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Some Features of K^0 Decay: The $\Delta S=\Delta Q$ Rule, and the $|\Delta I|=\frac{1}{2}$ Rules for Leptonic and Three-Pion Decays*

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In a sample of ~36 000 K^0 and $\bar{K}^0$ mesons produced in antiproton annihilations in a liquid-hydrogen chamber, the K^0 leptonic decay rate and the time distribution of K^0 leptonic decays as well as the $K^0\to\pi^++\pi^-+\pi^0$ decay rates were studied. These results permit tests of the $\Delta S=\Delta Q$ rule and CP violation in leptonic decays, as well as tests of the $\Delta I=\frac{1}{2}$ rule for both leptonic and nonleptonic decays when compared with published results on other charge channels. The data, within their experimental error, are found to be consistent with all three rules.

I. INTRODUCTION

A. The $\Delta S=\Delta Q$ Rule and CP Conservation

THERE exist models[1] of the weak interaction formulated under the hypothesis that the strangeness-changing current of the strongly interacting particles contains only terms corresponding to a change in strangeness algebraically equal to the change in charge. This is usually referred to as the $\Delta S=\Delta Q$ selection rule. This hypothesis was questioned in 1962 in an experiment[2] which found a large violation of the rule in leptonic decays of K^0 mesons.

We report here an experiment with increased sensitivity to check the validity of the $\Delta S=\Delta Q$ rule in K^0 decays. A brief account of preliminary results of this experiment, which shows no discrepancy with the predictions of the rule, has already been presented.[3] This result is substantiated by that of another group[4] which has also found no violation of the $\Delta S=\Delta Q$ selection rule in K^0 decays in an experiment very similar to that of Ref. 2. The present paper contains a more complete description of our experiment and more extensive results.

Under the assumption of the $\Delta S=\Delta Q$ rule, the only allowed leptonic decays of K^0 mesons are

$$K^0\to\pi^-+L^++\nu \quad (\text{Amplitude}=f),$$
$$\bar{K}^0\to\pi^++L^-+\bar{\nu} \quad (\text{Amplitude}=f^*),^5 \qquad (1a)$$

where L stands for lepton.

If $\Delta S\neq\Delta Q$ transitions are allowed, we have also the processes

$$K^0\to\pi^++L^-+\bar{\nu} \quad (\text{Amplitude}=g),$$
$$\bar{K}^0\to\pi^-+L^++\nu \quad (\text{Amplitude}=g^*). \qquad (1b)$$

CP conservation in leptonic K^0 decay would require that the relative phase of f and g be zero.

We are interested in the time distribution of leptonic decays following the creation of K^0 and $\bar{K}^0$ mesons. Again, assuming CPT invariance, the short- and long-lived modes can be written[6]

$$K_1^0=(1+|r|^2)^{-1/2}(|K^0\rangle+r|\bar{K}^0\rangle)$$

and

$$K_2^0=(1+|r|^2)^{-1/2}(|K^0\rangle-r|\bar{K}^0\rangle).$$

The parameter r is not known at present. However, from the smallness of the CP violation in the experiment of Christenson *et al.*,[7] it is extremely likely that the difference between r and unity is of the same order, so that if $r=1+\epsilon$, then $|\epsilon|\approx10^{-3}$. Define

$$X=g^*/f,$$
$$\Gamma^+=\text{Rate}(K^0\to L^++\nu+\pi^-),$$
$$\Gamma^-=\text{Rate}(K^0\to L^-+\bar{\nu}+\pi^+),$$
$$\bar{\Gamma}^+=\text{Rate}(\bar{K}^0\to L^++\nu+\pi^-),$$
$$\bar{\Gamma}^-=\text{Rate}(\bar{K}^0\to L^-+\bar{\nu}+\pi^+),$$
$$\Gamma_1=\text{Total rate of } K_1^0,$$
$$\Gamma_2=\text{Total rate of } K_2^0,$$
$$\Delta m=K_1-K_2 \text{ mass difference},$$
$$\epsilon=|\epsilon|e^{i\beta},$$
$$X=|X|e^{i\delta}.$$

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[1] See, for example, R. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); R. E. Marshak and E. Sudarshan, *Proceedings Padua-Venice Conference on Mesons and Newly Discovered Particles, 1957* (Soc. Italiana de Fisica, Padua-Venice, 1958); N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).
[2] R. P. Ely *et al.* (California-Padua-Wisconsin Collaboration), Phys. Rev. Letters **8**, 132 (1962).
[3] L. Kirsch, R. J. Plano, J. Steinberger, and P. Franzini, Phys. Rev. Letters **13**, 35 (1964).
[4] B. Aubert, L. Behr, J. P. Lowys, P. Mittner, and C. Pascaud, Phys. Letters **10**, 215 (1964).
[5] This is true only if CPT is conserved. See Ref. 6.
[6] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **106**, 340 (1957).
[7] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964). See also W. Galbraith *et al.*, *ibid.* **14**, 383 (1965).

Retaining terms linear in ϵ, the time dependence of the four rates will be:

$$\Gamma^+ \propto \{[|1+X|^2+2|\epsilon|(|X|\cos(\beta+\delta)+|X|^2\cos\beta)]e^{-\Gamma_1 t}+[|1-X|^2-2|\epsilon|(|X|\cos(\beta+\delta)-|X|^2\cos\beta)]e^{-\Gamma_2 t} +2[(1-|X|^2-2|\epsilon||X|^2\cos\beta)\cos\Delta mt-(2|X|\sin\delta+2|\epsilon||X|\sin(\beta+\delta))\sin\Delta mt]e^{-(\Gamma_1+\Gamma_2)t/2}\},$$

$$\Gamma^- \propto \{[|1+X|^2+2|\epsilon|(\cos\beta+|X|\cos(\beta+\delta))]e^{-\Gamma_1 t}+[|1-X|^2+2|\epsilon|(\cos\beta-|X|\cos(\beta+\delta))]e^{-\Gamma_2 t} +2[(-1+|X|^2-2|\epsilon|\cos\beta)\cos\Delta mt-(2|X|\sin\delta+2|\epsilon||X|\sin(\beta+\delta))\sin\Delta mt]e^{-(\Gamma_1+\Gamma_2)t/2}\},$$

$$\bar{\Gamma}^+ \propto \{[|1+X|^2-2|\epsilon|(\cos\beta+|X|\cos(\beta-\delta))]e^{-\Gamma_1 t}+[|1-X|^2-2|\epsilon|(\cos\beta-|X|\cos(\beta-\delta))]e^{-\Gamma_2 t} -2[(1-|X|^2-2|\epsilon|\cos\beta)\cos\Delta mt-(2|X|\sin\delta+2|\epsilon||X|\sin(\beta-\delta))\sin\Delta mt]e^{-(\Gamma_1+\Gamma_2)t/2}\},$$

$$\bar{\Gamma}^- \propto \{[|1+X|^2-2|\epsilon|(|X|\cos(\beta-\delta)+|X|^2\cos\beta)]e^{-\Gamma_1 t}+[|1-X|^2+2|\epsilon|(|X|\cos(\beta-\delta)-|X|^2\cos\beta)]e^{-\Gamma_2 t} -2[(-1+|X|^2-2|\epsilon||X|^2\cos\beta)\cos\Delta mt-(2|X|\sin\delta+2|\epsilon||X|\sin(\beta-\delta))\sin\Delta mt]e^{-(\Gamma_1+\Gamma_2)t/2}\}.$$

In the present experiment we are interested in the sum of the four rates:

$$\Gamma^++\Gamma^-+\bar{\Gamma}^++\bar{\Gamma}^- \propto 4\{[|1+X|^2-2|\epsilon||X|\sin\beta\sin\delta]e^{-\Gamma_1 t}+[|1-X|^2+2|\epsilon||X|\sin\beta\sin\delta]e^{-\Gamma_2 t} -4|\epsilon||X|\cos\beta\sin\delta\sin\Delta mt\, e^{-(\Gamma_1+\Gamma_2)t/2}\} \quad (2)$$

and in the difference:

$$\Gamma^++\Gamma^--(\bar{\Gamma}^++\bar{\Gamma}^-) \propto 4\{|\epsilon|[(1+|X|^2)\cos\beta+2|X|\cos\beta\cos\delta]e^{-\Gamma_1 t}+|\epsilon|[(1+|X|^2)\cos\beta-2|X|\cos\beta\cos\delta]e^{-\Gamma_2 t} -2[|\epsilon|(1+|X|^2)\cos\beta\cos\Delta mt+2(|\epsilon||X|\sin\beta\cos\delta+|X|\sin\delta)\sin\Delta mt]e^{-(\Gamma_1+\Gamma_2)t/2}\}. \quad (3)$$

Actually our experiment is not sufficiently sensitive to permit the hope that the terms in ϵ can be detected. Neglecting these terms we obtain the more transparent expressions (2′) and (3′):

$$\Gamma^++\Gamma^-+\bar{\Gamma}^++\bar{\Gamma}^- \propto 4[|1+X|^2e^{-\Gamma_1 t}+|1-X|^2e^{-\Gamma_2 t}], \quad (2')$$

$$\Gamma^++\Gamma^--(\bar{\Gamma}^++\bar{\Gamma}^-) \propto -16|X|\sin\delta\sin\Delta mt\, e^{-(\Gamma_1+\Gamma_2)t/2}. \quad (3')$$

The sum (2′) of the four rates is the superposition of two exponentials with the short and long K^0 lifetimes. The relative amplitudes of these two exponentials is $\alpha=|(1+X)/(1-X)|^2$ and can be a measure of $\Delta S=\Delta Q$ violation. If $\Delta S\neq\Delta Q$ transitions are forbidden, $\alpha=1$. The difference (3′) can be used to investigate CP violation in the K^0 leptonic decays. If the amplitudes (1a) and (1b) conserve CP, then $\delta=0$ and the difference (3′) of K^0 and $\bar{K}^0$ leptonic decay rates is zero.

B. The $|\Delta I|=\frac{1}{2}$ Rule for Leptonic Decays

The weak interaction responsible for the nonleptonic decay of strange particles appears to satisfy also the selection rule $|\Delta I|=\frac{1}{2}$, where I is the total isospin. The predictions of this rule are in fact quite well verified in Λ^0 decay,[8] in the decay of K^0 into two pions[9] and in $\Sigma^\pm$ decay.[10]

Okubo, Marshak, and Sudarshan[11] have proposed to extend the rule $|\Delta I|=\frac{1}{2}$ to leptonic decays of decays of strange particles. In particular, in the current-current interaction scheme, one way of ensuring the $\Delta S=\Delta Q$ rule is to assume that the strangeness-changing current of the strongly interacting particles transforms like a spinor under rotation in charge space. Under this hypothesis it follows that $\Gamma_{2L}=2\Gamma_{+L}$, where Γ_{+L} is the rate for the process $K^+\rightarrow\pi^0+L^++\nu$.

The experimental situation is not so clear. The analysis of Luers *et al.*[12] of K-meson decays concludes that the agreement is satisfactory; however, the Berkeley group[13,14] reports disagreement.

C. The $|\Delta I|=\frac{1}{2}$ Rule for Pionic Decays

The assumption that the strangeness-changing current transforms as an isospinor does not guarantee that nonleptonic decays satisfy the $|\Delta I|=\frac{1}{2}$ rule, since in the current-current scheme terms corresponding to $|\Delta I|=\frac{3}{2}$ will appear in general. If, however, it is assumed that the strangeness-changing part of the weak Lagrangian transforms like an isospinor, the rule $|\Delta I|=\frac{1}{2}$ follows for all nonleptonic decays of strange particles. For the case of K-meson decays we obtain in particular the relation

$$\Gamma_2(+-0)=2\Gamma_+(+00),$$

where $\Gamma_2(+-0)$ is the decay rate of the $K_2{}^0$ to $\pi^+\pi^-\pi^0$, and $\Gamma_+(+00)$ is the K^+ decay rate to $\pi^+\pi^0\pi^0$.

Recently, attempts to put the above rules on a more fundamental basis have been made by Cabibbo[15] and

[8] W. H. Humphrey and R. R. Ross, Phys. Rev. **127**, 1305 (1962).
[9] M. Roos, Rev. Mod. Phys. **35**, 314 (1963).
[10] P. Franzini and D. Zanello, Phys. Letters **5**, 254 (1963).
[11] S. Okubo, R. E. Marshak, and E. C. G. Sudarshan, Phys. Rev. Letters **2**, 12 (1959).
[12] D. Luers, I. S. Mittra, W. J. Willis, and S. S. Yamamoto, Phys. Rev. **133**, B1267 (1964).
[13] J. A. Anderson, F. S. Crawford, Jr., R. L. Golden, D. Stern, T. O. Binford, and V. G. Lind, Phys. Rev. Letters **14**, 475 (1965).
[14] G. Alexander, S. P. Almeida, and F. S. Crawford, Jr., Phys. Rev. Letters **9**, 69 (1962).
[15] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

others, assuming

(1) that the current itself transforms like some components of an SU_3 octet,

and

(2) that the Lagrangian contains only terms corresponding to the eight-dimensional representation of SU_3.

Under this assumption the $\Delta S=\Delta Q$ and the $|\Delta \mathbf{I}|=\frac{1}{2}$ rules follow for all $\Delta S=\pm 1$ processes.

The experimental situation for the three-pion decay of K mesons was also not so clear up to a short while ago. A direct measurement[14] of $\Gamma_2(+-0)$ was in agreement with the prediction of the rule and the experimental rate for $\Gamma_+(+00)$, but the same authors find disagreement with this prediction if instead they use their own value for the K^0 charged leptonic decay rate and the ratio

$$\frac{\Gamma_2(+-0)}{\Gamma_2(\text{all charged decays})}$$

obtained by Luers *et al.*[12]

A more complete analysis of the problem has been carried through by Luers *et al.*[12] These authors conclude that within the experimental uncertainties the rule $|\Delta \mathbf{I}|=\frac{1}{2}$ for the three-pion decay modes of the K meson is satisfied. Dalitz[16] has discussed this problem with similar conclusions. A very recent[13] measurement of the $K_2^0 \to \pi^+\pi^-\pi^0$ rate is also in agreement with the rule.

D. Outline of Experiment

In our experiment K^0 mesons are produced in the annihilation of antiprotons stopped in the 30-in. Columbia-BNL hydrogen chamber. In approximately 650 000 pictures, containing $\sim$735 000 annihilations, some 36 000 K^0 and $\bar{K}^0$ mesons are produced in various reactions, such as

$$\begin{aligned} \bar{p}+p &\to K^0+\bar{K}^0+\pi^0 \\ &\to K^0+K^\pm+\pi^\mp \\ &\to K^0+K^\pm+\pi^\mp+\pi^0 . \end{aligned}$$

The experimental problem is that of selecting the leptonic decays. Of the $\sim$36 000 K^0's produced, $\sim$24 000 escape detection, 12 000 decay as K_1^0's into two charged pions, approximately 100 are expected to undergo leptonic decay within the fiducial region which will be defined presently, and another $\sim$40 are expected to decay into other three-body modes, specifically $K^0 \to \pi^++\pi^-+\pi^0$ and $K^0 \to \pi^++\pi^-+\gamma$. We proceed by measuring all V's in a certain fiducial region surrounding stopped antiprotons, and then rejecting the dominant $K^0 \to 2\pi$ mode if a kinematic fit to this is possible. This is done independently of the possible origin of the V. The remaining events are either three-body decays or background events of various sorts (Λ decays, charged K decays). The latter are in large measure eliminated by inspection. The remaining events, now chiefly three-body decays, are kinematically analyzed for compatibility of any of the three-body decays with the observations, using the directions specified by all antiproton annihilations within the fiducial volume. In general, more than one decay hypothesis may fit the event, and each decay hypothesis may fit with two different values of the K^0 momentum. The resultant K momenta, together with the information available at the supposed production vertex, such as the momenta of the charged tracks which are produced, are checked for compatibility with the assumption that the vertex is in fact responsible for producing the K^0. The remaining events contain the leptonic K^0 decays of known origin and constitute the basic data of the experiment.

II. EXPERIMENTAL PROCEDURE

A. Selection of Events

1. Scanning. The pictures were scanned twice in three views. It was required that the V be visible in all three views. Scanners were instructed to accept all "V's" within a radius of 15 cm, in any view, of a $\bar{p}$ annihilation vertex, whether or not the V points to the vertex.

2. Measurement. Measurements of 12 556 V's were made in three stereoscopic views and 10 530 passed the geometrical reconstruction program. No attempt was made to remeasure events which failed.

3. Fiducial Volume. After measurement all events are rejected unless they can be associated with at least one annihilation such that the distance from the annihilation is greater than 0.2 and less than 15 cm and the dip angle is less than 60°. In addition, events are rejected unless the dip angle of either track of the V is less than 70°. In this selection 8881 events survived. In the following we restrict ourselves to these events.

4. Removal of $K^0 \to \pi^++\pi^-$ Decays from Sample. Most of the observed V's are examples of two-pion decay. All V's are tested for a fit to this hypothesis without reference to the possible source of production (one-constraint fit). This fitting procedure, as well as all subsequent ones, uses the GRIND program developed at CERN.[17] An event is classified as $K^0 \to \pi^++\pi^-$ and removed from the sample if the χ^2 for this hypothesis is <15. The probability of χ^2 exceeding 15 is less than 0.0001, so that approximately one $K_1^0 \to \pi^++\pi^-$ in our sample should statistically be able to survive the rejection; 8560 events are classified as $K_1^0 \to \pi^++\pi^-$. The momentum distribution for 7786 of the $K^0 \to \pi^++\pi^-$ events which have also a clear production

[16] R. Dalitz, *International Conference on Fundamental Aspects of Weak Interactions* (Brookhaven National Laboratory, Upton, New York, 1963).

[17] R. Böck, CERN Report No. 61-29, 1961 (unpublished).

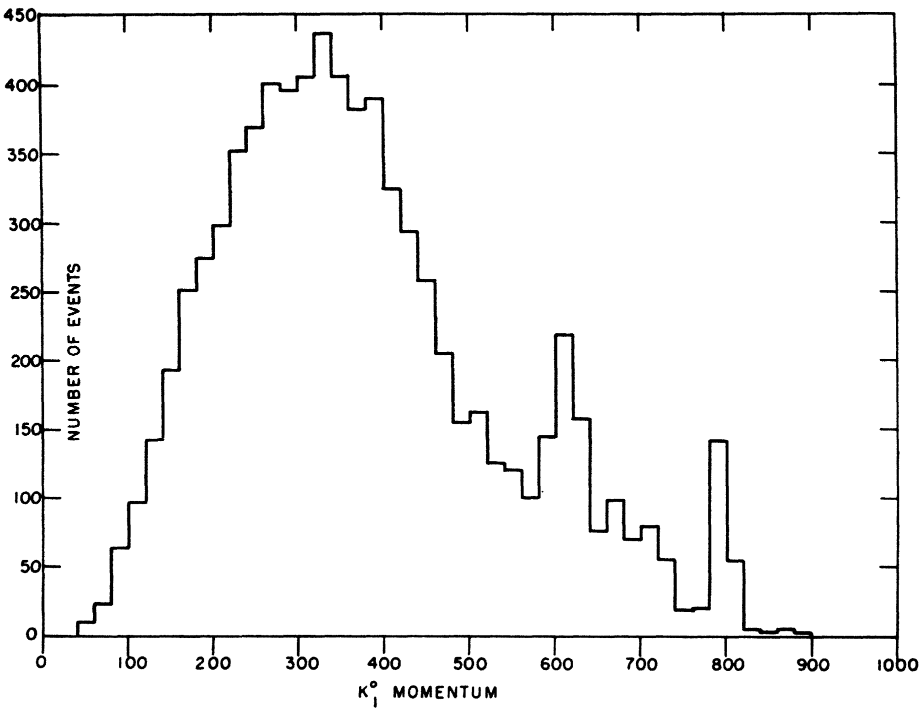

FIG. 1. Momentum distribution of 7786 $K_1^0 \to \pi^+ + \pi^-$ events. Units are MeV/*c*.

origin is shown in Fig. 1. The mean lifetime of the K_1^0 on the basis of these events is $\tau_1 = (0.848 \pm 0.014) \times 10^{-10}$ sec (Fig. 2). The error quoted is statistical.

5. Events are rejected if after measurement and inspection they are found to fit the hypothesis of Λ^0 decay or charged K, π, or μ decays. There were 59 events classified as Λ^0 decays, 31 as charged K decays, and 33 as $\pi \to \mu$ or $\mu \to e$ decays.

6. π-p Scattering. Some of the V's represent examples of π-p scattering with such small momentum transfer that the recoil proton is unobservable. To avoid this background we eliminate all V's for which the vector sum of the two measured momenta is less than 60 MeV/*c*. There were 37 such cases.

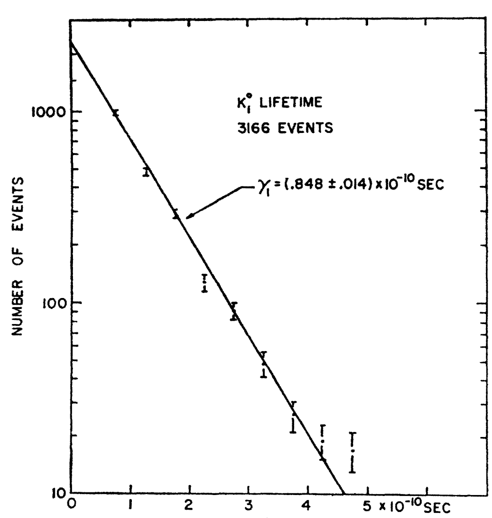

FIG. 2. Decay-time distribution in the K_1^0 center of mass of 3166 $K^0 \to \pi^+ + \pi^-$ events.

There remain 161 cases. These are selected, except for the condition on length, without bias of lifetime. They contain the bulk of the three-body decays and some background. The following additional selection criteria involve association of the decay with a particular annihilation vertex.

7. Decay and Production Fits. Given the directions with respect to all $\bar{p}$ annihilations within the fiducial volume of the V, we calculate possible (zero constraint) fits to the six hypothesis $K^0 \to \mu^\pm + \pi^\mp + \nu$, $K^0 \to e^\pm + \pi^\mp + \nu$, $K^0 \to \pi^+ + \pi^- + \pi^0$ (τ decay), and $K^0 \to \pi^+ + \pi^- + \gamma$ (radiative decay). An event may fit one or more hypothesis, or none. The fitting also yields K^0 momenta, in general two momenta per fitting hypothesis. These momenta are checked for compatibility with the production vertex. There were 33 events which had either no decay fit, or were incompatible with the production vertex; 128 events are left as candidates for three-body K^0 decay.

8. Rejection of $K^0 \to \pi^+ + \pi^- + \pi^0$. From these events we delete all which fit the τ hypothesis. This will also lose a small number of leptonic decays, as we will see later in the efficiency calculation. There are 14 such events.

9. Rejection of $K^0 \to \pi^+ + \pi^- + \gamma$. There is a not insubstantial background of K_1^0 radiative decay which will be discussed in the following. In order to control this background we reject at a small loss of leptonic detection efficiency all events which permit a radiative fit, and for which the γ-ray momentum in the center of mass of the K^0 is less than 50 MeV. Three events were rejected on this basis. In addition, 2 events are rejected for the following reasons: The first has an uncertainty in the momentum as large as the momentum itself, the second is the only event in the sample compatible with two origins.

We are left with 109 events. Many of these fit to more than one hypothesis with more than one momentum. There are, however, 88 events for which all permissible momenta are within ± 50 MeV/c of a central value.

In Fig. 3 we plot the decay-time distribution in the center of mass of the K^0 for these 88 events with unambiguous momentum. In Fig. 4 we present the histogram of all events. In this figure, for those events with a multiple solution, each momentum is weighted in proportion to the K^0 momentum spectrum of Fig. 1.

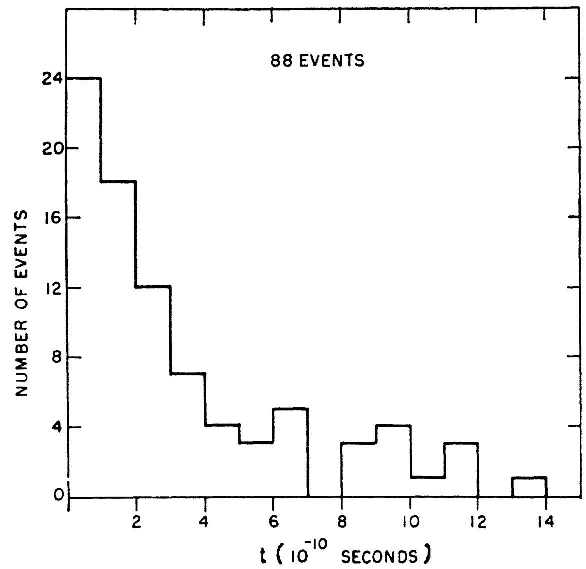

FIG. 3. Decay-time distribution for 88 three-body K^0 decay with well-defined momentum.

B. Detection Efficiency

We estimate our scanning efficiency to be of the order of 95% for all V's. However, in the following we assume only that the efficiency of the scan is the same for leptonic as for $\pi^+\pi^-$ decay. Some of the decays are lost, however, by subsequent selection criteria. We have performed a Monte Carlo calculation on 2000 muonic and electronic K^0 decays, distributed in the decay according to the $V-A$ theory (with $f_-=0$ and $f_+=\text{const}$) and in momentum according to Fig. 1. The positive and negative tracks were assigned typical errors, and criteria 4, 6, 8, and 9 were applied. Of the 2000 events, 1982 survived criterion 6. Of these, 1710 survived criterion 4. Of these, 1654 survived criterion 8, and of these, 1629 survived criterion 9. The leptonic decay detection is therefore $\epsilon=0.815\pm0.01$.

C. Available Path Length and Decay Time

About 75% of the events have a potential available path length greater than the 15-cm fiducial requirement. However, some antiprotons stop in a region of the chamber such that not all decay directions have 15 cm available.

In order to determine the efficiency $\epsilon(t)$ for observing a K^0 living a time t in its rest frame, we use our sample of $K^0\to\pi^++\pi^-$. Presumably the potential path and the momentum distributions for these events are the same as for the leptonic decays. For each $K\to 2\pi$ event we calculate the potential path and using the measured momentum, the corresponding potential time. Since the average decay length for the $K_1{}^0$'s in this experiment is about 1.8 cm, very few decay outside of the chamber. In this manner we obtain the potential time distribution $F(t)$. The probability $\epsilon_1(t)$ of observing a K^0 decay at time t is given by

$$\epsilon_1(t)=\int_t^\infty F(t)dt\bigg/\int_0^\infty F(t)dt.$$

Furthermore, the requirement of a K^0 length greater than 2 mm results in a reduction in the detection of short-lived decays. This has the form:

$$\epsilon_2(t)=\int_{ml/t}^\infty f(p)dp\bigg/\int_0^\infty f(p)dp,$$

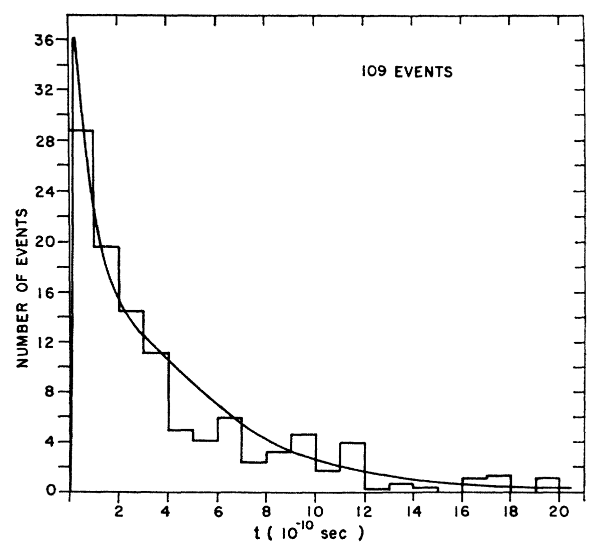

FIG. 4. Decay-time distribution for all 109 three-body K^0 decays. For events with multiple solutions, each momentum value has been weighted according to the momentum spectrum of Fig. 1.

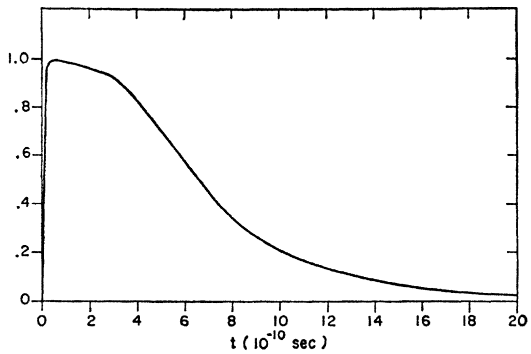

FIG. 5. Geometrical detection efficiency as a function of time in the K^0 rest system.

where $f(p)$ is the $K \to 2\pi$ momentum distribution and $l=2$ mm. The total efficiency is

$$\epsilon(t)=\epsilon_1(t)\times\epsilon_2(t).$$

$\epsilon(t)$ is shown in Fig. 5. The method is not quite correct; a correction for the finite lifetime of the K_1^0 should be applied to the K_1^0 momentum spectrum. However, the average available path is several times greater than the average K_1^0 path and we have spared ourselves this correction.

D. Background

Some of the 109 events finally selected may be random associations of three-body K^0 decays and antiproton annihilations. In an effort to estimate this number, we assume that the 33 decays which are found, but fail either a decay fit or a production fit, are due to background K_2 decays randomly associated. We now find the probability that a randomly oriented K_2^0 will give a production and a decay fit by associating a sample of accepted leptonic decays in turn with each of the production vertices of some randomly selected annihilations. The number of fits divided by the number of tries is 1/7. We, therefore, estimate the random background at approximately $(33)(1/7)/(6/7)=5.5$ events. No attempt is made to correct for this contamination.

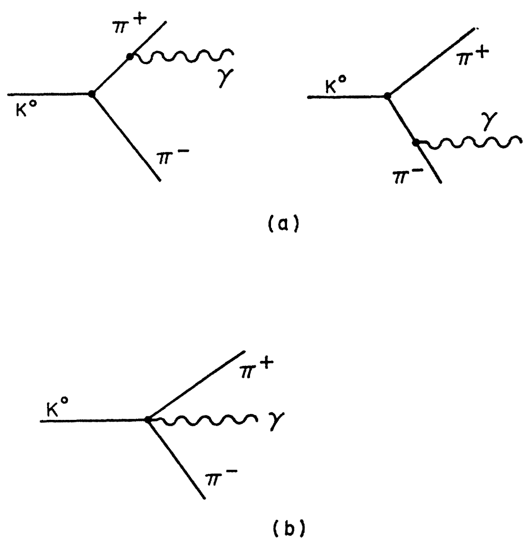

FIG. 6. (a) Feynman diagram for inner bremsstrahlung. (b) Feynman diagram for "direct" radiative decay.

III. ANALYSIS OF EXPERIMENTAL RESULTS

A. The $\Delta S=\Delta Q$ Selection Rule

Before the distribution of the form (2′) can be compared with the experimental result, it is first necessary to augment the distribution with the expectation for the radiative decay $K^0 \to \pi^+ + \pi^- + \gamma$. Here we must distinguish two mechanisms for the radiative decay: (a) inner bremsstrahlung illustrated by the diagrams of Fig. 6(a), and (b) direct photon production illustrated by the diagram of Fig. 6(b). The inner bremsstrahlung can be calculated, giving the result[18]:

$$d\Gamma_1(\pi^+\pi^-\gamma)=\frac{\alpha}{\pi}\frac{dk}{k}\left(1-\frac{2k}{m_K}\right)\frac{\beta}{\beta_{\pi\pi}}\times\left[\frac{1+\beta^2}{\beta}\ln\frac{1+\beta}{1-\beta}-2\right]\Gamma_1(\pi^+\pi^-), \quad (4)$$

where

$\beta_{\pi\pi}$ = pion velocity in the center-of-mass system for the decay $K^0 \to \pi^+ + \pi^-$,
β = pion velocity in the π-π center-of-mass system for the decay $K_1^0 \to \pi^+ + \pi^- + \gamma$,
k = photon momentum in the rest system of the K^0,
m_K = K^0 mass, and
$\Gamma_1(\pi^+\pi^-)$ = transition probability for $\pi^+\pi^-$ decay of the K_1^0.

This is a radiative correction to the $\pi^+\pi^-$ decay mode, and is therefore a decay mode of the K_1^0, and not the K_2^0. The distribution (4) is peaked as $1/k$ at low photon momenta, and this is the reason for excluding from the experimental sample those events which are compatible with radiative decay with small photon momenta (see Sec. II).

No reliable theory exists for estimating the direct photon production. There is, however, an experimental upper limit which can be placed on this mode. Eisler finds,[19] in an examination of K_2^0 decays into two charged prongs plus Dalitz pair, 25 such decays. These are all compatible with the π decay hypothesis: $K_2^0 \to \pi^+ + \pi^- + \pi^0$, $\pi^0 \to \gamma + e^+ + e^-$. At most, one of these is compatible with the decay: $K^0 \to \pi^+ + \pi^- + e^+ + e^-$. Using the branching ratio of Luers *et al.*[12] for the decay $K_2^0 \to \pi^+ + \pi^- + \pi^0$, an upper limit of 1% can be put

[18] We are indebted to Professor M. A. B. Bég, Dr. R. Friedberg, and Dr. J. Schultz for providing us with the quoted result.

[19] Private Communication. We wish to thank Professor F. Eisler for making this result available to us.

on the decay $K_2^0 \to \pi^+ + \pi^- + \gamma$. We are therefore justified in ignoring this possibility and correct (1) for the bremsstrahlung (2), to obtain

$$F(t) = \tfrac{1}{2}N\{(\alpha A + B)e^{-\Gamma_1 t} + Ae^{-\Gamma_2 t}\}\epsilon(t), \quad (5)$$

where

$$A = \epsilon\Gamma_{2L} = 0.815\Gamma_{2L},$$

$$B = \epsilon^1 \int_{50}^{k_{\max}} d\Gamma_{1\pi\pi\gamma} = 15.9\times10^6 \text{ sec}^{-1},$$

and

$$\epsilon^1 = 0.865,$$

according to a Monte Carlo calculation similar to that described in Sec. II.

In our experiment we observed in all 8560 $K^0 \to \pi^+ + \pi^-$, which when corrected for short time losses correspond to 9611 $K^0 \to \pi^+ + \pi^-$. From this the total K^0 flux is

$$2\times9611/0.694 = 27\,700.$$

The K_2^0 leptonic rate can be estimated from other experiments as follows: The ratio $\Gamma_2(\pi^+\pi^-\pi^0)/\Gamma_{2(+-0)} + \Gamma_{2L}$ is given by Luers *et al.*[12] to be 0.171 ± 0.02. The ratio $\Gamma_2(\pi^0\pi^0\pi^0)/\Gamma_{2L}+\Gamma_{2(+-0)}$ is given by Anikina *et al.*[20] to be 0.24 ± 0.08. From these two results $\Gamma_{2L}/\Gamma_2 = 0.67 \pm 0.05$. The most recent result[21] on the K_2^0 decay rate is $\Gamma_2 = (18.5\pm1.7)\times10^6$ sec^{-1}. This gives us then $\Gamma_{2L} = (12.4\pm1.46)\times10^6$ sec^{-1}.

The distribution (5) with α as the only free parameter is fitted to the experimental distribution (Fig. 4), using the maximum-likelihood method. The likelihood function yields $\alpha = 0.75_{-0.70}^{+0.80}$. The experimental distribution is therefore in good agreement with the value $\alpha = 1$ expected on the basis of the $\Delta S = \Delta Q$ rule. The expected distribution for $\alpha = 1$, $\Delta S = \Delta Q$ is shown in Fig. 4. In noting the agreement of this theoretical distribution with the experiment, it should be kept in mind that the theoretical curve has no free parameters.

Alternatively, we can analyze our data in terms of the distribution (5) keeping both α and Γ_{2L} as free parameters. We then find $\Gamma_{2L} = (9.4\pm1.3)\times10^{+6}$ sec^{-1} and $\alpha = 1.64_{-0.88}^{+1.26}$. The likelihood contour is shown in Fig. 7.

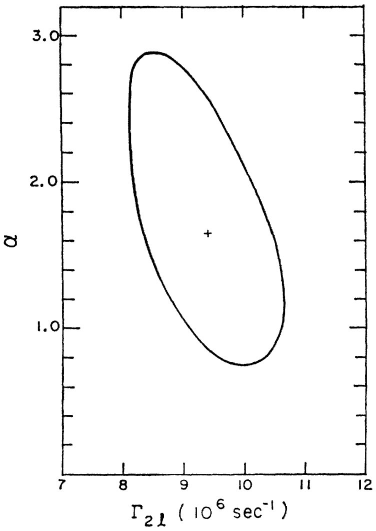

FIG. 7. Log likelihood contour: $\ln(L_{\max}/L) = 0.5$ for 109 events. $\alpha = 1.64_{-0.88}^{+1.26}$. $\Gamma_{2L} = (9.4\pm1.3)\times10^6$ sec^{-1}.

B. *CP* Violation in K^0 Leptonic Decay

In general, it is not possible in this technique to separate events in which the decaying meson is initially a K^0 from those in which it is $\bar{K}^0$. However, it is sometimes possible to tell on the basis of the information at the production vertex when the accompanying meson is a K^- or K^+. On this basis we have identified 22 $\bar{K}^0$ and 23 K^0 events. The time distributions of these events as well as their differences are plotted in Fig. 8. The predictions of (3′) for two cases of substantial *CP* violation ($X = 1.0$, $\delta = \pi/2$; $X = 0.5$, $\delta = \pi/2$) are also shown using $\Delta m = 0.79\Gamma_1$. In fact, the experimental result does not deviate in a significant way from zero, so that the data are certainly compatible with no maximal *CP* violation. We cannot, however, rule out Sachs'[22] proposal that $\delta = \pi/2$.

C. The K_2^0 Leptonic Decay Rate and the $|\Delta I| = \frac{1}{2}$ Rule in Leptonic Decays

These data can also furnish the rate for K_2^0 leptonic decay. Setting $\alpha = 1$ in formula (5), we have computed the likelihood function with Γ_{2L} as a parameter. The result gives as the best value

$$\Gamma_{2L} = (9.85_{-1.05}^{+1.15})\times10^6 \text{ sec}^{-1}.$$

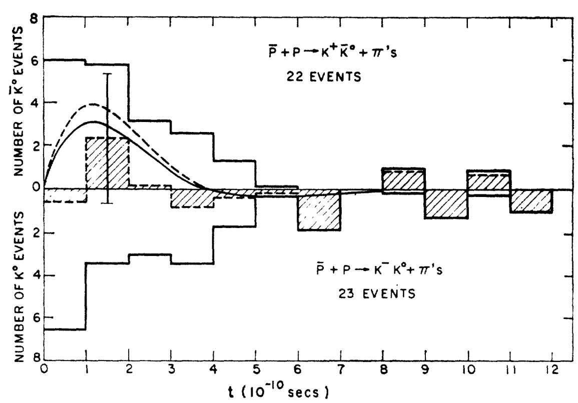

FIG. 8. Shaded area represents $N_{\bar{K}^0} - N_{K^0}$. The smooth curves are given by (3′) normalized as in (5) and multiplied by $(N_{\bar{K}^0}+N_{K^0})/109$. The dashed curve assumes $X = 1.0$, $\delta = \pi/2$. The solid curve assumes $X = 0.5$, $\delta = \pi/2$.

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[20] See Ref. 16, p. 395.

[21] T. Fujii, J. Jovanovich, F. Turkot, and G. Zorn, Bull. Am. Phys. Soc. 9, 442 (1964).

[22] R. G. Sachs, Phys. Rev. Letters 13, 286 (1964).

This is in agreement with the one used in the previous analysis. Combining our value for Γ_{2L} with the independent value $\Gamma_{2L}=(12.40\pm1.46)\times10^6$ sec^{-1} derived above from independent data, one obtains

$$\Gamma_{2L}=(10.8\pm0.9)\times10^6\ \text{sec}^{-1}.$$

The corresponding rate for K^+ leptonic decay is $\Gamma_{+L}=6.2\pm0.85$ on the basis of the new results of Shaklee *et al.*[23] The ratio Γ_{2L}/Γ_{+L} expected from the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule is 2. Experimentally we obtain

$$\Gamma_{2L}/\Gamma_{+L}=(10.8\pm0.9)/(6.2\pm0.85)=1.7\pm0.3.$$

D. The Rate $K_1^0\to\pi^++\pi^-+\pi^0$ and the $|\Delta I|=\frac{1}{2}$ Rule in Three-Pion Decay

Fourteen of our events are kinematically compatible with τ decay. They are also compatible with leptonic decay modes. However, it is kinematically substantially improbable that a leptonic decay has a configuration compatible with τ decay. The argument has been presented in some detail by Luers *et al.*[12] and we omit it here. On this basis the 14 events are accepted as examples of the decay $K^0\to\pi^++\pi^-+\pi^0$. Assuming CP invariance, the angular-momentum barrier is expected to suppress the τ decay rate of the K_1^0 with respect to the K_2^0, and this is consistent with the time distribution of these events. Using the same geometric detection factor and flux determination as before, these 14 events yield $\Gamma_{2(+-0)}=(1.4\pm0.4)\times10^6$ sec^{-1}. Another recent determination by Anderson *et al.*[13] has yielded $\Gamma_{2(+-0)}=(3.26\pm0.77)\times10^6$ sec^{-1} on the basis of 18 events.

Independently, $\Gamma_{2(+-0)}$ can be found using the branching ratio

$$\Gamma_{2(+-0)}/\Gamma_{2L}=0.206\pm0.028$$

of Luers *et al.*[12] and $\Gamma_{2L}=(10.8\pm0.9)\times10^6$ sec^{-1} found previously. In this way $\Gamma_{2(+-0)}=(2.22\pm0.35)\times10^6$ sec^{-1}, and combining these three independent determinations,

$$\Gamma_{2(+-0)}=(2.22\pm0.24)\times10^6\ \text{sec}^{-1}.$$

[23] F. S. Shaklee, G. L. Jensen, B. P. Roe, and D. Sinclair, Bull. Am. Phys. Soc. **9**, 34 (1964).

TABLE I. Number of events classified according to strangeness and lepton sign.

	K^0	Unidentified	$\bar{K}^0$
Negative leptons	3	12	4
Unidentified	13	43	14
Positive leptons	7	9	4

The corresponding charged K decay rate $\Gamma_{+(+00)}$ for the process $K^+\to\pi^++\pi^0+\pi^0$ is $(1.38\pm0.11)\times10^6$ sec^{-1} on the basis of the branching ratio of 1.69 ± 0.13 for τ' decay of Alexander *et al.*[24] Birge *et al.*,[25] Taylor *et al.*,[26] and Roe *et al.*[27]

The ratio $\Gamma_{2(+-0)}/\Gamma_{+(+00)}$ predicted by the $|\Delta I|=\frac{1}{2}$ rule is 2 while experimentally we obtain

$$\Gamma_{2(+-0)}/\Gamma_{+(+00)}=1.6\pm0.22.$$

The agreement is not as good as may perhaps be expected, but such that in our opinion there is no substantial evidence here against the validity of the rule.

Note added in proof. A more detailed analysis has been performed to determine the values of $|x|$ and δ. Each of the 109 events was examined to determine both the sign of the lepton and whether a K^0 or $\bar{K}^0$ had been produced (see Sec. IIIB). The results of this analysis are listed in Table I. For each of the nine categories in this table, a likelihood function was constructed from appropriate combinations of the four rates in Sec. IA. The products of these likelihood functions was maximized as a function of $|x|\sin\delta$ and $|x|\cos\delta$ for selected values of the $K_1^0-K_2^0$ mass difference. We find

$$\Delta m=0.1,\quad |x|=0.51_{-0.51}^{+0.60},\quad \delta=114°_{-36°}^{+51°},$$
$$\Delta m=0.5,\quad |x|=0.25_{-0.25}^{+0.38},\quad \delta=107°_{-42°}^{+69°},$$
$$\Delta m=0.8,\quad |x|=0.24_{-0.24}^{+0.33},\quad \delta=109°_{-43°}^{+70°}.$$

[24] G. Alexander, R. H. W. Johnston, and C. O'Ceallaigh, Nuovo Cimento **6**, 478 (1957).
[25] R. W. Birge, D. H. Perkins, T. E. Peterson, D. H. Stork, and M. W. Whitehead, Nuovo Cimento **4**, 834 (1959).
[26] S. Taylor, G. Harris, T. Orear, J. Lee, and P. Baumel, Phys. Rev. **114**, 359 (1959).
[27] B. P. Roe, D. Sinclair, J. L. Brown, D. A. Glaser, J. A. Kadyk, and G. H. Trilling, Phys. Rev. Letters **2**, 349 (1961).