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Annihilation of Antiprotons in Hydrogen at Rest. III. The Reactions $\bar{p}+p \rightarrow \omega^0+\pi^++\pi^-$ and $\bar{p}+p \rightarrow \omega^0+\rho^0$†

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The reactions (a) $\bar{p}+p \rightarrow \omega^0+\pi^++\pi^-(\pi^+\pi^-$ nonresonating), and (b) $\bar{p}+p \rightarrow \omega^0+\rho^0$ have been studied for antiprotons at rest. It is found that reaction (a) proceeds from the ^3S $\bar{p}p$ state, whereas reaction (b) is allowed only for the ^1S state. Reaction (a) accounts for 0.039 ± 0.005 of all annihilations, and reaction (b) for 0.007 ± 0.003 of all annihilations.

I. INTRODUCTION

It has been observed by Chadwick *et al.*[1] that a substantial fraction of $\bar{p}p$ annihilation into four charged and one neutral pion proceeds through intermediate ω^0 formation. It may be noted that this is the reaction, albeit for antiprotons in flight, in which the ω^0 was discovered.[2] Continuing our study of $\bar{p}p$ annihilation at rest[3] we present here a phenomenological analysis of the $\omega\pi\pi$ channel.[4]

II. EXPERIMENTAL RESULTS

From an exposure of the 30-in. Columbia-BNL hydrogen chamber to the separated low-energy antiproton beam at the BNL AGS, 16 700 "4-prong" events representing 35 600 stopped antiprotons have been analyzed. All events fitting the reaction $\bar{p}+p \rightarrow 2\pi^+ +2\pi^-$ were rejected. From the remaining 14 560 events, 7859 events could be fitted to the reaction $\bar{p}+p \rightarrow 2\pi^+ +2\pi^-+\pi^0$ (1-*C* fit). In Fig. 1 the distribution of the square of the missing mass $(M_m)^2$ of these events calculated from the unfitted quantities is presented. For the following analysis events with $(M_m)^2$ outside the interval -0.082 to 0.118 BeV^2 were rejected. We remain with a sample of 6353 events which is reasonably free from contaminations (less than 3%) and biases. Using the data of Fig. 1, and assuming that the asymmetry is due to multi-π^0 contaminations, the accepted mass interval contains 0.875 of the events corresponding to this channel.

Figure 2 shows the $\pi^+\pi^-\pi^0$ invariant mass ($M_{\pi^+\pi^-\pi^0}$) distribution around the ω^0 mass, where a large accumulation of events occurs. In order to determine the amount of $\omega\pi\pi$ production the experimental distribution of Fig. 2 has been fitted to a smoothly varying background plus a Gaussian of adjustable width. The mass of the ω^0 was taken to be 784.5 MeV. A best fit was obtained with 18.7-MeV half-width plus a second-order polynomial. To this fit there corresponds a total of $1250\pm95\omega^0\pi^+\pi^-$ events.

The partial rate for $\omega^0\pi^+\pi^-$ is then $1.14\times(1250\pm95)/(0.875\times35\,600)=0.046\pm0.0045$ per stopped antiproton. The factor 1.14 accounts for the neutral decay modes of the ω.[5]

The isometric Dalitz plot in the region $770\leq M_{\pi^+\pi^-\pi^0}\leq800$ is shown in Fig. 3. Figure 4(b) shows the projection of the experimental distribution on the T_{ω^0} axis of the Dalitz plot. Figures 4(a) and 4(c) show corresponding projections obtained from two control regions with $735\leq M_{\pi^+\pi^-\pi^0}\leq765$ MeV and $805\leq M_{\pi^+\pi^-\pi^0}\leq835$ MeV.

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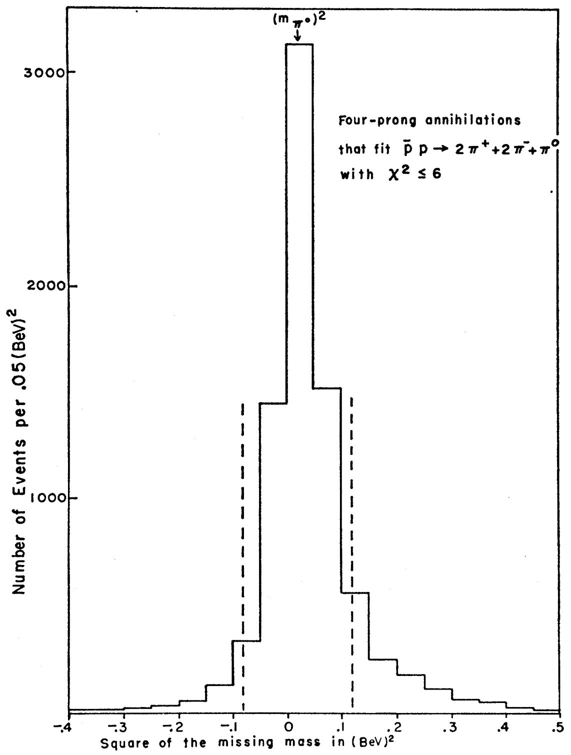

FIG. 1. $\bar{p}p \to 2\pi^+2\pi^-\pi^0$. Distribution in the square of the missing mass calculated from the unfitted momenta of the charged pions.

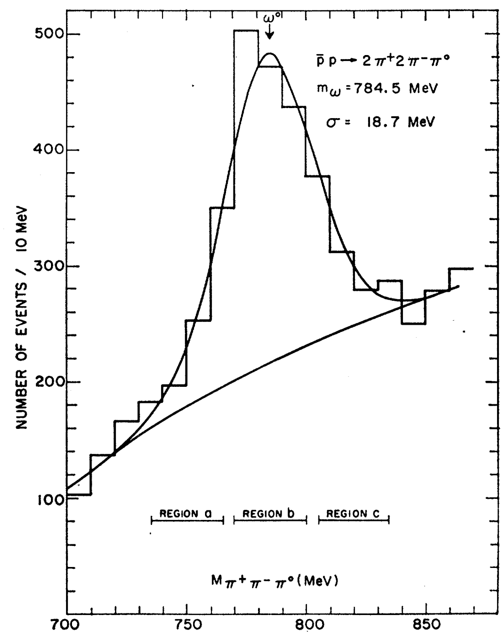

FIG. 2. $\bar{p}p \to \pi^+\pi^- + (\pi^+\pi^-\pi^0)$. Invariant mass distribution of $(\pi^+\pi^-\pi^0)$ between 700 and 870 MeV.

III. ANALYSIS

Only two of the four initial S states contribute to the process of interest, namely: ^1S $p\bar{p}$ with $T=1$ and ^3S $p\bar{p}$ with $T=0$. The internal variables distribution for the $\omega^0\pi^+\pi^-$ final state can be used to determine how much each of the two initial states contributes. The simplest Lorentz-invariant amplitudes for transitions from the two $p\bar{p}$ states to the $\omega^0\pi^+\pi^-$ state are

$$^1S(p\bar{p}) \to \omega^0\pi^+\pi^-, \quad M_a = \epsilon_{\alpha\beta\gamma\delta} S_\alpha^{\omega} p^{\omega} p_\gamma^{+} p_\delta^{-};$$

$$^3S(p\bar{p}) \to \omega^0\pi^+\pi^-, \quad M_b = S_\alpha^{(p\bar{p})} S_\beta^{\omega} A_{\alpha\beta},$$

where S_μ is the polarization four-vector for $J^P=1^-$ state, p_μ the four-momentum. The upper indices refer to a particle (or $p\bar{p}$ system). $A_{\alpha\beta}$ is a tensor symmetric in the two-pion variables since the two pions in the reaction $^3S(p\bar{p}) \to \omega^0\pi^+\pi^-$ are in $T=0$ state. Including terms linear in the pion momenta the tensor $A_{\alpha\beta}$ can be written as $A_{\alpha\beta} = a\delta_{\alpha\beta} + b(p_\alpha^+ p_\beta^- + p_\alpha^- p_\beta^+)$, where a and b are arbitrary complex coefficients.

Since there is evidence for a ρ resonance in the $\omega^0\pi^+\pi^-$ events we also consider the amplitude for $\bar{p}p \to \rho^0\omega^0$. C conservation restricts this reaction to the ^1S $p\bar{p}$ state. Thus

$$^1S(p\bar{p}) \to \rho\omega M_c = \epsilon_{\alpha\beta\gamma\delta} p_\alpha^{(p\bar{p})} p_\beta^{\omega} S_\gamma^{\omega} S_\delta^{\rho} \times \frac{i\Gamma}{(M_{\pi^+\pi^-} - M_\rho) + \frac{1}{2} i\Gamma} S_\mu^{\rho} (p_\mu^+ - p_\mu^-),$$

where M_ρ and Γ are the ρ mass and width, $M_{\pi^+\pi^-}$ is the invariant mass of the $\pi^+\pi^-$ system and the other symbols have the same meaning as before. We do not include in our analysis the production of the $\pi^\pm\omega^0$ resonance[6] at 1220 MeV because the $\omega\pi^\pm$-invariant mass distribution for our events indicates that only $\sim 3\pm3\%$ of the ω mesons resonate with a pion in this mass region. From the above amplitudes we obtain the

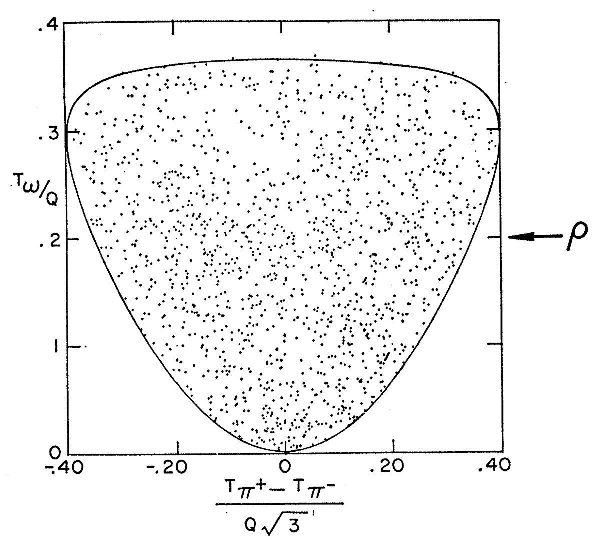

FIG. 3. $\bar{p}p \to \pi^+\pi^- + (\pi^+\pi^-\pi^0)$. Dalitz plot for events with $770 \le M_{\pi^+\pi^-\pi^0} \le 800$ MeV.

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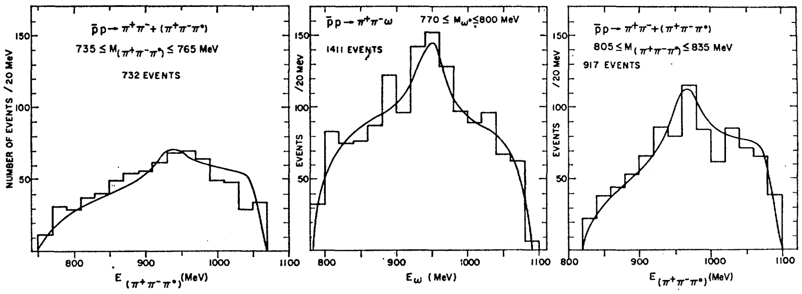

FIG. 4. $\bar{p}p \rightarrow \pi^+\pi^- + (\pi^+\pi^-\pi^0)$; (a) energy distribution of $(\pi^+\pi^-\pi^0)$ $735 \leq M_{\pi^+\pi^-\pi^0} \leq 765$ MeV; (b) energy distribution of $(\pi^+\pi^-\pi^0)$ $770 \leq M_{\pi^+\pi^-\pi^0} \leq 800$ MeV; (c) energy distribution of $(\pi^+\pi^-\pi^0)$ $805 \leq M_{\pi^+\pi^-\pi^0} \leq 835$ MeV.

following distribution functions:

$$^1S(p\bar{p}) \rightarrow \omega\pi\pi: \quad \sum_{\text{pol}} |M_a|^2 \propto (\mathbf{p}_+ \times \mathbf{p}_-)^2 \tag{1}$$

$$^3S(p\bar{p}) \rightarrow \omega\pi\pi: \quad \sum_{\text{pol}} |M_b|^2 \propto C_1 + C_2(E_+E_- - \mathbf{p}_+ \cdot \mathbf{p}_-) + C_3(E_+E_- - \mathbf{p}_+ \cdot \mathbf{p}_-)^2 \tag{2}$$

$$^1S(p\bar{p}) \rightarrow \omega\rho: \quad \sum_{\text{pol}} |M_c|^2 \propto \frac{(\mathbf{p}_+ \times \mathbf{p}_-)^2}{(M_{\pi^+\pi^-} - M_\rho)^2 + (\frac{1}{2}\Gamma)^2}. \tag{3}$$

The coefficients C_1 and C_3 are positive numbers while C_2 can be positive as well as negative. The distributions (1) to (3) can be written in terms of the coordinates of the isometric Dalitz plot (Q is defined as $2m_p - 2m_\pi - M_{\pi^+\pi^-\pi^0}$),

$$X = (T_+ - T_-)/Q\sqrt{3},$$
$$Y = T_{\omega^0}/Q.$$

After folding the experimental distribution of Fig. 3 around the y axis we have divided it into 50 sections of height 0.1 $y_{\max}$ and width 0.2 $x_{\max}$. Best fits to the data have been obtained using the χ^2 method, varying the free coefficients of the distributions (1)–(3). The fits that were obtained with a probability greater than 1% are presented in the first part of Table I.

The numbers in the table represent the fractional contribution of the corresponding term to the total rate. Fit a is the best fit possible with expressions (1) to (3), while for fits b to d various coefficients in (1) to (3) have been arbitrarily set to zero. There is no evidence for any contribution to the $(\mathbf{p}_+ \times \mathbf{p}_-)^2$ and the $(p_\mu{}^+ \cdot p_\mu{}^-)$ terms. For simplicity, we therefore use solution d in the following, in which these terms are dropped. In order to assess the significance of the results we must subtract the background due to the reactions without ω production. In the "ω region" (region b in Fig. 2) there are about 55% $\omega\pi\pi$ and 45% five-pion events without ω^0. We assume that interference effects between $\omega\pi\pi$ and five-pion production can be ignored. The behavior of the background is estimated by studying two control regions selected according to $735 < M_{\pi^+\pi^-\pi^0} < 765$ MeV (region a in Fig. 2) and to $805 < M_{\pi^+\pi^-\pi^0} < 835$ MeV (region c in Fig. 2).

Again, there is no evidence for the necessity of the $(\mathbf{p} \times \mathbf{p}_-)^2$ term. We choose the solution b from the second part of Table I, and solution b from the third part of the table, for the purpose of subtracting that part of the distribution d in the first part, which is not

TABLE I. Fractional contributions of the distribution functions to the total rate.

Fit No.	Constant	$\rho\omega$	$E^+E^- - \mathbf{p}^+ \cdot \mathbf{p}^-$	$(E^+E^- - \mathbf{p}^+ \cdot \mathbf{p})^2$	$(\mathbf{p}^+ \times \mathbf{p}^-)^2$	Number of degrees of freedom	χ^2	Prob. %
			$770 \leq M_{(\pi^+\pi^-\pi^0)} \leq 800$ MeV					
a	0.68±0.03	0.16±0.03	−0.06±0.26	0.28±0.16	−0.06±0.06	45	55	15
b	0.67±0.09	0.13±0.03	−0.14±0.25	0.34±0.15	···	46	56	15
c	0.66±0.05	0.14±0.03	···	0.26±0.03	−0.06±0.06	46	55	17
d	0.62±0.04	0.13±0.02	···	0.25±0.03	···	47	56	17
			$735 \leq M_{(\pi^+\pi^-\pi^0)} \leq 765$ MeV					
a	0.52±0.14	0.01±0.05	0.55±0.40	−0.21±0.23	0.13±0.11	45	38	83
b	0.92±0.04	0.08±0.03	···	···	···	48	51	37
			$805 \leq M_{(\pi^+\pi^-\pi^0)} \leq 835$ MeV					
a	0.78±0.13	0.12±0.04	−0.37±0.3	0.30±0.18	0.17±0.09	45	44	50
b	0.84±0.04	0.16±0.03	···	···	···	48	55	23
			$\bar{p}p \rightarrow \omega^0\pi^+\pi^-$ (Background subtracted)					
	0.27±0.1	0.15±0.06	···	0.58±0.25				

attributable to ω^0 production. Taking into account the tails of the ω^0 peak in the two control regions the $\omega\pi\pi$ and $\omega\rho$ fractions can be obtained by the expression:

$$2.34\times(\text{distribution d, first part})-0.67 \times(\text{distribution b, second part} + \text{distribution b, third part}).$$

The result is given in the fourth part of Table I. In Figs. 4(a), 4(b), and 4(c) the energy distributions calculated from the best fits chosen above are shown as solid lines.

IV. CONCLUSIONS

We wish to draw the following conclusions[7]:

1. The channel $\omega^0+\pi^++\pi^-$ contains both nonresonant pions and pions resonating as ρ. The nonresonant state dominates. Production of ρ mesons accounts for $(15\pm6)\%$ of the channel. We recall that $\omega^0\rho^0$ production must be from the ^1S state.

2. The smallness of the $(\mathbf{p}_+\times\mathbf{p}_-)^2$ term shows that the nonresonant production is dominantly from the ^3S state.

3. The fraction of all $\bar{p}p$ annihilations which are attributable to these reactions is $\bar{p}p\rightarrow\omega^0+\pi^++\pi^-$ (nonresonant): 0.039 ± 0.005 of all annihilations, $\bar{p}p\rightarrow\omega^0+\rho$: 0.007 ± 0.003 of all annihilations.

[7] Our conclusions 1 and 3 are in good agreement with the results of M. Cresti, A. Grigoletto, S. Limentani, A. Loria, L. Peruzzo, R. Santangelo, B. Chadwick, W. T. Davies, M. Derrick, C. J. B. Hawkins, P. M. D. Gray, J. H. Mulvey, P. B. Jones, D. Radojicic, and C. A. Wilkinson, in *Proceedings of the Sienna International Conference on Elementary Particles, 1963*, edited by G. Bernadini and G. D. Puppi (Societá Italiana di Fisica, Bologna, 1963), p. 263.

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Shmushkevich's Method for a Charge-Independent Theory*

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The Shmushkevich method for deducing consequences of charge independence is explained and discussed. This method, which avoids entirely the use of Clebsch-Gordan coefficients, generates linear equalities and inequalities among cross sections using only a simple counting procedure. A comprehensive list of such relations, applying to most elementary-particle reactions of interest which involve at least one pair of isospin-$\frac{1}{2}$ particles, is presented. A discussion of the various uses of these relations is given.

I. INTRODUCTION

By assuming that a set of elementary-particle reactions exhibits invariance under a given symmetry group, we are enabled to deduce consequences of this invariance in the form of relations among cross sections. One class of relations, which is particularly easy to deduce is the class of relations linear in the cross sections. Linear relations, because of their simplicity, are also of greater use. In what follows, consideration will be restricted to consequences of charge independence[1–5]; similar methods[6–9] may be applied for reactions which display invariance under other compact groups.

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