PHYSICAL REVIEW VOLUME 140, NUMBER 5B 6 DECEMBER 1965

Precise Hyperon Masses*

P. SCHMIDT
Columbia University, New York, New York
(Received 21 July 1965)

Nine independent measurements have been performed on hyperons produced in K^- capture, and a set of precise hyperon mass measurements has been obtained. The following mass values are reported (in MeV): $m_{\Sigma^+}=1189.64\pm0.11$, $m_{\Sigma^0}=1192.41\pm0.14$, $m_{\Sigma^-}=1197.43\pm0.11$, $m_{\Lambda^0}=1115.61\pm0.07$; $m_{\Sigma^0}-m_{\Lambda^0}=76.61\pm0.28$, $m_{\Sigma^-}-m_{\Sigma^+}=7.89\pm0.12$, $m_{\Sigma^-}-m_{\Sigma^0}=4.99\pm0.12$.

I. INTRODUCTION

We have performed a number of measurements on the products of the capture of K^- mesons in the hydrogen bubble chamber, in order to find precise values of the masses of the Σ and Λ hyperons. All previously available precision measurements rely on range determinations for the precision in the mass, and most of these are performed in nuclear emulsions. Recently some questions on the reliability of our knowledge of the energy-range relations were raised.[1] It may not be useless, therefore, to present some new measurements based on different techniques.

II. EXPERIMENTAL PROCEDURE, IN GENERAL

All measurements are made in an exposure of the Columbia-BNL 30-in. H_2 chamber to a separated K^- beam at the Brookhaven AGS. The beam has a transport momentum of 700 MeV/c and passes through 800 g/cm^2 of carbon so that the bulk of the K^- mesons stop in the center of the chamber. The chamber has a magnetic field of 14 000 G with nonuniformities which attain 5% at the edge of the illuminated region. The field distribution has been measured with a precision of the order of 0.1% and these measurements are incorporated in the geometrical reconstruction program. We point out at once that the spatial variation, but not the absolute value of the field is used in the subsequent analysis. The stopped K^- mesons interact to produce hyperons in every case. In Table I we present the reactions which were studied, the number of examples included in the final compilation, the quantity which is tabulated in order to arrive at the mass value, and the mass which is measured. As can be seen, there are 11 independent measurements. The unknown quantities are the four hyperon masses, the absolute value of the magnetic field and the density of the liquid, which enters into the conversion of range to momentum. For the latter we use the table in the GRIND program of CERN. This table has been checked roughly by this group in connection with another experiment.[2] The K^-, π^-, π^+, π^0, n, and p masses are assumed to be known.

TABLE I. Reactions analyzed and corresponding measured quantities.

	Reaction	No. of events	Measurement[a]
1a	$K^-+p\to\Sigma^-+\pi^+$	3002	$p(\pi^+)=$ 173.09 ±0.11
1b	$\Sigma^-\to n+\pi^-$	2919	$p(\pi^-)=$ 193.41 ±0.11
2a	$K^-+p\to\Sigma^++\pi^-$	3824	$p(\pi^-)=$ 181.70 ±0.13
2b	$\Sigma^+\to n+\pi^+$	3825	$p(\pi^+)=$ 185.74 ±0.12
3	$K^-+p\to\Sigma^0+\pi^0$, $\Sigma^0\searrow\Lambda^0+e^++e^-$	208	$p(\Sigma^0)=$ 181.12 ±0.23
4	$\Sigma^0\to\Lambda^0+e^++e^-$	208	$m(\Sigma^0)=$1191.97 ±0.28
5	$K^-+p\to\Sigma^++\pi^-$, $\Sigma^+\searrow p+\pi^0$	381	$p(p)=$ 189.35 ±0.16
6	$\Sigma^-+p\to\Sigma^0+n$, $\Sigma^0\searrow\Lambda^0+e^++e^-$	12	$p(\Sigma^0)=$ 62.91 ±1.06
7	$\Sigma^-+p\to\Lambda^0+n$	85	$p(\Lambda^0)=$ 289.17 ±0.25
8	$\Lambda^0\to p+\pi^-$	488	$m(\Lambda^0)=$1115.505±0.05
9	$K^-+p\to\Sigma^-+\pi^+$	355	$p(\Sigma^-)=$ 173.98 ±0.11

[a] The values in this column are based on "zero values" of the masses and the hydrogen density. (Masses are in MeV, momenta in MeV/c.)

III. EXPERIMENTAL PROCEDURE AND RESULTS FOR THE 11 REACTIONS

Reactions 1 and 2

Events are selected for measurement only if the K^- ends in two collinear tracks and the decay of the Σ is clear so that no ambiguity exists on the sign of the Σ which decays. In particular very short Σ tracks and decays in which the Σ decay pion is emitted backwards with respect to the Σ are often ambiguous and are not accepted. After measurement, events are rejected unless each pion has a length greater than 10 cm, dips less than 60°, and the Σ length is in the interval 0.2 cm$<L_{\Sigma^-}<1.1$ cm, 0.2 cm$<L_{\Sigma^+}<0.95$ cm. The upper limit on the Σ track length is introduced to eliminate Σ's which stop before decay.

For each accepted event, the capture-pion momentum is measured, as well as the momentum of the decay pion after transformation to the Σ rest system. This transformation depends, although insensitively, on assumed masses of the nucleons, the K^-, the Σ, as well

* Work supported in part by the U. S. Atomic Energy Commission.

[1] W. H. Barkas, J. N. Dyer, and H. H. Heckman, Phys. Rev. Letters 11, 26 (1963).

[2] N. Gelfand, D. Miller, M. Nussbaum, J. Ratau, J. Schultz, J. Steinberger, T. H. Tan, L. Kirsch, and R. Plano, Phys. Rev. Letters 11, 436 (1963).

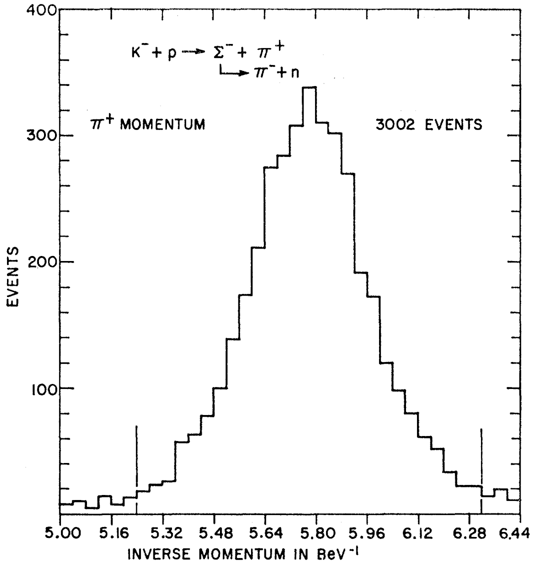

FIG. 1. Inverse momentum distribution of π^+ in $K^-+p\to\Sigma^-+\pi^+$, $\Sigma^-\to n+\pi^-$.

as on the density of the liquid. For this purpose we use what we may call "zero values" of these quantities. The following zero values are used[3] (masses in MeV):

$$m_{\pi^\pm}{}^0= 139.58,\quad m_{\pi^0}{}^0= 135.0,$$
$$m_n{}^0= 939.507,\quad m_p{}^0= 938.213,$$
$$m_\Lambda{}^0=1115.36,\quad m_{\Sigma^-}{}^0=1196.0,$$
$$m_{\Sigma^0}{}^0=1191.5,\quad m_{\Sigma^+}{}^0=1189.4,$$
$$m_{K^-}{}^0= 493.98,$$

$\rho^0=1.063\times$ (density assumed for range-energy table in GRIND). The subsequent analysis is by a linear expansion in the deviations from these zero values. The $\pi^\pm$-π^0 mass difference is assumed to be so well known that it is not varied. We calculate the momentum of the decay pion after the transformation by means of the "zero values," as well as the derivatives of these momenta with respect to changes in these "zero values".

In Figs. 1–4 we present the momentum distributions so obtained. The error in each measurement is typically 6 MeV/c.

A function of the form

$$A+B\exp[-(1/p-1/\bar{p})^2/2\sigma^2] \quad \text{(A)}$$

is fitted to the experimental distributions to determine the best values of A, B, $1/\bar{p}$ and σ. We find:

Reaction 1a, $\bar{p}_{\pi^+}=173.054\pm0.107$ MeV/c;
1b, $\bar{p}_{\pi^-}=193.407\pm0.111$ MeV/c;
2a, $\bar{p}_{\pi^-}=181.698\pm0.127$ MeV/c;
2b, $\bar{p}_{\pi^+}=185.741\pm0.116$ MeV/c.

[3] M. Roos, Rev. Mod. Phys. **35**, 314 (1963).

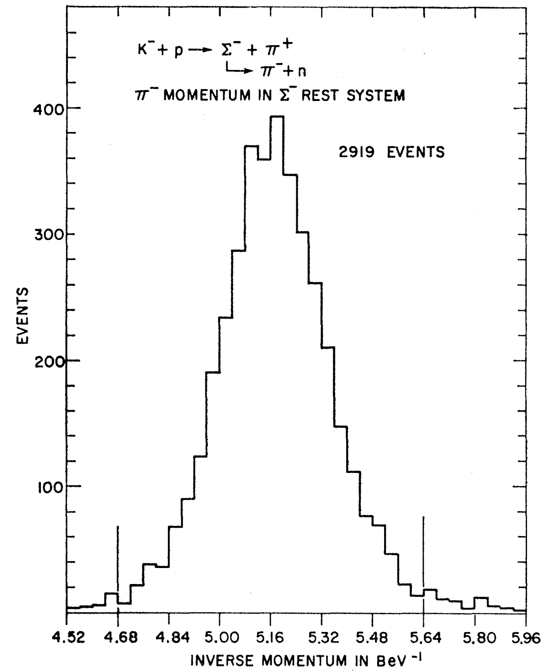

FIG. 2. Inverse momentum distribution of π^- in the Σ^- rest system for $K^-+p\to\Sigma^-+\pi^+$, $\Sigma^-\to n+\pi^-$.

The sums of the two momenta in 1a and 1b as well as those in 2a and 2b are essentially independent of the Σ masses. They depend on the masses which we assume known and serve as calibration of the magnetic field. The absolute value of the field is then known with a precision substantially better than 0.1%. This uncertainty contributes a negligible amount to the errors in all other reactions. The reaction most sensitive to the field is reaction 5, where the error due to the field

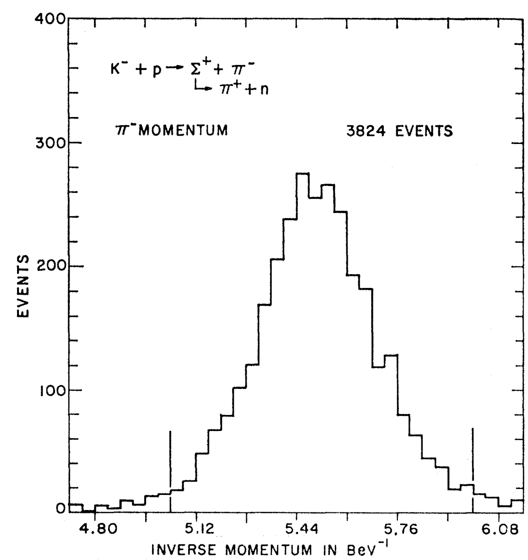

FIG. 3. Inverse momentum distribution of π^- in $K^-+p\to\Sigma^++\pi^-$, $\Sigma^+\to n+\pi^+$.

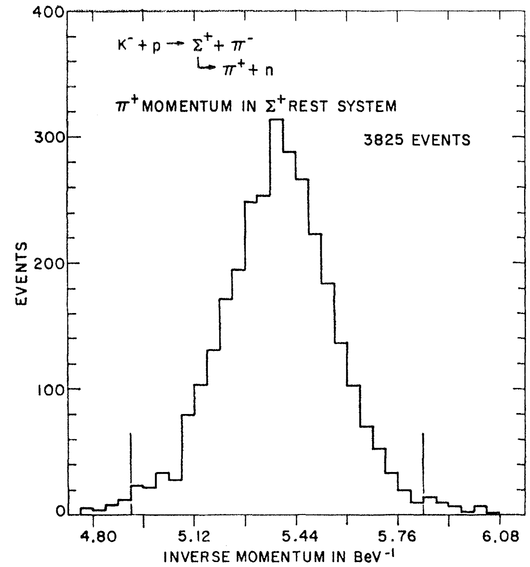

FIG. 4. Inverse momentum distribution of π^+ in the Σ^+ rest system for $K^- + p \to \Sigma^+ + \pi^-$, $\Sigma^+ \to n + \pi^+$.

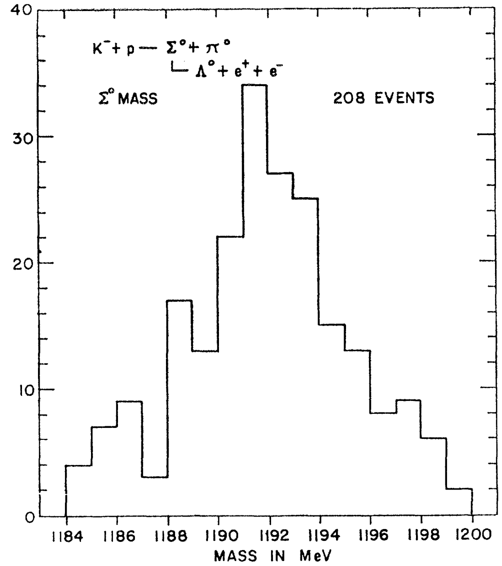

FIG. 6. Combined mass distribution of Λe^+e^-.

uncertainty is approximately one-third of the error contributed by other sources. In the following the magnetic field is therefore considered as known.

For the mass determinations we use the ratios of the pion momenta of the a and b reactions. Reaction 1 yields then the Σ^- mass and Reaction 2, the Σ^+ mass. We will express the masses in terms of deviations from the zero values: $m = m^0 + \Delta m$. The results are

$$\Delta m_{\Sigma^-} = 1.529 \pm 0.078\,,$$
$$\Delta m_{\Sigma^+} = 0.241 \pm 0.084\,.$$

The errors do not include uncertainties in the masses of the kaon, pion, and nucleon. For the purpose of the subsequent analyses, we also state here the results for these two experiments as linear expansions in the deviations from the zero values of the masses:

$$-3.013 \pm 0.154 = 1.08\,\Delta m_K - 1.97(\Delta m_{\Sigma^-} - \Delta m_p) - 0.148\,\Delta m_\pi\,; \quad (1)$$

$$-0.489 \pm 0.171 = 1.06\,\Delta m_K - 2.03(\Delta m_{\Sigma^+} - \Delta m_p) - 0.052\,\Delta m_\pi\,. \quad (2)$$

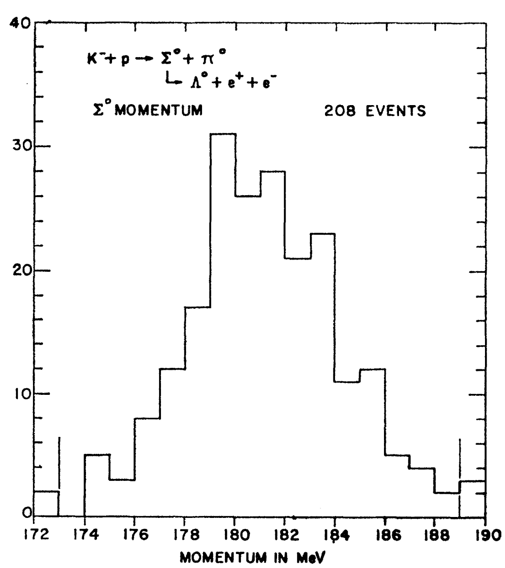

FIG. 5. Combined momentum distribution of Λe^+e^-.

Reactions 3: $K^- + p \to \Sigma^0 + \pi^0$ and 4: $\Sigma^0 \to e^+ + e^- + \Lambda^0$

There are 1309 measured events for which the K^- ending is accompanied by a Dalitz pair and Λ^0. The requirement that the errors in the combined momentum of Λ^0, e^+, and e^- and the combined mass of Λ^0, e^+, and e^- be each less than 10 MeV reduces the sample to 928 events. Most of these are background; the Dalitz pair is from π^0 decay, not from Σ^0 decay. We then plot in Fig. 5 the distribution in the combined $\Lambda^0e^+e^-$ momentum, for 208 events in the mass interval $m_{\Lambda^0e^+e^-} = 1192.0 \pm 7.0$. This momentum measurement, as well as those of the Λ's in Eq. (7), rests chiefly on the proton range. In Fig. 6, we present the distribution in the combined $\Lambda^0e^+e^-$ mass for those events in the interval of combined momentum

$$|\mathbf{p}_\Lambda + \mathbf{p}_{e^+} + \mathbf{p}_{e^-}| = 181 \pm 8.0\,.$$

The average momentum and Σ^0 mass are extracted from the data of Figs. 5 and 6 by expressions of the type (A). The results can be presented in the form of the linearized equations (3) and (4).

$$-1.175 \pm 0.23 = 1.07(\Delta m_K + \Delta m_p - \Delta m_{\Sigma^0}) + 0.40\Delta m_\Lambda - 54.8\Delta p - 1.04\Delta m_\pi\,, \quad (3)$$

$$+0.473 \pm 0.28 = \Delta m_{\Sigma^0} - \Delta m_\Lambda - 3.4\Delta p\,. \quad (4)$$

Reaction 5: $K^- + p \to \Sigma^+ + \pi^-$, $\Sigma^+ \to p + \pi^0$

Clear collinear K-capture events are selected in which the hyperon decays to a heavy positive track which stops in the chamber. The Σ track is required to have a length in the interval $0.2 < L_\Sigma < 1.1$ and the proton to have a length in excess of 0.4 cm. The measurement rests chiefly on the range of the proton. The capture π^- meson is measured also and furnishes the angles of the Σ^+ more precisely than this short track could. The momentum of the proton, found on the basis of the range, is transformed to the Σ^+ rest frame. The resulting distribution is shown in Fig. 7. Again we fit a distribution of flat background and Gaussian of the type (A) to find the average momentum. The result is

$$0.250 \pm 0.155 = \Delta m_{\Sigma^+} - \Delta m_p - 48.6\ \Delta\rho - 0.59\ \Delta m_\pi. \quad (5)$$

Reaction 6: $\Sigma^- + p \to \Sigma^0 + n$

Events are selected in which a Σ^- from a collinear capture ends in a Dalitz pair and Λ^0. The Σ^- has the characteristic range, $0.95 < R_{\Sigma^-} < 1.20$, of a Σ^- which comes to rest without decay. The events are rare, but free of ambiguity. The distribution in the Σ^0 momentum is shown in Fig. 8. The result for the Σ^--Σ^0 mass difference, weighting each event inversely with the square of the error, is given in Eq. (6):

$$0.541 \pm 0.12 = \Delta m_{\Sigma^-} - 0.955\ \Delta m_{\Sigma^0} + 0.046\ \Delta m_\Lambda - 0.081\ \Delta m_\pi - 4.4\ \Delta\rho. \quad (6)$$

Reaction 7: $\Sigma^- + p \to \Lambda^0 + n$

Events are selected in which a Σ^-, emitted in a collinear capture, stops with the length characteristic of a Σ^- coming to rest, $0.95 < R_{\Sigma^-} < 1.20$, and the Λ is associated. The distribution in Λ^0 momentum obtained by fitting with the GRIND program is shown in Fig. 9. The typical error in the Λ^0 momentum is 3 MeV. The linearized result based on the distribution, each event weighted inversely as the square of the associated error, is written in Eq. (7):

$$1.895 \pm 0.153 = 1.81\ \Delta m_{\Sigma^-} - 1.30\ \Delta m_\Lambda - 20.2\ \Delta\rho - 1.127\ \Delta m_\pi. \quad (7)$$

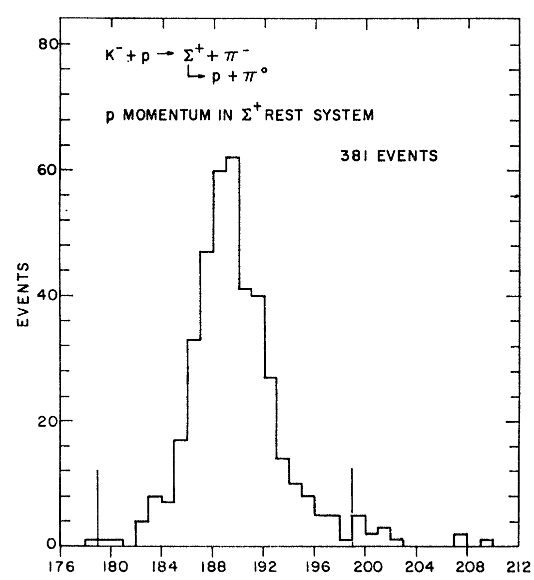

FIG. 7. Momentum distribution of the proton in the Σ^+ rest system for $K^- + p \to \Sigma^+ + \pi^-$, $\Sigma^+ \to p + \pi^0$.

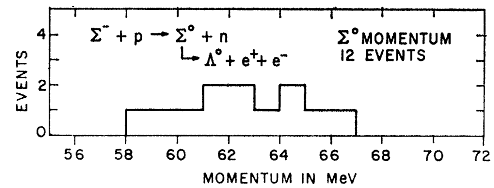

FIG. 8. Combined momentum distribution of Λe^+e^- for $\Sigma^- + p \to \Sigma^0 + n$, $\Sigma^0 \to \Lambda^0 + e^+ + e^-$.

Reaction 8: $\Lambda^0 \to \pi^- + p$

We have measured several Λ's for which both the pion and proton end in the chamber. The Λ^0 may be produced either in K^- capture or Σ^- capture, but nothing is assumed about its momentum. The experiment is used to find a relationship between the Λ^0 mass and the hydrogen density which is necessary in order to calibrate the range-energy relation. Assuming the "zero-value" density, the result can be presented in terms of a distribution in the Λ^0 mass (Fig. 10). Typical errors in the Λ^0 mass are 0.4 MeV per event. The linearized result for the average of the distribution, after weighting each event with the inverse of the square of the error, is

$$0.145 \pm 0.052 = (\Delta m_\Lambda - \Delta m_p) - 20.2\ \Delta\rho - 1.13\ \Delta m_\pi. \quad (8)$$

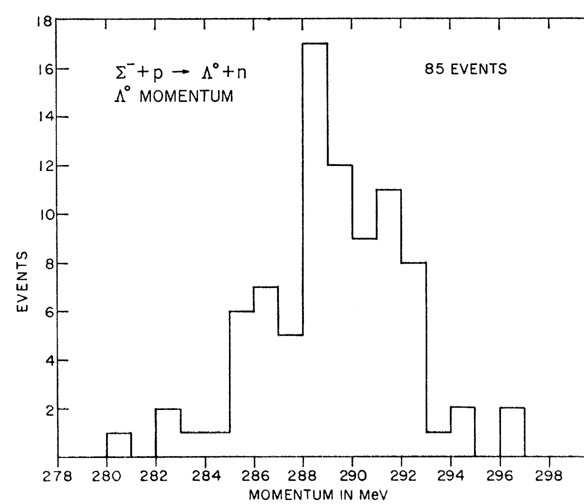

FIG. 9. Momentum distribution of Λ^0 in $\Sigma^- + p \to \Lambda^0 + n$.

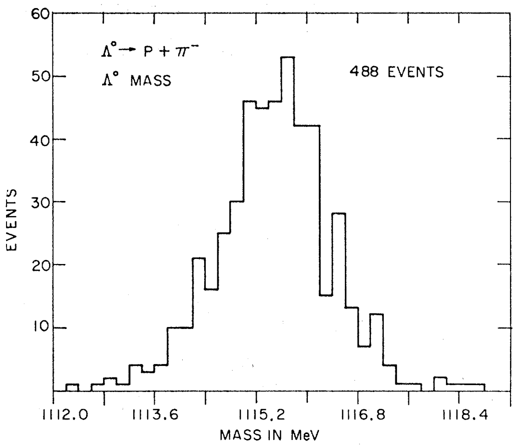

FIG. 10. Mass distribution for $\Lambda^0 \rightarrow p+\pi^-$ where both the proton and π^- stop in the chamber.

Reaction 9: $K^-+p \rightarrow \Sigma^-+\pi^-$

Here we measure the range distribution of Σ^-'s which are produced in collinear capture events, and which come to rest in the chamber. The range interval is chosen as $0.95<R_{\Sigma^-}<1.20$. The distribution is shown in Fig. 11. The resulting linearized equation is

$$-0.595\pm0.11=-1.17\,\Delta m_{\Sigma^-}+1.08(\Delta m_K+\Delta m_p)-0.676\,\Delta m_\pi-44.2\,\Delta\rho. \quad (9)$$

IV. HYPERON MASSES

We have performed 11 independent measurements. Two of these serve to calibrate the field. The remaining nine results have been written as linear expansions in deviations from "zero-value" parameters. The seven parameters are the Σ, Λ, K^-, and π masses and the density ρ of the hydrogen. The parameters should now be picked to give the best possible fit. However, the resultant errors in the masses are large unless the pion and kaon masses are assumed known.

For the pion mass, we use the recent measurement of Shafer *et al.*[4]:

$$m_{\pi^-}=139.58\pm0.015.$$

This value and the Q value[5] (75.11±0.14) for the decay $K^+ \rightarrow \pi^++\pi^++\pi^-$ yields the K^+ mass, $m_{K^+}=493.85\pm0.16$. We combine this with the measurement of Barkas *et al.*,[1] $m_{K^-}=493.7\pm0.3$, to find the value for the $K^\pm$ mass which is used in the following:

$$m_{K^\pm}=493.80\pm0.15.$$

For the nucleon masses we use the new values according to the compilation of Rosenfeld *et al.*[6]:

$$m_p=938.256\pm0.005,$$
$$m_n=939.550\pm0.005.$$

It should be noted these values are not the same as the "zero mass values" used in the Eqs. (1)–(9).

Of the nine experimental results, reactions 1, 2, 4, and 6 do not depend on the range-energy relation. For these we find

from (1), $\Delta m_{\Sigma^-}=(1.529\pm0.078)+0.548\,\Delta m_K-0.075\,\Delta m_\pi+\Delta m_p;$

from (2), $\Delta m_{\Sigma^+}=(0.241\pm0.084)+0.522\,\Delta m_K-0.026\,\Delta m_\pi+\Delta m_p;$

from (4), $m_{\Sigma^0}-m_{\Lambda^0}=76.61\pm0.28;$

from (6), $m_{\Sigma^-}-m_{\Sigma^0}=4.99\pm0.12.$

These equations with the above masses yield

$$m_{\Sigma^-}=1197.47\pm0.11,$$
$$m_{\Sigma^+}=1189.59\pm0.11,$$
$$m_{\Sigma^0}=1192.47\pm0.16,$$
$$m_{\Lambda^0}=1115.86\pm0.32.$$

Subjecting all equations except number 9 (Σ^- range) to a least-squares fit, a good fit ($\chi^2=2.9$, 3 constraints) is obtained with the following mass relations:

$$\Delta m_\Lambda=(0.242\pm0.058)+0.199\,\Delta m_K+0.763\,\Delta m_\pi+\Delta m_p,$$
$$\Delta m_{\Sigma^+}=(0.288\pm0.077)+0.510\,\Delta m_K-0.069\,\Delta m_\pi+\Delta m_p,$$
$$\Delta m_{\Sigma^0}=(0.965\pm0.113)+0.534\,\Delta m_K+0.106\,\Delta m_\pi+\Delta m_p,$$
$$\Delta m_{\Sigma^-}=(1.490\pm0.069)+0.548\,\Delta m_K+0.004\,\Delta m_\pi+\Delta m_p.$$

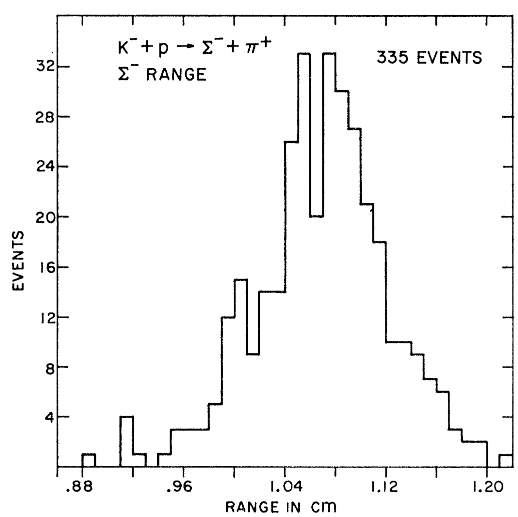

FIG. 11. Range distribution of Σ^- in $K^-+p \rightarrow \Sigma^-+\pi^+$.

[4] R. E. Shafer, K. M. Crowe, and D. A. Jenkins, Phys. Rev. Letters **14**, 923 (1965).
[5] E. R. Cohen, K. M. Crowe, and J. W. M. DuMond, *Fundamental Constants of Physics* (Interscience Publishers, Inc., New York, 1957), Vol. I, p. 65.
[6] A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **36**, 977 (1964).

These relations give the following masses:

$$m_\Lambda = 1115.61 \pm 0.07,$$
$$m_{\Sigma^+} = 1189.64 \pm 0.11,$$
$$m_{\Sigma^0} = 1192.41 \pm 0.14,$$
$$m_{\Sigma^-} = 1197.43 \pm 0.11.$$

When Eq. (9) is included in the fit, the χ^2 increases to 63 for four constraints.

The Σ^- mass, as calculated from Eq. (9) alone, is $m_{\Sigma^-} = 1196.53 \pm 0.24$. A parallel to this discrepancy has been pointed out by Barkas.[1] In emulsion studies the measurement of the Σ^- mass based on the Σ^- range in reaction (9) yields $m_{\Sigma^-} = 1195.96 \pm 0.30$,[2] whereas the value based on the pion range in reaction (1a) yields the value $m_{\Sigma^-} = 1197.6 \pm 0.5$.[1] The discrepancy is attributed to a possible difficulty in the range-energy relation of slow negative particles in emulsions. We seem to find a similar problem in the case of hydrogen. In any case we follow the example of Barkas and exclude the Σ^--range measurements from the compilation until the discrepancy may become clearer.

V. CONCLUSION

We have found the following mass values:

$$m_\Lambda = 1115.61 \pm 0.07,$$
$$m_{\Sigma^+} = 1189.64 \pm 0.11,$$
$$m_{\Sigma^0} = 1192.41 \pm 0.14,$$
$$m_{\Sigma^-} = 1197.43 \pm 0.11,$$

and the following mass differences:

$$m_{\Sigma^0} - m_\Lambda = 76.61 \pm 0.28,$$
$$m_{\Sigma^-} - m_{\Sigma^0} = 4.99 \pm 0.12,$$
$$m_{\Sigma^-} - m_{\Sigma^+} = 7.89 \pm 0.12.$$

These can be compared with previously reported measurements[7–16] summarized in Table II.

TABLE II. Previous hyperon mass measurements, in MeV.

	Λ^0 mass	
1115.42±0.19	Bogdanowicz *et al.*[a]	Emulsion, stop K^-
1115.42±0.15	Bhowmik *et al.*[b]	Emulsion, K^- beam; p, π^- stop
1115.55±0.15	Lodge *et al.*[c]	Emulsion, K^-; p, π^- ranges
1115.36±0.14	Barkas *et al.*[d]	Emulsion; p, π^- ranges
1115.42±0.1	Average	
	$m_{\Sigma^-} - m_{\Sigma^0}$	
4.75±0.10	Burnstein *et al.*[e]	Chamber: $\Sigma^- + p \to \Sigma^0 + n$ at rest
4.45±0.4	Barkas and Rosenfeld[f]	Berge, Rosenfeld, Ross, Solmitz, and Tripp[l]
4.87±0.12	Dosch *et al.*[g]	Chamber: $\Sigma^- + p \to \Sigma^0 + n$
4.79±0.08	Average	
	Σ^+ mass	
1189.6 ±0.4	Swami[h]	Emulsion; $\Sigma^+(\text{rest}) \to p$ stop
1189.2 ±0.5	Evans *et al.*[i]	Emulsion; $\Sigma^+(\text{rest}) \to p$ stop
1189.35±0.15	Barkas *et al.*[j]	Emulsion; $\Sigma^+ \to p + \pi^0$, $n + \pi^+$
1189.5 ±0.5	Burnstein *et al.*[e]	Chamber: $\Sigma^+(\text{rest}) \to p$ stop. Used as check in density.
1189.33±0.22	Barkas *et al.*[d]	Emulsion; $\Sigma^+(\text{rest}) \to p$ stop
Also		
1190.3 ±0.5	White[k]	$\Sigma^+(\text{rest}) \to p$ stop; emulsion
1189.35±0.11	Average	
	Σ^- mass	
1197.6 ±0.5	Barkas *et al.*[j]	$K^- + p \to \Sigma^\pm + \pi^\pm$; π ranges $+\Sigma^+$ mass; emulsion
1196.9 ±0.36	Burnstein *et al.*[e]	Chamber: Σ^- range
1197.0 ±0.24	Burnstein *et al.*[e]	Chamber: $\Sigma^- + p \to \Lambda^0 + n$
1197.1 ±0.2	Average	
	$m_{\Sigma^-} - m_{\Sigma^+}$	
8.25±0.25	Dosch *et al.*[g]	Chamber: $K^- + p \to \Sigma^\pm + \pi^\mp$

[a] See Ref. 7. [b] See Ref. 8. [c] See Ref. 9. [d] See Ref. 10. [e] See Ref. 11. [f] See Ref. 12. [g] See Ref. 13. [h] See Ref. 14. [i] See Ref. 15. [j] See Ref. 1. [k] See Ref. 16.
[l] University of California Radiation Laboratory Report No. 8030 (unpublished), and private communication.

Our mass values are generally higher than previous values by approximately 0.2–0.3 MeV. This is not a large amount, but, nevertheless, statistically significant. The differences should, therefore, probably be laid to systematic origins.

In this connection we may be permitted to note that at least some of our measurements are independent of uncertainties in the shape of the range-energy relation.

[7] J. Bogdanowicz, M. Danysz, A. Filipkowski, E. Marquit, E. Skrzypczak, A. Wroblewski, and J. Zakrzewski, Nuovo Cimento 11, 727 (1959).
[8] B. Bhowmik, D. P. Goyal, and N. K. Yamdagni, Nuovo Cimento 22, 296 (1961).
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[10] W. H. Barkas, J. N. Dyer, C. J. Mason, N. A. Nickols, and F. M. Smith, Phys. Rev. 124, 1209 (1961).
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ACKNOWLEDGMENTS

We would like to express our gratitude to Professor Jack Steinberger for his constant encouragement, advice, and help in this experiment, and Dr. Dan Tycko for the least-squares fitting program and several interesting discussions. The invaluable efforts of the Nevis Scanning and Measuring Staff are well appreciated. Further thanks go to Dr. U. Nauenberg, Dr. C. Alff-Steinberger, Dr. C. Y. Chang, and S. Marateck for providing some of the events used in their experiments for this one. It is a pleasure to acknowledge the help of Dr. A. Prodell and of the 30-in. hydrogen-bubble-chamber operating crew. The assistance of the A.G.S. staff is also greatly appreciated.

PHYSICAL REVIEW VOLUME 140, NUMBER 5B 6 DECEMBER 1965

Measurement of the K^0 Mass and the K^0-K^- Mass Difference*

J. K. Kim, L. Kirsch, and D. Miller†
Columbia University, New York, New York

(Received 21 July 1965)

From the decay of the K_1^0 in a hydrogen bubble chamber, we have measured the mass of the K^0 to be 497.44±0.33 MeV. From the reaction

$$K^-p \to K^0 n, \quad K^0 \to \pi^+\pi^-$$

we have measured the K^0-K^- mass difference to be 3.71±0.35 MeV.

I. INTRODUCTION

The K^0 mass was measured by two independent methods in the Columbia-BNL 30-in. hydrogen bubble chamber exposed to $\bar{p}$ and K^- from the low-energy separated beam at the Brookhaven alternating gradient synchrotron (AGS). The first procedure was based on the study of a large sample of two-body decays of K^0 mesons produced by stopping antiprotons. The second procedure was to determine Δ, the K^0-K^- mass difference, from the charge exchange reaction

$$K^-p \to K^0 n, \quad K^0 \to \pi^+\pi^-$$

followed by elastic scattering of the neutron.

A discussion is included of the tests used to determine an absolute scale factor for the conversion of curvature measurements to momenta, and to check the validity of the error assignments.

II. K^0 MASS FROM DECAY

A sample of V's was fitted to the three-constraint hypothesis $K^0 \to \pi^+\pi^-$ using the measured directions of the neutral K^0 meson and the measured momenta and directions of the pions. The fits were performed using the GRIND kinematics program.[1] Since the program is unable to treat the mass as a variable, we fit each event using five values of the K^0 mass, ranging from 496.5 to 498.5 MeV in steps of 0.5 MeV. Near the minimum, the curve of χ^2 versus K^0 mass is a parabola with the minimum located at the best value of the mass. For each event, the lowest three values of χ^2 are fitted to a parabola to determine the value of χ^2 at the minimum, and if this value exceeds 5.0, the event is rejected. The accepted sample contains 2223 events. The best value of the K^0 mass from these events is determined by fitting the curve $\sum_{i=1}^{2223}\chi_i^2$ versus mass to a parabola as shown in Fig. 1. The result is $m_{K^0}=497.44\pm0.23$ MeV, where the error is obtained from the values of the mass for which $\sum_i\chi_i^2=(\sum_i\chi_i^2)_{\min}+1.0$. This error reflects the measurement and multiple-scattering errors assigned to each track.

We have two independent checks on the error assignments. The experimental width of the momentum distribution for a set of events produced at a fixed momentum should equal the width of the resolution function for the same sample. The resolution function is obtained by multiplying the assigned momentum error for each event by $\sqrt{2}$ and then plotting a Gaussian ideogram about the fixed momentum. The pion momenta from the reactions $K^-p \to \Sigma\pi$ and $\bar{p}p \to \pi^+\pi^-$ are used to perform this test. Good agreement is achieved in both cases, indicating not only a correct magnitude, but also a correct momentum dependence for the errors. The curves for the reaction $\bar{p}p \to \pi^+\pi^-$ are plotted in Fig. 2.

The second check is whether or not the errors which are determined by the fitting program based on the assigned errors are correct for the three-constraint fit.

* Work supported in part by U. S. Atomic Energy Commission.
† Present address: Massachusetts Institute of Technology, Cambridge, Massachusetts.
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