Study of the K^+ Decay Probability*

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We have measured the mean life of the K^+ meson with particular attention paid to whether the decay obeys the exponential law. We find no evidence for nonexponential decay. The data give a χ^2 of 365 with 379 degrees of freedom when fitted to a pure exponential between 2.5 and 7.3 mean lives from production. The mean life is found to be 12.443 ± 0.038 nsec.

I. INTRODUCTION

We describe here an experiment to measure the mean life of the K^+ meson with particular attention to the question of whether or not the exponential decay law is obeyed. This question has recently been raised by the experimental observation of the long-lived neutral K meson decaying to two π mesons.[1] The effect observed in that experiment corresponds to the expected K_1^0 decay rate after about six mean lives from production while the detector was actually about 300 mean lives from the point of production. There is also the theoretical observation by Goldberger and Watson[2] that when one makes the usual association of a resonant state with a singularity in the S matrix there are no known physical principles dictating that this singularity be a simple pole. In the decay of a resonant state, the presence of a pole of order greater than unity could be reflected in deviations from a pure exponential law. They show, in general, that at a time t after production of an unstable state the probability that the system is still in the initial state is given by

$$A_r(t) = e^{-t/2\tau} \sum_{i=0}^{r-1} \alpha_i t^i \qquad (1)$$

where τ is the mean life of the resonant state, r is the order of the singularity, and the α_i are time-independent functions dependent on the production mechanisms.

In the present experiment the decay curve of the K^+ meson was observed from ~ 1.9 to ~ 7.8 mean lives. The data were analyzed assuming $r=1$ to obtain a more precise value of the K^+ mean life. The acceptability of the assumption of $r=1$ is based on a χ^2 test of the goodness of fit. Because of the good fit obtained for $r=1$, an attempt was made to fit only one higher term. A limit on α_1 for $r=2$ is found.

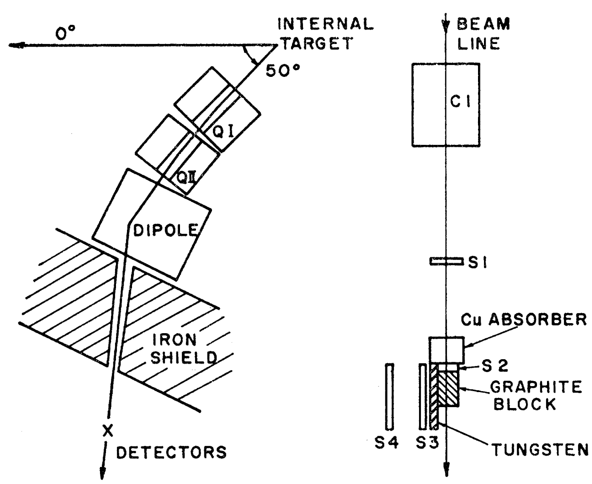

Fig. 1. Beam layout and detail of detectors.

II. APPARATUS

The experiment was performed at the Princeton-Pennsylvania 3-BeV proton accelerator (PPA). A plan view of the experimental arrangement—beam and detector—is shown in Fig. 1. A system of two quadrupole focusing magnets and one dipole bending magnet served to define a beam of charged particles with a momentum of 475 MeV/c. The magnet arrangement for the beam differs from the conventional arrangement in one important respect, namely, it was found to be desirable to bend the beam in the quadrupoles as well as in the dipole magnet. A troublesome source of background in previous stopping K^+ experiments has been protons in the beam with the wrong momentum but with the right velocity to trigger the Čerenkov counter. These protons arise, in part at least, from protons and neutrons which come directly down the beam through the quadrupole magnets and scatter on the pole tips of the dipole magnet. By deflecting the beam in the quadrupoles, the direct line of sight from the target to the bending magnet

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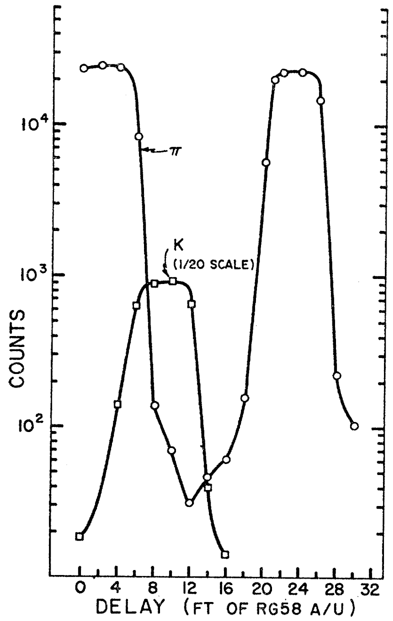

FIG. 2. rf timing curve. For the π curve, the counts are $S1$ $S2$ rf coincidences; for the K curve, $C1$ $S1$ $S2$ rf coincidences are plotted. In the π curve, the K mesons appear as a small shoulder on the trailing edge of the π peak. The width of the peak corresponds to the resolution of the electronic circuitry.

pole faces was avoided. The bending in the quadrupoles was accomplished by rotating them with respect to the 50° beam line from the target and displacing them off center. For the beam layout shown in Fig. 1, the central ray was deflected 7.5° in the quadrupoles and 27° in the dipole magnet. The solid angle accepted by the system was 4×10^{-3} Sr, and the momentum slice was $\pm1.5\%$ when a 1-in. wide$\times$4-in. high scintillation counter was used to define the K^+.

The K^+ mesons in the beam were electronically selected using velocity, range, and dE/dx, and were brought to rest in 8.6 g/cm^2 of graphite at a distance of 6.95 m from the target. In addition to Čerenkov counters, velocity selection was accomplished by using a unique feature of the PPA. It was first observed by Professor P. Piroué that the protons striking the target during the normal targeting procedure are highly bunched in time and synchronized with the 30 Mc/sec radio frequency (rf) voltage. This fact was used in this experiment to time the arrival of particles at the detector relative to their time of production in the target. Specifically, this time of flight from the target was accomplished by feeding the rf signal provided by the accelerator control room to a discriminator, properly delaying this signal for a K^+ meson, a π^+ meson, or a proton, and placing it in coincidence with a signal from a detector downstream from the target. The discriminator threshold was set at low voltage relative to the rf signal so that it served as a zero cross detector and thereby made the timing insensitive to amplitude variations in the rf voltage. Two examples of rf timing curves which demonstrate the degree of secondary particle bunching are shown in Fig. 2. Significantly, the use of time of flight from the target completely eliminated the need for any anticoincidence counter in the selection of the K^+. Thus, a stopping K^+ was signalled by a coincidence $C1$ $S1$ $S2$ (rf) where $C1$ is a velocity-selective Čerenkov counter, $S1$ and $S2$ are beam defining scintillation counters, and rf is the properly delayed radio-frequency signal discussed above. This master coincidence gated a time-to-amplitude converter (TAC) which was turned on by a fast $C1$ $S2$ coincidence. The decay of the K^+ was signalled by a coincidence between the two scintillation counters $S3$ and $S4$ which turned off the TAC. One-quarter inch of tungsten was placed between $S3$ and the graphite block in order to convert gamma rays from neutral pions arising from $K_{\pi2}$, τ, and τ' decay modes and to insure that no K^+ could stop in $S3$. When it was desired to look only at the $K_{\mu2}^+$ decay mode, an additional 11/16 in. of tungsten was placed between $S3$ and $S4$.

III. ANALYSIS AND RESULTS

A plot of a single sample of data is shown in Fig. 3. It is clear from this plot that the background is structured with peaks appearing with a period corresponding to the rf. Since the background peaks (typically about 4.0% of the peak of the decay curve) are so well defined and since the rf frequency can be known well, we have used these background peaks as timing calibration marks. The technique has the enormous advantage of providing timing calibration under running conditions concurrent with the collection of the data.

Assuming a pure exponential decay law, the data for each of 19 runs were fit to a distribution function $dN(n)$ which comes about from the following considerations:

Given a turn-on of the TAC by a valid K^+, the probability that turn-off (count) will occur in the interval

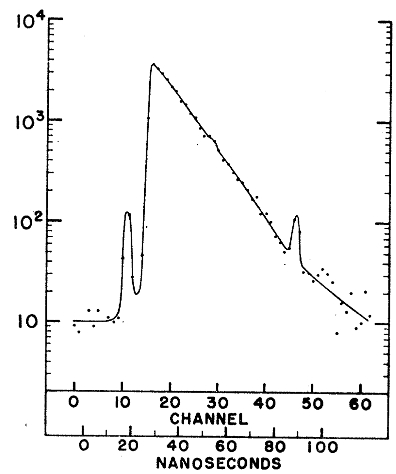

FIG. 3. Sample lifetime run. The curve is sketched in as a guide and does not necessarily represent the best fit.

t to $t+dt$ is the sum of two terms: The first term is the product of the probability that the K^+ has survived to t, that the K^+ has decayed in t to $t+dt$ and is detected, and that no background count has occurred before t; the second term is the product of the probability that a background count comes in dt, that no background count has come before t, and that either the K^+ has survived to t or if it decayed before t went undetected. Explicitly,

$$dN(n)=\{Ae^{-sgn/\tau}+B[1-\delta(1-e^{-sgn/\tau})]\}hdn\,, \quad (2)$$

where $dN(n)$ represents the number of counts in the nth channel, τ is the meanlife, A is a normalization constant, B is the average number of background counts per channel before the K^+ decay peak, typically about 0.3% of the number of counts in the decay peak, s is the time per channel, approximately 2 nsec/channel, δ is the efficiency for detecting a K^+ decay given a valid turn-on of the TAC and was typically about 0.06; and g and h are two functions of the channel number which allow for any small nonlinearity in the system.

For each run, the data were analyzed from channels 20 to 26 and 31 to 44. We started the analysis well away from the region of the sharp rise to avoid any bias due to finite timing resolution or prompt background. Prompt background arises from particles in the beam which although they fulfill all the requirements for being a K^+ meson do not stop in the graphite block but rather scatter through $S3$ and $S4$ simulating valid but prompt decays. A gap was left in the data from channel 26 to channel 30 in order to avoid the rf background peak in this region. Analysis was terminated at channel 44 for the same reason.

The background counts per channel prior to the K^+ decay peak, B, were determined for each run. Likewise, δ, which is essentially just the number of counts in the K^+ decay region divided by the total number of turn-ons of the TAC, was found for each run. The error in the mean life due to errors in B and δ was typically about 0.25% while the statistical error in each run was about 1%.

The time per channel s and the linearity functions g and h were determined by three independent calibrations of the system. As has been mentioned, the rf structure in the background provided an excellent built-in time calibration. The average channel width over the analysis region was found for each run by precisely locating the rf peaks bracketing the region and measuring the average frequency of the accelerator during the time the beam was on target. The error in the mean life due to uncertainty in the calibration using the background peaks was typically about 0.4% for each run. In addition, there was a systematic error of 0.15% due to uncertainty in the average rf of the accelerator from run to run. Also, at the beginning and end of each run, calibration data were obtained using fixed lengths of RG 58 A/U cable between the turn-on and turn-off signal. The cables had, in turn, been calibrated by comparing them to the time of flight of relativistic pions and to a crystal-controlled oscillator. The third method of calibration which yielded information on the linearity of the system as well as provided still a further check on the average channel width consisted of turning on the system with a repetitive pulse of fixed frequency and turning off the system with a random pulse from a scintillation counter exposed to a Co^{60} source. Counting the turn-on pulses, the turn-off pulses, and the number of counts per channel in a measured time provides a direct determination of the time interval covered by each channel. Nine such measured runs were made, and it was found that the linearity function h could be fitted well by a quadratic function:

$$h=1.0+(1.62\pm0.25)\times10^{-4}n+(6.8\pm0.3)\times10^{-6}n^2.$$

The function g is just given by $(d/dn)(ng)=h$. The values for the average channel width obtained from each of the three methods of calibration were in agreement.

Analyzing our data for the case $r=1$, we have determined from separate runs the following mean lives:

$$\tau(K^+)=(12.443\pm0.038)\text{ nsec},$$
$$\tau(K^+_{\mu2})=(12.501\pm0.087)\text{ nsec}.$$

In the sample of data referred to as K^+, the $K^+_{\mu2}$ detection sensitivity is reduced relative to the other modes because gamma rays from π^0 secondaries are also detected. These results agree within statistics with previous measurements of Fitch and Motley[3] who found $\tau(K^+)=(12.21\pm.26)$ nsec, $\tau(K^+_{\mu2})=(11.7_{-0.7}^{+0.8})$ nsec, Alvarez *et al.*,[4] who measured $\tau(K^+)=(12.27\pm0.15)$ nsec, $\tau(K^+_{\mu2})=(14.0\pm2)$ nsec, and Boyarski *et al.*,[5] who found $\tau(K^+)=(12.31\pm0.11)$ nsec, and $\tau(K^+_{\mu2}$ $=(12.59\pm0.18)$ nsec.

In order to determine how well our data were fitted with the assumption of a pure exponential decay law, a χ^2 test of goodness of fit was used. For the K^+ lifetime data, χ^2 is 365, whereas a value of 379 ± 28 is expected on the basis of the number of degrees of freedom.

Although the good fit obtained for the case $r=1$ gives strong support for the assumption of a pure exponential decay law, there is the question as to how large α_1 could be on the assumption that $r=2$. Our data were fitted with a distribution function obtained from Eq. (1) with $r=2$ with both τ and α_1 as parameters. In this analysis we found that the lifetime remained essentially unchanged from that found for $r=1$ and the average α_1 was $(-2\pm220)\times10^{-6}$/nsec.

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IV. CONCLUSION

We conclude that there is no evidence for nonexponential behavior in the decay and that the mean life of the K^+ meson is 12.443 ± 0.038 nsec.

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Normalization of Bethe-Salpeter Wave Functions

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In this paper is presented a brief and transparent derivation, which does not depend on the existence of a conserved quantity, of the normalization condition for Bethe-Salpeter bound-state wave functions. A comment on the structure of the condition is made. Application of the condition to the Bethe-Salpeter wave-function description of physical meson states in ordinary pseudoscalar-meson theory is also described in detail.

1. INTRODUCTION

EVER since the introduction into quantum-field theory of the Bethe-Salpeter equation,[1] treatment of the normalization of the bound state or Bethe-Salpeter wave function has presented considerable difficulty. Early derivations[2] required the existence of a conserved quantity such as baryon number or electric charge and are therefore inapplicable to neutral meson bound states, for example. Later authors, notably Allcock[3] and Cutkosky and Leon,[4] obtained normalization conditions without assuming the existence of a conserved quantity and showed[4] that their results were in agreement with the previous results. In the present paper, we give (a) a new method of derivation of the normalization of Bethe-Salpeter wave functions, which does not depend on the existence of a conserved quantity and which appears to be much more direct and transparent than those of Refs. 3 and 4; (b) a demonstration of the fact that, although the normalization condition obtained seems of somewhat odd appearance, its structure is very similar to that of the normalization conditions commonly used for one-particle wave functions in quantum field theory; (c) an application of the normalization condition obtained to the Bethe-Salpeter wave function description of the physical meson states in ordinary pseudoscalar meson theory. The discussion here is similar in spirit to but of more general nature than that given earlier by Okubo and Feldman,[5] and quite closely related to some recent work of Rowe.[6]

The material of the paper has been organized as follows. In Sec. 2, we present the work associated with (a) and (b) above, while treatment of (c) is to be found in Sec. 3.

2. NORMALIZATION OF BETHE-SALPETER WAVE FUNCTIONS

We illustrate our procedure by consideration of a convenient example, that of two fermion fields, ψ_A and ψ_B, which describe distinguishable particles of the same mass, interacting with a neutral scalar meson field. This allows easy comparison with many important papers[1–3] on the Bethe-Salpeter formalism, as well as with the introduction to the subject given by Schweber.[7] Similar treatment of other interesting cases follows readily.

We begin with a brief review of those portions of

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