# Antiproton-Nucleon Annihilation at Rest into Two Mesons and $SU(6)$ Symmetry

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We discuss extensively two-meson production processes in antiproton-nucleon annihilation at rest on the $SU(6)$ model. For the orbital angular momenta in the initial and final states we take the $s$ and the $p$ wave, respectively. The effects of the $SU(3)$-invariant interaction and of the first-order $SU(3)$-breaking interaction are taken into account. All the transition amplitudes are expressed in terms of five $SU(6)$-invariant amplitudes [seven in the case when the $SU(3)$-breaking effect is included]. These expressions could give a very convenient tool for a systematic analysis of the experimental results. We derive the relations between transition amplitudes and discuss several useful relations such as

$$\sigma(\bar{p}n \to \pi^0\pi^-)/k_\pi = 2\sigma(\bar{p}n \to K^0K^-)/k_K,$$
$$\sigma(\bar{N}N \to \eta\omega)/k_{\eta\omega} = (9/25)\sigma(\bar{N}N \to \eta\rho)/k_{\eta\rho},$$

and

$$\sigma(\bar{N}N \to \pi\varphi) = \sigma(\bar{N}N \to \eta\varphi) = \sigma(\bar{N}N \to \rho\varphi) = \sigma(\bar{N}N \to \omega\varphi) = 0$$

(where use is made of the usual $\omega$-$\varphi$ mixing parameter and $k$ is the relative momentum of the final two mesons).

## 1. INTRODUCTION

IN this article an extensive analysis of the annihilation of antiprotons on nucleons at rest into two mesons

$$\bar{p} + N \to M_1 + M_2 \qquad (1)$$

is given in the $SU(6)$ model.[1] This process is of particular interest for the following reasons:

(i) There are many experimentally allowed final states to relate in this framework, so that one can hope to obtain and to be able to check several significant predictions upon transition amplitudes to these states.

(ii) The kinematics is "frozen" in the initial states and the total energy is constant, so that ambiguities usually associated with the interpretation of unitary symmetry predictions for dynamical processes are strongly reduced.

Before entering into details, it is worthwhile to review briefly the predictions of the $SU(3)$ symmetry scheme and of the $\tilde{U}(12)$ extension[2] of $SU(6)$. In $SU(3)$ there are no significant relations for annihilation at rest into two mesons,[3] unless some dynamical assumptions are imposed.[4] In pure $\tilde{U}(12)$ the process is forbidden,[5] and there are no clear reasons as to what kind of breaking ("kinetons") one has to consider for letting it occur.[6] (In any case, agreement with experiments is not achieved.)

The process is forbidden also in the full symmetry limit by $SU(6)$ but in this case the hypothesis of "minimal" breaking, which is very naturally suggested by the expression of the matrix element in the helicity formalism (see Sec. 2), turns out to be very likely for the discussion of the annihilation at rest. In a previous work[7] this approach gave a very encouraging result for the particular case of two-pseudoscalar-meson production in antiproton annihilation on proton. In this work we exploit it completely, considering all the possible initial antinucleon-nucleon states and all the possible final states which are composed of two 35-plet mesons. Although the annihilation of antiproton on neutron is experimentally much less known than that on proton, we find it to be of the greatest interest for a check of the validity of the model since in this case several simple relations, of direct physical meaning, can be obtained.

## 2. CONSTRUCTION OF THE $S$ MATRIX

Throughout this paper, we take the initial state to be pure $s$ wave consistent with the experimental

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data.[8–10] By two mesons we mean two "primary" mesons, including all the resonances which belong to the 35-plet. As $s$-wave nucleonium has odd spatial parity and all the relevant mesons have odd intrinsic parity, parity conservation requires that the final state be pure $p$ wave.

Our approach is purely phenomenological, so we are interested only in the form of the $S$ matrix which refers to process (1) in the momentum representation. For the spatial part it must behave like a vector in order to give the required $|\Delta\mathfrak{L}|=1$ selection rule for the transition. So it must be proportional to the meson relative momentum $\mathbf{q}$, which is the only vector to hand. Conservation of total angular momentum requires it to be contracted with a spin one (no other vectors are available), constructed from the spinorial part of the field operators of the four particles involved in the reaction (regardless of whether the final mesons do have spin or not). By indicating this spinor by $\chi_j{}^i$, one gets

$$S=\Omega\chi_j{}^i(\boldsymbol{\sigma}(\mathbf{q}/|q|))_i{}^j, \tag{2}$$

where $\Omega$ includes coupling constants and the $SU(3)$ part of the field operators.

It is convenient to introduce the "spurion" $Q_\mu{}^\nu$ defined by

$$Q_\mu{}^\nu=\delta_M{}^N(\boldsymbol{\sigma}\cdot(\mathbf{q}/|q|))_m{}^n, \tag{3}$$

where $\mu$ and $\nu$, $M$ and $N$, and $m$ and $n$ refer to $SU(6)$, $SU(3)$, and $SU(2)$ indices, respectively, the correspondence being given, as usual, by $\mu=(M,m)$, etc. The hypothesis of "minimal" breaking is very naturally suggested by Eqs. (2) and (3) and can be stated as requiring that the $SU(6)$ part of $\Omega\chi_j{}^i$ behave like a 35-plet, so that formally $SU(6)$ invariance is recovered. Thus, the part of the $S$ matrix including this spurion is given by

$$S^{(0)}=\textstyle\sum_i a_i I_{i\nu}{}^\mu Q_\mu{}^\nu, \tag{4}$$

where the $I_{i\nu}{}^\mu$ are the $SU(6)$ 35-plet formed out of the $SU(6)$ part of the field operators of the particles involved in reaction (1) and the $a_i$ are $SU(6)$-invariant amplitudes. The summation index $i$ is understood to run over all the possible terms.

Treating the 35 meson states of the adjoint representation $M_\alpha{}^\beta$ as different states of the same "particle," $S^{(0)}$ must be even under the exchange of the two mesons. As $Q_\mu{}^\nu$ is odd under this operation and meson field operators commute, the $I_{i\nu}{}^\mu$ must also be antisymmetrical for the exchange. Therefore, in general, $M_\alpha{}^\beta(1)$ and $M_\gamma{}^\delta(2)$, which describe the two mesons, are contained in it in the form

$$M_\alpha{}^\beta(1)M_\gamma{}^\delta(2)-M_\gamma{}^\delta(1)M_\alpha{}^\beta(2).$$

Up to here parity conservation, total angular-momentum conservation, and Bose statistics requirements are satisfied. As far as charge-conjugation invariance is concerned, let us observe that an $SU(6)$ multiplet, in general, has no simple transformation properties under $C$. If, on the contrary, we take the operator $R$ to be defined by[11]

$$R=Ce^{i\pi S_2}, \tag{5}$$

where $S_2$ is the second component of ordinary spin, we see that any $SU(6)$ multiplet goes into its transpose under $R$ except for a possible appearance of phase factor for the baryon field which can be disregarded in the present case. The relevant part of $I_{i\nu}{}^\mu$ in Eq. (4) is an $SU(3)$ scalar times an $SU(2)$ triplet $\chi_n{}^m$. In order to satisfy charge-conjugation invariance, the $SU(3)$ part must be invariant under $C$. The $SU(2)$ part $\chi_n{}^m$, on the other hand, goes into $-\chi_m{}^n$ under $e^{i\pi S_2}$, so that $C$ invariance is satisfied[11] by the $I_{i\nu}{}^\mu$ which go into their transpose with a change of sign under $R$.

There are eighteen 35-plet which can be constructed out of the product of the 56-plet $\bar{B}_{\alpha\beta\gamma}$ (to which the antiproton belongs), the 56-plet $B^{\alpha\beta\gamma}$ (to which the nucleon belongs), and the two meson 35-plets $M_\beta{}^\alpha(1)$, $M_\beta{}^\alpha(2)$. Imposing the condition that they are odd under the exchange of the mesons and odd under transposition of the $SU(6)$ indices, we have the following five terms $I_{i\mu}{}^\nu$ which contribute to Eq. (4):

$$I_{1\mu}{}^\nu=\bar{B}_{\alpha\beta\gamma}B^{\alpha\beta\gamma}(M_\mu{}^\delta(1)M_\delta{}^\nu(2)-M_\delta{}^\nu(1)M_\mu{}^\delta(2)),$$
$$I_{2\mu}{}^\nu=[\bar{B}_{\alpha\beta\mu}B^{\alpha\beta\gamma}(M_\gamma{}^\delta(1)M_\delta{}^\nu(2)-M_\delta{}^\nu(1)M_\gamma{}^\delta(2)) -\bar{B}_{\alpha\beta\gamma}B^{\alpha\beta\nu}(M_\delta{}^\gamma(1)M_\mu{}^\delta(2)-M_\mu{}^\delta(1)M_\delta{}^\gamma(2))],$$
$$I_{3\mu}{}^\nu=\bar{B}_{\alpha\beta\gamma}B^{\alpha\beta\delta}(M_\mu{}^\gamma(1)M_\delta{}^\nu(2)-M_\delta{}^\nu(1)M_\mu{}^\gamma(2)),$$
$$I_{4\mu}{}^\nu=\bar{B}_{\alpha\beta\mu}B^{\alpha\gamma\nu}(M_\delta{}^\beta(1)M_\gamma{}^\delta(2)-M_\gamma{}^\delta(1)M_\delta{}^\beta(2)),$$

and

$$I_{5\mu}{}^\nu=[\bar{B}_{\alpha\beta\mu}B^{\alpha\gamma\delta}(M_\gamma{}^\beta(1)M_\delta{}^\nu(2)-M_\delta{}^\nu(1)M_\gamma{}^\beta(2)) -\bar{B}_{\alpha\beta\gamma}B^{\alpha\delta\nu}(M_\delta{}^\beta(1)M_\mu{}^\gamma(2)-M_\mu{}^\gamma(1)M_\delta{}^\beta(2))]. \tag{6}$$

The term $I_1$ is the contribution of a singlet 1 baryon pair and a 35-plet meson pair. The terms $I_2$, $I_3$, $I_4$, and $I_5$ include the contributions of initial **35** and final **35**, initial **35** and final **405**, initial **405** and final **35**, and initial **405** and final **405**, respectively. Here one should mention that, to obtain contributions from pure irreducible multiplets, the trace parts must be subtracted.

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TABLE I. $SU(2)$-invariant transition amplitudes ${}^{S}(M_1M_2)_{I,J}$ expressed in terms of the $SU(6)$-invariant amplitudes $a_i$ and $b_i$. Use is made of $\omega$ and $\varphi$ defined in Eq. (15). The listed coefficients have been multiplied by the factor $9\sqrt{2}$.

| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $b_1$ | $b_3$ |
|---|---|---|---|---|---|---|---|
| ${}^0(\pi\pi)_{11}$ | 0 | 20 | $-10$ | 2 | 0 | 0 | $-10$ |
| ${}^0(K\bar{K})_{11}$ | 0 | $-10$ | 5 | $-1$ | 0 | 0 | $-10$ |
| ${}^0(K\bar{K})_{01}$ | 0 | 6 | $-3$ | $-3$ | 0 | 0 | 6 |
| ${}^1(\pi\varphi)_{11}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ${}^1(\pi\omega)_{11}$ | 0 | 0 | 10 | 0 | $-2$ | 0 | 10 |
| ${}^1(\pi\rho)_{10}$ | 0 | $-12\sqrt{3}$ | $6\sqrt{3}$ | $2\sqrt{3}$ | 0 | 0 | $6\sqrt{3}$ |
| ${}^1(\pi\rho)_{01}$ | 0 | 0 | $6\sqrt{3}$ | 0 | $2\sqrt{3}$ | 0 | $6\sqrt{3}$ |
| ${}^1(\eta\varphi)_{01}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ${}^1(\eta\omega)_{01}$ | 0 | 0 | $-2\sqrt{3}$ | 0 | $-2\sqrt{3}$ | 0 | $-2\sqrt{3}$ |
| ${}^1(\eta\rho)_{11}$ | 0 | 0 | $10/\sqrt{3}$ | 0 | $10/\sqrt{3}$ | 0 | $10/\sqrt{3}$ |
| ${}^1(K\bar{K}^*)_{11}$ | 0 | 0 | $5\sqrt{2}$ | 0 | 0 | 0 | $-10\sqrt{2}$ |
| ${}^1(K\bar{K}^*)_{10}$ | 0 | $6\sqrt{3}$ | $-3\sqrt{3}$ | $-\sqrt{3}$ | 0 | 0 | $6\sqrt{3}$ |
| ${}^1(K\bar{K}^*)_{01}$ | 0 | 0 | $-3\sqrt{2}$ | 0 | 0 | 0 | $6\sqrt{2}$ |
| ${}^1(K\bar{K}^*)_{00}$ | 0 | $-18\sqrt{3}$ | $9\sqrt{3}$ | $-3\sqrt{3}$ | 0 | $-108\sqrt{3}$ | $-18\sqrt{3}$ |
| ${}^1(\bar{K}K^*)_{11}$ | 0 | 0 | $5\sqrt{2}$ | 0 | 0 | 0 | $-10\sqrt{2}$ |
| ${}^1(\bar{K}K^*)_{10}$ | 0 | $-6\sqrt{3}$ | $3\sqrt{3}$ | $\sqrt{3}$ | 0 | 0 | $-6\sqrt{3}$ |
| ${}^1(\bar{K}K^*)_{01}$ | 0 | 0 | $3\sqrt{2}$ | 0 | 0 | 0 | $-6\sqrt{2}$ |
| ${}^1(\bar{K}K^*)_{00}$ | 0 | $-18\sqrt{3}$ | $9\sqrt{3}$ | $-3\sqrt{3}$ | 0 | $-108\sqrt{3}$ | $-18\sqrt{3}$ |
| ${}^1(\varphi\varphi)_{00}$ | $-72\sqrt{3}$ | 0 | 0 | 0 | 0 | $144\sqrt{3}$ | 0 |
| ${}^1(\omega\varphi)_{00}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ${}^1(\omega\omega)_{00}$ | $-72\sqrt{3}$ | $-36\sqrt{3}$ | $-18\sqrt{3}$ | $-6\sqrt{3}$ | $12\sqrt{3}$ | $-72\sqrt{3}$ | $-18\sqrt{3}$ |
| ${}^1(\rho\varphi)_{10}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ${}^1(\rho\omega)_{10}$ | 0 | $12\sqrt{3}$ | $6\sqrt{3}$ | $-2\sqrt{3}$ | $4\sqrt{3}$ | 0 | $6\sqrt{3}$ |
| ${}^0(\rho\rho)_{11}$ | 0 | $20\sqrt{3}$ | $50/\sqrt{3}$ | $2\sqrt{3}$ | $16\sqrt{3}$ | 0 | $50/\sqrt{3}$ |
| ${}^2(\rho\rho)_{11}$ | 0 | 0 | $20(5/3)^{1/2}$ | 0 | $4\sqrt{15}$ | 0 | $20(5/3)^{1/2}$ |
| ${}^1(\rho\rho)_{01}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ${}^1(\rho\rho)_{00}$ | $-216$ | $-108$ | $-54$ | $-18$ | $-60$ | $-216$ | $-54$ |
| ${}^0(K^*\bar{K}^*)_{11}$ | 0 | $-10\sqrt{3}$ | $-25/\sqrt{3}$ | $-\sqrt{3}$ | 0 | 0 | $50/\sqrt{3}$ |
| ${}^1(K^*\bar{K}^*)_{11}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ${}^2(K^*\bar{K}^*)_{11}$ | 0 | 0 | $-10(5/3)^{1/2}$ | 0 | 0 | 0 | $20(5/3)^{1/2}$ |
| ${}^1(K^*\bar{K}^*)_{10}$ | 0 | $6\sqrt{6}$ | $3\sqrt{6}$ | $-\sqrt{6}$ | 0 | 0 | $-6\sqrt{6}$ |
| ${}^1(K^*\bar{K}^*)_{01}$ | 0 | 0 | $-6$ | 0 | 0 | 0 | 12 |
| ${}^1(K^*\bar{K}^*)_{00}$ | $-72\sqrt{6}$ | $-18\sqrt{6}$ | $-9\sqrt{6}$ | $-3\sqrt{6}$ | 0 | $36\sqrt{6}$ | $18\sqrt{6}$ |

All these terms induce full symmetry in $SU(3)$. A first-order breaking of the $SU(3)$ part can be taken into account substituting $\delta_M{}^N$ in Eq. (3) by

$$T_M{}^N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}. \tag{7}$$

Defining, as previously,

$$\mathcal{T}_\mu{}^\nu = T_M{}^N(\boldsymbol{\sigma}\cdot(\mathbf{q}/|q|))_m{}^n, \tag{8}$$

and writing

$$S^{(1)} = \sum_i b_i I_{i\mu}{}^\nu \mathcal{T}_\nu{}^\mu, \tag{9}$$

it is easily found that the allowed $I_{i\mu}{}^\nu$ are the same as in Eq. (6). The total $S$ matrix is

$$S = S^{(0)} + S^{(1)}. \tag{10}$$

### 3. THE TRANSITION AMPLITUDES

By taking matrix elements of $S$, Eq. (10), we obtain the transition amplitudes for the process (1) expressed as linear combinations of ten $SU(6)$-invariant amplitudes $a_i$ and $b_i$. The terms in $b_2$, $b_4$, and $b_5$, however, are easily seen to give the same contributions as $a_2$, $a_4$, and $a_5$ and can be disregarded.

The initial state can have isospin $I=0$, 1 and spin $S_{\rm in}=0$, 1 (for the $s$ wave, these are also the values of the total angular momentum $J$). Since $I$ and $J$ are conserved, it is convenient to label the transition amplitudes with these quantum numbers. They completely specify the initial state but not, in general, the final one, for which the final spin $S$ must also be given. Therefore we use the $SU(2)$-invariant transition amplitude

$$^{S}(M_1M_2)_{I,J},$$

which refers to the production of the mesons $M_1$ and $M_2$ in the channel $I$ and $J$ with total final spin $S$.

In Table I, the $SU(2)$-invariant transition amplitudes are given in terms of the $SU(6)$-invariant amplitudes. One can derive from them the transition amplitude referring to particular charge, spin, and orbital angular-momentum final states by using standard $SU(2)$ techniques with any consistent phase convention. (See Appendix A for our phase convention.)

The total rate for the production of $M_1$, $M_2$, on the other hand, is most simply obtained by squaring and summing the appropriate $SU(2)$-invariant transition amplitudes as listed in Table I over all allowed quantum-number eigenstates (without any additional factors for final spin and isotopic-spin dependence apart from those coming from the decomposition of initial $\bar{p}p$ state into isotopic-spin eigenstates), and then multiplying by the final-state density. If we are interested in relative magnitudes rather than in absolute rates, the phase space can be effectively assumed to be the final momentum.[12]

## 4. USEFUL RELATIONS AND DISCUSSIONS

From Table I it is possible to derive all the relations between the $SU(2)$-invariant transition amplitudes—or transition amplitudes for particular charge, spin, and orbital-angular-momentum states. As there are 33 $SU(2)$-invariant transition amplitudes, depending on just five $SU(6)$-invariant amplitudes (seven if breaking terms are taken into account), one can easily work out 28 relations (26 with the breaking terms) between them.

Here we list only the simplest ones:

$$^{0}(\pi\pi)_{11}=-2\,^{0}(K\bar{K})_{11}, \tag{11}$$

$$^{1}(\eta\omega)_{01}=-\tfrac{3}{5}\,^{1}(\eta\rho)_{11}, \tag{12}$$

$$^{1}(\pi\varphi)_{11}=^{1}(\rho\varphi)_{10}=^{1}(\eta\varphi)_{01}=0, \tag{13}$$

$$^{1}(\omega\varphi)_{00}=0. \tag{14}$$

[Use is made of the usual $\omega\varphi$ mixing parameters[13,14]

$$\begin{aligned}\omega&=(1/\sqrt{3})\omega_8+(\tfrac{2}{3})^{1/2}\varphi_1,\\ \varphi&=-(\tfrac{2}{3})^{1/2}\omega_8+(1/\sqrt{3})\varphi_1,\end{aligned} \tag{15}$$

where $\omega_8$ and $\varphi_1$ are the members of an $SU(3)$ octet and singlet, respectively.] The relation (11) is valid only in the case of complete $SU(3)$ symmetry, while the relations (12), (13), and (14) are satisfied even including the $SU(3)$-breaking terms $b_1$ and $b_3$.

All the transition amplitudes appearing in Eqs. (11)–(14) have a direct physical meaning in the sense that the transition amplitude referring to definite isotopic-spin state is completely described by just one $SU(2)$-invariant transition amplitude (final spin and total angular momentum are uniquely fixed) so that proportionality between the amplitudes directly gives proportionality between cross sections. For the other processes this is not the case, in general, as they are described by various $SU(2)$-invariant transition amplitudes. For completeness we give the relations among them in Appendix B.

The proportionality (11) includes, for example, the triangular relation

$$\langle\pi^+\pi^-|\bar{p}p\rangle-\langle K^+K^-|\bar{p}p\rangle+\langle K^0\bar{K}^0|\bar{p}p\rangle=0 \tag{11a}$$

and the relation

$$\langle\pi^0\pi^-|\bar{p}n\rangle=-\sqrt{2}\langle K^0K^-|\bar{p}n\rangle, \tag{11b}$$

which can be rewritten as

$$\frac{\sigma(\bar{p}n\to\pi^0\pi^-)}{\sigma(\bar{p}n\to K^0K^-)}=2\frac{k_\pi}{k_K}. \tag{11c}$$

Equation (11a) seems to be well satisfied by the experimental data as was discussed already.[7] The proportionality (11c) should be particularly useful for a more direct experimental check of the relation (11). The relevant data, not available now, are strongly desirable.

The relations (12) and (13) lead us to

$$\frac{\sigma(\bar{N}N\to\eta\omega)}{\sigma(\bar{N}N\to\eta\rho)}=\frac{9}{25}\frac{k_{\eta\omega}}{k_{\eta\rho}}, \tag{12a}$$

$$\sigma(\bar{N}N\to\pi\varphi)=\sigma(\bar{N}N\to\rho\varphi)=\sigma(\bar{N}N\to\eta\varphi)=0. \tag{13a}$$

These also could be easily checked experimentally. Present data,[15] however, are very ambiguous: $\pi\varphi$ production has been observed, thus showing a certain violation of the above prediction; the upper limit for the ratio $\sigma(\bar{N}N\to\pi\omega)/\sigma(\bar{N}N\to\pi\varphi)$, on the other hand, is much larger than one due to the large background of continuously distributed $3\pi$ events, although no $\omega$'s have been detected with certainty. Equation (14) is not useful, since $\sigma(\bar{N}N\to\omega\varphi)$, just above the threshold, is practically suppressed by phase space.

As a last remark we want to stress that the validity of the predictions of the present paper can hardly be checked by expressing them as relations among $SU(2)$-

[12] This is related to the explicit form of Eqs. (3) and (8). If one uses $\mathbf{q}$ instead of $\mathbf{q}/|q|$, an extra factor $q^2$ appears. On $SU(6)$ grounds only, one cannot decide what explicit form of phase space is to be used.

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[14] In the $SU(6)$ model if we treat the mass operator as the sum of (**1**·**1**) in **1** and (**8**·**1**) in **35**, we obtain exactly the same mixing parameters as those of Eq. (15). The (**8**·**1**) contribution in **35**×**35** gives a shifting from these values. T. K. Kuo and T. Yao, Phys. Rev. Letters **13**, 415 (1964); M. A. B. Bég and V. Singh, *ibid.* **13**, 418 (1964).

[15] V. E. Barnes, K. W. Lai, P. Anninos, L. Gray, P. Hagerty, E. Harth, T. Kalogeropoulos, S. Zenone, V. Dore, G. Moneti, and V. Valente, *Proceedings of the 12th Annual International Conference on High-Energy Physics, Dubna 1964*, (Atomizdat, Moscow, 1965); L. Bertanza (private communication).

invariant transition amplitudes except for very simple cases. This is due to the large experimental errors, to the difficulty in deducing $SU(2)$-invariant transition amplitudes from cross sections, and to the arbitrariness in fixing their relative phases. We think that more definite data, which are anyhow needed, and a systematic $\chi^2$ test using Table I on them would unambiguously decide to what extent the present scheme is supported by experiment.

After the completion of this work, we became aware of another paper[16] on this subject, in which a part of the results presented above for $\bar{p}p$ annihilation is reported. The spin dependence of the transition amplitudes, however, is not completely specified: If allowance is made for suitable multiplication factors, their results agree with ours.

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## APPENDIX A

Although the decomposition of the most widely used $SU(6)$ multiplets in terms of particle field operators is well known, no explicit phase convention is at present available in the literature, so that we feel it worthwhile to cover this point.

In the $SU(6)$ symmetry we find the explicit relations between components of tensors $T_{\mu\nu\cdots}{}^{\alpha\beta\cdots}$ and the field operators of the particles in any irreducible representation, in the following way. We use isotopic spin $I$, $U$ spin, ordinary spin $S$, hypercharge, and charge operators defined by

$$\begin{aligned} &I_+=A_{1i}{}^{2i}, \quad I_-=A_{2i}{}^{1i}, \quad I_3=\tfrac{1}{2}(A_{1i}{}^{1i}-A_{2i}{}^{2i}),\\ &U_+=A_{2i}{}^{3i}, \quad U_-=A_{3i}{}^{2i}, \quad U_3=\tfrac{1}{2}(A_{2i}{}^{2i}-A_{3i}{}^{3i}),\\ &S_+=A_{N1}{}^{N2}, \; S_-=A_{N2}{}^{N1}, \; S_3=\tfrac{1}{2}(A_{N1}{}^{N1}-A_{N2}{}^{N2}),\\ &Y=\tfrac{1}{3}(A_{1i}{}^{1i}+A_{2i}{}^{2i}-2A_{3i}{}^{3i}),\\ &Q=\tfrac{1}{3}(2A_{1i}{}^{1i}-A_{2i}{}^{2i}-A_{3i}{}^{3i}), \end{aligned} \tag{A1}$$

where $A_{Rr}{}^{Ss}$ with $R, S=1, 2, 3$ and $r, s=1, 2$ represent the generators of $SU(6)$ and satisfy the commutation relations

$$[A_\sigma{}^\rho, A_\mu{}^\lambda]=\delta_\mu{}^\rho A_\sigma{}^\lambda-\delta_\sigma{}^\lambda A_\mu{}^\rho,$$

and $A_\lambda{}^\lambda=0$ [as usual, $\rho=(R,r)$ etc. and summation is understood over repeated indices].

Generally, tensors must satisfy suitable symmetry and trace conditions in order to correspond to irreducible representations: for 35-plet and 56-plet these are

$$\begin{aligned} &M_\lambda{}^\lambda=0,\\ &B^{\mu\rho\sigma}=B^{\rho\mu\sigma}=B^{\mu\sigma\rho}. \end{aligned} \tag{A2}$$

Let us write state vectors and field operators as $|\Psi_{i,m}\rangle$ and $\psi_{i,m}$, where $i$ and $m$ correspond to the magnitude and the component of some generalized angular momentum—say $I$—relating to an $SU(2)$ subgroup of $SU(6)$. We have

$$\langle\Psi_{i,m}|I_3=\langle\Psi_{i,m}|m \tag{A3}$$

and

$$\langle\Psi_{i,m}|I_+=\langle\Psi_{i,m-1}|[(i+m)(i-m+1)]^{1/2}. \tag{A4}$$

The relation (A4) fixes the relative phases of the state vectors in accord with the Condon-Shortley[17] phase convention.

These relations can be rewritten as

$$[I_3,\psi_{i,m}]=-m\psi_{i,m}, \tag{A5}$$

and

$$[I_+,\psi_{i,m}]=-[(i+m)(i-m+1)]^{1/2}\psi_{i,m-1}. \tag{A6}$$

The commutation relations between generators $A_\sigma{}^\rho$ and arbitrary tensors $T_{\mu\nu\cdots}{}^{\alpha\beta\cdots}$ are given by

$$\begin{aligned} [A_\sigma{}^\rho, T_{\mu\nu\cdots}{}^{\alpha\beta\cdots}]=&\delta_\mu{}^\rho T_{\sigma\nu\cdots}{}^{\alpha\beta\cdots}+\delta_\nu{}^\rho T_{\mu\sigma\cdots}{}^{\alpha\beta\cdots}+\cdots\\ &-\delta_\sigma{}^\alpha T_{\mu\nu\cdots}{}^{\rho\beta\cdots}-\delta_\sigma{}^\beta T_{\mu\nu\cdots}{}^{\alpha\rho\cdots}-\cdots. \end{aligned} \tag{A7}$$

The starting point for building up the required correspondence between the field operators $\varphi_j$ ($j$ now runs over all the members of the multiplet) and the tensor components $T_{\mu\nu\cdots}{}^{\alpha\beta\cdots}$ is to take some nondegenerate weight (one of which for instance is the highest weight[18]) and then to pick up the corresponding field operator and tensor component which are simply proportional. A proper use of Eqs. (A1) and (A5)–(A7) enables us to find out all the other relations between $\varphi_j$ and $T_{\mu\nu\cdots}{}^{\alpha\beta\cdots}$. The normalization is fixed by the condition

$$\sum_{\mu\nu\cdots}\sum_{\alpha\beta\cdots}|T_{\mu\nu\cdots}{}^{\alpha\beta\cdots}|^2=\sum_j|\varphi_j|^2. \tag{A8}$$

By using such a method one can explicitly express $M_\sigma{}^\rho$ and $B^{\mu\rho\sigma}$ in terms of their $SU(6)$ content. We define the field operators of antibaryons by

$$\bar{B}_{\mu\rho\sigma}=B^{\mu\rho\sigma\dagger}, \tag{A9}$$

where the right-hand side stands for the Hermitian conjugate of $B^{\mu\rho\sigma}$. Again using the same method, we find the following result on phase convention.

[16] W. Alles, E. Borchi, G. Martucci, and R. Gatto, Phys. Letters 17, 328 (1965).

[17] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, New York, 1935). The widely used table of Clebsch-Gordan coefficients by A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirt, and M. Ross [Rev. Mod. Phys. 36, 977 (1964)] follows the phase convention in this book.

[18] See, for example, M. Konuma, K. Shima, and M. Wada, Progr. Theoret. Phys. (Kyoto), Suppl. 28 (1963), Sec. 5.

The phase convention for these tensors coincides with that of Condon-Shortley for isotopic spin, $U$ spin, and ordinary spin when we take $\{|B\rangle\}$ and

$$\{(-1)^{I_3+Y+S_3}|\bar{B}\rangle\}$$

as the bases of baryon and antibaryon states. Fixing the contents of the field operators of the meson 35-plet as

$$M=\begin{bmatrix} \dfrac{P+V_0}{\sqrt{2}} & V_{-1} \\ V_1 & \dfrac{P-V_0}{\sqrt{2}} \end{bmatrix}$$

with

$$P=\begin{bmatrix} \dfrac{\pi^0}{\sqrt{2}}+\dfrac{\eta}{\sqrt{6}} & \pi^- & K^- \\ \pi^+ & -\dfrac{\pi^0}{\sqrt{2}}+\dfrac{\eta}{\sqrt{6}} & \bar{K}^0 \\ K^+ & K^0 & -\dfrac{2}{\sqrt{6}}\eta \end{bmatrix}$$

$$V_i=\begin{bmatrix} \dfrac{\rho_i^0}{\sqrt{2}}+\dfrac{\omega_{8i}}{\sqrt{6}}+\dfrac{\varphi_{1i}}{\sqrt{3}} & \rho_i^- & K_i^{*-} \\ \rho_i^+ & -\dfrac{\rho_i^0}{\sqrt{2}}+\dfrac{\omega_{8i}}{\sqrt{6}}+\dfrac{\varphi_{1i}}{\sqrt{3}} & \bar{K}_i^{*0} \\ K_i^{*+} & K_i^{*0} & -\dfrac{2}{\sqrt{6}}\omega_{8i}+\dfrac{\varphi_{1i}}{\sqrt{3}} \end{bmatrix}$$

(where the lower index for vector nonet stands for the spin component), the phase convention of mesons coincides with that of Condon-Shortley when we take the bases $\{|K^+\rangle,|K^0\rangle\}$, $\{-|\pi^+\rangle, |\pi^0\rangle, |\pi^-\rangle\}$, $\{|\eta\rangle\}$ and $\{-|\bar{K}^0\rangle, |K^-\rangle\}$ for the isotopic-spin multiplet states and $\{|\pi^-\rangle,|K^-\rangle\}$, $\{|K^0\rangle, \frac{1}{2}(|\pi^0\rangle-\sqrt{3}|\eta\rangle), -|\bar{K}^0\rangle\}$, $\{-\frac{1}{2}(\sqrt{3}|\pi^0\rangle+|\eta\rangle)\}$ and $\{|K^+\rangle,|\pi^+\rangle\}$ for the $U$-spin multiplet states. For vector mesons, we can simply substitute $K\rightarrow K^*$, $\bar{K}\rightarrow\bar{K}^*$, $\pi\rightarrow\rho$, $\eta\rightarrow\omega_8$ and no extra phases are required for ordinary spin.

These considerations are a direct extension of the case of the $SU(3)$ model[19] and can be generalized to any higher symmetry scheme.

## APPENDIX B

In Sec. 4 six relations [Eqs. (11)–(14)] are listed. In this Appendix we derive and discuss briefly the other relations.

*Class (i)* Charge-conjugation relations:

$$^1(\bar{K}K^*)_{11}={}^1(K\bar{K}^*)_{11}, \qquad \text{(B1)}$$

$$^1(\bar{K}K^*)_{10}=-{}^1(K\bar{K}^*)_{10}, \qquad \text{(B2)}$$

$$^1(\bar{K}K^*)_{01}=-{}^1(K\bar{K}^*)_{01}, \qquad \text{(B3)}$$

$$^1(\bar{K}K^*)_{00}={}^1(K\bar{K}^*)_{00}. \qquad \text{(B4)}$$

*Class (ii)* Vanishing amplitudes:

$$^1(K^*\bar{K}^*)_{11}={}^1(\rho\rho)_{01}=0. \qquad \text{(B5)}$$

*Class (iii)* Relations among amplitudes which can be easily determined from the experimental data. (This means that the final two-meson states have only one set of values of $J$ and $S$ for definite $I$.)

$$5\,{}^1(\pi\omega)_{11}-5\sqrt{3}\,{}^1(\pi\rho)_{01}+4\sqrt{3}\,{}^1(\eta\rho)_{11}=0, \qquad \text{(B6)}$$

$$^1(\pi\rho)_{10}-2\,{}^1(\pi\rho)_{01}+{}^1(\rho\omega)_{10}=0, \qquad \text{(B7)}$$

$$3\,{}^0(\pi\pi)_{11}+8\,{}^0(K\bar{K})_{01}+3\,{}^1(\pi\rho)_{10}=0. \qquad \text{(B8)}$$

There is another relation which involves $^1(\varphi\varphi)_{00}$. As the $\varphi\varphi$ production is, however, energetically forbidden and the relation is implicitly included in Table I, we do not list it.

*Class (iv)* Proportionality between amplitudes in the previous class (iii) and others:

$$2\,{}^1(K\bar{K}^*)_{10}=-{}^1(\pi\rho)_{10}. \qquad \text{(B9)}$$

*Class (v)* Triangular relations which involve two amplitudes in the class (iii):

$$^1(K\bar{K}^*)_{00}=2\sqrt{3}\,{}^0(K\bar{K})_{11}+(1/\sqrt{3})\,{}^0(K\bar{K})_{01}, \qquad \text{(B10)}$$

$$^2(\rho\rho)_{11}=\tfrac{2}{3}(5/3)^{1/2}\,{}^1(\pi\omega)_{11}+\tfrac{4}{3}(5)^{1/2}\,{}^1(\eta\rho)_{11}, \qquad \text{(B11)}$$

$$\begin{aligned}(1/5\sqrt{2})\,{}^1(K\bar{K}^*)_{11}&=-(1/3\sqrt{2})\,{}^1(K\bar{K}^*)_{01}\\ &=-\tfrac{1}{6}\,{}^1(K^*\bar{K}^*)_{01}=-\tfrac{1}{10}(\tfrac{3}{5})^{1/2}\,{}^2(K^*\bar{K}^*)_{11}\\ &=\tfrac{1}{12}\,{}^1(\pi\omega)_{11}+(1/20\sqrt{3})\,{}^1(\eta\rho)_{11}. \qquad \text{(B12)}\end{aligned}$$

There are five other relations involving the amplitudes $^0(K^*\bar{K}^*)_{11}$, $^1(K^*\bar{K}^*)_{10}$, $^1(K^*\bar{K}^*)_{00}$, $^0(\rho\rho)_{11}$, and $^1(\rho\rho)_{00}$. These are more complicated and seem to be rather useless. As these relations are included in Table I implicitly, we omit them.

Of the above relations, Eqs. (12)–(14), (B1)–(B7), (B10), (B11), and the part of the proportionalities between strange-particle channels in Eq. (B12) are valid even in the case of an $SU(3)$ breaking interaction. The other relations become weaker and more complicated with the breaking terms and we refer again to Table I for them.

[19] M. Konuma and Y. Tomozawa, Nuovo Cimento 33, 250 (1964).