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Weak Electromagnetic Decays of Hyperons in $SU(3)$*

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Two-body and three-body weak radiative decays of hyperons are studied in context of the $SU(3)$ scheme. The various sum rules that follow from T-L symmetries as well as those due to R invariance are derived. A pole model is set up for the two-body decays of both radiative and pionic type and is used to predict the amplitudes and decay rates for the photonic modes. In particular, the rate for $\Sigma^+ \to p+\gamma$ calculated in this model agrees with the recent experimental value.

I. INTRODUCTION

THE recent experimental information on the two- and three-body weak electromagnetic (WE) decays[1] together with the prospect of further information in the not too distant future has prompted this study of the problem. We analyze these decays in the light of $SU(3)$ symmetry,[2] first using pure symmetry arguments and then combining $SU(3)$ symmetry with a pole model as a dynamical assumption.

Over the past few years, along with the continuing support for strong interactions being approximately $SU(3)$-invariant, there has been increasing evidence that the principle of octet dominance[3] holds for weak, electromagnetic, and medium-strong interactions. According to this principle the weak, electromagnetic, and medium-strong interactions transform under $SU(3)$ predominantly like the members of an eightfold representation. Among other things octet dominance implies the $\Delta I=\frac{1}{2}$ rule.

In studying the symmetry properties of the WE decays we assume that the effective interaction Hamiltonian transforms like a product of two octet tensors as $T_1^1 \times T_2^3 + \text{H.c.}$ in Okubo's[4] notation. By imposing time-reversal invariance, T-L symmetries,[5] and R invariance for this Hamiltonian, we are able to derive sum rules among the various WE decay amplitudes.

Since symmetry arguments alone are not sufficient to determine the WE decay rates we are led to dynamical considerations. Several years ago it was shown[6] that the pole model could be profitably applied to the nonleptonic decay of baryons. Recently the pole model has been applied to nonleptonic decays[7] with definite $SU(3)$ properties assumed for the vertices. In view of the success of the pole model, and of $SU(3)$ in general, we are led to apply a pole model to the two-body WE decays, in which all vertices are assumed to have definite $SU(3)$ transformation properties. Several earlier attempts[8] at a pole model for the WE decays have been made in which no symmetry higher than isospin is assumed. This part of the present work then, represents an updating of the earlier attempts in light of $SU(3)$.

We assume that the weak vertex in our pole model for the WE decays is exactly the same as the one appearing in the pole model for the nonleptonic decays of hyperons. Therefore, we first set up and solve the pole model for the pionic decays of hyperons. The method employed is very close to that of Sugawara and of Lee and Swift; we will give the pole-model solution for the parity-conserving (p.c.) and parity-violating (p.v.) amplitudes for completeness and to establish our notation. In the process it is found that the effective weak Hamiltonian must be $TL(1)\times P$ invariant if we wish to treat the p.v. amplitude by means of baryon poles.

The parameters describing the effective weak vertex are determined from the experimental data on pionic decays and are then used in the pole model for WE

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decays to evaluate the WE decay amplitudes and decay rates. The value we obtain for the decay rate of $\Sigma^+ \to p+\gamma$ agrees well with the recent experimental value. We also predict decay rates for $\Xi \to \Sigma+\gamma$ and $\Lambda \to n+\gamma$. The former are the only energetically allowed decays of Ξ into Σ.

In Sec. II we examine the consequences of assuming specific $SU(3)$ properties for the WE decays from a pure symmetry point of view. In Sec. III the pole model is applied to the pionic decays of hyperons. The solution of this model is applied to a pole model for the two-body WE decays in Sec. IV.

II. PURE SYMMETRY APPROACH

In this section we shall use pure symmetry arguments to obtain relations between different decay amplitudes for both the two-body and the three-body weak electromagnetic decays of the hyperons. Throughout this work we have taken the point of view that the effective nonleptonic weak interaction is $TL(2)\times P$-invariant when written in a nonderivative form of coupling. The motivation for using this symmetry comes from the demonstration by Rosen[5] that this is the weakest symmetry which predicts Lee's sum rule[9]

$$\sqrt{3}\langle\Sigma^+|p\pi^0\rangle-\langle\Lambda|p\pi^-\rangle=2\langle\Xi^-|\Lambda\pi^-\rangle \qquad \text{(II.1)}$$

for both p.c. and p.v. amplitudes. This requirement leads to restrictions on the other weak couplings. The two-body weak vertex to be used in the pole model in Sec. III is found to be $TL(1)\times P$-invariant. (The difference in symmetry properties between the two-and three-body weak Hamiltonians is traceable to the odd parity of the pion.) The effective interaction for the two-body WE decays of the type

$$B\to B'+\gamma \qquad \text{(II.2)}$$

is also $TL(1)\times P$ invariant, whereas the effective interaction for the three-body WE decays of the type

$$B\to B'+\pi+\gamma \qquad \text{(II.3)}$$

is $TL(2)\times P$-invariant.

Two-Body Decays

There are six possible decay modes of this kind:

$$\begin{aligned}\Sigma^+&\to p+\gamma\,,\\ \Xi^-&\to\Sigma^-+\gamma\,,\\ \Xi^0&\to\Sigma^0+\gamma\,,\\ \Xi^0&\to\Lambda+\gamma\,,\\ \Lambda&\to n+\gamma\,,\\ \Sigma^0&\to n+\gamma\,.\end{aligned} \qquad \text{(II.4)}$$

[9] B. W. Lee, Phys. Rev. Letters **12**, 83 (1964). H. Sugawara, Progr. Theoret. Phys. (Kyoto) **31**, 212 (1964).

We assume for the effective coupling, a space-time structure of the form

$$H_{\mathrm{WE}(2)}=i/2\bar{\Psi}_{B'}(C+D\gamma_5)\sigma_{\mu\nu}\Psi_B F_{\mu\nu} \qquad \text{(II.5)}$$

which is gauge invariant. We further assume: (a) that the electromagnetic current transforms as the $T_1{}^1$ component of an octet, (b) that the weak interaction transforms as the $T_2{}^3$ component of an octet, and (c) CP invariance.

(a) and (b) imply that the interaction transforms as the $T_{21}{}^{31}$ component of representations contained in the product $\mathbf{8}\otimes\mathbf{8}$. Under these assumptions there are four independent parameters for p.c. as well as for p.v. amplitudes. The interaction, suppressing the space time structure, is of the form

$$\begin{aligned}H_{\mathrm{WE}(2)}(\text{p.c.})=&\,\alpha(\bar{B}_2{}^3B_1{}^1+\text{H.c.})\\ &+3\beta(\bar{B}_1{}^3B_2{}^1-\tfrac{1}{3}\bar{B}_i{}^3B_2{}^i+\text{H.c.})\\ &+\gamma(B_1{}^1B_2{}^3+\text{H.c.})\\ &+3\delta(\bar{B}_2{}^1B_1{}^3-\tfrac{1}{3}\bar{B}_2{}^iB_i{}^3+\text{H.c.})\,.\end{aligned} \qquad \text{(II.6)}$$

The p.v. part has the same $SU(3)$ structure with coupling constants $\alpha', \beta', \gamma', \delta'$. The requirement of $TL(1)\times P$ invariance, which is consistent with our basic assumption about weak-interaction symmetry, leads to

$$\alpha=\gamma$$

and

$$\alpha'=\gamma'. \qquad \text{(II.7)}$$

These conditions lead to the following three sum rules, valid for both p.c. and p.v. amplitudes:

$$\sqrt{3}\langle\Lambda|n\gamma\rangle-\langle\Sigma^0|n\gamma\rangle=2\langle\Xi^0|\Sigma^0\gamma\rangle\,, \qquad \text{(II.8a)}$$

$$2\langle\Lambda|n\gamma\rangle-\langle\Xi^0|\Lambda\gamma\rangle=\sqrt{3}\langle\Xi^0|\Sigma^0\gamma\rangle\,, \qquad \text{(II.8b)}$$

$$\langle\Sigma^+|p\gamma\rangle+\langle\Xi^-|\Sigma^-\gamma\rangle=-2(\sqrt{\tfrac{2}{3}})\{\langle\Xi^0|\Lambda\gamma\rangle+\langle\Lambda|n\gamma\rangle\}\,. \qquad \text{(II.8c)}$$

We next withdraw the requirement of $TL(1)\times P$ invariance and impose, instead, R-conjugation invariance. Although the strong interactions are not invariant under R conjugation, it is hoped that this symmetry may have some validity for the weak interactions (e.g., the prediction of $\alpha_\Lambda=-\alpha_{\Xi^-}$).[5] Under R invariance, then, we find

$$\begin{aligned}\alpha&=-\gamma\,, & \alpha'&=\gamma'\,,\\ \beta&=-\delta\,, & \beta'&=\delta'\,.\end{aligned} \qquad \text{(II.9)}$$

(Note that the photon must be odd under R conjugation if the electromagnetic interaction is to be R-invariant.) We now find for the p.c. amplitudes

$$\langle\Lambda|n\gamma\rangle=\langle\Xi^0|\Lambda\gamma\rangle\,, \qquad \text{(II.10a)}$$

$$\langle\Xi^0|\Sigma^0\gamma\rangle=\langle\Sigma^0|n\gamma\rangle\,, \qquad \text{(II.10b)}$$

$$\langle\Sigma^+|p\gamma\rangle=\langle\Xi^-|\Sigma^-\gamma\rangle\,, \qquad \text{(II.10c)}$$

and for the p.v. amplitudes

$$\langle\Lambda|n\gamma\rangle=\frac{1}{\sqrt{3}}\langle\Xi^0|\Sigma^0\gamma\rangle=-\frac{1}{\sqrt{3}}\langle\Sigma^0|n\gamma\rangle$$
$$=-\langle\Sigma^0|\Lambda\gamma\rangle\,, \qquad \text{(II.11a)}$$
$$\langle\Sigma^+|p\gamma\rangle=-\langle\Xi^-|\Sigma^-\gamma\rangle\,. \qquad \text{(II.11b)}$$

If we impose both $TL(1)\times P$ and R invariance on the two-body WE decay interaction, then for the p.v. amplitudes the results are the same as for R invariance alone, while for the p.c. interaction we have

$$\alpha=\gamma=0$$
$$\beta=-\delta \qquad \text{(II.12)}$$

and so

$$\langle\Sigma^+|p\gamma\rangle=\langle\Xi^-|\Sigma^-\gamma\rangle=-2(\tfrac{2}{3})^{1/2}\langle\Lambda|n\gamma\rangle$$
$$=-2(\tfrac{2}{3})^{1/2}\langle\Sigma^0|\Lambda\gamma\rangle=-2\sqrt{2}\langle\Xi^0|\Sigma^0\gamma\rangle \qquad \text{(II.13)}$$
$$=-2\sqrt{2}\langle\Sigma^0|n\gamma\rangle\,.$$

We would like to compare the above discussion with that of Hara[10] who uses $TL(1)$ invariance. This leads to the same results for the p.c. amplitudes which we get by imposing $TL(1)\times P$, but for the p.v. amplitudes $TL(1)$ invariance gives

$$\alpha'=-\gamma'\,,$$
$$\beta'=\delta'=0\,, \qquad \text{(II.14)}$$

and we obtain

$$\langle\Sigma^+|p\gamma\rangle=\langle\Xi^-|\Sigma^-\gamma\rangle=0\,, \qquad \text{(II.15)}$$
$$\langle\Lambda|n\gamma\rangle=\langle\Xi^0|\Lambda\gamma\rangle=-\sqrt{3}\langle\Sigma^0|n\gamma\rangle=\sqrt{3}\langle\Xi^0|\Sigma^0\gamma\rangle\,. \qquad \text{(II.16)}$$

In this case R invariance coupled with $TL(1)$ invariance requires all p.v. amplitudes to vanish.

Three-Body Decays

There are seven different decay processes of the type in Eq. (II.3) which are obtained by simply adding a photon to the seven observable nonleptonic hyperon decays. They are

$$\Sigma^-\to n+\pi^-+\gamma\,,$$
$$\Sigma^+\to n+\pi^++\gamma\,,$$
$$\Sigma^+\to p+\pi^0+\gamma\,,$$
$$\Lambda\to p+\pi^-+\gamma\,, \qquad \text{(II.17)}$$
$$\Lambda\to n+\pi^0+\gamma\,,$$
$$\Xi^-\to\Lambda+\pi^-+\gamma\,,$$
$$\Xi^0\to\Lambda+\pi^0+\gamma\,.$$

Since the $\Delta I=\frac{1}{2}$ rule is not applicable all seven have independent amplitudes.

We assume a space-time structure of the form

$$H_{\mathrm{WE}(3)}=i\Psi_{B_2}(g+g'\gamma_5)\sigma_{\mu\nu}\Psi_{B_1}\varphi F_{\mu\nu} \qquad \text{(II.18)}$$

[10] Y. Hara, Phys. Rev. Letters **12**, 378 (1964).

for the effective coupling. This is not the only form of interaction possible but is the simplest gauge-invariant term and we use it to illustrate our approach. We also make the assumptions (a), (b), and (c) as before. There are 24 possible coupling terms, of which eight do not contribute because they involve only K or η^0 mesons. The remaining 16 are

$$H_{\mathrm{WE}(3)}(\text{p.c.})$$
$$=f_1\bar{B}_i{}^3B_2{}^iP_1{}^1+f_2\bar{B}_i{}^3B_1{}^1P_2{}^i+3f_3\{\bar{B}_i{}^3B_1{}^iP_2{}^1$$
$$-\tfrac{1}{3}\bar{B}_i{}^3B_j{}^iP_2{}^j\}+3f_4\{\bar{B}_i{}^3B_2{}^1P_1{}^i-\tfrac{1}{3}\bar{B}_i{}^3B_2{}^jP_j{}^i\}$$
$$+3f_5\{\bar{B}_2{}^1B_i{}^3P_1{}^i-\tfrac{1}{3}\bar{B}_2{}^jB_i{}^3P_j{}^i\}+f_6\bar{B}_1{}^1B_i{}^3P_2{}^i$$
$$+f_7\bar{B}_2{}^iB_i{}^3P_1{}^1+3f_8\{\bar{B}_2{}^iB_1{}^3P_i{}^1-\tfrac{1}{3}\bar{B}_2{}^iB_j{}^3P_i{}^j\}$$
$$+3f_9\{\bar{B}_i{}^1B_1{}^3P_2{}^i-\tfrac{1}{3}\bar{B}_i{}^jB_j{}^3P_2{}^i\}$$
$$+3f_{10}\{\bar{B}_i{}^1B_2{}^3P_1{}^i-\tfrac{1}{3}\bar{B}_i{}^jB_2{}^3P_i{}^j\}$$
$$+3f_{11}\{\bar{B}_2{}^3B_1{}^iP_i{}^1-\tfrac{1}{3}\bar{B}_2{}^3B_j{}^iP_i{}^j\}$$
$$+3f_{12}\{\bar{B}_2{}^3B_i{}^1P_1{}^i-\tfrac{1}{3}\bar{B}_2{}^3B_i{}^jP_j{}^i\}$$
$$+3f_{13}\{\bar{B}_1{}^3B_2{}^iP_i{}^1-\tfrac{1}{3}\bar{B}_j{}^3B_2{}^iP_i{}^j\}$$
$$+3f_{14}\{\bar{B}_1{}^3B_i{}^1P_2{}^i-\tfrac{1}{3}B_j{}^3B_i{}^jP_2{}^i\}$$
$$+3f_{15}\{\bar{B}_1{}^iB_i{}^3P_2{}^1-\tfrac{1}{3}\bar{B}_j{}^iB_i{}^3P_2{}^j\}$$
$$+3f_{16}\{\bar{B}_1{}^iB_2{}^3P_i{}^1-\tfrac{1}{3}\bar{B}_j{}^iB_2{}^3P_i{}^j\}+\text{H.c.} \qquad \text{(II.19)}$$

We have omitted the space-time structure in Eq. (II.19) and so identical expressions can be written for p.v. amplitudes. Since there are only seven decay modes and 16 independent couplings in Eq. (II.19) there are no relations between any decay amplitudes. If we impose $TL(2)\times P$ invariance which follows from our assumption about the effective weak interaction, then we are left with ten independent amplitudes and hence no relations among the decay amplitudes are obtained. We turn to R invariance to provide more restrictions on the amplitudes.

Since R invariance distinguishes between p.c. and p.v amplitudes we consider them separately. For p.c. amplitudes R invariance alone gives the following relations:

$$\langle\Xi^-|\Lambda\pi^-\gamma\rangle=\langle\Lambda|p\pi^-\gamma\rangle\,, \qquad \text{(II.20a)}$$
$$\langle\Xi^0|\Lambda\pi^0\gamma\rangle=-\langle\Lambda|n\pi^0\gamma\rangle\,. \qquad \text{(II.20b)}$$

Imposing both $TL(2)\times P$ invariance and R invariance on the interaction leaves five independent p.c. amplitudes. It would seem that apart from Eq. (II.20) there should be no sum rules. However, it turns out there is a sum rule among the five p.c. matrix elements which is

$$(\sqrt{6})\{\langle\Lambda|p\pi^-\rangle+\sqrt{2}\langle\Lambda|n\pi^0\gamma\rangle\}$$
$$=\{\langle\Sigma^-|n\pi^-\gamma\rangle+\langle\Sigma^+|n\pi^+\gamma\rangle-\sqrt{2}\langle\Sigma^+|p\pi^0\gamma\rangle\}\,. \qquad \text{(II.21)}$$

R invariance alone gives the following relations for the p.v. amplitudes:

$$\langle\Xi^-|\Lambda\pi^-\gamma\rangle=-\langle\Lambda|p\pi^-\gamma\rangle\,, \qquad \text{(II.22a)}$$
$$\langle\Xi^0|\Lambda\pi^0\gamma\rangle=\langle\Lambda|n\pi^0\gamma\rangle\,. \qquad \text{(II.22b)}$$

Restriction to invariance under both $TL(2)\times P$ and R reduces the number of independent amplitudes to five again. This time we find a different sum rule for the p.v. amplitudes

$$(\sqrt{6})\{\langle\Lambda|p\pi^-\gamma\rangle+\sqrt{2}\langle\Lambda|n\pi^0\gamma\rangle\} =\{\langle\Sigma^+|n\pi^+\gamma\rangle-7\langle\Sigma^-|n\pi^-\gamma\rangle -5\sqrt{2}\langle\Sigma^+|p\pi^0\gamma\rangle\}\,. \quad \text{(II.23)}$$

Summarizing, we have found using $TL(2)\times P$ invariance and R invariance for the three-body WE decays of hyperons, (i) the sum rule (II.21) for the p.c amplitudes, (ii) the sum rule (II.23) for the p.v. amplitudes and (iii) the relations

$$\langle\Xi^-|\Lambda\pi^-\gamma\rangle_\pm=\pm\langle\Lambda|p\pi^-\gamma\rangle_\pm\,, \quad \text{(II.24a)}$$

$$\langle\Xi^0|\Lambda\pi^0\gamma\rangle_\pm=\mp\langle\Lambda|n\pi^0\gamma\rangle_\pm\,, \quad \text{(II.24b)}$$

where $+$ and $-$ stand for p.c. and p.v. amplitudes, respectively. Thus with these assumptions we expect parity-violating effects in $\langle\Xi^-|\Lambda\pi^-\gamma\rangle$ and $\langle\Xi^0|\Lambda\pi^0\gamma\rangle$ to have opposite signs from the corresponding effects in $\langle\Lambda|p\pi^-\gamma\rangle$ and $\langle\Lambda|n\pi^0\gamma\rangle$, respectively.

For completeness we would like to mention the consequences of making the interaction $TL(1)$ invariant. The reason for interest in $TL(1)$ invariance is that it follows from a current-current theory[11] of weak interactions with Cabibbo[12]-type currents and CP invariance. With $TL(1)$ invariance everything said above about p.v amplitudes remains unchanged; for the p.c. amplitudes we lose the sum rule (II.21) since the number of amplitudes contributing becomes six.

III. POLE MODEL FOR NONLEPTONIC DECAYS

We will assume here that the baryon nonleptonic weak decay amplitudes are dominated by baryon and meson poles.[6] The effective, nonleptonic weak Hamiltonian describing the process $B\to B'+\pi$ is given by

$$H_{NL}=\bar\Psi_{B'}(A-B\gamma_5)\Psi_B\Phi_\pi\,. \quad \text{(III.1)}$$

The pole model supposes that the amplitudes, A and B, receive contributions only from pole diagrams in which there is an effective, weak (baryon-baryon or meson-meson) vertex and a strong vertex (see, e.g., Fig. 1). We will see that the weak vertices depend on five parameters. It is the purpose of this section to determine these parameters from the experimental information available on nonleptonic decays. We will then use these same parameters to describe the weak vertices appearing in our pole model for the WE decays in the next section.

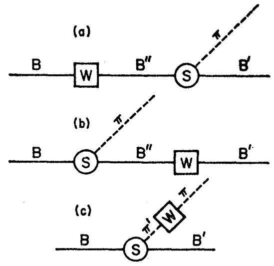

FIG. 1. Typical pole diagrams contributing to nonleptonic decays that are considered in Sec. III.

We make the usual assignment of the total weak Hamiltonian to the octet representation of $SU(3)$, and hence automatically satisfy the $\Delta I=\frac{1}{2}$ rule. We also assume time-reversal invariance for all Hamiltonians used here. We are led to describe the weak two-body vertices by the effective Hamiltonian

$$H_w=a(\bar B_i^3B_2^i+\bar B_i^2B_3^i)+b(\bar B_2^iB_i^3+\bar B_3^iB_i^2) +a'(\bar B_i^3\gamma_5B_2^i-\bar B_i^2\gamma_5B_3^i) +b'(\bar B_2^i\gamma_5B_i^3-\bar B_3^i\gamma_5B_i^2) -c(P_i^3P_2^i+P_i^2P_3^i)\,, \quad \text{(III.2)}$$

where a, b, a', b', and c are real constants.[13] H_w is manifestly a member of an $SU(3)$ octet and is, necessarily, $TL(1)\times P$-invariant.[14] Our assumption of $TL(1)\times P$ invariance for H_w allows us to evaluate the p.v. amplitudes in the same way that we evaluate the p.c. amplitudes.

The strong baryon-meson Hamiltonian to be used in the following is:

$$H_s=(1-f)\sqrt{2}g\,\mathrm{Tr}(\bar BPB+\bar BBP) +f\sqrt{2}g\,\mathrm{Tr}(\bar BPB-\bar BBP)\,, \quad \text{(III.3)}$$

where $g=-g_{PP\pi^0}$ is taken to have the value[15] 13.5 and f is a parameter, to be determined and governing the mixture of F- and D-type couplings of the mesons to the baryons.

By making use of Eqs. (III.2) and (III.3) and the appropriate diagrams of the type shown in Fig. 1 we may now express the p.c. amplitudes B in terms of the four parameters a, b, c, and f and the p.v. amplitudes A in terms of a', b', and f. To determine these parameters from experiment we will work with the decays: $\Lambda\to p+\pi^-$, $\Xi^-\to\Lambda+\pi^-$, $\Sigma^+\to n+\pi^+$, $\Sigma^-\to n+\pi^-$. These decays are represented below by the notation Λ_-^0, Ξ_-^-, Σ_+^+, and Σ_-^-, respectively. The amplitudes for the decays are found to be

$$A(\Lambda_-^0)=\frac{-2g(1-f)}{\sqrt{3}}\frac{b'}{\Sigma+N}-\frac{g}{\sqrt{3}}\frac{(b'-2a')}{\Lambda+N}\,, \quad \text{(III.4)}$$

$$B(\Lambda_-^0)=\frac{2g(1-f)}{\sqrt{3}}\frac{b}{\Sigma-N}-\frac{g}{\sqrt{3}}\frac{(b-2a)}{\Lambda-N} +\frac{g(1+2f)}{\sqrt{3}}\frac{c}{K^2-\pi^2}\,, \quad \text{(III.5)}$$

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[11] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[12] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[13] We assume that the matrix elements of H_w are obtained by replacing the B's by Dirac spinors. This amounts to assuming that the effective weak vertex does not depend on the momentum transfer across it.

[14] Lee and Swift (Ref. 7) choose their effective weak Hamiltonian to transform like λ_6 in Gell-Mann's notation (Ref. 2). This makes their Hamiltonian $TL(1)$-invariant, and when combined with time-reversal invariance makes the p.v. part to vanish identically. To survive, the p.v. part must be $TL(2)$-invariant.

[15] This value is based on the calculation of J. Hamilton and H. Woolcock, Rev. Mod. Phys. **35**, 737 (1963).

$$A(\Xi_-^-)=\frac{g}{\sqrt{3}}\frac{(1-2f)(2b'-a')}{\Xi+\Lambda}-\frac{2g}{\sqrt{3}}\frac{(1-f)a'}{\Xi+\Sigma}, \quad \text{(III.6)}$$

$$B(\Xi_-^-)=\frac{-q}{\sqrt{3}}\frac{(1-2f)(2b-a)}{\Xi-\Lambda}-\frac{2g}{\sqrt{3}}\frac{(1-f)}{\Xi-\Sigma}-\frac{g(4f-1)}{\sqrt{3}}\frac{c}{K^2-\pi^2}, \quad \text{(III.7)}$$

$$A(\Sigma_+^+)=\frac{\sqrt{2}g(1+f)b'}{\Sigma+N}+\frac{\sqrt{2}g(1-f)(b'-2a')}{3(\Lambda+N)}, \quad \text{(III.8)}$$

$$B(\Sigma_+^+)=\frac{\sqrt{2}g(1-f)b}{\Sigma-N}-\frac{\sqrt{2}g(1-f)(b-2a)}{3(\Lambda-N)}, \quad \text{(III.9)}$$

$$A(\Sigma_-^-)=-\frac{\sqrt{2}gfb'}{\Sigma+N}+\frac{\sqrt{2}g(1-f)(b'-2a')}{3(\Lambda+N)}, \quad \text{(III.10)}$$

$$B(\Sigma_-^-)=\frac{\sqrt{2}gfb}{\Sigma-N}-\frac{\sqrt{2}g(1-f)(b-2a)}{3(\Lambda-N)}-\sqrt{2}g(2f-1)\frac{c}{K^2-\pi^2}. \quad \text{(III.11)}$$

In the above equations we have written the particle symbol for its mass. In our calculations we have taken this mass to be the mean mass of the isospin multiplet to which the particle belongs.

Our pole model for the non-leptonic decays is essentially the same as that of Sugawara[7] and of Lee and Swift.[7] However, unlike Sugawara we have kept the K-pole terms and unlike Lee and Swift we have been able to use the baryon pole model for p.v. amplitudes because of our use of $TL(1)\times P$ invariance.

The experimental data[16] available on nonleptonic decays is used to evaluate amplitudes A and B as summarized in Table I. We assume that Σ_+^+ is pure p-wave and Σ_-^- is pure s-wave. It has been shown before[17] that the alternative assignment is inconsistent with the experimental data in a pole model of this kind.

For the p.c. amplitudes we have four parameters a, b, c, and f to be fitted to four experimental numbers $B(\Sigma_+^+)$, $B(\Sigma_-^-)$, $B(\Lambda_-^0)$, and $B(\Xi_-^-)$. The solutions are given in Table II. The value of f we obtain, $f=0.358$, is in good agreement with other estimates.[18] For p.v. amplitudes we use the value of f determined as above and fit a' and b' to the four experimental numbers $A(\Sigma_+^+)$, $A(\Sigma_-^-)$, $A(\Lambda_-^0)$, and $A(\Xi_-^-)$. The values of a' and b' chosen represent a best fit to the data, which is within 20%. The discrepancy can be accounted for by Σ_+^+ and Σ_-^- not being in pure orbital angular momentum channels and by other contributions that we have neglected. In any case, this discrepancy is not important as far as this work is concerned, because it is the p.c. amplitudes that make the dominant contribution to the rates of the WE decays. The values of a, b, f, and a', and b' are listed in Table II.

TABLE I. Decay amplitudes for nonleptonic decays derived from experiment assuming that Σ_+^+ is pure p-wave and Σ_-^- is pure s-wave.

Decay	A	B
$\Xi^-\to\Lambda+\pi^-$	4.69×10^{-7}	-1.66×10^{-6}
$\Lambda\to p+\pi^-$	-3.54×10^{-7}	-2.36×10^{-6}
$\Sigma^+\to n+\pi^+$	0	4.29×10^{-6}
$\Sigma^-\to n+\pi^-$	4.1×10^{-7}	0

TABLE II. Solution for the five parameters a, b, f, a', b' from Eqs. (III.4) to (III.11) using the data of Table I.

a	-3.51×10^{-4} MeV
b	8.36×10^{-4} MeV
f	0.358
a'	-9.04×10^{-5} MeV
b'	-2.54×10^{-5} MeV

IV. POLE MODEL FOR WEAK ELECTROMAGNETIC DECAYS

The weak electromagnetic decays of hyperons, $B\to B'+\gamma$, are described by an effective Hamiltonian of the form

$$H_{\rm WE}=\tfrac{1}{2}i\bar{\Psi}_{B'}(C+D\gamma_5)\sigma_{\mu\nu}\Psi_B F_{\mu\nu}. \quad \text{(IV.1)}$$

From this the decay rate is found to be[19]

$$R=\frac{1}{8\pi}\left\{\frac{m_B^2-m_{B'}^2}{m_B}\right\}^3(|C|^2+|D|^2) \quad \text{(IV.2)}$$

and the angular distribution of the final baryon B' in the rest system of the initial baryon B is

$$w(\theta)d\theta\propto(1+\alpha P_B\cos\theta), \quad \text{(IV.3)}$$

where

$$\alpha=2\,\mathrm{Re}(C^*D)/(|C|^2+|D|^2). \quad \text{(IV.4)}$$

P_B' is the polarization of initial baryon B and θ is the angle between the direction of polarization of B and the momentum of B'.

It is our purpose now to relate C and D, through a pole model to the pionic decay parameters determined in the last section. We assume that the WE decays are dominated by baryon poles and typical pole diagrams are shown in Fig. 2. The weak vertices are described by the same effective Hamiltonian, Eq. (III.2), used in the

[16] M. L. Stevenson *et al.*, Phys. Letters **9**, 349 (1964).
[17] See Ref. 7. See also M. Sugawara and T. Sakuma, Phys. Rev. **135**, B260 (1964).
[18] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963); R. E. Cutkosky, Ann. Phys. **23**, 415 (1963); G. Murtaza and M. A. Rashid, Phys. Letters **8**, 370 (1964); W. Willis *et al.*, Phys. Rev. Letters **13**, 291 (1964). See also Lee and Swift Ref. 7.
[19] R. E. Behrends, Phys. Rev. **111**, 1691 (1958).

TABLE III. Partial decay rates for weak electromagnetic decays expressed in terms of hyperon magnetic moments.

Decay	R_+ (in $\sec^{-1}$)	R_- (in $\sec^{-1}$)
$\Sigma^+ \to p+\gamma$	$(8.85\mu_{\Sigma^+}-20.2)^2\times 10^6$	$(31.7\mu_{\Sigma^+}+72)^2$
$\Xi^- \to \Sigma^- + \gamma$	$(9.07\mu_{\Sigma^-}-8.22\mu_{\Xi^-})^2\times 10^5$	$(36.3\mu_{\Sigma^-}+33\mu_{\Xi^-})^2$
$\Lambda \to n+\gamma$	$(19.5\mu_\Lambda+12.4\mu_T+44.2)^2\times 10^5$	$(53.5\mu_\Lambda+19.9\mu_T-121)^2$
$\Xi^0 \to \Sigma^0+\gamma$	$(18.5\mu_{\Xi^0}-20.4\mu_{\Sigma^0}-4.57\mu_T)^2\times 10^4$	$(23.2\mu_{\Xi^0}+25.5\mu_{\Sigma^0}+6.9\mu_T)^2$
$\Xi^0 \to \Lambda+\gamma$	$(23.5\mu_{\Xi^0}-27.7\mu_\Lambda-13.1\mu_T)^2\times 10^5$	$(11.9\mu_{\Xi^0}+14.1\mu_\Lambda+51.7)^2$

pole model for the pionic decay modes in the last section. We describe the electromagnetic vertex by[20]

$$H_E = ie\bar{\Psi}_B\gamma_\mu\Psi_B A_\mu + \tfrac{1}{2}ie\bar{\Psi}_B(\mu_B/M_B)\sigma_{\mu\nu}\Psi_B F_{\mu\nu}. \quad \text{(IV.5)}$$

We assume that it transforms as T_1^1 member of an $SU(3)$ octet.[21]

Now, from Eqs. (II.5) and (III.2) and the pole diagrams appropriate to various decays, we find for the WE decay amplitudes, C and D, the expressions:

$$C(\Sigma^+\to p\gamma) = e\left(\frac{\mu_{\Sigma^+}}{2\Sigma}-\frac{\mu_p}{2N}\right)\frac{b}{\Sigma-N}, \quad \text{(IV.6)}$$

$$D(\Sigma^+\to p\gamma) = e\left(\frac{\mu_{\Sigma^+}}{2\Sigma}+\frac{\mu_p}{2N}\right)\frac{b'}{\Sigma+N}, \quad \text{(IV.7)}$$

$$C(\Lambda\to n\gamma) = e\left(\frac{\mu_\Lambda}{2\Lambda}-\frac{\mu_n}{2N}\right)\frac{(b-2a)/\sqrt{6}}{\Lambda-N} + \frac{e\mu_T}{\Sigma+\Lambda}\frac{b/\sqrt{2}}{(\Sigma-N)}, \quad \text{(IV.8)}$$

$$D(\Lambda\to n\gamma) = e\left(\frac{\mu_\Lambda}{2\Lambda}+\frac{\mu_n}{2N}\right)\frac{(b'-2a')/\sqrt{6}}{\Lambda+N} + \frac{e\mu_T}{\Sigma^0+\Lambda}\frac{b'}{\sqrt{2}(\Sigma+N)}, \quad \text{(IV.9)}$$

$$C(\Xi^0\to \Sigma^0\gamma) = e\left(\frac{\mu_{\Sigma^0}}{2\Sigma}-\frac{\mu_{\Xi^0}}{2\Xi}\right)\frac{a/\sqrt{2}}{\Xi-\Sigma} - \frac{e\mu_T}{\Sigma+\Lambda}\frac{(2b-a)/\sqrt{6}}{\Xi-\Lambda}, \quad \text{(IV.10)}$$

$$D(\Xi^0\to \Sigma^0\gamma) = e\left(\frac{\mu_{\Sigma^0}}{2\Sigma}+\frac{\mu_{\Xi^0}}{2\Xi}\right)\frac{a'/\sqrt{2}}{\Xi+\Sigma} + \frac{e\mu_T}{\Sigma+\Lambda}\frac{(2b'-a')/\sqrt{6}}{\Xi+\Lambda}, \quad \text{(IV.11)}$$

$$C(\Xi^0\to \Lambda\gamma) = e\left(\frac{\mu_{\Xi^0}}{2\Xi}-\frac{\mu_\Lambda}{2\Lambda}\right)\frac{(2b-a)/\sqrt{6}}{\Xi-\Lambda} + \frac{e\mu_T}{\Sigma+\Lambda}\frac{a/\sqrt{2}}{\Xi-\Sigma}, \quad \text{(IV.12)}$$

$$D(\Xi^0\to \Lambda\gamma) = e\left(\frac{\mu_{\Xi^0}}{2\Xi}+\frac{\mu_\Lambda}{2\Lambda}\right)\frac{(2b'-a')/\sqrt{6}}{\Xi+\Lambda} - \frac{e\mu_T}{\Sigma+\Lambda}\frac{a'/\sqrt{2}}{\Xi+\Sigma}, \quad \text{(IV.13)}$$

$$C(\Xi^-\to \Sigma^-\gamma) = e\left(\frac{\mu_{\Sigma^-}}{2\Sigma}-\frac{\mu_{\Xi^-}}{2\Xi}\right)\frac{a}{\Xi-\Sigma}, \quad \text{(IV.14)}$$

$$D(\Xi^-\to \Sigma^-\gamma) = e\left(\frac{\mu_{\Sigma^-}}{2\Sigma}+\frac{\mu_{\Xi^-}}{2\Xi}\right)\frac{a'}{\Xi+\Sigma}. \quad \text{(IV.15)}$$

Here μ_T is the transition magnetic moment $\mu(\Sigma^0-\Lambda)$ describing the vertex $\Sigma^0\to\Lambda+\gamma$. We have not considered $\Sigma^0\to n+\gamma$ since it is expected to be swamped by the large rate for $\Sigma^0\to\Lambda+\gamma$.

Using the values of the parameters a, b, a', and b' determined in the last section (see Table II) and Eq. (IV.2) we have found the WE decay rates as functions of the relevant magnetic moments. These are shown in Table III. R_+ is the partial decay rate due to the p.c. amplitude C and R_- is due to the p.v. amplitude D.

If we assume that the magnetic moment of Σ^+ is given by its $SU(3)$ predicted value[22] we find the branching ratio

$$B \equiv \frac{R(\Sigma^+\to p+\gamma)}{R(\Sigma^+\to p+\pi^0)} = 0.28\% \quad \text{(IV.16)}$$

which is in good agreement with the recent experi-

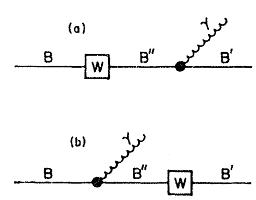

FIG. 2. Typical pole diagrams for weak electromagnetic decays which are considered in Sec. IV.

[20] In this expression, as well as in Eqs. (IV.6) to (IV.15) and in Table III, $e\mu_B/2M_B$ is the anomalous magnetic moment of the baryon B. The vertex Σ^0-Λ^0-γ is described by the interaction $(ie/2)\bar{\Psi}_{\Sigma^0}\mu_T/M_T\sigma_{\mu\nu}\Psi_\Lambda F_{\mu\nu}$+H.c., where we have taken M_T to be given by $\frac{1}{2}(M_\Sigma+M_\Lambda)$.

[21] We are using "mass-corrected" values for the magnetic moments, see, e.g., M.A.B. Bég and A. Pais, Phys. Rev. 137, B1514 (1965).

[22] S. Coleman and S. L. Glashow, Phys. Rev. Letters 6, 423 (1961).

TABLE IV. Decay rates and asymmetry parameters for WE decays in our model.

Decay	Rate (in sec^{-1})	α	Branching ratio (%) $\frac{R(B\to B'+\gamma)}{R(B\to B''+\pi)}$
$\Sigma^+\to p+\gamma$	1.79×10^7	$+0.061$	2.8×10^{-1}
$\Sigma^+\to p+\pi^0$	6.47×10^9		
$\Xi^-\to\Sigma^-+\gamma$	1.12×10^3	-0.37	2×10^{-5}
$\Xi^-\to\Lambda+\pi^-$	5.75×10^9		
$\Lambda\to n+\gamma$	2.38×10^6	$+0.25$	2.1×10^{-1}
$\Lambda\to n+\pi^0$	1.11×10^9		
$\Xi^0\to\Sigma^0+\gamma$	3.77×10^6	$+0.031$	1.1×10^{-1}
$\Xi^0\to\Lambda+\pi^0$	3.27×10^9		
$\Xi^0\to\Lambda+\gamma$	9.91×10^5	-0.24	3×10^{-2}
$\Xi^0\to\Lambda+\pi^0$	3.27×10^9		

mental result[1] $B=0.37\pm0.08\%$. In Table IV we have listed the WE decay rates obtained if all magnetic moments take on their $SU(3)$ values. Here we have also compared these decay rates with the appropriate nonleptonic decay rates in the form of branching ratios. Note that the assumption of $SU(3)$ values for the magnetic moments forces the decay $\Xi^-\to\Sigma^-+\gamma$ to be very slow. The neutral decays $\Lambda\to n+\gamma$ and $\Xi^0\to\Sigma^0+\gamma$ are predicted by our model to have some chance of measurement.

We have also listed the values for α predicted on our model in Table IV. The values given should not be taken literally but are indicative of the relative signs and of orders of magnitude.

V. CONCLUSION

We first studied the two- and three-body weak electromagnetic baryon decays from the standpoint of $SU(3)$ symmetry alone and found sum rules among the decay amplitudes when various assumptions about R conjugation and TL invariances were made. We have then used the weak vertex from a pole model for the baryon nonleptonic decays to construct a pole model for the weak-electromagnetic decays. If the baryon magnetic moments are assumed to take on their $SU(3)$ predicted values we find a value for the decay rate of $\Sigma^+\to p+\gamma$ in good agreement with a recently reported experimental value for this rate. The p.c. amplitudes calculated on the pole model in Sec. IV are found to be roughly consistent with the sum rules (II.8). According to our model the decay $\Xi^-\to\Sigma^-+\gamma$ proceeds very slowly and will not be easily seen. However, the decays $\Lambda\to n+\gamma$ and $\Xi^0\to\Sigma^0+\gamma$ will have a good chance of being seen, if our model provides a good description of the two-body weak-electromagnetic decays. Thus observation of $\Lambda\to n+\gamma$ or the measurement of asymmetry parameter α in $\Sigma^+\to p+\gamma$ would provide a test for this model and an indication about $SU(3)$ predictions of baryon magnetic moments.

Note added in proof. (i) After this work was submitted, unpublished reports by S. Y. Lo, K. Tanaka, and C. Iso on the same subject have reached us. Lo derives sum rules using RP-invariance which do not agree with our results. Iso, using a different kind of pole model gets predictions for the rate of $\Lambda\to n+\gamma$ similar to ours. However, he has to use the rate of $\Sigma^+\to p+\gamma$ as input in contrast to our treatment. Tanaka assumes a pure octet property for the effective interaction which does not seem justified to us. Other papers on the subject: A. P. Contogouric and How-Sen Wong, Nucl. Phys. **40**, 34 (1963); S. K. Bose and R. E. Marshak, Nuovo Cimento **23**, 556 (1962).

(ii) The relation due to R invariance between $\langle\Sigma^+|p\gamma\rangle$ and $\langle\Xi^-|\Sigma^-\gamma\rangle$ in Eq. (II.10) and (II.11) was derived in global symmetry by S. P. Rosen, Phys. Rev. Letters **9**, 186 (1962). This at once gives

$$R(\Xi^-\to\Sigma^-\gamma)/R(\Sigma^+\to p\gamma)\sim0.1.$$

The pole model used by us in the paper does not agree with this.

(iii) Our assumption of Σ_+^+ being p wave and Σ_-^- being s wave has now received some support from the experiment on $\Sigma^\pm\to n\pi^\pm\gamma$ by M. Bazin *et al.*, Phys. Rev. Letters (to be published).

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