trajectory $\alpha(t)$ remains close to $\alpha=1$ in the same limit. The coupling strength of the dominant pole determines the power law for the amplitude, as in Serber's analysis, since the eikonal has logarithmic, small b dependence in such a case. This form for the residue β conforms to what one expects on the basis of a dispersion relation,[23] where t_0 is determined by the behavior of $\mathrm{Im}\beta(t)$ above threshold.

The conjecture of Regge-pole dominance for χ thus enables us to understand, with simple computation methods and on the basis of well-known properties of a few poles (mainly the Pomeranchuk pole), features of high-energy amplitudes which even in a phenomenological pole analysis (of the amplitude itself) were difficult to interpret; i.e., the large $-t$ dependence, departure from logarithmic behavior of $d\sigma/d\Omega$, importance of absorptive corrections in ρ-dominated charge exchange, and the appreciable real part of the amplitude near $t=0$ at moderate and high energies.[24,25] A decisive empirical test of this conjecture would be possible if a convincing calculation of the pole parameters (utilizing two-body t-channel states) could be carried out. Unfortunately, this does not seem to be within reach at the moment, since even the decay widths of two-body resonances such as ρ cannot yet be correctly calculated from first principles.

[23] H. Cheng and D. Sharp, Phys. Rev. **132**, 1854 (1963).

[24] In our picture, the ratio of real to imaginary parts of the forward amplitude varies only logarithmically with energy, if we include only the pomeranchon. This may be compared with the prediction of Phillips and Rarita [Phys. Rev. Letters **14**, 502 (1965)] who use three-pole fits for the amplitudes, and obtain a rapidly decreasing function.

[25] K. J. Foley, R. S. Gilmore, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. H. Willen, R. Yamada, and L. C. L. Yuan, Phys. Rev. Letters **14**, 862 (1965).

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Simultaneous Bootstrap of the ρ and f^0 in $\pi\pi$ Scattering

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A self-consistent calculation of the parameters of the ρ and f^0 resonances is carried out, using an effective-potential approach. The exchange of these resonances in $\pi\pi$ scattering is used to construct a lowest order effective potential at each energy. This is sufficient to produce these same resonances in the scattering process. The requirement that the masses and reduced widths of these output resonances be the same as those of the exchanged resonances can then be used to fix these parameters. With 10% self-consistency, we obtained mass $\approx$600 MeV, width $\approx$100 MeV for the ρ, and mass $\approx$1100 MeV, width $\approx$90 MeV for the f^0. The width of the ρ thus comes out considerably narrower than in the usual calculations which ignore the f^0.

NUMEROUS calculations have been made of the parameters of the ρ meson, using the bootstrap approach.[1] In this approach one considers $\pi\pi$ scattering in which the force (input) is due to the exchange of the ρ. This force is sufficient to produce an $I=1$, $J=1$ resonance (output), which can then be identified with the ρ. If one requires that the mass and width be the same for both the input and output resonances one can determine these parameters.

Most bootstrap calculations have used the N/D method, which is simply a device for unitarizing the input. In these calculations, however, the reduced width (which, except for a simple numerical factor, is just the square of the $\rho\pi\pi$ coupling constant) invariably comes out several times larger than the experimental value. It has also been argued that the shape of the $I=1$, $J=1$ cross section is not correctly given by the N/D method. The cross section above the position of the resonance does not fall off as rapidly as would seem to be suggested by experiment.[2]

Recently one of us has proposed an effective-potential approach for making strong-interaction calculations.[3] This again is just a device for unitarizing an input. We shall see that the aforementioned defects of the N/D approach can be removed in this method.

In the lowest approximation, the method turns out to be quite elementary.[4] Suppose $m=\rho$ mass (with pion mass$=1$), $s=$square of the total c.m. energy, and $t=-2q^2(1-\cos\theta)$ where $q^2=\frac{1}{4}s-1$ and $\theta=$c.m. scattering angle. Then we first evaluate Fig. 1(a) as a dispersion diagram, i.e., we compute it as a Feynman graph, express it as a function of s and t and replace t by m^2 everywhere except in the denominator of the ρ

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[1] For a review of bootstrap methods, see F. Zachariasen, in *Strong Interactions and High Energy Physics*, edited by R. G. Moorhouse (Plenum Press, New York, 1964).

[2] J. R. Fulco, G. L. Shaw, and D. Y. Wong, Phys. Rev. **137**, B1242 (1965).

[3] L. A. P. Balázs, Phys. Rev. **137**, B1510 (1965).

[4] Actually this approximation breaks down near $s=0$. Since, however, there are no zero-mass particles in strong-interaction physics, we can simply assume some lowest mass $m_{\min}$ (say, the mass of the pion) and never look at $s<m_{\min}^2$.

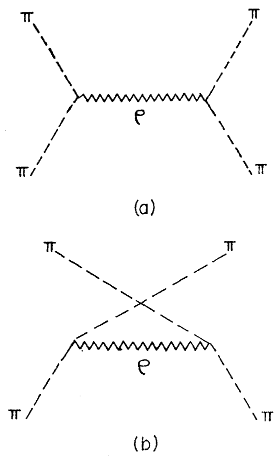

FIG. 1. Diagrams for ρ exchange.

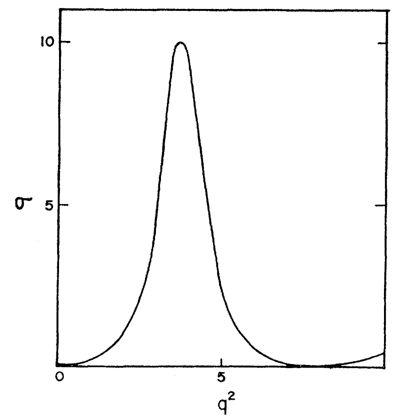

FIG. 2. The $I=1$, $l=1$ partial-wave cross section in the ρ-f^0 bootstrap. We are using pion mass units.

propagator. This gives a contribution to the relativistic amplitude

$$B^I(s,t)=[3\beta_{I1}\Gamma_1{}^1/(m^2-t)](m^2-4+2s), \quad (1)$$

where $\Gamma_1{}^1$ is the reduced width of the ρ, and β_{I1} (which is an element of the crossing matrix) has the values $\beta_{01}=1$, $\beta_{11}=\frac{1}{2}$, and $\beta_{21}=-\frac{1}{2}$. In our lowest approximation the effective potential at any given energy is simply the potential which, in Born approximation, leads to the same amplitude as is given by Eq. (1). It therefore has the form

$$V^I(r,q^2)=-12\beta_{I1}\Gamma_1{}^1 s^{-1/2}(m^2-4+2s)r^{-1}e^{-mr}, \quad (2)$$

where we have added an extra factor of 2 to take into account Fig. 1(b). The factor of $2s^{-1/2}$ comes from the fact that the relativistic amplitude is $\frac{1}{2}s^{1/2}f$, where f is the usual physical amplitude. Now we just slove the Schrödinger equation

$$\nabla^2\psi+[q^2-V^I(r,q^2)]\psi=0, \quad (3)$$

and plot

$$\Phi_l{}^I(q^2)=2s^{-1/2}q^{2l+1}\cot\delta_l{}^I, \quad (4)$$

where $\delta_l{}^I$ is the phase shift for angular momentum l and isotopic spin I. If we have a resonance at $q^2=\nu_R$, then $\Phi_l{}^I(\nu_R)=0$ and the reduced width is

$$\Gamma_l{}^I=-1/\Phi_l{}^{I\prime}(\nu_R). \quad (5)$$

The requirement that the mass and width of the ρ resonance in the $I=1$, $l=1$ state as calculated by the above procedure be equal to the corresponding values assumed in Eqs. (1) and (2), is sufficient for fixing these parameters. We obtain $m=4.2$, $\Gamma_1{}^1=0.47$. The corresponding experimental values are $m=5.5$, $\Gamma_1{}^1=0.18$.

The above lowest-order calculation does not depend on any arbitrary parameters, such as cutoffs or subtraction points. The reduced width, however, is still large compared with the experimental value. It was therefore decided to include the exchange of the f^0 along with the ρ, and calculate the (output) mass and reduced widths of both these resonances. The input values of these parameters were varied until they were equal to the output values to within about 10%. Thus if we take as inputs $m=4.0$, $\Gamma_1{}^1=0.25$ and $m=8$, $\Gamma_2{}^0=0.007$ for the masses and reduced widths of the ρ and f^0, respectively, we obtain the outputs $m=4.4$, $\Gamma_1{}^1=0.24$ and $m=7.3$, $\Gamma_2{}^0=0.0064$. The corresponding experimental values[5] are $m=5.5$, $\Gamma_1{}^1=0.18$, and $m'=9.0$, $\Gamma_2{}^0=0.0045\pm0.0011$.

We thus see that the inclusion of the f^0 leads to a considerable narrowing of the width of the ρ.[6] Moreover, if we plot the $I=1$, $l=1$ partial-wave cross section

$$\sigma_1{}^1(q^2)=12\pi q^{-2}/(1+\cot^2\delta_1{}^1). \quad (6)$$

We see from Fig. 2 that it does fall off fairly rapidly above the position of the resonance. In fact the cross section is more or less symmetric about its maximum, and gives a width which is about the same as was obtained above from Eq. (5). Our calculation therefore seems to be capable of reproducing, at least roughly, the main features of the ρ resonance.

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[5] We are taking mass = 765 MeV, width = 110 MeV for the ρ, and mass = 1250 MeV width = 100±25 MeV for the f^0. See A. H. Rosenfield *et al.*, Rev. Mod. Phys. **36**, 977 (1964).

[6] This does not seem to happen in N/D calculations. See, for instance, L. A. P. Balázs, Phys. Rev. **129**, (1963).