simultaneously a number of the coupling constants: vector, axial vector, weak-magnetism, induced pseudoscalar, and (if one admits the possibility of interactions of the second kind[1]), scalar and tensor.

(2) The expression for the capture rate involves a sum of products of these coupling constants with nuclear matrix elements with the usual difficulties attendant on obtaining reliable evaluations of the latter.

(3) In the one case where the nuclear matrix elements are readily evaluated, namely capture in hydrogen, the capture takes place largely in H-μ-H molecules, and only recently is reliable information becoming available concerning the muon-molecular orbital wave function.[2,3]

(4) In muon capture the momentum transfer is sufficiently great that one must consider nucleon-recoil effects which can be taken reliably into account only to first order in the reciprocal nucleon mass.

The principal quantitative successes toward establishing a universal theory for weak interactions are those associated with the conserved-vector-current theory (CVC) and these give one considerable confidence in the present understanding of the vector part of the weak-interaction coupling.[4] Notable here is the success in quantitatively relating the $f\tau$ value for the beta decay of O^{14} to the vector coupling constant in muon decay, and more recently, the experiments of Lee, Mo, and Wu[5,6] establishing the existence of the weak-magnetism term in beta decay through the comparison of the beta spectra of N^{12} and B^{12}. The high-energy neutrino experiments also give strong indications that the weak-magnetism term is present in muon processes, since a large fraction of the total high-energy "elastic" cross section comes from this term.[7] The magnitude of the muon coupling to the weak-magnetism term is only poorly known from these last experiments, however.

With respect to the axial vector coupling, the fact that the predicted ratio $(\pi\to e+\nu)/(\pi\to\mu+\nu)$ agrees so well with experiment indicates that the muon and the electron are coupled equally strongly to the one-pion matrix element $\langle\pi|J_\mu^5|0\rangle$ of the axial vector current operator, and thus one has good evidence that insofar as this matrix element is concerned, $F_A^{\mu}/F_A^{\beta}=1\pm0.05$.[8]

Up to now there is relatively little *direct* evidence in nuclear capture processes concerning the equality of the electron and muon couplings to matrix elements $\langle N|J_\mu^5|N\rangle$, although calculations based on this last assumption (in combination with others) are generally

* Supported in part by the U. S. Air Force through Air Force Office of Scientific Research Contract AF 49(638)-1389.
† Supported in part by U. S. Atomic Energy Commission under Contract No. AT-11-1-720.
‡ A. P. Sloan Foundation Fellow.

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[2] W. Roy Wessel and P. Phillipson, Phys. Rev. Letters **13**, 23 (1964).
[3] A. Halpern, Phys. Rev. Letters **13**, 660 (1964); P. K. Kabir, Phys. Letters **14**, 257 (1965).
[4] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); S. S. Gerschtein and J. B. Zel'dovich, Zh. Eksperim. i Teor. Fiz. **29**, 698 (1955) [English transl.: Soviet Phys.—JETP **2**, 576 (1957)].
[5] Y. K. Lee, L. W. Mo, and C. S. Wu, Phys. Rev. Letters **10**, 253 (1963).
[6] C. S. Wu, Rev. Mod. Phys. **36**, 618 (1964).
[7] Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) **23**, 1117 (1960); J. Løvseth (private communication).
[8] H. P. C. Rood, thesis, University of Groningen, Groningen, Holland, 1964 (unpublished).

consistent with experimental information. Notable in this connection are the results for the capture reactions:

$$\text{(a)}\quad \mu^- + p \to n + \nu_\mu,^{3}$$
$$\text{(b)}\quad \mu^- + \mathrm{He}^3 \to \mathrm{H}^3 + \nu_\mu,^{9-11}$$

where the computed capture rates are sufficiently close to the experimental values to suggest that these assumptions are not too far from the truth. However, it is necessary to remark that for both these processes the predicted rates depend somewhat sensitively on the magnitude of the induced pseudoscalar coupling constant which is very poorly known. This constant can be related to the pion lifetime and pion-nucleon coupling constant in the approximation of keeping only the one-pion exchange pole as was first done by Goldberger and Treiman.[12] Unfortunately, the result obtained by the Goldberger-Treiman relation differs by a factor of about 2 from that obtained in recent experiments on radiative muon capture.[13] Furthermore, reaction (a) involves the μ-molecular wave function mentioned earlier while reaction (b) involves nuclear matrix elements with their uncertainties. In this last case, the situation is better than usual in that information concerning elastic electron scattering form factors for the nuclei involved is available and can be employed to minimize the nuclear matrix element uncertainties. Still, accurate quantitative conclusions concerning the coupling constants are difficult to obtain. The quantitative situation is even worse for total capture rates in nuclei such as He^4, C^{12}, O^{16}, and Ca^{40} as well as other nuclei, where nuclear-structure information is even less reliable.[14]

The purpose of the present communication is to show that there exists one capture reaction, namely,

$$\mu^- + \mathrm{C}^{12} \to \mathrm{B}^{12}(\text{ground state}) + \nu_\mu, \qquad (1)$$

which is particularly favorable as a means of obtaining quantitative results on coupling constants for two reasons:

(1) Sufficient information is available from other experiments to determine nuclear matrix elements with relatively high accuracy.

(2) The capture rate in this case is relatively insensitive to the magnitude of the induced pseudoscalar coupling constant (and to the vector and tensor coupling constants as well) and independent of a possible scalar coupling.

Thus, if one assumes that CVC determines the weak-magnetism coupling constant, one can establish the equality of the muon and electron axial vector coupling constants to an accuracy of about 7% by use of available experimental information. Or, if one prefers to assume the equality of these last coupling constants, one can consider the results as evidence for weak magnetism in muon capture, thereby establishing its magnitude as consistent with CVC to the same order of accuracy as it is presently established in beta decay (i.e., 30%).

The experimental data which are employed to determine the requisite nuclear matrix elements in this case are:

(i) The recent precise measurements of Gudden[15] (combined with some earlier measurements of Dudelzak and Taylor[16] and Barber and Goldemberg[17]) of the transverse electromagnetic form factor for inelastic electron scattering to the 15.1-MeV $J^\pi = 1^+$ level in C^{12} which is the isobaric analog of the ground state of B^{12}.

(ii) The $M1$ lifetime measurement of Schmid and Scholz[18] for the same 15.1-MeV level in C^{12}.

(iii) The $f\tau_{1/2}$ value (one of the best known in nuclear physics) measured by Fisher[19] for the beta transition:

$$\mathrm{B}^{12} \to \mathrm{C}^{12}(\text{g.s.}) + e^- + \bar{\nu}_e,$$

(g.s. = ground state), namely,

$$f\tau_{1/2} = 11\,700 \pm 120\ \text{sec}. \qquad (2)$$

These are then combined with the recent measurement of the partial muon capture rate for the reaction (1):

$$\Lambda_{\mu c}[\mathrm{C}^{12} \to \mathrm{B}^{12}(\text{g.s.})] = [6.75_{-0.75}^{+0.30}] \times 10^{+3}\ \text{sec}^{-1} \qquad (3)$$

by Maier, Edelstein, and Siegel[20] to obtain information concerning coupling constants.

The essential idea is that this μ capture, with $|T=0, J=0^+\rangle \to |T=1, J=1^+\rangle$, takes place predominantly through the axial vector and weak-magnetism couplings, since the vector and scalar matrix elements vanish except for small nucleon-recoil terms in the case of the former. The computed rate is also very insensitive to the assumed value for the induced pseudoscalar (and tensor) coupling constants. Thus it is primarily the Gamow-Teller nuclear spin-matrix elements at substantial momentum transfer which are required to compute the major contribution to the capture rate. On the other hand, the matrix element of the transverse magnetic dipole operator in inelastic electron scattering (leading from the ground state to the 15.1-MeV level in C^{12}) involves both a spin and an orbital contribution (apart from possible exchange-moment contributions, which are here neglected) where the former is multiplied by a relatively large factor 4.71, the isotopic vector mag-

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[13] M. Conversi, R. Diebold, and L. diLella, Phys. Rev. **136**, B1077 (1964).
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[18] H. Schmid and W. Scholz, Z. Physik **175**, 430 (1963).
[19] T. R. Fisher, Phys. **130**, 2388 (1963).
[20] E. J. Maier, R. M. Edelstein, and R. T. Siegel, Phys. Rev. **133**, B663 (1964).

netic moment of the nucleon. A comparison of the limit of this form factor for small momentum transfer, or equivalently, the $M1$ radiative width of the 15.1-MeV level in C^{12}, with the $f\tau$ value for the beta decay of B^{12} allows one to separate the orbital and spin contributions to the matrix element by use of isospin invariance. It is found that the matrix element is dominated by the spin contribution in agreement with the theoretical analysis of Wiedenmüller.[21] Simple nuclear models strongly suggest that the form factors describing the momentum-transfer dependence of the orbital and spin parts of the matrix element are very similar, thus allowing one to calculate the required spin-matrix element at the requisite momentum transfer (apart from some small theoretical corrections to take into account interference terms with second-forbidden contributions) which is needed to evaluate the axial vector contribution to muon capture. In addition, the orbital matrix element which is also determined in this process is useful for the evaluation of one of the small nucleon-recoil correction terms.

The capture reaction (1) has also been studied in the past by Wolfenstein,[22] Fujii and Primakoff,[23] Morita and Fujii,[24] and Flamand and Ford[25] with the same end in mind. These authors normalize to the $f\tau$ value for the B^{12} beta decay but then must rely on nuclear models to evaluate the required matrix elements. In particular they require the models to determine the necessary matrix elements at the momentum transfer appropriate to the capture process. The essential difference between our results and those obtained previously is our use of experimental data (particularly the inelastic electron scattering data) to eliminate nearly all the dependence on theoretical nuclear models for evaluation of the matrix elements. The only respects in which nuclear models enter our calculation are the following:

(a) We assume that the spin and orbital parts of the transverse magnetic dipole matrix element have the same momentum-transfer dependence. This result is valid in the intermediate coupling model over the whole range from pure j-j to pure L-S coupling and depends essentially on the fact that the nucleon involved in the transition is a $1p$ nucleon. In this respect this assumption is almost model-independent.

(b) We again must use the intermediate-coupling model to evaluate one of the small nucleon-recoil matrix elements. Even in this case we attribute a liberal error to the result by using the extreme limits of j-j and L-S coupling as a measure of the uncertainties.

The remaining assumptions that enter into the present work (as well as previous work) are:

(i) The nuclear operators for muon capture, beta decay, and electron scattering can be obtained by the usual nonrelativistic reduction of the free-nucleon interaction. This means that we neglect possible exchange currents in nuclei as contributors to either the electromagnetic currents or the weak interaction currents.

(ii) Correction terms of order M^{-2}, where M is the nucleon mass, are negligible. They are presumably small ($\sim 1\%$) but have not been explicitly calculated.

(iii) We can use charge independence (isospin invariance) to go back and forth between B^{12} and C^{12}. There is some room for doubt here because of the 10% difference[6] in the $f\tau$ values for the beta decays of B^{12} and N^{12} to the ground state of C^{12}. On the basis of exact charge symmetry, these should be equal. However, there is an open channel for $N^{12}\to C^{11}+p$ only 0.49 MeV above the ground state of N^{12} while the corresponding open channel $B^{12}\to B^{11}+n$ lies 3.37 MeV above the ground state of B^{12}. Friar,[26] and independently, Eichler, Tombrello, and Bahcall[27] have shown that this can account for the discrepancy in $f\tau$ values by modifying the radial overlap of the wave functions involved in the beta-decay matrix elements. On the other hand, the open channel $C^{12}\to B^{11}+p$ lies 0.85 MeV above the 15.1 state in C^{12} while the open channel $C^{12}\to C^{11}+n$ lies 3.61 MeV above the same state. This may have some effect on the isotopic purity of the 15.1-MeV C^{12} level, and we are forced to disregard this.

The reader interested in the results rather than the details of calculation may proceed directly to the section entitled Discussion.

METHOD OF CALCULATION

The muon capture rate summed over nuclear orientation to go from an initial nuclear state $|a\rangle$ to a final nuclear state $|b\rangle$ is given by[28] [we use units $\hbar=c=1$].

$$\Lambda_{\mu c}(a\to b)=\frac{\nu^2}{2\pi}\frac{1}{(1+\nu/AM)}\sum_{M_f}\sum_{M_i}\frac{1}{2J_i+1}\int\frac{d\hat{\nu}}{4\pi}\Bigg\{\left|G_V\int 1\right|^2+\left|G_A\int\boldsymbol{\sigma}\right|^2+(|G_p|^2-2\,\mathrm{Re}G_pG_A^*)\left|\hat{\nu}\cdot\int\boldsymbol{\sigma}\right|^2$$
$$-\left[\frac{G_V^*g_V}{M}\left(\int 1\right)^*\left(\hat{\nu}\cdot\int\mathbf{p}\right)+\frac{g_A(G_A^*-G_p^*)}{M}\left(\hat{\nu}\cdot\int\boldsymbol{\sigma}\right)^*\left(\int\mathbf{p}\cdot\boldsymbol{\sigma}\right)+\frac{G_Ag_V^*}{M}i\hat{\nu}\cdot\left(\int\boldsymbol{\sigma}\right)\times\left(\int\mathbf{p}\right)^*+\mathrm{c.c.}\right]\Bigg\},\quad(4)$$

[21] H. Weidenmüller, Nucl. Phys. **21**, 397 (1960).
[22] L. Wolfenstein, Nuovo Cimento **13**, 319 (1959).
[23] A. Fujii and H. Primakoff, Nuovo Cimento **12**, 327 (1959).
[24] M. Morita and A. Fujii, Phys. Rev. **118**, 606 (1960).
[25] G. Flamand and K. W. Ford, Phys. Rev. **116**, 1591 (1959).
[26] J. Friar (unpublished).
[27] J. Eichler, T. A. Tombrello, and J. N. Bahcall, Phys. Letters **13**, 146 (1964).
[28] J. R. Luyten, H. P. C. Rood, and H. A. Tolhoek, Nucl. Phys. **41**, 236 (1963).

where we have used the following definitions:

$$\nu = E_a + m_\mu + \text{B.E.}(\mu) - E_b \quad \text{(neutrino energy)},$$

$$M = \text{nucleon mass},$$

$$\begin{aligned}
\int 1 &= \langle b | \sum_{i=1}^{A} \tau_i^{(-)} \exp[-i\boldsymbol{\nu}\cdot\mathbf{r}_i]\phi_\mu(r_i) | a\rangle, \\
\int \boldsymbol{\sigma} &= \langle b | \sum_{i=1}^{A} \tau_i^{(-)} \exp[-i\boldsymbol{\nu}\cdot\mathbf{r}_i]\phi_\mu(r_i)\boldsymbol{\sigma}_i | a\rangle, \\
\int \mathbf{p} &= \langle b | \sum_{i=1}^{A} \tau_i^{(-)} \exp[-i\boldsymbol{\nu}\cdot\mathbf{r}_i]\phi_\mu(r_i)\mathbf{p}_i | a\rangle, \\
\int \mathbf{p}\cdot\boldsymbol{\sigma} &= \langle b | \sum_{i=1}^{A} \tau_i^{(-)} \exp[-i\boldsymbol{\nu}\cdot\mathbf{r}_i]\phi_\mu(r_i)\mathbf{p}_i\cdot\boldsymbol{\sigma}_i | a\rangle,
\end{aligned} \tag{5}$$

$$\phi_\mu(r_i) = \text{muon wave function at } r_i,$$

$$\tau^{(-)}|p\rangle = |n\rangle.$$

The coupling constants follow from the general form of the nucleon vertex entering into muon capture (Fig. 1).[14]

$$\begin{aligned}
\langle P' | J_\mu(0) + J_\mu{}^5(0) | P\rangle = \bar{u}(P')\{&F_1(q^2)\gamma_\mu + F_2(q^2)\sigma_{\mu\nu}q_\nu \\
&+ iF_S(q^2)q_\mu + F_A(q^2)\gamma_5\gamma_\mu - iF_P(q^2)q_\mu\gamma_5 \\
&- F_T(q^2)\sigma_{\mu\nu}q_\nu\gamma_5\}\tau^{(-)}u(P),
\end{aligned} \tag{6}$$

$$\begin{aligned}
G_V/G &= F_1[1+(\nu/2M)] + m_\mu F_S, \\
G_A/G &= F_A - (\nu/2M)(F_1 + 2MF_2), \\
G_P/G &= (m_\mu F_p - F_A - [F_1 + 2MF_2 + 2MF_T])\nu/2M, \\
g_V/G &= F_1, \\
g_A/G &= F_A, \\
G &= 1.02\times 10^{-5}/M^2,
\end{aligned} \tag{7}$$

[all form factors evaluated for $q^2 = (2\nu - m_\mu)m_\mu$]. We have included the possibility of second-class currents ($F_S, F_T \neq 0$) for generality.

If we look at a transition $|T=0, J=0^+\rangle \to |T=1, J=1^+, 2^-, 3^+$, etc., then $\int 1 \equiv 0$ and the capture formula simplifies

$$\begin{aligned}
\Lambda_{\mu c}(a\to b) = (|\phi_\mu|^2)_{\text{av}} \frac{\nu^2}{2\pi} \frac{1}{(1+\nu/AM)} \sum_{M_f} \int \frac{d\hat{\nu}}{4\pi} \Bigg\{ |G_A|^2 \left|\int \boldsymbol{\sigma}\right|^2 + (|G_p|^2 - 2\,\text{Re}G_pG_A{}^*) \left|\hat{\nu}\cdot\int \boldsymbol{\sigma}\right|^2 \\
- \left[\frac{g_A(G_A{}^* - G_p{}^*)}{M} \left(\hat{\nu}\cdot\int\boldsymbol{\sigma}\right)^* \left(\int \mathbf{p}\cdot\boldsymbol{\sigma}\right) + \frac{G_A g_V{}^*}{M} i\hat{\nu}\cdot\left(\int\boldsymbol{\sigma}\right)\times\left(\int\mathbf{p}\right)^* + \text{c.c.}\right] \Bigg\}.
\end{aligned} \tag{8}$$

We have assumed that we can remove the average value of the square of the muon wave function from the nuclear matrix elements, since the muon wave function differs only by one to two percent from constancy over the nuclear volume. We have from Sens[29]

$$(|\phi_\mu|^2)_{\text{av}} = \frac{(Z\alpha m_\mu)^3}{\pi} \left(\frac{1}{1+(m_\mu/AM)}\right)^3 R, \tag{9}$$

$$R = 0.86 \pm 1\%. \tag{10}$$

The value of R is the same to within this error if the wave function is averaged only over the p-shell nucleons, instead of the whole nucleus.[30]

Specializing to the case $|T=0, 0^+\rangle \to |T=1, 1^+\rangle$, we can write in spherical tensor notation[31]

$$\sum_{M_f} \left|\int\boldsymbol{\sigma}\right|^2 = |(1^+\| \sum_{i=1}^{A} \tau^{(-)}(i) j_0(\nu x_i)\sigma(i) \| 0^+)|^2 + 5|(1^+\| \sum_{i=1}^{A} \tau^{(-)}(i) j_2(\nu x_i)[C_2\odot\sigma]_1 \| 0^+)|^2, \tag{11}$$

$$\sum_{M_f} \left|\hat{\nu}\cdot\int\boldsymbol{\sigma}\right|^2 = \tfrac{1}{3} |(1^+\| \sum_{i=1}^{A} \tau^{(-)}(i)\{ j_0(\nu x_i)\sigma(i) + (\sqrt{10}) j_2(\nu x_i)[C_2\odot\sigma]_1\} \| 0^+)|^2, \tag{12}$$

$$\begin{aligned}
-\frac{i}{\nu} \sum_{M_f} \hat{\nu}\cdot\left(\int\boldsymbol{\sigma}\right)\times\left(\int\mathbf{p}\right)^* = \frac{1}{3}\left(1^+ \left\| \sum_{i=1}^{A} \tau^{(-)}(i) \left[\frac{3j_1(\nu x_i)}{\nu x_i}\right] l(i) \right\| 0^+\right)^* \\
\times (1^+\| \sum_{i=1}^{A} \tau^{(-)}(i)\{ j_0(\nu x_i)\sigma(i) - (\sqrt{\tfrac{5}{2}}) j_2(\nu x_i)[C_2\odot\sigma]_1\} \| 0^+),
\end{aligned} \tag{13}$$

[29] J. C. Sens, Phys. Rev. **113**, 679 (1958).

[30] G. Ravenhall (private communication). This result suggests that the average value of the muon wave function required is not too sensitive to the other factors present in the radial integrals involved in matrix elements. Actually the major contributions come from those radial integrals in Eqs. (11) to (14) which contain $j_0(\nu x_i)$ and hence the use of the same average value of the muon wave function in all integrals would be expected to involve errors no larger than that already allowed for in Eq. (10).

[31] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957).

$$\sum_{M_f}\left(\hat{p}\cdot\int\boldsymbol{\sigma}\right)^*\left(\int\mathbf{p}\cdot\boldsymbol{\sigma}\right)=-i(1^+\|\sum_{i=1}^{A}\tau^{(-)}(i)j_1(\nu x_i)C_1(\Omega_i)(\mathbf{p}_i\cdot\boldsymbol{\sigma}_i)\|0^+)$$
$$\times(1^+\|\sum_{i=1}^{A}\tau^{(-)}(i)\{j_0(\nu x_i)\sigma(i)+(\surd 10)j_2(\nu x_i)[C_2\odot\sigma]_1\}\|0^+),\quad(14)$$

where we have defined:

$$C_{lm}(\Omega)=(4\pi/2l+1)^{1/2}Y_{lm}(\Omega),$$
$$[C_l\odot\sigma]_{JM}=\sum_{mq}(lm1q|l1JM)C_{lm}(\Omega)\sigma_{1q}.\quad(15)$$

We can write the $f\tau$ value for the β decay of B^{12} to the ground state of C^{12} as

$$f\tau_{1/2}(B^{12}\to C^{12}(\text{g.s.})+e^-+\bar{\nu}_e)$$
$$=\ln 2\left[\frac{|GF_A{}^\beta|^2 m_e{}^5}{2\pi^3}\tfrac{1}{3}|(0^+\|\sum_{i=1}^{A}\tau_i{}^{(+)}\sigma_i\|1^+)|^2\right]^{-1}.\quad(16)$$

This is an allowed Gamow-Teller transition. The corrections to the calculated transition probability coming from forbidden contributions can be estimated from the work of Morita[32] to be less than $\frac{1}{2}\%$. We can therefore use the measured value of $f\tau_{1/2}$ as given in Eq. (2) to compute

$$|(0^+\|\sum_{i=1}^{A}\tau_i{}^{(+)}\sigma_i\|1^+)|^2.$$

If one looks at electrons which are scattered through 180° as they excite C^{12} from the ground state to the 1^+, $T=1$, 15.1-MeV level, then one measures the reduced matrix elements of the transverse-magnetic-dipole operator[17]

$$\frac{d\sigma}{d\Omega}(\theta=180°)=\frac{\pi\alpha^2}{k_1{}^2}\frac{1}{[1+(k_1+k_2)/AM]}$$
$$\times|(1^+\|T_1{}^{\text{mag}}(k_1+k_2)\|0^+)|^2\quad(17)$$

where k_1 is the incident-electron wave number, $\alpha=1/137$, and

$$T_{1M}{}^{\text{mag}}(q)=\int d\mathbf{x}[\boldsymbol{\mathfrak{u}}_N(\mathbf{x})\cdot(\nabla\times j_1(qx)\boldsymbol{\mathfrak{Y}}_{111}{}^M(\Omega_x))$$
$$+\mathbf{j}_N(\mathbf{x})\cdot j_1(qx)\boldsymbol{\mathfrak{Y}}_{111}{}^M(\Omega_x)]\quad(18)$$

$e\boldsymbol{\mathfrak{u}}_N(\mathbf{x})$ and $e\mathbf{j}_N(\mathbf{x})$ are the nuclear magnetization and current-density operators. Reducing the vector spherical harmonics[31] and keeping only the term linear in τ_3 for the transition in mind, we see that electron scattering measures[33–35]

$$|(1^+\|T_1{}^{\text{mag}}(q)\|0^+)|^2=\frac{1}{12\pi}\left(\frac{q}{2M}\right)^2|(1^+\|\sum_{i=1}^{A}\frac{\tau_i{}^{(3)}}{\sqrt{2}}$$
$$\times\{l(i)[j_0(qx_i)+j_2(qx_i)]+(\lambda_p-\lambda_n)[j_0(qx_i)\sigma(i)$$
$$-\sqrt{\tfrac{5}{2}}j_2(qx_i)[C_2\odot\sigma]_1]\}\|0^+)|^2.\quad(19)$$

According to the conserved-vector-current theory,[4] this is the same matrix element which governs muon capture through the vector interaction. Since the transition is allowed Gamow-Teller, this is not very useful in trying to compute the total capture rate. We shall see, however, that the above expression gives us valuable information on the axial vector matrix elements also.

If we take the long-wavelength limit of the above expression, we can get the partial width of the 15.1-MeV level for $M1$ radiation to the ground state,

$$|(1^+\|T_1{}^{\text{mag}}\|0^+)|^2\underset{q\to 0}{=}\frac{1}{12\pi}\left(\frac{q}{2M}\right)^2$$
$$\times|(1^+\|\sum_{i=1}^{A}\frac{\tau_i{}^{(3)}}{\sqrt{2}}\{l(i)+(\lambda_p-\lambda_n)\sigma(i)\}\|0^+)|^2,\quad(20)$$

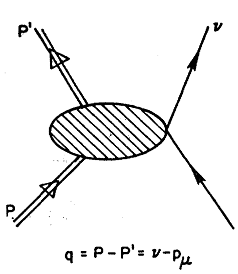

FIG. 1. General form of the nucleon vertex entering into muon capture. $q=P-P'=\nu-p_\mu$ is the four-momentum transfer.

[32] M. Morita, Phys. Rev. **113**, 1584 (1959).

[33] From the work of McVoy and Van Hove, (Ref. 34) we can write the matrix elements of the nuclear-current operator to order $1/M^2$ as

$$\hat{e}_{\mathbf{q}\lambda}\cdot\left\langle f\middle|e\int\mathbf{J}_N(\mathbf{x})\exp(i\mathbf{q}\cdot\mathbf{x})d\mathbf{x}\middle|i\right\rangle$$
$$=\hat{e}_{\mathbf{q}\lambda}\cdot\left\langle f\middle|\sum_{j=1}^{A}\exp[i\mathbf{q}\cdot\mathbf{x}(j)]\left\{\frac{e_j\mathbf{p}(j)}{M}+\frac{\mu_j}{2M}i\boldsymbol{\sigma}(j)\times\left[\mathbf{q}-\frac{\omega}{2M}\mathbf{p}(j)\right]\right\}\middle|i\right\rangle,$$

where $\omega=E_f-E_i$, and $\hat{e}_{\mathbf{q}\lambda}$ is a unit vector perpendicular to $\mathbf{q}$ [we need only the transverse part of the current—this allows us to make partial integrations, and to use an unsymmetrized form with respect to the momentum operators $\mathbf{p}(j)$]. Thus the $1/M$ *correction* to the terms we are discussing goes as $[\mathbf{q}-(\omega/2M)\mathbf{p}(j)]$. If we say $|\mathbf{p}(j)|\sim q$ we get a correction factor of only $1-(\omega/2M)=0.992$. Even if we let $|\mathbf{p}(j)|\leqslant 3q$ (~300 MeV in the present case), we only have a 2% correction to the matrix element. We shall therefore drop this correction. There are two other corrections to worry about, as indicated by Willey. One is the center-of-mass effect which enters if we try to make a single-particle model of the state as in Eq. (23), and the other involves the form factors for the individual nucleons. These effects tend to cancel (we evaluate the center-of-mass correction in an oscillator well). One needs (Ref. 35) $F_1(q^2)e^{\eta^2/4A}=F_1(q^2)e^{\eta^2/48}=0.995$ in the present case: We will therefore also drop these terms.

[34] K. W. McVoy and L. Van Hove, Phys. Rev. **125**, 1034 (1962).

[35] R. S. Willey, Nucl. Phys. **40**, 529 (1963).

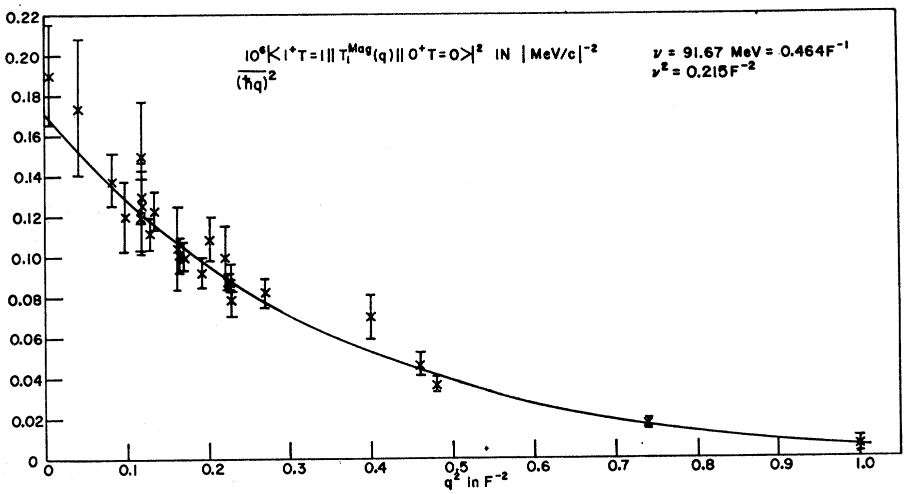

FIG. 2. Experimental points (Refs. 15–18) and best fit as described in the discussion of Eqs. (24) and (25) for the reduced matrix element of the transverse magnetic dipole operator taken between the ground state and the 1^+ $T=1$ state at 15.11 MeV in C^{12}. q^2 is the square of the three-momentum transferred to the nucleus. The value of ν, the momentum transfer appropriate to muon capture, is indicated in the figure.

and the transition probability is

$$\omega(C^{12*}\to C^{12}(\text{g.s.})+\gamma)=\tfrac{2}{3}\alpha k\left(\frac{k}{2M}\right)^2$$

$$\times\tfrac{1}{3}|(0^+\|\sum_{i=1}^{A}\frac{\tau_i^{(3)}}{\sqrt{2}}\{l(i)+(\lambda_p-\lambda_n)\sigma(i)\}\|1^+)|^2. \quad (21)$$

NUMERICAL CALCULATION

For the particular transition in question we have[6,14]

$$\nu=(105.655-13.887-0.102)\text{ MeV}=91.67\text{ MeV}. \quad (22)$$

Therefore this much momentum must be transferred to the system while making the transition, and we need the transition form factors for this value of q.

If one calculates the reduced matrix element of the transverse-magnetic dipole operator by assigning the 1^+ state to the $p_{3/2}^{-1}p_{1/2}$ configuration and making the ground state of C^{12} a closed $p_{3/2}$ shell, one gets[17]

$$|(p_{3/2}^{-1}p_{1/2}1^+T=1\|T_1^{\text{mag}}(q)\|0^+T=0)|^2$$
$$=\frac{4}{9\pi}\left(\frac{q}{2M}\right)^2[(\lambda_p-\lambda_n-\tfrac{1}{2})(1-\tfrac{1}{6}\eta^2)e^{-\eta^2/4}$$
$$+(\tfrac{1}{4}[\lambda_p-\lambda_n]-\tfrac{1}{2})\tfrac{1}{6}\eta^2e^{-\eta^2/4}]^2 \quad (23)$$

where we have computed the radial matrix elements using $1p$ oscillator wave functions and we write $\eta=qb_{\text{osc}}=[(\hbar^2q^2/M)/\hbar\omega_{\text{osc}}]^{1/2}$. We have indicated the terms coming from $j_0(qx_i)$ [first group] and those from $j_2(qx_i)$ [second group]. A more detailed intermediate-coupling treatment of the states involved by Kurath cuts down the over-all strength [the above formula is about 3.5 times too large] but does not change the shape appreciably.[36] We have therefore attempted to make a best fit to the electron-scattering data summarized by Gudden[15] using b_{osc} as a parameter and a form corresponding to

$$\frac{|q^{-1}(1^+\|T_1^{\text{mag}}(q)\|0^+)|^2}{|q^{-1}(1^+\|T_1^{\text{mag}}(q)\|0^+)|^2_{q=0}}=[1-\tfrac{1}{6}\eta^2+\tfrac{1}{6}\rho\eta^2]^2e^{-\eta^2/2}, \quad (24)$$

where ρ was taken from the above as $\rho=0.16$. [ρ does not change much over the whole range of jj to LS coupling.[36]] The fit is obtained by treating all quoted experimental errors as statistical and is shown in Fig. 2.[37] The two-parameter fit to the data is better than one has a right to expect on a purely statistical basis. One finds

$$b_{\text{osc}}=1.92\pm0.03\text{ F} \quad (25)$$

and the values of the reduced matrix element indicated in Table I. Thus if we believe the purely statistical errors, then we can determine the γ width to $\pm2.5\%$ and the required form factor to $\pm3.8\%$.[38,39] The oscillator parameter one gets from this analysis is considerably larger than that ($b_{\text{osc}}=1.62$ F) obtained from fits to elastic electron scattering or Coulomb energies.[40,41] We assume that this reflects the inadequacy of simple harmonic oscillator wave functions for computing the transition matrix element. [We also note that the calculations of Flamand and Ford[25] were very sensitive to this parameter, as they pointed out.] We now proceed to compute the matrix elements for μ capture:

(a) $\sum_{M_f}|\int\boldsymbol{\sigma}|^2$: If we return to Eqs. (23) and (24)

[36] D. Kurath, Phys. Rev. **134**, B1025 (1964).

[37] We are very greatly indebted to C. Sheppey of the Computor Division of CERN for carrying out the statistical fit for us.

[38] The errors we quote are taken to be root-mean-square, or standard deviations. In assigning theoretical errors we shall assume they are the same. Thus in adding two numbers, we find the new standard deviation by, $z=x+y\Rightarrow\sigma_z^2=\sigma_x^2+\sigma_y^2$ and when multiplying two numbers by, $z=xy\Rightarrow\sigma_z^2=x_0^2\sigma_y^2+y_0^2\sigma_x^2$. The confidence limit of finding the desired number within $\pm\sigma$ is then 68.3%. We treat all error distributions as uncorrelated and symmetrical (Ref. 39).

[39] L. G. Parratt, *Probability and Experimental Errors in Science* (John Wiley & Sons, Inc., New York, 1961).

[40] U. Meyer-Berkhout, K. W. Ford, and A. E. S. Green, Ann. Phys. (N. Y.) **8**, 119 (1959).

[41] B. C. Carlson and I. Talmi, Phys. Rev. **96**, 436 (1954).

TABLE I. Values of the reduced matrix element of the transverse-magnetic-dipole operator taken between the ground state and the 1^+, $T=1$ state at 15.11 MeV in C^{12}. The values and errors are obtained from our best fit to the experimental data (Refs. 15–18) in Fig. 1. The fit is described in the discussion of Eqs. (24) and (25).

q in MeV	0	15.11	91.67	Value (91.67)/ Value (0)
$\frac{10^6}{q^2}\lvert\langle 1^+\Vert T_1^{\mathrm{mag}}(q)\Vert 0^+\rangle\rvert^2$ in $[\mathrm{MeV}]^{-2}$	0.171 ± 0.0035	0.170 ± 0.004	0.091 ± 0.004	0.53 ± 0.02

we find that the ratio of second-forbidden to allowed terms goes as

$$\frac{\text{second forbidden}}{\text{allowed}}=\rho\frac{\frac{1}{6}\eta^2}{1-\frac{1}{6}\eta^2}. \tag{26}$$

Using the best fit to the electron-scattering data we are interested in, we have

$$\eta=(1.92\mathrm{F})\left(\frac{91.67}{197.5}\mathrm{F}^{-1}\right)=0.891. \tag{27}$$

Thus the above ratio is

$$\frac{\text{second forbidden}}{\text{allowed}}=0.15\rho. \tag{28}$$

For our fit to the data, we used $\rho=0.16$. Kurath shows the number ρ varies only from 0.16 to 0.26 over the whole range of intermediate coupling.[36] Thus the second-forbidden contributions represent only a change of 2.5 to 4% in the matrix element. We shall therefore use the *theory to extract the small correction of the second-forbidden term*, which can be done by the above considerations to within 3% *in the square of the matrix element.* We note that since nuclear orientations are summed over, the allowed and second forbidden terms contribute incoherently to $|\int\boldsymbol{\sigma}|^2$, and from the above, the square of the second-forbidden contribution is completely negligible.

By using the $f\tau_{1/2}$ value (2) and Eq. (16) we get a value for $|(0^+\Vert\sum_{i=1}^A\tau_i^{(+)}\sigma_i\Vert 1^+)|^2$. The long-wavelength limit of the matrix element of T_1^{mag} gives $|(1^+\Vert\sum\tau_i^{(-)}\times[l(i)+(\lambda_p-\lambda_n)\sigma(i)]\Vert 0^+)|^2$. By comparing these two values, and using a sign for the matrix elements of σ which agrees with that of j-j and intermediate coupling, we can solve to get

$$\begin{aligned}(1^+\,T=1\Vert\sum_{i=1}^{A}\tau_i^{(-)}l(i)\Vert 0^+\,T=0)&=-0.28\pm0.15,\\ (1^+\,T=1\Vert\sum_{i=1}^{A}\tau_i^{(-)}\sigma(i)\Vert 0^+\,T=0)&=1.07\pm0.03,\end{aligned} \tag{29}$$

where the corresponding results for j-j coupling are $-2/\sqrt{3}$ and $+4/\sqrt{3}$, respectively. Thus, since the σ matrix element is to be multiplied by $\lambda_p-\lambda_n=4.71$, it is responsible for 94% of the $M1$ matrix element, in agreement with estimates of Wiedenmüller.[21] We therefore write

$$\frac{|GF_A{}^\beta|^2}{2\pi^2}\sum_{M_f}\left|\int\boldsymbol{\sigma}\right|^2=\frac{3\pi}{m_e{}^5}\frac{\ln 2}{(f\tau_{1/2})}\frac{|q^{-1}(1^+\Vert T_1^{\mathrm{mag}}(q)\Vert 0^+)|^2_{q=91.67\,\mathrm{MeV}}}{|q^{-1}(1^+\Vert T_1^{\mathrm{mag}}(q)\Vert 0^+)|^2_{q=0}}\times\left[1-2\rho\frac{\frac{1}{6}\eta^2}{1-\frac{1}{6}\eta^2}\right]_{q=91.67\,\mathrm{MeV}}. \tag{30}$$

We first normalize the matrix element to give the correct $f\tau_{1/2}$ value for B^{12} and then we use the electron scattering to measure the form factor of the matrix element of σ, putting in the small correction to take out the second-forbidden contribution. The $T_1^{\mathrm{mag}}(q)$ operator still contains a 6% contribution of $\mathbf{l}$; however, both $\mathbf{l}$ and $\boldsymbol{\sigma}$ are now multiplied by the *same* Bessel function $j_0(qx_i)$ so in any model in which the radial wave functions factor [i.e., any intermediate coupling in the $1p$ shell], the above *is exactly* the form factor. In any event, we assume that the relative change of the form factors of the $\mathbf{l}$ and $\boldsymbol{\sigma}$ terms is a higher order correction. Thus, using the $f\tau_{1/2}$ value in the Introduction, the ratio of $T_1^{\mathrm{mag}}(q)$ matrix elements in Table I, and the correction factor above

$$\left[1-2\rho\frac{\frac{1}{6}\eta^2}{1-\frac{1}{6}\eta^2}\right]_{q=91.67\,\mathrm{MeV}}=0.95\pm3\% \tag{31}$$

one finds

$$\frac{\nu^2 m_\mu{}^3}{2\pi^2}|GF_A{}^\beta|^2\sum_{M_f}\left|\int\boldsymbol{\sigma}\right|^2=8.08\times10^7\,\mathrm{sec}^{-1}\pm5\%. \tag{32}$$

Thus we have determined the square of the dominant nuclear matrix element to $\pm5\%$.

(b) $\sum_{M_f}|\int\hat{\nu}\cdot\boldsymbol{\sigma}|^2$: We use the same procedure as above. The only difference is that now we must compute only the longitudinal components of $\boldsymbol{\sigma}$ and the allowed and second-forbidden terms will thus interfere. Again we shall use our theoretical model to correct the matrix elements of $\sum_{M_f}|\int\boldsymbol{\sigma}|^2$ to get just the longitudinal components

$$\left(\frac{\sum_{M_f}|\int\hat{\nu}\cdot\boldsymbol{\sigma}|^2}{\frac{1}{3}\sum_{M_f}|\int\boldsymbol{\sigma}|^2}\right)_{j-j\text{ coupling}}=\frac{|(1-\frac{1}{6}\eta^2)-\frac{1}{12}\eta^2|^2}{|(1-\frac{1}{6}\eta^2)|^2+2|\eta^2/24|^2}$$

$$\cong\left[\frac{1-\frac{1}{4}\eta^2}{1-\frac{1}{6}\eta^2}\right]^2=0.86\pm7\%\quad[\text{at }\eta=0.891].\quad(33)$$

The error is estimated from our uncertainty in the knowledge of the *correction term* due to our uncertainty in the knowledge of the coupling scheme. [This uncertainty is not particularly important since the term going as $\sum_{M_f}|\int\hat{\nu}\cdot\boldsymbol{\sigma}|^2$ makes only about a 20% contribution to the total capture rate.]

$$\frac{\nu^2 m_\mu^3}{2\pi^2}|GF_A{}^\beta|^2\sum_{M_f}\left|\int\hat{\nu}\cdot\boldsymbol{\sigma}\right|^2=2.32\times10^7\text{ sec}^{-1}\pm9\%.\quad(34)$$

(c) $-(i/\nu)\sum_{M_f}\hat{\nu}\cdot(\int\boldsymbol{\sigma})\times(\int\mathbf{p})^*$: This term enters the total capture rate multiplied by a factor ν/M and represents one of the "nucleon recoil corrections." By Eq. (13), we can relate this to matrix elements of $\mathbf{l}$ and $\boldsymbol{\sigma}$. Since this is already a correction term, we will use the computed form factor of the $\mathbf{l}$ contribution $[\sim e^{-\eta^2/4}]$. The appropriate matrix elements of $\int\boldsymbol{\sigma}$ have been discussed above. We can thus write [noting that the nuclear matrix elements involved make a real contribution]

$$\frac{\nu^2 m_\mu^3}{2\pi^2}|G_A{}^\beta|^2\sum_{M_f}\frac{-i\hat{\nu}}{\nu}\cdot\left(\int\boldsymbol{\sigma}\right)\times\left(\int\mathbf{p}\right)^*$$

$$=-0.83\times10^{-7}\text{ sec}^{-1}\pm50\%.\quad(35)$$

The coefficient of this term in the capture rate becomes

$$2(\nu/M)\,\text{Re}G_A g_v{}^*=0.196\,\text{Re}G_A g_v{}^*.$$

(d) $1/\nu\sum_{M_f}(\hat{\nu}\cdot\int\boldsymbol{\sigma})^*(\int\mathbf{p}\cdot\boldsymbol{\sigma})$: For this we need the matrix elements of $(\int\mathbf{p}\cdot\boldsymbol{\sigma})$ about which we have no direct experimental information. This term also enters multiplied by ν/M and represents the other "nucleon recoil correction" to the total capture rate. We shall use the results of Flamand and Ford who calculated it in intermediate coupling and shall use the limiting L-S and j-j coupling cases to assign an error to it (which should thus be quite generous). The relevant combination of nuclear matrix elements is again real so we can look at the combination

$$y=\frac{2}{\nu}\sum_{M_f}\left(\hat{\nu}\cdot\int\boldsymbol{\sigma}\right)^*\left(\int\mathbf{p}\cdot\boldsymbol{\sigma}\right)\Big/\sum_{M_f}\left|\hat{\nu}\cdot\int\boldsymbol{\sigma}\right|^2,\quad(36)$$

$$y=\frac{6}{i\nu}\frac{(1^+\|\sum_{i=1}^{A}\tau^{(-)}(i)j_1(\nu x_i)c_1(\Omega_i)(\mathbf{p}_i\cdot\boldsymbol{\sigma}_i)\|0^+)}{(1^+\|\sum_{i=1}^{A}\tau^{(-)}(i)\{j_0(\nu x_i)\sigma(i)+(\sqrt{10})j_2(\nu x_i)[c_2\odot\sigma]_1\}\|0^+)}.\quad(37)$$

For the numerator, we have in j-j coupling

$$\frac{6}{i\nu}(1^+\|\sum_{i=1}^{A}\tau^{(-)}(i)j_1(\nu x_i)c_1(\Omega_i)(\mathbf{p}_i\cdot\boldsymbol{\sigma}_i)\|0^+)_{j-j\text{ coupling}}=\left(\frac{4\pi}{3}\right)^{1/2}\frac{5}{\sqrt{\pi}}\left[1-\frac{\eta^2}{10}\right]e^{-(\eta^2/4)}$$

$$=4.34\quad(\text{at }\eta=0.891).\quad(38)$$

Now in the notation of Flamand and Ford:

$$\frac{6}{\nu i}(1^+\|\sum_{i=1}^{A}\tau^{(-)}(i)j_1(\nu x_i)c_1(\Omega_i)(\mathbf{p}_i\cdot\boldsymbol{\sigma}_i)\|0^+)\equiv\left(\frac{4\pi}{3}\right)^{1/2}\frac{2}{i\nu}\langle g_1Y_1\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\rangle.\quad(39)$$

They give a formula which agrees with the above in the j-j limit. They also give

$$\frac{2}{i\nu}\langle g_1Y_1\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\rangle=2.23,\quad j\text{-}j\text{ coupling};$$

$$=1.91,\quad\text{best intermediate coupling};\quad(40)$$

$$=1.35,\quad L\text{-}S\text{ coupling};$$

where they have used an oscillator parameter fit to elastic scattering and include the muon wave function in their matrix element. We shall therefore reduce our number in the ratio of Flamand and Ford to get the best intermediate coupling value and then assign errors as *the extreme limits in coupling*

$$\frac{6}{i\nu}(1^+\|\sum_{i=1}^{A}\tau^{(-)}(i)j_1(\nu x_i)c_1(\Omega_i)$$

$$\times(\mathbf{p}_i\cdot\boldsymbol{\sigma}_i)\|0^+)|_{\eta=0.891,\text{ intermediate coupling}}=3.70\pm25\%.\quad(41)$$

Combining this with our experimental result for the matrix element in the denominator as discussed pre-

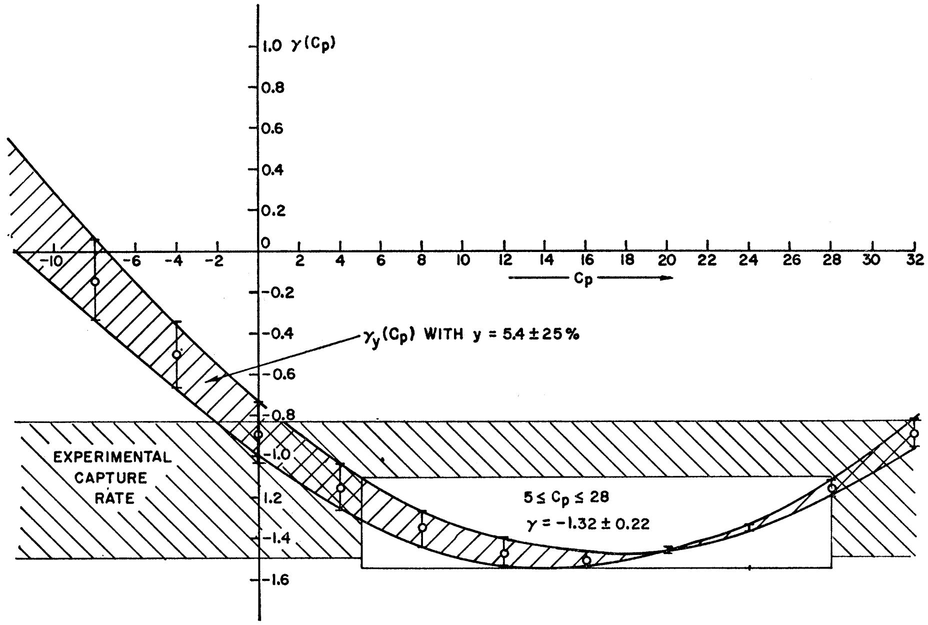

FIG. 3. The quantity $\gamma_y(C_p)$ defined in Eq. (44) is plotted against $C_p = m_\mu F_p/F_A{}^\mu$. y is discussed in Eqs. (36)–(42) and the estimated uncertainty in y is indicated in the figure. The value of $\gamma(C_p)$ as determined from Eq. (44) and the experimental value of the partial muon capture rate given in Eq. (3) [using the "canonical values" of the other coupling constants (Ref. 14)] is also shown.

viously, we find

$$y = 5.4 \pm 25\%. \tag{42}$$

Combining Eqs. (8), (9), (32), (34), (35), and (36), we find the muon capture rate to be

$$\begin{aligned}\Lambda_{\mu c}(\mu^- + C^{12} \rightarrow B^{12} + \nu_\mu) &= (5.73\times 10^3\ \text{sec}^{-1} \pm 5\%)\left|\frac{G_A}{GF_A{}^\beta}\right|^2 \\ &+ (1.65\times 10^3\ \text{sec}^{-1} \pm 9\%)\left|\frac{F_A{}^\mu}{F_A{}^\beta}\right|^2 \gamma \\ &+ (-0.12\times 10^3\ \text{sec}^{-1} \pm 50\%)\frac{\text{Re}G_A g_v{}^*}{|GF_A{}^\beta|^2},\end{aligned} \tag{43}$$

where we have defined

$$\gamma \equiv \frac{1}{|GF_A{}^\mu|^2}\left\{|G_p|^2 - 2\,\text{Re}G_p G_A{}^* - y\frac{\nu}{M}\text{Re}[g_A(G_A{}^* - G_p{}^*)]\right\}. \tag{44}$$

COUPLING-CONSTANT ANALYSIS

We shall assume that the coupling constants are real and that we only have first-class currents [$F_S = F_T = 0$; note that F_S does not contribute to the capture rate for this transition], returning to the possible role of F_T in the discussion.

Our main uncertainty in the μ-capture rate now lies in γ. We have a large uncertainty in the nuclear matrix element y and of course in the induced pseudoscalar coupling constant

$$m_\mu F_p/F_A{}^\mu \equiv C_p. \tag{45}$$

(The Goldberger-Treiman value is $C_p = 7.5$ while Conversi *et al.*,[13] give $C_p = 13.3 \pm 2.7$.) If we give the other coupling constants their "canonical values" as discussed in Foldy and Walecka,[14] we find [for $\nu = 91.67$ MeV]:

$$\nu/2M = 0.0488, \tag{46}$$

$$\begin{aligned}G_A/GF_A{}^\mu &= 1 - (\nu/2M)\mu(\nu^2)/F_A{}^\mu, \\ G_p/GF_A{}^\mu &= \{C_p - 1 - [\mu(\nu^2)/F_A{}^\mu]\}\nu/2M, \\ g_A/GF_A{}^\mu &= 1,\end{aligned} \tag{47}$$

where we have defined $\mu(\nu^2) = (\lambda_p - \lambda_n)F_m(\nu^2) = (4.71) \times (0.978) = 4.60$ and we take $F_A{}^\mu \equiv F_A{}^\beta$. In this case we can plot $\gamma_y(C_p)$ versus C_p and this is done in Fig. 3. The shaded area indicates our uncertainty in y. We see that γ_y has a *broad minimum* as a function of C_p in the region of interest for C_p. We have also plotted in Fig. 3 the values of γ corresponding to the experimental capture rate as determined from Eq. (43) and the "canonical values" of the coupling constants given above. The indicated uncertainty is taken from the *experimental error on the capture rate alone.* Any attempt to determine C_p from the partial capture rate is clearly impossible in this case. We can, however, turn this insensitivity of γ to C_p to our advantage. We see from Fig. 3 that even if we take a wide range of values, say

$$5 \leqslant C_p \leqslant 28, \tag{48}$$

we still know γ fairly accurately:

$$\gamma = -1.32 \pm 0.22. \tag{49}$$

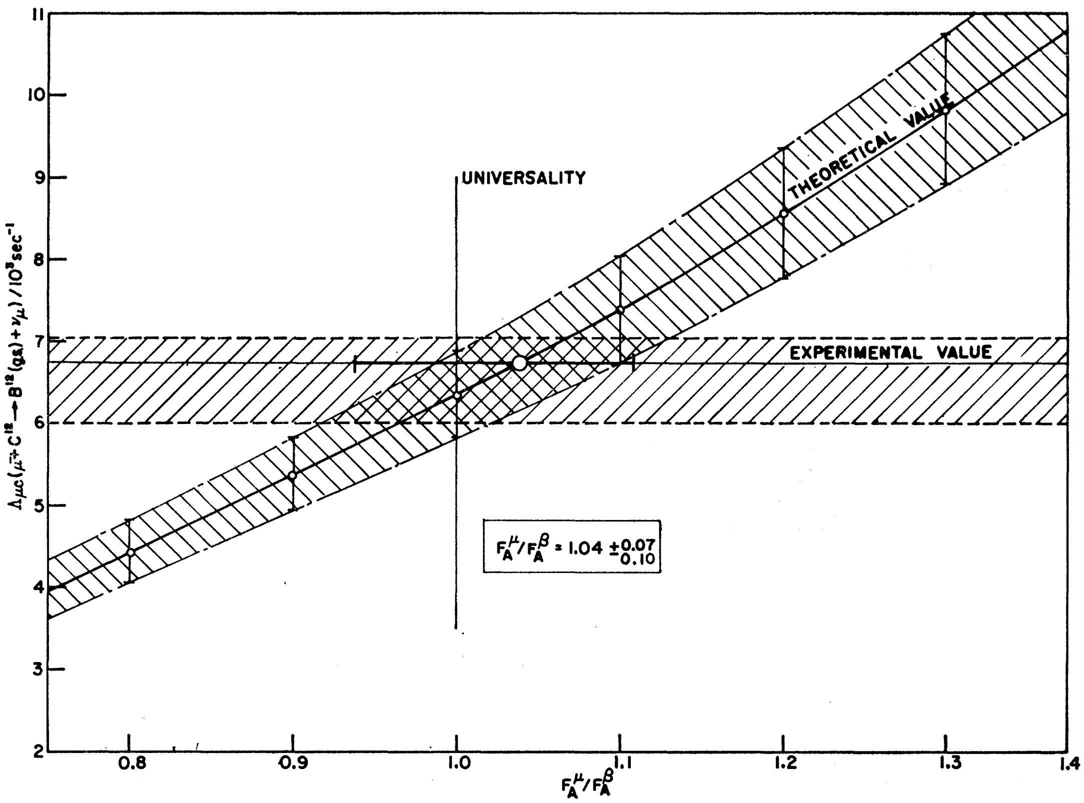

FIG. 4. Experimental and theoretical values of the partial capture rate for the process (1) plotted against $F_A{}^\mu/F_A{}^\beta$, assuming $\mu(\nu^2)=\mu_{\rm CVC}(\nu^2)$.

If we write out γ in detail, we have from Eqs. (44), (47),

$$\gamma=-2\left(\frac{\nu}{2M}\right)\left\{y\left[1-\left(\frac{\nu}{2M}\right)(C_p-1)\right]+\left(1-\frac{\nu}{2M}x\right)(C_p-1-x)-\frac{1}{2}\left(\frac{\nu}{2M}\right)(C_p-1-x)^2\right\} \quad (50)$$

and we see that it only depends on the ratio

$$x\equiv\mu(\nu^2)/F_A{}^\mu. \quad (51)$$

A further useful property of γ is

$$\frac{\partial\gamma}{\partial x}=2\left(\frac{\nu}{2M}\right)\left(1-\frac{\nu}{2M}x\right). \quad (52)$$

Thus the change in γ with x is independent of both y and C_p. Therefore to get γ and its error for other values of x, we have only to shift the boxed-in region in Fig. 3 up or down. Because of the factors $(\nu/2M)$ in Eq. (52), γ is quite insensitive to x.

We can now analyze Eq. (43) from two different points of view:

1. We can assume $\mu(\nu^2)$ and $g_v(\nu^2)$ are given by their conserved-vector-current theory values

$$g_v/G=F_1(\nu^2)=0.978$$
$$\mu_{\rm CVC}(\nu^2)=(\lambda_p-\lambda_n)F_m(\nu^2)=4.60 \quad (53)$$

and plot the resulting capture rate as a function of $F_A{}^\mu/F_A{}^\beta$. This is done in Fig. 4. To evaluate all of the terms in Eq. (43) we also need to assume a value for $F_A{}^\beta$, for which we take[42,43]

$$F_A{}^\beta=-1.18\pm0.03\,, \quad (54)$$

but since our leading terms involve only the *ratio* $F_A{}^\mu/F_A{}^\beta$, we are quite insensitive to the actual value of this number. We have also plotted the experimental result (3) in Fig. 4. From the intersection of the two curves we obtain

$$\frac{F_A{}^\mu}{F_A{}^\beta}=1.04_{-0.10}{}^{+0.07}\,, \quad \text{assuming CVC and } 5\leqslant C_p\leqslant 28 \quad (55)$$

where we have assumed the usual value of the sign. The errors are obtained by combining the errors in Eq. (43) and the experimental errors in (3).[44] This ratio is certainly consistent with unity and the accuracy is almost as good as that obtained from the ratio $(\pi\rightarrow e+\nu)/(\pi\rightarrow\mu+\nu)$ which is $F_A{}^\mu/F_A{}^\beta=1\pm0.05$.[8]

2. We can assume $F_A{}^\mu/F_A{}^\beta=1$ and plot the resulting

[42] F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. **135**, B467 (1964).

[43] C. P. Bhalla, Bull. Am. Phys. Soc. **10**, 544 (1965).

[44] We use the usual formulas for combining errors even though the experimental error is unsymmetrical.

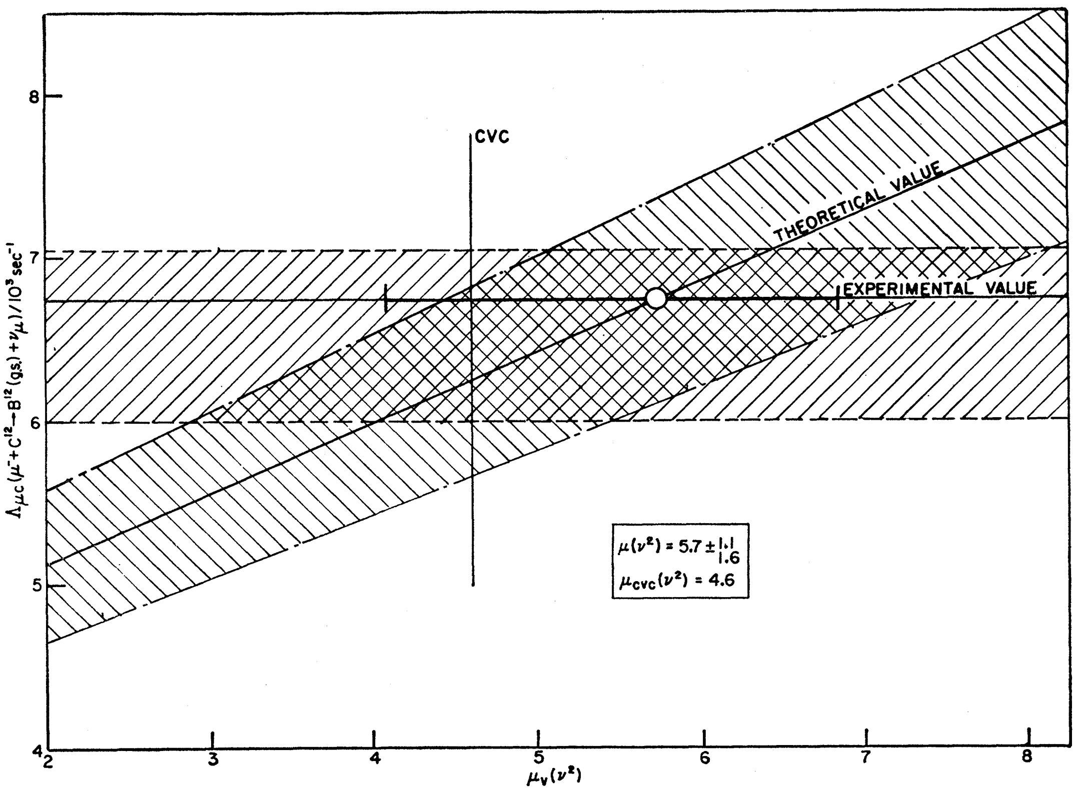

FIG. 5. Experimental and theoretical values of the partial capture rate for the process (1) plotted against $\mu(\nu^2)$, assuming $F_A{}^{\mu}/F_A{}^{\beta}=1$.

capture rate as a function of $\mu(\nu^2)$. This is done in Fig. 5. Comparing with the experimental result, we obtain

$$\mu(\nu^2)=5.7_{-1.6}^{+1.1},$$

$$\text{assuming } F_A{}^{\mu}/F_A{}^{\beta}=1 \quad\text{and}\quad 5<C_p<28. \quad (56)$$

This is to be compared with the CVC value

$$\mu_{\mathrm{CVC}}(\nu^2)=4.60.$$

Under this assumption we clearly have evidence for the presence of the weak-magnetism term with the right sign and with a magnitude which agrees within the errors with the CVC. The accuracy here is comparable to that obtained by comparing the beta spectra of N^{12} and B^{12}.[5,6]

DISCUSSION

The basic idea of the present work is that by combining the results for the $f\tau$ value for the beta decay of B^{12} to the ground state of C^{12} and the transverse form factor for electron excitation of the $T=1$, $J^{\pi}=1^{+}$, 15.1-MeV level in C^{12}, which is the isobaric partner of the ground-state B^{12}, one has enough *experimental* information to get accurate values for almost all the nuclear matrix elements relevant to the process

$$\mu^{-}+C^{12}\rightarrow B^{12}(\mathrm{g.s.})+\nu_{\mu}.$$

This allows one to draw some conclusions about the weak-coupling constants in muon capture in a way that is almost independent of any nuclear model. There are of course some very basic nuclear physics assumptions which go into this work. These are:

1. We know the nuclear operators for muon capture, beta decay, and electron scattering, and these are obtained by making the usual nonrelativistic reduction of the free nucleon interaction, and then summing over nucleons. This means we neglect possible meson exchange currents.

2. Nucleon-recoil correction forms of order $1/M^2$, where M is the nucleon mass, are negligible.

3. We can use charge independence to go back and forth between B^{12} and C^{12}.

We need a nuclear model for the states involved in order to correct for some second-forbidden contributions and to calculate one nucleon-recoil correction term about which we have no direct experimental information. We essentially use the *j-j* and *L-S* coupling models to bracket these quantities and therefore our estimate of these correction terms is almost model-independent.

Actually, information on just one partial capture rate is still not very illuminating, since there are several coupling constants involved in muon capture. The particular transition considered here has the advantage of being almost independent of C_p, the induced-pseudoscalar coupling constant which is not very well known. It is also independent of the vector coupling constant except for a small recoil-correction form. Therefore the calculated capture rate essentially involves two coupling constants, $F_A{}^\mu/F_A{}^\beta$, the ratio of the axial-vector coupling constant in μ capture to that in beta decay, and $\mu(\nu^2)$, the weak-magnetism coupling constant. Our conclusions are either

$$F_A{}^\mu/F_A{}^\beta=1.04_{-0.10}^{+0.07} \quad [\text{assuming CVC and } 5<C_p<28]$$

or

$$\mu(\nu^2)=5.7_{-1.6}^{+1.1} \quad [\text{assuming } F_A{}^\mu/F_A{}^\beta=1 \text{ and } 5<C_p<28].$$

The conserved-vector-current theory value of this last quantity is

$$\mu_{\mathrm{CVC}}(\nu^2)=4.60.$$

The accuracy of the first conclusion is comparable to that obtained by looking at the vacuum to one-pion matrix elements of the axial-vector current where, from the branching ratio of $\pi\rightarrow e+\nu/\pi\rightarrow\mu+\nu$, one has $F_A{}^\mu/F_A{}^\beta=1\pm0.05$.[8] The accuracy of the second conclusion is comparable to that obtained in beta decay by comparing the spectra of N^{12} and B^{12}. Assuming $F_A{}^\mu/F_A{}^\beta=1$ clearly shows the need for a weak-magnetism term in muon capture of the sign and, within the quoted errors, of the magnitude predicted by CVC.

It is not within our province to go into a detailed discussion of the accuracy of the experimental results which we use. We have used the published data with the quoted errors taken to be purely statistical. The value for $\Lambda_{\mu c}(\mu^-+C^{12}\rightarrow B^{12}(\text{g.s.})+\nu_\mu)$ has fluctuated in the past. This is discussed by Mair, Edelstein, and Siegel.[20] We take their value

$$\Lambda_{\mu c}=(6.75_{-0.75}^{+0.30})\times10^3\ \text{sec}^{-1}$$

which is the latest one, and the most accurate. The data for the transverse-electromagnetic form factor of the 15.1-MeV 1^+ state in C^{12} comes from several groups[15–18] as well as from different experiments (electron scattering[15–17] and lifetime measurement[18]). Thus a best fit to all the data, as we have carried it out, would seem to be reliable. We use the $f\tau$ value for

$$B^{12}\rightarrow C^{12}(\text{g.s.})+e^-+\bar{\nu}_e$$

to normalize our matrix elements in Eq. (30) and this value is very well known.[19]

We can ask about the possible effect of second-class currents. They are very easily included in the preceding analysis. The scalar coupling F_S of Eq. (7) does not contribute to this process since it is a $0^+\rightarrow1^+$ transition. The tensor coupling and induced pseudoscalar coupling always enter in the same combination as seen in Eq. (7). Our previous considerations indicate that we are insensitive to this combination. We can thus simply translate our bounds on C_p to bounds on this combination

$$5\leqslant C_p-\frac{2MF_T{}^\mu}{F_A{}^\mu}\leqslant28$$

in our conclusions stated above. If we used the Goldberger-Treiman value of $C_p=7.5$ we would get

$$-2.5\leqslant\frac{2MF_T{}^\mu}{F_A{}^\mu}\leqslant20.5,$$

for example, which is a wide range of tensor couplings.

In analyzing Eq. (43) one could also easily include the possibility of purely imaginary couplings for the second class currents, leading to time-reversal violation, as suggested by Cabibbo[45]; however, recent experiments on $K_{\mu3}$ decays indicate that this effect is absent for the strangeness-changing currents[46] and therefore such an analysis is probably not very profitable at the present time.

Note added in proof. After completion of this manuscript, the authors received an unpublished report by C. W. Kim and H. Primakoff which considers the present problem from a similar point of view but with a different treatment of the induced pseudoscalar interaction. Their results are in general agreement with ours. The work of Kim and Primakoff is scheduled for publication in The Physical Review.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the gracious hospitality extended to them by CERN at the time this work was initiated.

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