Of course the $\boldsymbol{\sigma}$'s in (6.5) fit in between $\mathbf{E}^*$ and $\mathbf{E}$ in (6.6)–(6.8).

If these three equations are multiplied by (5.7) and averaged over b spin, it is found that only terms linear in $\mathbf{a}''$ survive. Hence, it is reasonable to look at the dependence of $\mathbf{Q}\cdot\boldsymbol{\Phi}$ and σ_{1M0} on $\mathbf{a}$, $\mathbf{a}''$, but not on $\mathbf{a}'$ by using (5.11) rather than (5.7). We obtain

$$\mathbf{Q}\cdot\boldsymbol{\Phi}=X^{(1)}|y+z|^2+X^{(2)}\,\mathrm{Re}y(y+z)^*+X^{(3)}|y|^2 \quad (6.9)$$

where, for the $\frac{1}{2}^+$ sequence,

$$X^{(1)}=8(j+1)(\mathbf{Q}\cdot\mathbf{a}'')-4(2j+1)(\mathbf{Q}\cdot\mathbf{a})\mathbf{a}''\cdot\mathbf{a}, \quad (6.10a)$$

$$X^{(2)}=(20j^2+30j+2)(\mathbf{Q}\cdot\mathbf{a}'')-(20j^2+30j-\tfrac{2}{3})(\mathbf{Q}\cdot\mathbf{a})(\mathbf{a}''\cdot\mathbf{a}), \quad (6.10b)$$

$$X^{(3)}=j^{-1}(-3j^2+22j+15)(\mathbf{Q}\cdot\mathbf{a})(\mathbf{a}''\cdot\mathbf{a})+j^{-1}(20j^2+16j-9)(\mathbf{Q}\cdot\mathbf{a}''), \quad (6.10c)$$

and, for the $\frac{1}{2}^-$ sequence,

$$X^{(1)}=-4(j+1)^3(\mathbf{Q}\cdot\mathbf{a}'')+2(j+1)^2(j+2)(\mathbf{Q}\cdot\mathbf{a})(\mathbf{a}''\cdot\mathbf{a}), \quad (6.11a)$$

$$X^{(2)}=(j+1)(j+2)[(10j^2+15j+6)(\mathbf{Q}\cdot\mathbf{a}'')-(10j^2+15j+14/3)(\mathbf{Q}\cdot\mathbf{a})(\mathbf{a}''\cdot\mathbf{a})], \quad (6.11b)$$

$$X^{(3)}=(j+2)[(2j^2+3j-\tfrac{1}{2})(\mathbf{Q}\cdot\mathbf{a}'')-(j^2+3j-\tfrac{7}{2})(\mathbf{Q}\cdot\mathbf{a})(\mathbf{a}''\cdot\mathbf{a})]. \quad (6.11c)$$

Let a basis be defined in the decay configuration by

$$\mathbf{a}=\mathbf{n}^3,$$
$$\mathbf{a}''=\mathbf{n}^1\sin\theta''+\mathbf{n}^3\cos\theta'', \quad \cos\theta''=\mathbf{a}''\cdot\mathbf{a}.$$

Then the projected cross sections for the $\frac{1}{2}^-$ sequence are

$$d\sigma_{100}=R\cos\theta''[4|y+z|^2+(8/3)\,\mathrm{Re}y(y+z)^*+j^{-1}(17j^2+38j+6)|y|^2]\,d\cos\theta'', \quad (6.12a)$$

$$d\sigma_{1-10}=-d\sigma_{110}=(R/\sqrt{2})\sin\theta''[8(j+1)|y+z|^2+(20j^2+30j+\tfrac{2}{3})\,\mathrm{Re}y(y+z)^*\times j^{-1}(20j^2+16j-9)|y|^2]\,d\cos\theta''. \quad (6.12b)$$

The formulas for the $\frac{1}{2}^-$ sequence are

$$d\sigma_{100}=R\cos\theta''[-2j(j+1)^2|y+z|^2+\tfrac{4}{3}(j+1)(j+2)\,\mathrm{Re}y(y+z)^*+(j+2)(j^2+3)|y|^2]\,d\cos\theta'', \quad (6.13a)$$

$$d\sigma_{1-10}=d\sigma_{110}=(R/\sqrt{2})\sin\theta''[-4(j+1)^3|y+z|^2+(j+1)(j+2)(10j^2+15j+9)\,\mathrm{Re}y(y+z)^*+\tfrac{1}{2}(j+2)(4j^2+6j-1)|y|^2]\,d\cos\theta''. \quad (6.13b)$$

The ratios of the $M=0$ to the $M=\pm1$ terms are independent of the production and give information on the decay constants. One may hope that the spin and parity determination need not rely on formulas of this complexity. Once the spin and parity are known, however, such formulas may be needed if one wishes to determine the decay constants. The reader will appreciate why we do not wish to present the complete moment analysis.

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CP Violation in Nonleptonic K^0 Decays*

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The nonleptonic K^0 decays are examined on the basis of the CPT theorem and unitary symmetry without the requirement of CP invariance. It is shown that the present model (based on the CPT theorem and unitary symmetry) is consistent with the various experimental branching ratios of $K\to 2\pi$ modes, if CP invariance is almost maximally violated. Further, the decays $K_1^0\to 3\pi^0$ and $K_2^0\to 3\pi^0$ are forbidden by unitary symmetry in the framework of the boson pole model, even if CP invariance is violated.

APPARENT violation of CP invariance which appears in the decay mode $K_2^0\to\pi^+\pi^-$ has been reported.[1] This led to a number of attempts to explain the experimental result without CP violation.[2] We examine the interrelation between CP invariance and unitary symmetry (SU_3 invariance) in which the nonleptonic Lagrangian behaves as a member of the **8** and **27** representations and the strong interactions are invariant under the transformations of the group SU_3. It is convenient to introduce spurions of $I=\frac{1}{2}$ and $I=\frac{3}{2}$ so as to express the K decay modes in terms of SU_3 channel

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[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).

[2] J. S. Bell and J. K. Perring, Phys. Rev. Letters **13**, 348 (1964); J. Bernstein, N. Cabibbo, and T. D. Lee, Phys. Letters **12**, 146 (1964); S. Weinberg, Phys. Rev. Letters **13**, 495 (1964); T. D. Lee, Columbia University (unpublished).

amplitudes for which the requirement of CP invariance is not imposed. We do not imply that unitary symmetry is more relevant than CP invariance for the $K\to 2\pi$ decays, but merely examine this "old" problem from the present different point of view.

The states of the K_1^0 and K_2^0 mesons may be written on the basis of the CPT theorem as[3–5]

$$K_1^0=N(K^0-r\bar{K}^0)\,,\qquad K_2^0=N(K^0+r\bar{K}^0)\,, \tag{1}$$

where N is a renormalization constant, r is related to the off-diagonal elements of the mass squared matrix, and $CP|K^0)=-|\bar{K}^0)$.

The spurions $S_{1/2}$ and $S_{3/2}$ of isotopic spin $I=\frac{1}{2}(I_3=\pm\frac{1}{2})$ and $I=\frac{3}{2}(I_3=\pm\frac{1}{2})$ that carry one unit of strangeness transform as[6]

$$S_{1/2}=a(K'+\bar{K}')+b(K'-\bar{K}')\,,\qquad S_{3/2}=a(K''+\bar{K}'')+b(K''-\bar{K}'')\,, \tag{2}$$

where $a^2+b^2=1$. Here K', $\bar{K}'$, K'', and $\bar{K}''$ belong to the **8**, **8**, **27**, and **27** representations of SU_3, and $(K'\pm\bar{K}')$ and $(K''\pm\bar{K}'')$ have $CP=\pm 1$. The form of the nonleptonic interaction is not assumed at this stage.

With the aid of the spurions, the $K^0\to\pi^+\pi^-$ amplitudes are expressible as

$$\begin{aligned}
(K^0\bar{K}'|\pi^+\pi^-)_0&=(1/40)A_{27}+(1/10)A_{8s}-(1/8)A_1+(1/2\sqrt{5})A_{as}\,,\\
(\bar{K}^0K'|\pi^+\pi^-)_0&=(1/40)A_{27}+(1/10)A_{8s}-(1/8)A_1-(1/2\sqrt{5})A_{as}\,,\\
(K^0\bar{K}''|\pi^+\pi^-)_2&=-(70^{1/2}/56)B_{27s}+(6^{1/2}/24)B_{27a}\,,\\
(\bar{K}^0K''|\pi^+\pi^-)_2&=-(70^{1/2}/56)B_{27s}-(6^{1/2}/24)B_{27a}\,,
\end{aligned}$$

where the subscripts denote the isotopic spin I, and A_i and B_i are the channel amplitudes. When one considers the transition from a (parity-violating) spurion to the $K\pi\pi$ system, the state is totally symmetric among the three particles, so that[7]

$$\begin{aligned}
(K^0\bar{K}'|\pi^+\pi^-)_0&=(\bar{K}^0K'|\pi^+\pi^-)_0=\tfrac{1}{2}A_{8s}\,,\\
(K^0\bar{K}''|\pi^+\pi^-)_2&=(\bar{K}^0K''|\pi^+\pi^-)_0=-(70^{1/2}/56)B_{27s}\,.
\end{aligned} \tag{3}$$

The Eqs. (2) and (3) lead to

$$\begin{aligned}
&s(K^0S_{1/2}|\pi^+\pi^-)_0\\
&\quad=-s(K^0S_{1/2}|\pi^0\pi^0)_0=(\bar{K}^0S_{1/2}|\pi^+\pi^-)_0\\
&\quad=-(\bar{K}^0S_{1/2}|\pi^0\pi^0)_0=\tfrac{1}{2}A_{8s}(a+b)\,,\\
&s(K^0S_{3/2}|\pi^+\pi^-)_2\\
&\quad=(\bar{K}^0S_{3/2}|\pi^+\pi^-)_2=2s(K^0S_{3/2}|\pi^0\pi^0)_2\\
&\quad=2(\bar{K}^0S_{3/2}|\pi^0\pi^0)_2=\tfrac{1}{3}\sqrt{2}s(K^+|\pi^+\pi^0)_2\\
&\quad=\tfrac{1}{3}\sqrt{2}(K^-|\pi^-\pi^0)_2=-(70^{1/2}/56)B_{27s}(a+b)\,,
\end{aligned} \tag{4}$$

where

$$s=(a+b)/(a-b)\,.$$

Then, one obtains from Eqs. (1) to (4)

$$s(K^0|\pi^+\pi^-)=(\bar{K}^0|\pi^+\pi^-)\,, \tag{5}$$

$$R=\left|\frac{(K_2^0|\pi^+\pi^-)}{(K_1^0|\pi^+\pi^-)}\right|^2=\left|\frac{1+rs-Z(1+rs)}{(1-rs)-Z(1-rs)}\right|^2=\left|\frac{1+rs}{1-rs}\right|^2, \tag{6}$$

$$\alpha_1\equiv\frac{R(K_1^0|\pi^+\pi^-)}{R(K_1^0|\pi^0\pi^0)}=\left|\frac{\sqrt{2}(1-rs)-\sqrt{2}Z(1-rs)}{(1-rs)+2Z(1-rs)}\right|^2=\left|\frac{\sqrt{2}-\sqrt{2}Z}{1+2Z}\right|^2=\frac{R(K_2^0|\pi^+\pi^-)}{R(K_2^0|\pi^0\pi^0)}\equiv\alpha_2, \tag{7}$$

$$\beta=\left|\frac{(K^+|\pi^+\pi^0)}{(K_1^0|\pi^+\pi^-)}\right|^2=\left|\frac{3Z(1+|r|^2)^{1/2}/\sqrt{2}}{(1-rs)(1-Z)}\right|^2, \tag{8}$$

where

$$Z=(70^{1/2}/28)B_{27s}/A_{8s}\,.$$

Equations (5) to (8) are consequences of the CPT theorem and unitary symmetry when CP invariance is not imposed. They are valid in the presence of final state interactions.

The relations that follow from the CPT theorem and CP invariance of nonleptonic interactions without unitary symmetry are

$$(K^0|\pi^+\pi^-)=-(\bar{K}^0|\pi^+\pi^-)\,, \tag{9}$$

and Eqs. (6) to (8) with the replacement $s=-1$. Equations (7) and (8) with $r=-s=1$ are equivalent to those given by Dalitz.[8]

We now assume that the nonleptonic interaction is formed from a sum of products of weak currents and their Hermitian conjugates. This requirement is equivalent to the invariance of the interaction under the exchange of indices 2 and 3 when matrices are written in terms of tensor notation.[9] For the spurion, this corre-

[3] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **106**, 340 (1957); R. G. Sachs, Ann. Phys. (N. Y.) **22**, 239 (1964).

[4] R. G. Sachs, Phys. Rev. Letters **13**, 286 (1964).

[5] T. N. Truong, Phys. Rev. Letters **13**, 358a (1964); T. T. Wu and C. N. Yang, *ibid.* **13**, 380 (1964).

[6] It is assumed that K' and $\bar{K}'$ belong to the same octet; K'' and $\bar{K}''$ to the same 27-plet; and, further, that the $CP=\pm 1$ parts of $S_{1/2}$ and $S_{3/2}$ have the same coefficients a and b, because the assumption that they belong to different multiplets does not add anything new. In the interaction that is considered here, $CK'=-\bar{K}'$ and $CK''=-\bar{K}''$, so that $CPK'=\bar{K}'$ and $CPK''=\bar{K}''$.

[7] The symmetric condition can be imposed by requiring that $(K^0\bar{K}'|\pi^+\pi^-)=(\pi^-\bar{K}'|\bar{K}^0\pi^-)=(\pi^+\bar{K}'|\pi^+\bar{K}^0)$ and $(\bar{K}^0K'|\pi^+\pi^-)=(\pi^+K'|\pi^+K^0)=(\pi^-K'|K^0\pi^-)$ from which follows that $A_{8s}=A_{27}=-\frac{1}{3}A_1$ and $A_{as}=0$. Corresponding relations for $(K^0\bar{K}''|\pi^+\pi^-)$ lead to $B_{27a}=0$.

[8] R. H. Dalitz, *International Conference on Fundamental Aspects of Weak Interactions* (Brookhaven National Laboratory, Upton, New York, 1963), p. 378.

[9] N. Cabibbo, Phys. Rev. Letters **12**, 62 (1964); M. Gell-Mann, *ibid.* **12**, 155 (1964); K. Itabashi, Phys. Rev. **136**, B221 (1964); Y. Hara, Phys. Rev. Letters **12**, 378 (1964).

sponds to invariance under $K' \to \bar{K}'(K'' \to \bar{K}'')$ or $b=0, s=1$.[10]

If we combine this $s=1$ with $s=-1$ that follows when CP invariance of nonleptonic interactions is imposed, the relation $A_{8_s}=B_{27_s}=0$ follows from Eq. (5) with $s=1$ and Eq. (9). This is an alternative proof that then all the $K\to 2\pi$ decay modes vanish.[9] In other words, the $K\to 2\pi$ decay modes owe their existence to the violation of CP invariance or unitary symmetry or both, provided current-current form of the Lagrangian is assumed. Therefore, a model of $K\to 2\pi$ decays with CP invariance and violation of unitary symmetry is closely related to that with unitary symmetry and CP violation. The following model assumes that CP is violated in nonleptonic interactions.

We now attempt to find the values of rs, $|r|^2$, and Z that are consistent with the experimental data,[1,9] $R\approx 2.6\times 10^{-6}$, $\alpha_1\approx 2$, and $\beta=2\times 10^{-3}$. The choice

$$rs=-1+\epsilon,$$
$$|r|^2=1$$

leads to

$$R=|(\epsilon)/(2-\epsilon)|^2,$$
$$\alpha_1=2|(1-Z)/(1+2Z)|^2,$$
$$\beta=|3Z/2(1-Z)|^2.$$

The values $|\epsilon|\sim 3.2\times 10^{-3}$ and $|Z|\sim 0.03$ would fit R, α_1, and β. The value of rs is approximately -1. Since unitary symmetry is assumed here $s=1$ which leads to $r\approx -1$. This value of r requires $CP|K^0)=|\bar{K}^0)$ rather than $CP|K^0)=-|\bar{K}^0)$ which has been assumed. In this sense, we have maximum violation of CP invariance. [This is evident also because $s=1$ is taken rather than $s=-1$ that one found in Eq. (9) from CP invariance.]

It is amusing that a model that requires unitary symmetry, CPT theorem, and almost maximal violation of CP invariance can be consistent with the experimental data of $K\to 2\pi$ decays. In view of the fact that strong interactions respect CP invariance to a higher degree than unitary symmetry,[11] the present model is of academic interest. So far as the $K_1{}^0\to 2\pi$ and $K_2{}^0\to 2\pi$ decays are concerned, they are not forbidden by unitary symmetry alone, so that the existence of these modes does not give information on the possible violations of unitary symmetry.

We now examine the decay modes $K_1{}^0\to 3\pi^0$ and $K_2{}^0\to 3\pi^0$ in a similar manner within the framework of the boson pole model in which the structure of the diagrams consists of a weak vertex $(P|P)$, a boson propagator D, and the amplitude of strong interaction $(P|PPP)$ or $(PP|PP)$, where P represents the pseudoscalar mesons. The weak vertex is assumed to transform like $(K'+\bar{K}')$ and is CP-invariant, i.e., $(K^0|\pi^0)=(\bar{K}^0|\pi^0)$. It was shown that, if the strong interaction is SU_3-invariant (and also CP-invariant) the $K\to 3\pi$ decays are forbidden.[12]

When the weak vertex is not CP-invariant, one has from Eq. (2)

$$(\bar{K}^0|\pi^0)=s(K^0|\pi^0).$$

It is assumed that the weak vertex is independent of momentum. From Eq. (1) decays $K_1{}^0\to 3\pi^0$ and $K_2{}^0\to 3\pi^0$ are then expressible as

$$N(K^0|\pi^0)(1\mp sr)[-(\pi^0\pi^0|\pi^0\pi^0)+(K^0\pi^0|K^0\pi^0)+(K^0\pi^0|\pi^0K^0)+(K^0K^0|\pi^0\pi^0)]D=0$$

because the sum of the amplitudes in the square bracket vanishes.[12] In a similar way, the modes $K_1{}^0\to\pi^+\pi^-\pi^0$ and $K_2{}^0\to\pi^+\pi^-\pi^0$ also vanish. In other words, SU_3 invariance alone forbids these decays even if CP invariance is violated. Moreover, the mode $K_1{}^0\to 3\pi^0$ is forbidden while the mode $K_2{}^0\to 3\pi^0$ is allowed by CP invariance. However, if CP invariance is violated in nonleptonic decays, $K_1{}^0\to 3\pi^0$ should not be suppressed compared to the mode $K_2{}^0\to 3\pi^0$. The mode $K_2{}^0\to 3\pi^0$ gives information on the extent of SU_3 violation in the framework of the boson pole model.

The branching ratios of the nonleptonic decay of hyperons are given by relations similar to those of $K_1{}^0$ in Eq. (7) and thus are uneffected by maximal violation of CP invariance.

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[10] This statement is equivalent to Gell-Mann's statement (Ref. 9) that the strangeness-changing part of this nonleptonic interaction is the sixth component of a unitary octet plus 27-plet.

[11] The present knowledge of time reversal in nuclear interactions is discussed by E. M. Henley and B. A. Jacobsohn, Phys. Rev. **113**, 225 (1959).

[12] K. Tanaka, Phys. Rev. **136**, B1813 (1964). References to special cases are contained in this article.