Regge Poles, Optical Model, and High-Energy Charge-Exchange Reactions*

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An optical-model viewpoint, with a Regge-pole form for the absorptive energy-dependent potential, is shown to imply an absorptive correction to Regge-pole formulas for charge-exchange scattering amplitudes. At the same time a closed form for high-energy elastic-scattering amplitudes is obtained incorporating cuts in the angular-momentum plane. Numerical estimates illustrate the difference between these predictions and the uncorrected poles. The power law for large-momentum-transfer behavior is shown to be easily obtainable.

RECENT fits to high-energy $\pi^- p$ charge-exchange cross sections have been carried out successfully,[1] with no explicit considerations of unitarity, using only ρ exchange treated as a single Regge pole. On the other hand, analyses of high-energy peripheral reactions such as $\pi^- p \to \rho N$ based on single-particle-exchange formulas have been successful only when appreciable corrections due to initial- and final-state diffraction scattering are included.[2,3] Such corrections applied to *elementary* vector-meson ρ exchange are not capable of fitting the $\pi^- p$ charge-exchange data.[4] The question has been raised by several authors[4,5] whether the application of "absorptive corrections," following the prescriptions of Sopkovich,[6] Durand and Chiu,[3] and Jackson and Gottfried,[3,15] to a Regge-pole approximation is theoretically consistent, or whether one should assume that these corrections are already contained in a Regge-pole expression. This question is closely connected with the foundation of the absorptive correction formula itself in a relativistic S-matrix point of view, which has been a subject of considerable discussion.[7]

In this paper we clarify these points and some properties of high-energy elastic scattering, on the basis of an optical-model point of view. The central feature of our discussion is the conjecture that at energies of a few BeV, the effective optical-model potential is to be given by (the Fourier-Bessel transform of) the leading Regge poles. This conjecture, together with the eikonal approximation for the elastic-scattering amplitudes, leads immediately to the absorptive-correction formula for $\pi^- p$ charge exchange with the ρ Regge pole as the uncorrected amplitude. We will first discuss bases of this conjecture, then exhibit the above charge-exchange result with a numerical example, and finally comment on consequences for the analysis of elastic scattering.

The optical model for high-energy elastic scattering[8,9] is based on the determination of an effective complex energy-dependent potential which, when used in the appropriate one-particle equation of motion [e.g., Dirac equation with potential] for the projectile particle reproduces accurately the exact scattering amplitude. Such a potential is determined in principle by eliminating explicit reference to the inelastic channels which are coupled by unitarity to the elastic-scattering channel; the exact equivalent potential operator is nonlocal in energy, but is approximated by a local operator at high energy. It is this latter which is used in the eikonal approximation,[9,10] a high-energy approximation for the scattering solution of the elastic-channel equation of motion. Omne's has shown[10] that at high energies an effective eikonal function can be constructed for any elastic-scattering amplitude which satisfies a Mandelstam representation, but the procedure for *a priori* calculation of the potential remains obscure.

An effective complex energy-dependent potential has been defined by Chew and Frautschi,[11] based on the properties of the Mandelstam representation spectral functions. This potential is defined such that Mandelstam's iterative construction process for the double spectral functions, using the analytically continued elastic unitarity condition,[12] will reproduce the exact amplitude when this potential is taken as input information. The analytic properties of this effective potential are well known, e.g., from inspection of the dispersion graphs which contribute; the longest range part is given simply in terms of the two-particle elastic-scattering amplitude in the t channel. Since the Chew-Frautschi (CF) potential plays the same physical role (at low energies) that the effective optical-model potential does (at high energies), it is plausible to assume that the high-energy limit of the CF potential

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[2] M. H. Ross and G. L. Shaw, Phys. Rev. Letters **12**, 627 (1964).
[3] J. D. Jackson, Rev. Mod. Phys. **37**, 484 (1965); L. Durand, III, and Y. T. Chiu, Phys. Rev. **139**, B646 (1965).
[4] V. Barger and M. Ebel, Phys. Rev. **138**, B1148 (1965).
[5] See, for example, J. D. Jackson, Ref. 3.
[6] N. J. Sopkovich, Nuovo Cimento **26**, 186 (1962).
[7] J. Ball and W. R. Frazer, Phys. Rev. Letters **14**, 746 (1965); R. Omnes, Phys. Rev. **137**, B649 (1965); E. Squires, Nuovo Cimento **34**, 1328 (1964); R. C. Arnold, Phys. Rev. **136**, B1388 (1964).
[8] K. M. Watson, Rev. Mod. Phys. **30**, 565 (1958); R. Serber, Phys. Rev. **72**, 114 (1957); S. Fernbach, R. Serber, and T. B. Taylor, *ibid.* **75**, 1352 (1949).
[9] R. J. Glauber, *Lectures Delivered at the Summer Institute for Theoretical Physics, University of Colorado, 1958-1959* (Interscience Publishers, Inc., New York, 1959), Vol. 1.
[10] R. Omnes, Phys. Rev. **137**, B653 (1965).
[11] G. F. Chew, *S-Matrix Theory of Strong Interactions* (W. A. Benjamin, Inc., New York, 1962), Chap. 7.
[12] S. Mandelstam, Phys. Rev. **112**, 1344 (1958).

is precisely the function whose Fourier-Bessel (FB) transform appears as the eikonal[9] for high-energy elastic scattering.[13]

For sufficiently large impact parameters,[14] the CF potential reduces to its longest range components, i.e., the nearest t-channel singularity contribution. The high s limit of this long-range component is given by the leading Regge-pole terms for elastic scattering in the t channel, at least if ladder-type diagrams dominate the t channel; this corresponds to the least coherent inelastic processes in the s channel, as desired in the optical-model viewpoint. Our conjecture is now that these terms dominate the potential—and hence the eikonal function—not only for large, but for *all* important impact parameters in high-energy s-channel elastic scattering. This is not the same as assuming that the *amplitude* approaches the Regge-pole terms; in nonrelativistic nuclear physics applications of the optical potential, the high-energy eikonal approximation resembles not at all the potential itself, unless the potential strength is very weak.[9]

The exact eikonal will contain other, short-range contributions χ_c, which may or may not be obtained from Regge poles. For regions of s and impact parameter b where the pole contributions χ_p have a large imaginary part, e.g., from the Pomeranchuk pole, the total eikonal function $\chi=\chi_c+\chi_p$ will also enjoy this property unless there is coherent cancellation between long- and short-range components. The optical-model viewpoint would seem to preclude such coherence, between pole and nonpole terms. (Within the pole terms there will be coherence, as implied by factorization of the residues for various states in the t channel.) The consequence of this observation can be seen from the elastic-scattering S-matrix element representation as a product of central and pole factors:

$$S(s,b^2)=\exp[i\chi(s,b^2)]=\exp[i\chi_c(s,b^2)]\exp[i\chi_p(s,b^2)].$$

For regions of small b in which χ_p is large and imaginary, the magnitude of the last factor will be small, independently of χ_c; while for large b, χ_c is small compared to χ_p. The net result is that because of such "shielding," the total cross section (and moderately small $-t$ scattering) need not depend appreciably on χ_c, since they are integrals of (products of slowly varying functions with) $S(b^2)$.

In $\bar{p}p$ reactions it seems reasonable that χ_c introduces a considerable extra damping at small b, from short-range annihilation contributions; but the analytic form one should use in this case for χ_c is not clear.

Elastic πp scattering may be analyzed in terms of definite isospin states, $T=\frac{3}{2}$ and $T=\frac{1}{2}$. The optical potentials will be different for these states, since there is a nearby singularity (ρ pole) in the t channel with isospin 1. Since the ρ trajectory lies lower than the P trajectory, at high energies there will be only a small difference between the eikonal function for $T=\frac{1}{2}$ elastic scattering and that for $T=\frac{3}{2}$ scattering. The charge-exchange scattering amplitude is the difference between the elastic-scattering amplitudes in the two isospin states; thus,

$$T_{\mathrm{CE}}(s,b^2)=\exp[i\chi_{3/2}(s,b^2)]-\exp[i\chi_{1/2}(s,b^2)]$$

which, if the difference of eikonals is very small, becomes

$$T_{\mathrm{CE}}(s,b^2)\cong i\exp[i\chi]\chi_\rho(s,b^2)\equiv iS(s,b^2)\chi_\rho(s,b^2). \quad (1)$$

Here $i\chi_\rho$ is the Fourier-Bessel transform of the ρ Regge pole, assumed to be the only important t-channel $T=1$ singularity. This is exactly the form of the absorptive-correction formula given by Sopkovich,[6] Durand and Chiu,[3] Gottfried and Jackson,[15] and others for inelastic reactions.

This derivation of the correction formula can be generalized to include other inelastic reactions, but only insofar as higher symmetries [e.g., $SU(3)$ or $SU(6)$] allow the representation of off-diagonal S-matrix elements as small differences of elastic-scattering eigenamplitudes.

Unless the elastic-scattering S-matrix element is sufficiently small, there will be little quantitative difference between the amplitude given by (1) and the "uncorrected" ρ Regge pole. In the case under discussion, we find significant—but not drastic—modifications, which in a *purely phenomenological approach* could be compensated by an alternate choice of ρ-pole parameters, but in an *a priori* calculation (based on 2-body t-channel states) would be very important for a comparison of theory with experiment. To see the effect of using (1) rather than the simple pole, we take Logan's fit[1] to π^-p charge exchange. For purposes of illustration, consider the ρ-pole amplitude at 10 BeV/c. Ignoring the slow t-dependence of the signature factor, we have

$$A_{\mathrm{CE}}{}^{\rho}(t)=a_1\exp[tR_1{}^2], \quad (2)$$

where $R_1{}^2=\alpha_\rho'\ln(E/\mu)\cong 4.22$ BeV^{-2}, and a_1 is independent of t. At the same energy, the elastic scattering (if fit with a purely imaginary amplitude) may be described by[16]

$$A_{\mathrm{EL}}(t)=ia_0\exp[tR_0{}^2], \quad (3)$$

where $R_0{}^2=4.20$ BeV^{-2}, and a_0 is independent of t.

[13] To raise this from the level of a plausibility argument, it would be necessary to show explicitly that the construction of Ref. 12 can be approximated by the eikonal expression for the amplitude, at high energies. By utilizing methods of Ref. 14, it is easy to see that branch points in t are present in A_{EL} which simulate the infinite number of many-particle thresholds in the t channel. Our amplitudes are, therefore, quite different from unitary one-meson exchange amplitudes.

[14] R. Blankenbecler and M. L. Goldberger, Phys. Rev. **126**, 766 (1962). See also R. C. Arnold, Ref. 7.

[15] K. Gottfried and J. D. Jackson, Nuovo Cimento **34**, 735 (1964).

[16] K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **11**, 425 (1963).

Performing Fourier-Bessel transforms of these, we obtain

$$T_{\mathrm{CE}}{}^{\rho}(b^2)=(a_1/2R_1{}^2)\exp[-b^2/4R_1{}^2]$$

and, with $S(b^2)=1+2i\rho T_{\mathrm{EL}}(b^2)$, where $\rho(s)$ is a suitable[14] phase-space function,

$$S(b^2)=1-C\exp[-b^2/4R_0{}^2], \tag{4}$$

where C is inversely proportional to $R_0{}^2$.

At this energy $C\cong 0.8$, which means that the small impact parameter components of the charge-exchange amplitude will be strongly affected. Explicitly, from (1),

$$T_{\mathrm{CE}}(b^2)=(a_1/2R_1{}^2)\times[\exp(-b^2/4R_1{}^2)-C\exp(-b^2/4R_2{}^2)], \tag{5}$$

where $R_2{}^{-2}=R_1{}^{-2}+R_0{}^{-2}$; the inverse FB transform of this yields

$$A_{\mathrm{CE}}(t)=a_1[\exp(tR_1{}^2)-\epsilon\exp(tR_2{}^2)], \tag{6}$$

where $\epsilon=CR_2{}^2/R_1{}^2$.

Now $R_2{}^2\cong\frac{1}{2}R_1{}^2$, so $\epsilon\cong 0.4$; the forward charge-exchange amplitude is reduced by 40%, and the second exponential in (5) will dominate the (original) first term when $-t>0.25$ BeV^2. These modifications are not far outside the errors in Logan's analysis in terms of the uncorrected pole. Such a cancellation (probably of the real part only) between pole and "correction" term may explain the minimum observed[17] in this cross section near $-t=0.5$ BeV^2, which is difficult to interpret[1] in terms of a single pole. Note that the second term does not correspond to a Regge pole, since its strength depends on the energy; it must be interpreted as containing contributions from cuts in the J plane.

In the elastic-scattering amplitude, if the eikonal is given by J-plane poles, we will apparently obtain J-plane cut contributions from the nonlinear terms in the expansion of $\exp(i\chi)$, in a similar way. We can estimate the range of impact parameter which in πp scattering is dominated by the pole (linear) approximation, by keeping the first nonlinear term in the expansion. If the exponential comes from a single pole, imaginary for $t=0$ and linearly proportional to s (i.e., pomeranchon), ignoring the variation of signature and reduced residue factors with t yields

$$\chi(b^2)\cong iC\exp(-b^2/4R_0{}^2),$$

where $R_0{}^2$ is logarithmically increasing with energy and C is inversely proportional to $R_0{}^2$; we find

$$A_{\mathrm{EL}}(t)\cong(i\,R_0{}^2C/2\rho)\times[\exp(tR_0{}^2)-(C/4)\exp(tR_0{}^2/2)+\cdots]. \tag{7}$$

The first term will be larger than the second term at 10 BeV/c only for $-t>0.60$ BeV.^{2} For sufficiently small t, we expect the phase of the amplitude will be given qualitatively by the uncorrected pole terms, but the relationship between the phase of the amplitude and phase of the potential is relatively complicated in general[18]; the signature factor enters the eikonal for all $t<0$ and not just for $t=0$ as in our simple estimate above. Clearly, then, if our conjecture is true one needs to repeat the detailed Regge-pole fits[19] in elastic scattering to redetermine the pole parameters. Note that for ultrahigh energies such that $(\log S)\to\infty$, for fixed t the expression (7) approaches the pure Pomeranchuk pole (first term), since $C\to 0$; but we have no reason for ignoring χ_c if χ_p becomes small. Thus we make no assertions about the ultrahigh energy limit.

These J-plane cuts evidently are a consequence of the eikonal approach, based in turn on the elastic-unitarity condition. The latter has been used directly by Amati, Stanghellini, and Fubini (ASF),[20] with a Regge-pole form for the potential term; they obtained a sequence of branch points in the angular-momentum plane which our eikonal formula approximates. Later work by Mandelstam[21] has shown that in fact the ASF cuts are absent in the particular class of field-theoretic diagrams retained by their two-body unitarity condition, but that other diagrams with essential many-body intermediate states in the S channel produce cuts in the same position as those of ASF. The strengths of these cuts were incalculable with available techniques.

Our point of view is that the complete amplitude has such J-plane cuts with strengths reasonably approximated by the results of the naive approach of elastic unitarity, considered as a generalization of potential theory in the sense of the optical model. This can be reconciled with Mandelstam's result if, for example, essential multiparticle states are brought in by a treatment of the S-channel particles (as well as t-channel exchanges) as Regge poles represented field-theoretically by ladders. Such ladders will contribute to the Mandelstam class of diagrams with three double spectral functions when they connect in such a way to yield a nonplanar diagram, and these could be a large fraction of the over-all number of diagrams topologically required by unitarity.

If our approach is valid, the off-diagonal "absorptive-correction" factor thus should be considered as a cut contribution associated with a given Regge pole, such as the ρ in πp charge exchange.

If we believe in Regge-pole dominance of χ even for very small impact parameters, at energies of a few BeV, we can reproduce Serber's power-law result for large-momentum-transfer scattering[22] if the leading pole residue has a large $(-t)$ dependence $(t_0-t)^{-1}$, and its

[17] (As quoted by Phillips and Rarita, Ref. 1): P. Astburg, G. Finocchiaro, and A. Michelini *et al.*, in *Proceedings of the 12th Annual International Conference on High Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965). A. V. Stirling *et al.*, Saclay report, 1965 (unpublished).

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[19] T. Binford and B. R. Desai, Phys. Rev. **138**, B1167 (1965).

[20] D. Amati, S. Fubini, and A. Stanghellini, Phys. Letters **1**, 29 (1962) and Nuovo Cimento **26**, 896 (1962).

[21] S. Mandelstam, Nuovo Cimento **30**, 1127, 1148 (1963).

[22] R. Serber, Phys. Rev. Letters **10**, 357 (1963).

trajectory $\alpha(t)$ remains close to $\alpha=1$ in the same limit. The coupling strength of the dominant pole determines the power law for the amplitude, as in Serber's analysis, since the eikonal has logarithmic, small b dependence in such a case. This form for the residue β conforms to what one expects on the basis of a dispersion relation,[23] where t_0 is determined by the behavior of $\mathrm{Im}\beta(t)$ above threshold.

The conjecture of Regge-pole dominance for χ thus enables us to understand, with simple computation methods and on the basis of well-known properties of a few poles (mainly the Pomeranchuk pole), features of high-energy amplitudes which even in a phenomenological pole analysis (of the amplitude itself) were difficult to interpret; i.e., the large $-t$ dependence, departure from logarithmic behavior of $d\sigma/d\Omega$, importance of absorptive corrections in ρ-dominated charge exchange, and the appreciable real part of the amplitude near $t=0$ at moderate and high energies.[24,25] A decisive empirical test of this conjecture would be possible if a convincing calculation of the pole parameters (utilizing two-body t-channel states) could be carried out. Unfortunately, this does not seem to be within reach at the moment, since even the decay widths of two-body resonances such as ρ cannot yet be correctly calculated from first principles.

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[24] In our picture, the ratio of real to imaginary parts of the forward amplitude varies only logarithmically with energy, if we include only the pomeranchon. This may be compared with the prediction of Phillips and Rarita [Phys. Rev. Letters **14**, 502 (1965)] who use three-pole fits for the amplitudes, and obtain a rapidly decreasing function.

[25] K. J. Foley, R. S. Gilmore, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. H. Willen, R. Yamada, and L. C. L. Yuan, Phys. Rev. Letters **14**, 862 (1965).

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Simultaneous Bootstrap of the ρ and f^0 in $\pi\pi$ Scattering

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A self-consistent calculation of the parameters of the ρ and f^0 resonances is carried out, using an effective-potential approach. The exchange of these resonances in $\pi\pi$ scattering is used to construct a lowest order effective potential at each energy. This is sufficient to produce these same resonances in the scattering process. The requirement that the masses and reduced widths of these output resonances be the same as those of the exchanged resonances can then be used to fix these parameters. With 10% self-consistency, we obtained mass $\approx$600 MeV, width $\approx$100 MeV for the ρ, and mass $\approx$1100 MeV, width $\approx$90 MeV for the f^0. The width of the ρ thus comes out considerably narrower than in the usual calculations which ignore the f^0.

NUMEROUS calculations have been made of the parameters of the ρ meson, using the bootstrap approach.[1] In this approach one considers $\pi\pi$ scattering in which the force (input) is due to the exchange of the ρ. This force is sufficient to produce an $I=1$, $J=1$ resonance (output), which can then be identified with the ρ. If one requires that the mass and width be the same for both the input and output resonances one can determine these parameters.

Most bootstrap calculations have used the N/D method, which is simply a device for unitarizing the input. In these calculations, however, the reduced width (which, except for a simple numerical factor, is just the square of the $\rho\pi\pi$ coupling constant) invariably comes out several times larger than the experimental value. It has also been argued that the shape of the $I=1$, $J=1$ cross section is not correctly given by the N/D method. The cross section above the position of the resonance does not fall off as rapidly as would seem to be suggested by experiment.[2]

Recently one of us has proposed an effective-potential approach for making strong-interaction calculations.[3] This again is just a device for unitarizing an input. We shall see that the aforementioned defects of the N/D approach can be removed in this method.

In the lowest approximation, the method turns out to be quite elementary.[4] Suppose $m=\rho$ mass (with pion mass$=1$), $s=$square of the total c.m. energy, and $t=-2q^2(1-\cos\theta)$ where $q^2=\frac{1}{4}s-1$ and $\theta=$c.m. scattering angle. Then we first evaluate Fig. 1(a) as a dispersion diagram, i.e., we compute it as a Feynman graph, express it as a function of s and t and replace t by m^2 everywhere except in the denominator of the ρ

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[1] For a review of bootstrap methods, see F. Zachariasen, in *Strong Interactions and High Energy Physics*, edited by R. G. Moorhouse (Plenum Press, New York, 1964).

[2] J. R. Fulco, G. L. Shaw, and D. Y. Wong, Phys. Rev. **137**, B1242 (1965).

[3] L. A. P. Balázs, Phys. Rev. **137**, B1510 (1965).

[4] Actually this approximation breaks down near $s=0$. Since, however, there are no zero-mass particles in strong-interaction physics, we can simply assume some lowest mass $m_{\min}$ (say, the mass of the pion) and never look at $s<m_{\min}^2$.