Baryon Octet Scattering in $\tilde{U}(12)$

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The predictions of $\tilde{U}(12)$ for the reactions $NN \to NN$ and the crossed processes $N\bar{N} \to N\bar{N}$ that follow from contact interactions in conjunction with pole terms are compared with the available experimental data, with particular reference to forward scattering and the threshold limit. The results in the annihilation channel are fairly satisfactory, whereas those in the direct channel are decidedly not so at the low energies where the comparison is effected. With structureless amplitudes the calculations represent the first step of a more complete dispersion treatment which correctly restores the unitarity of the amplitude.

1. INTRODUCTION

THE successful application of $\tilde{U}(12)$ to three-point functions[1] has recently prompted similar calculations for four-point functions such as meson-baryon scattering[2] in the direct and crossed channels (processes which involve meson and baryon isobars still await analysis) and led to the belief that the predictions would be equally good. Apart from certain limiting situations such as forward scattering and annihilations at rest, this hope has been largely disappointed. However, as originally proposed, $\tilde{U}(12)$ was only expected to provide the starting approximation to any calculation, perturbation-theoretic or otherwise; for instance, the core and Born terms could be used as the basis of an N/D S-matrix computation of the scattering amplitude. In this view the theory cannot fail to be unitary.

It is this first-step approximation for the scattering of two baryon octets (N) that we shall investigate here. In addition to the nonderivative (regular) contact interaction, there are the derivative (irregular or kineton) couplings[3] which appear from the propagators of the pole terms, and which follow directly from ideas of the inhomogeneous $\tilde{U}(12)$ group.[4] We shall only consider the regular interactions with the modification of replacing the coupling constants by unknown form factors. Already at this level the calculations involved, being relativistic, are complicated enough to make the task of including all irregular interaction terms (of which there are well over a hundred) seem truly formidable. However, the neglect of these amplitudes is not serious for it so happens that in the special kinematical situation of forward scattering they reduce to regular form. At the same time we disregard the baryon-decuplet scattering processes which together with NN scattering form an integral part of the $\tilde{U}(12)$ program.

With this apology we direct our attention to those NN reactions that have direct physical interest: pN elastic and inelastic scattering of which the former will also represent $p\bar{p} \to N\bar{N}$ by crossing symmetry. In Sec. 2 we set down the $\tilde{U}(12)$ conventions that are relevant to our problem and in particular the currents of the 143 multiplet and of the singlet. The calculation of the elastic scatterings pp, pn, $p\Lambda$, $p\Sigma^{\pm}$, $p\Xi^{-}$, is carried out in Sec. 3; from these amplitudes we deduce the $SU(3)$-invariant amplitudes and thereby all other NN couplings.

These are presented in the conventional relativistic form[5] of G.G.M.W. in Sec. 4, where the connection with the nonrelativistic expressions[6] is also stated, as well as an approximate procedure for dealing with mass differences between isospin multiplets. Section 5 contains comparison and discussion of the results for forward NN scattering with the experimental data, while Sec. 6 is devoted to the crossed amplitudes describing $p\bar{p} \to N\bar{N}$ where experimental results are more readily available. Our general conclusion is that $\tilde{U}(12)$ works fairly well in the annihilation channel, analogously to $p\bar{p} \to$ mesons, but is definitely unsatis-

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[1] Abdus Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1956), referred to as S.D.S.; R. Delbourgo, M. A. Rashid, A. Salam, and J. Strathdee, Proc. Roy. Soc. (London) **A285**, 312 (1965); M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 267 (1965); B. Sakita and K. C. Wali, *ibid.* **14**, 404 (1965); Phys. Rev. **139**, B1355 (1965).

[2] R. Blankenbecler, M. L. Goldberger, K. Johnson, and S. B. Treiman, Phys. Rev. Letters **14**, 518 (1965); J. M. Cornwall, P. G. O. Freund, and K. T. Manhanthappa, *ibid.* **14**, 515 (1965); R. Delbourgo, Y. C. Leung, M. A. Rashid and J. Strathdee, *ibid.* **14**, 609 (1965); Y. Hara, *ibid.* **14**, 603 (1965); N. Chang and J. M. Shpiz, *ibid.* **14**, 617 (1965); The problem has been studied at the $SU(6)$ level by K. Johnson and S. B. Treiman, *ibid.* **14**, 189 (1965); J. C. Carter, J. J. Coyne, and S. Meshkov, *ibid.* **14**, 523, and 850 (E)(1965). F. J. Dyson and N. Xuong, *ibid.* **14**, 655 (1965); V. Barger and H. Rubin, *ibid.* **14**, 713 (1965); M. Konuma and E. Remiddi, *ibid.* **14**, 1082 (1965).

[3] J. M. Charap and P. T. Matthews, Phys. Letters **16**, 95 (1965). R. Oehme, Phys. Rev. Letters **14**, 664 (1965); *ibid.* **14**, 866 (1965); R. J. Rivers, Phys. Rev. **139**, B1587 (1965). The same conclusion has been arrived at by H. Harari and H. J. Lipkin, using W-spin formalism, Phys. Rev. (to be published).

[4] J. M. Charap, P. T. Matthews, and R. F. Streater, Proc. Roy. Soc. (London) (to be published). W. Rühl, Nuovo Cimento (to be published). Abdus Salam, R. Delbourgo, M. A. Rashid, and J. Strathdee, in Proceedings of the Seminar on High-Energy Physics and Elementary Particles, International Centre for Theoretical Physics, Trieste, 1965 [I.A.E.A., Vienna (to be published)].

[5] M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. **120**, 2250 (1960), referred to in the text as G.G.M.W.

[6] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952). The relations to physical quantities of these parameters have been given in the excellent review article by M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Ann. Rev. Nucl. Sci. **10**, 291 (1960).

factory in the direct channel—the most striking bad prediction is the equality of triplet and singlet np scattering lengths. However, the great sensitivity to possible bound states does not necessarily imply large $\tilde{U}(12)$ symmetry breakdown.

There are some important observations concerning the validity of our results which deserve special emphasis. These stem from the fact that the pole approximation preserves the inhomogeneous $\tilde{U}(12)$ structure and even with the irregular couplings such quantities as total cross sections remain easy to compute. This situation applies with force to all peripheral mechanisms where only the exchange (Born) amplitude is considered, e.g., $p\bar{p}\to Y\bar{Y}$ with meson exchange. However, any improvement of the scheme which comes to terms with unitarity, such as using the Born term as the potential input of an S-matrix N/D calculation, will necessarily destroy this last vestige of $\tilde{U}(12)$ symmetry. Part of these unitarity corrections again fall into $\tilde{U}(12)$ form; the remainder we have disregarded in this paper as a starting approximation. The extent to which this neglect is justified can only be judged by comparing the consequences with experiment and these are none too striking. Nonetheless this investigation should provide the first step of a correct dynamical treatment.

2. THE $\tilde{U}(12)$ AMPLITUDES

The inhomogeneous $\tilde{U}(12)$ theory classifies particles through the compact little group[7] $U(6)\otimes U(6)$. The 56 baryons and isobars belong to the (56,1) multiplet and when boosted to momentum p will fall in the 364 $\tilde{U}(12)$ representation.[1] In the notation of S.D.S.[1] which we follow throughout, the explicit decomposition of the 364 is

$$2m\Psi_{ABC}(p)=(\tfrac{3}{2})^{1/2}[(\not{p}+m)\gamma_\mu C]_{\alpha\beta}D_{\mu\gamma,pqr}+(6)^{-1/2}\times\{[(\not{p}+m)\gamma_5 C]_{\alpha\beta}\epsilon_{pqs}N_{\gamma r}{}^{s}+\text{cyclic in } ABC\}. \quad (1)$$

The modification to take account of mass differences between the multiplet members is described later.

According to the by now standard $\tilde{U}(12)$ prescriptions, the amplitudes which describe the scattering $N_1+N_2\to N_3+N_4$ are given by

$$\begin{aligned}M(s,t,u)&=\mathcal{A}(s,t,u)\bar{\Psi}^{ABC}(p_3)\Psi_{ABC}(p_1)\bar{\Psi}^{DEF}(p_4)\Psi_{DEF}(p_2)\\&-\mathcal{A}'(s,t,u)\bar{\Psi}^{ABC}(p_3)\Psi_{ABC}(p_2)\bar{\Psi}^{DEF}(p_4)\Psi_{DEF}(p_1)\\&+\mathcal{B}(s,t,u)\bar{\Psi}^{ABC}(p_3)\Psi_{DBC}(p_1)\bar{\Psi}^{DEF}(p_4)\Psi_{AEF}(p_2)\\&-\mathcal{B}'(s,t,u)\bar{\Psi}^{ABC}(p_3)\Psi_{DBC}(p_2)\bar{\Psi}^{DEF}(p_4)\Psi_{AEF}(p_1),\end{aligned} \quad (2)$$

where $s=(p_1+p_2)^2$, $t=(p_1-p_3)^2$, $u=(p_1-p_4)^2$.

Applying the generalized Pauli principle to the amplitude (2) we obtain the symmetry relations

$$\mathcal{A}(s,t,u)=\mathcal{A}'(s,u,t),\quad \mathcal{B}(s,t,u)=\mathcal{B}'(s,u,t).$$

It proves more convenient to cast (2) in the form

$$\begin{aligned}M(s,t,u)=&\mathcal{A}J_A{}^A(p_3,p_1)J_B{}^B(p_4,p_2)\\&-\mathcal{A}'J_A{}^A(p_3,p_2)J_B{}^B(p_4,p_1)+\mathcal{B}J_B{}^A(p_3,p_1)\\&\times J_A{}^B(p_4,p_2)-\mathcal{B}'J_B{}^A(p_3,p_2)J_A{}^B(p_4,p_1),\end{aligned} \quad (3)$$

where $J_B{}^A(p',p)=\bar{\Psi}^{ACD}(p')\Psi_{BCD}(p)$. The expressions for $J^R{}_p{}^q=(\gamma^R)_\beta{}^\alpha J_{\alpha p}{}^{\beta q}$ have already been worked out by S.D.S. and facilitate calculation[8] of

$$J_B{}^AJ_A{}^B=\tfrac{1}{4}\,\mathrm{Tr}(J^RJ^R).$$

As they are crucial to our work we list the baryon-octet contributions to the J^R:

$$\begin{aligned}J_p{}^q(p',p)&=(P^2/4m^2)[(\bar{N}N)_{p,F}{}^q+\delta_p{}^q\,\mathrm{Tr}(\bar{N}N)]+\cdots,\\J_{5,p}{}^q(p',p)&=(P^2/4m^2)[(\bar{N}\gamma_5N)_{p,D+(2/3)F}{}^q-\tfrac{1}{3}\delta_p{}^q\,\mathrm{Tr}(\bar{N}\gamma_5N)]+\cdots,\\J_{\mu5,p}{}^q(p',p)&=(P^2/4m^2)[i(\bar{N}\gamma_\mu\gamma_5N)_{p,D+(2/3)F}{}^q-\tfrac{1}{3}i\delta_p{}^q\,\mathrm{Tr}(\bar{N}\gamma_\mu\gamma_5N)]+\cdots,\\J_{\mu,p}{}^q(p',p)&=(P_\mu/2m)[(\bar{N}N)_{p,(1/3)F-D}{}^q+\tfrac{4}{3}\delta_p{}^q\,\mathrm{Tr}(\bar{N}N)]+(P^2/4m^2)[(\bar{N}\gamma_\mu N)_{p,D+(2/3)F}{}^q-\tfrac{1}{3}\delta_p{}^q\,\mathrm{Tr}(\bar{N}\gamma_\mu N)]+\cdots,\\J_{\mu\nu,p}{}^q(p',p)&=(i/4m^2)(P_\mu q_\nu-P_\nu q_\mu)[(\bar{N}N)_{p,(1/3)F-D}{}^q+\tfrac{4}{3}\delta_p{}^q\,\mathrm{Tr}(\bar{N}N)]\\&\qquad+(P^2/4m^2)[(\bar{N}\sigma_{\mu\nu}N)_{p,D+(2/3)F}{}^q-\tfrac{1}{3}\delta_p{}^q\,\mathrm{Tr}(\bar{N}\sigma_{\mu\nu}N)]+\cdots.\end{aligned} \quad (4)$$

Here $q=p-p'$, $P=p+p'$, $N=N(p)$, $\bar{N}=\bar{N}(p')$, and

$$\begin{aligned}&\mathrm{Tr}(\bar{N}N)=\bar{N}_q{}^pN_p{}^q,\\&(\bar{N}N)_{p,F}{}^q=\bar{N}_r{}^qN_p{}^r-\bar{N}_p{}^rN_r{}^q,\\&(\bar{N}N)_{p,D}{}^q=\bar{N}_r{}^qN_p{}^r+\bar{N}_p{}^rN_r{}^q.\end{aligned} \quad (5)$$

We make use of the $SU(3)$ identifications

$$\begin{aligned}&N_1{}^1=(1/\sqrt{2})[\Sigma^0+(1/\sqrt{3})\Lambda],\\&N_2{}^2=(1/\sqrt{2})[-\Sigma^0+(1/\sqrt{3})\Lambda],\quad N_3{}^3=-(2/\sqrt{6})\Lambda,\\&N_1{}^2=\Sigma^+,\quad N_2{}^1=\Sigma^-,\quad N_1{}^3=p,\\&N_3{}^1=\Xi^-,\quad N_2{}^3=n,\quad N_3{}^2=-\Xi^0.\end{aligned} \quad (6)$$

Some remarks about irregular and other amplitudes before closing this section: We know that the amplitude (2) as it stands blatantly violates unitarity[9]; in addition to (2) we should include spurion terms,[10] firstly, irregular terms constructed by $P_A{}^B=(\not{p})_\alpha{}^\beta\delta_p{}^q$ insertions that are necessarily present in the framework of inhomo-

[7] K. Bardackci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters **14**, 48 (1965). R. F. Dashen and M. Gell-Mann, Phys. Letters **17**, 142 (1965).

[8] We define F^RF^R as $FF-F_5F_5+F_\mu F_\mu-F_{\mu5}F_{\mu5}+\frac{1}{2}F_{\mu\nu}F_{\mu\nu}$ so that $\frac{1}{4}\sum_R(\gamma_R)_\alpha{}^\beta(\gamma_R)_\gamma{}^\delta=\delta_\gamma{}^\beta\delta_\alpha{}^\delta$. Note carefully the opposite sign of our metric, for the pseudoscalar and axial-vector parts in relation to Ref. 5.

[9] M. A. Bég and A. Pais, Phys. Rev. Letters **14**, 509 (1965); R. Blankenbecler *et al.*, Ref. 2.

[10] R. Oehme (see Ref. 4).

geneous $\tilde{U}(12)$, and secondly, higher spurions[10] of the type $\sum_R C_R(\gamma_R\otimes\gamma_R)$ that make up the totality of $SU(3)\otimes$ Poincaré-group amplitudes. In this paper we shall be testing the assumption that the regular amplitudes of the initial $\tilde{U}(12)$ approximation by and large dominate the processes in question. Even if it turns out that we need irregular amplitudes at the very minimum to explain certain experimental results, our conclusions will still apply to forward-scattering situations when these kineton terms reduce to the homogeneous $\tilde{U}(12)$ form.[4]

3. ELASTIC SCATTERING OF PROTONS BY THE BARYON OCTET

Relations (4) in conjunction with the amplitude (3) make it an easy matter to pick out any desired process. We shall naturally focus our attention to processes which can be physically realized, those in which the proton plays an active role. It is sufficient to consider the elastic scatterings pp, pn, $p\Lambda$, $p\Sigma^\pm$, $p\Xi^-$, as all other processes can be deduced from these. Except for $p\Xi^-$ scattering, the other cross sections are all observable as the interactions of hyperons with protons can be studied in the same pictures which record their production.[11]

At this stage we express our results for each case as linear combinations of the basic entities:

$$\begin{aligned}
S&=\bar{N}(p_3)N(p_1)\bar{N}(p_4)N(p_2)\,,\\
P&=\bar{N}(p_3)\gamma_5N(p_1)\bar{N}(p_4)\gamma_5N(p_2)\,,\\
A&=\bar{N}(p_3)i\gamma_\mu\gamma_5N(p_1)\bar{N}(p_4)i\gamma_\mu\gamma_5N(p_2)\,,\\
V&=\bar{N}(p_3)\gamma_\mu N(p_1)\bar{N}(p_4)\gamma_\mu N(p_2)\,,\\
T&=\tfrac{1}{2}\bar{N}(p_3)\sigma_{\mu\nu}N(p_1)\bar{N}(p_4)\sigma_{\mu\nu}N(p_2)\,,
\end{aligned}\qquad(7)$$

as well as the interchanged $(1\leftrightarrow 2)$ quantities $\tilde{S}$, $\tilde{P}$, $\tilde{A}$, $\tilde{V}$, $\tilde{T}$. In writing down the amplitude we have occasion to make use of the following very useful identities[12]

$$\begin{aligned}
&\bar{N}(p_4)N(p_2)\bar{N}(p_3)[(p_2+p_4)/2m]N(p_1)\\
&\quad=\tfrac{1}{2}[((s-u)/4m^2)(S-P)+V-(t/4m^2)T]\,,\\
&i\bar{N}(p_4)N(p_2)[(p_2+p_4)_\mu(p_2-p_4)_\nu-(p_2+p_4)_\nu(p_2-p_4)_\mu]\\
&\qquad\times\bar{N}(p_3)\sigma_{\mu\nu}N(p_1)/4m^2\\
&\quad=[((s-u)/4m^2)(-S-P)+V-(t/4m^2)T]\,.
\end{aligned}\qquad(8)$$

Also we introduce the abbreviations

$$x=s/4m^2\,,\quad y=t/4m^2\,,\quad z=u/4m^2\,;\quad x+y+z=1\,,$$

and note that

$$J_A{}^A(p',p)=\mathrm{Tr}J(p',p)=3(1-q^2/4m^2)\mathrm{Tr}(\bar{N}(p')N(p))\,.$$

Compiling the results:

$$\begin{aligned}
M_{pp}=&\,9\mathcal{A}(1-y)^2S-9\mathcal{A}'(1-z)^2\tilde{S}+\tfrac{1}{4}\mathcal{B}\{(1-y)^2[5S+(17/9)(V+T-A-P)]+(x-z)(1+y)(20/9)S\\
&+(8/9)(1-y)[V-yT-(x-z)P]\}-\tfrac{1}{4}\mathcal{B}'\{(1-z)^2[5\tilde{S}+(17/9)(\tilde{V}+\tilde{T}-\tilde{A}-\tilde{P})]\\
&+(x-y)(1+z)(20/9)\tilde{S}+(8/9)(1-z)[\tilde{V}-z\tilde{T}-(x-y)\tilde{P}]\}\,;
\end{aligned}\qquad(9)$$

$$\begin{aligned}
M_{pn}=&\,9\mathcal{A}(1-y)^2S+\tfrac{1}{4}\mathcal{B}\{(1-y)^2[4S-(8/9)(V+T-A-P)]+(x-z)(1+y)(16/9)S+(28/9)(1-y)\\
&\times[V-yT-(x-z)P]\}-\tfrac{1}{4}\mathcal{B}'\{(1-z)^2[\tilde{S}+(25/9)(\tilde{V}+\tilde{T}-\tilde{A}-\tilde{P})]+(x-y)(1+z)(4/9)\tilde{S}\\
&+(20/9)(1-z)[\tilde{V}-z\tilde{T}-(x-y)\tilde{P}]\}\,;
\end{aligned}\qquad(10)$$

$$\begin{aligned}
M_{p\Lambda}=&\,9\mathcal{A}(1-y)^2S+\tfrac{1}{4}\mathcal{B}\{3(1-y)^2S+2(x-z)(1+y)S+(1-y)[V-yT-(x-z)P]\}\\
&-\tfrac{1}{4}\mathcal{B}'\tfrac{3}{2}(1-z)^2[\tilde{S}+\tilde{V}+\tilde{T}-\tilde{A}-\tilde{P}]\,;
\end{aligned}\qquad(11)$$

$$\begin{aligned}
M_{p\Sigma^+}=&\,9\mathcal{A}(1-y)^2S+\tfrac{1}{4}\mathcal{B}\{(1-y)^2[4S+(16/9)(V+T-A-P)]+(x-z)(1+y)(4/9)S\\
&+(16/9)(1-y)[V-yT-(x-z)P]\}-\tfrac{1}{4}\mathcal{B}'\{(1-z)^2[\tilde{S}+\tfrac{1}{9}(\tilde{V}+\tilde{T}-\tilde{A}-\tilde{P})]+(x-y)(1+z)(16/9)\tilde{S}\\
&-(8/9)(1-z)[\tilde{V}-z\tilde{T}-(x-y)\tilde{P}]\}\,;
\end{aligned}\qquad(12)$$

$$\begin{aligned}
M_{p\Sigma^-}=&\,9\mathcal{A}(1-y)^2S+\tfrac{1}{4}\mathcal{B}\{(1-y)^2[2S-(4/9)(V+T-A-P)]+(x-z)(1+y)(8/9)S\\
&+(14/9)(1-y)[V-yT-(x-z)P]\}\,;
\end{aligned}\qquad(13)$$

$$\begin{aligned}
M_{p\Xi^-}=&\,9\mathcal{A}(1-y)^2S+\tfrac{1}{4}\mathcal{B}\{(1-y)^2[S+\tfrac{1}{9}(V+T-A-P)]+(x-z)(1+y)(16/9)S\\
&-(8/9)(1-y)[V-yT-(x-z))P]\}\,.
\end{aligned}\qquad(14)$$

4. CONVENTIONAL RELATIVISTIC AND NONRELATIVISTIC FORMS FOR *NN* SCATTERING

Before casting our results in the standard G.G.M.W. form,[5] we first extend the considerations given in their paper from $SU(2)$ (nucleon-nucleon scattering) to $SU(3)$ space. This represents a trivial generalization: in place of the $I=0$, $I=1$ amplitudes, we construct the $SU(3)$ amplitudes corresponding to 27, 10, $\overline{10}$, 8_D, 8_F, and 1 intermediate states for the direct channel. It is the Pauli principle which allows us to eliminate the transition amplitude $8_D\leftrightarrow 8_F$, leaving us with six independent reactions.[13] Making use of the Clebsch-

[11] P. D. SeSouza, G. A. Snow, and S. Meshkov, Phys. Rev. **135**, 565 (1964).

[12] D. Amati, E. Leader, and B. Vitale, Nuovo Cimento **17**, 68 (1960).

[13] K. Itabashi and K. Tanaka, Phys. Rev. **135**, 452 (1964).

TABLE I. $M^{(27)}$, $M^{(8D)}$, and $M^{(1)}$ amplitudes deduced from $\tilde{U}(12)$; $x=s/4m^2$, $y=t/4m^2$, $z=u/4m^2$

$F_i \backslash I$	27	8_D	1
F_1	$((9/2\mathcal{A}-(8/9)\mathcal{B})(1-y)^2+(1/18)$ $\times\mathcal{B}[5(1+y)(x-z)+2(1-y)]+(y\leftrightarrow z)$	$((9/2)\mathcal{A}+(1/2)\mathcal{B})(1-y)^2+(1/4)$ $\times\mathcal{B}(1-y)+(y\leftrightarrow z)$	$((9/2)\mathcal{A}+2\mathcal{B})(1-y)^2+(1/2)$ $\times\mathcal{B}[(1+y)(x-z)-2(1-y)]+(y\leftrightarrow z)$
F_2	$((9/4)\mathcal{A}+(7/9)\mathcal{B})(1-y)^2+(1/36)$ $\times\mathcal{B}[(7+3y)(x-z)-4(1-y)]-(y\leftrightarrow z)$	$((9/4)\mathcal{A}-(1/8)\mathcal{B})(1-y)^2+(1/8)$ $\times\mathcal{B}(x-z-2)(1-y)-(y\leftrightarrow z)$	$((9/4)\mathcal{A}-\mathcal{B})(1-y)^2+(1/4)$ $\times\mathcal{B}[(-1+3y)(x-z)+4(1-y)]-(y\leftrightarrow z)$
F_3	$-(1/9)\mathcal{B}(1-y)+(y\leftrightarrow z)$	$-(1/4)\mathcal{B}(1-y)+(y\leftrightarrow z)$	$\mathcal{B}(1-y)+(y\leftrightarrow z)$
F_4	$((9/4)\mathcal{A}+(2/3)\mathcal{B})(1-y)^2+(1/36)$ $\times\mathcal{B}[(3+7y)(x-z)+4(1-y)]-(y\leftrightarrow z)$	$((9/4)\mathcal{A}-(3/8)\mathcal{B})(1-y)^2+(1/8)$ $\times\mathcal{B}(2-x+z)(1-y)-(y\leftrightarrow z)$	$(9/4)\mathcal{A}(1-y)^2+(1/4)$ $\times\mathcal{B}[(3-y)(x-z)-4(1-y)]-(y\leftrightarrow z)$
F_5	$(1/9)\mathcal{B}(1-y)(3+2y-2z)+(y\leftrightarrow z)$	$(1/4)\mathcal{B}(1-y)(3+2y-2z)+(y\leftrightarrow z)$	$-\mathcal{B}(1-y)(3+2y-2z)+(y\leftrightarrow z)$

Gordan tables[14] for $SU(3)$ these are linearly related to the six processes considered in Sec. 3 as follows:

$$\begin{pmatrix} M^{(27)} \\ M^{(\bar{10})} \\ M^{(10)} \\ M^{(8_D)} \\ M^{(8_F)} \\ M^{(1)} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 2 & 0 & 0 \\ 1 & 0 & -\frac{5}{2} & -5/4 & 15/4 & 0 \\ -1 & -2 & \frac{9}{2} & \frac{1}{4} & -\frac{3}{4} & 0 \\ 1 & 4 & -8 & 0 & -4 & 8 \end{pmatrix} \begin{pmatrix} M_{pp} \\ M_{pn} \\ M_{p\Lambda} \\ M_{p\Sigma^+} \\ M_{p\Sigma^-} \\ M_{p\Xi^-} \end{pmatrix} . \quad (15)$$

Notice that $M^{(27)}$ and $M^{(\bar{10})}$ correspond to the isotriplet and isosinglet parts of nucleon-nucleon scattering.

We are now able to write all NN scattering amplitudes under the assumption of $SU(3)$ invariance in terms of the

$$M^{(I)}=F_1^{(I)}(S-\tilde{S})+F_2^{(I)}(T+\tilde{T})-F_3^{(I)}(A-\tilde{A}) +F_4^{(I)}(V+\tilde{V})-F_5^{(I)}(P-\tilde{P}). \quad (16)$$

We infer from the Pauli principle that

$$F_i^{(27,8_D,1)}(s,t,u)=(-1)^{i+1}F_i^{(27,8_D,1)}(s,u,t),$$
$$F_i^{(10,\bar{10},8_F)}(s,t,u)=(-1)^{i}F_i^{(10,\bar{10},8_F)}(s,u,t), \quad (17)$$

which conditions provide a useful check on our work. In casting our amplitudes in the form (16) from (15) we must make use of the crossing matrix

$$\begin{pmatrix} S+\tilde{S} \\ V-\tilde{V} \\ T-\tilde{T} \\ A+\tilde{A} \\ P+\tilde{P} \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ -2 & 0 & 0 & 1 & -2 \\ -3 & 0 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} S-\tilde{S} \\ V+\tilde{V} \\ T+\tilde{T} \\ A-\tilde{A} \\ P-\tilde{P} \end{pmatrix} . \quad (18)$$

The final expressions are collected in Tables I and II.

In addition to those pN amplitudes already given in equations (9) to (14), we may now evaluate some inelastic amplitudes, e.g.,

$$\langle\Lambda\Lambda|M|p\Xi^-\rangle=(9/40)M^{(27)}-\tfrac{1}{10}M^{(8_D)}-\tfrac{1}{8}M^{(1)},$$
$$\langle\Sigma^0\Sigma^0|M|p\Xi^-\rangle=(1/40)M^{(27)}+\tfrac{1}{10}M^{(8_D)}-\tfrac{1}{8}M^{(1)},$$
$$\langle\Lambda^0\Sigma^0|M|p\Xi^-\rangle=\tfrac{1}{10}\sqrt{3}(M^{(27)}-M^{(8_D)}) +\tfrac{1}{12}\sqrt{3}(M^{(\bar{10}}-M^{(10}),$$
$$\langle\Sigma^+\Sigma^-|M|p\Xi^-\rangle=-(1/40)M^{(27}-\tfrac{1}{12}(M^{(10)}+M^{(\bar{10})}) -\tfrac{1}{10}M^{(8_D)}+\tfrac{1}{6}M^{(8_F)}+\tfrac{1}{8}M^{(1)},$$
$$\langle p\Sigma^0|M|p\Lambda\rangle=(\sqrt{3}/20)(M^{(27)}-M^{(8_D)}) +\tfrac{1}{12}\sqrt{3}(M^{(8_F)}-M^{(\bar{10})}).$$

The last process is particularly interesting from the point of view of crossing to $p\bar{p}\rightarrow\Lambda\bar{\Sigma}^0$ so we present the

TABLE II. $M^{(\bar{10})}$, $M^{(10)}$ and $M^{(8F)}$ amplitudes deduced from $\tilde{U}(12)$; $x=s/4m^2$, $y=t/4m^2$, $z=u/4m^2$.

$F_i \backslash I$	$\bar{10}$	10	8_F
F_1	$((9/2)\mathcal{A}+(2/3)\mathcal{B})(1-y)^2+(1/6)$ $\times\mathcal{B}[(1+y)(x-z)+4(1-y)]$ $-(y\leftrightarrow z)$	$((9/2)\mathcal{A}-(5/3)\mathcal{B})(1-y)^2+(1/6)$ $\times\mathcal{B}[-(1+y)(x-z)-10(1-y)]$ $-(y\leftrightarrow z)$	$((9/2)\mathcal{A}+(5/6)\mathcal{B})(1-y)^2+(1/12)$ $\times\mathcal{B}[4(1+y)(x-z)-5(1-y)]$ $-(y\leftrightarrow z)$
F_2	$((9/4)\mathcal{A}+(1/6)\mathcal{B})(1-y)^2+(1/12)$ $\times\mathcal{B}[(5-3y)(x-z)-8(1-y)]$ $+(y\leftrightarrow z)$	$((9/4)\mathcal{A}+(5/6)\mathcal{B})(1-y)^2+(1/12)$ $\times\mathcal{B}[(1-3y)(x-z)-4(1-y)]$ $+(y\leftrightarrow z)$	$((9/4)\mathcal{A}-(7/24)\mathcal{B})(1-y)^2+(1/24)$ $\times\mathcal{B}[(-1+9y)(x-z)+10(1-y)]$ $+(y\leftrightarrow z)$
F_3	$-(2/3)\mathcal{B}(1-y)-(y\leftrightarrow z)$	$-(1/3)\mathcal{B}(1-y)-(y\leftrightarrow z)$	$(5/12)\mathcal{B}(1-y)-(y\leftrightarrow z)$
F_4	$((9/4)\mathcal{A}-(1/2)\mathcal{B})(1-y)^2+(1/12)$ $\times\mathcal{B}[(-3+5y)(x-z)+8(1-y)]$ $+(y\leftrightarrow z)$	$((9/4)\mathcal{A}+(1/2)\mathcal{B})(1-y)^2+(1/12)$ $\times\mathcal{B}[(-3+y)(x-z)+4(1-y)]$ $+(y\leftrightarrow z)$	$((9/4)\mathcal{A}+(1/8)\mathcal{B})(1-y)^2+(1/24)$ $\times\mathcal{B}[(9-y)(x-z)-10(1-y)]$ $+(y\leftrightarrow z)$
F_5	$(2/3)\mathcal{B}(1-y)(3+2y-2z)-(y\leftrightarrow z)$	$(1/3)\mathcal{B}(1-y)(3+2y-2z)-(y\leftrightarrow z)$	$-(5/12)\mathcal{B}(1-y)(3+2y-2z)-(y\leftrightarrow z)$

[14] P. McNamee, S. J. Chilton, and F. Chilton, Rev. Mod. Phys. 36, 1005 (1964).

result in detail,

$$\sqrt{3}M(p\Sigma|p\Lambda) =\tfrac{1}{4}\mathcal{B}\{(5/3)(1-y)^2(V+T-A-P)+\tfrac{2}{3}(x-z)(1+y)S -(7/3)(1-y)[V-yT-(x-z)P]\} -\tfrac{1}{4}\mathcal{B}'\{\tfrac{1}{2}(1-z)^2(3\tilde{S}-\tilde{V}-\tilde{T}+\tilde{A}+\tilde{P}) +(1-z)[\tilde{V}-z\tilde{T}-(x-y)\tilde{P}]\}. \quad (19)$$

While it is certainly possible to express physically interesting quantities such as cross sections $d\sigma$, polarizations $\langle P\rangle$, etc., directly in terms of the F_i (for instance through the helicity amplitudes listed in G.G.M.W.), we prefer first to pass into the conventional nonrelativistic parametrization[6] where the connections with $d\sigma$ and $\langle P\rangle$ are widely known and have been extensively tabulated.[15] This nonrelativistic reduction is straightforwardly, if tediously, carried out by using the Pauli representation of the γ matrices in (7) and the expansion

$$4M=(3F_1+6F_2-4F_3+4F_4-F_5)S +(-F_1-2F_3+2F_4+F_5)V+(-F_1+2F_2-F_5)T +(F_1-6F_3-2F_4-F_5)A +(F_1-6F_2-4F_3+4F_4-3F_5)P. \quad (20)$$

We work in the center-of-mass frame with $\mathbf{p}$ and $\mathbf{p}'$ denoting the initial and final relative momenta and, following standard practice, define the unit vectors

$$\hat{q}=\frac{\mathbf{p}-\mathbf{p}'}{|\mathbf{p}-\mathbf{p}'|},\quad \hat{P}=\frac{\mathbf{p}+\mathbf{p}'}{|\mathbf{p}+\mathbf{p}'|},\quad \hat{n}=\frac{\mathbf{p}\times\mathbf{p}'}{|\mathbf{p}\times\mathbf{p}'|},$$

and the projections

$$\sigma_{q,P,n}=\boldsymbol{\sigma}\cdot\hat{q},\,\hat{P},\,\hat{n}.$$

The conventional nonrelativistic expansion then reads[16]

$$M=\alpha 1\otimes 1+\beta\sigma_n\otimes\sigma_n+i\gamma(\sigma_n\otimes 1+1\otimes\sigma_n) +\delta\sigma_q\otimes\sigma_q+\epsilon\sigma_P\otimes\sigma_P, \quad (21)$$

M being understood to act between the two-component spinors χ, viz. $Q\otimes Q'\rightarrow \chi_3{}^\dagger Q\chi_1\chi_4{}^\dagger Q'\chi_2$. $\alpha, \beta, \gamma, \delta, \epsilon$ are, of course, complex functions of the center-of-mass energy E $[=(\mathbf{p}^2+m^2)^{1/2}=\tfrac{1}{2}s^{1/2})]$ and the scattering angle θ $[\cos\theta=\mathbf{p}\cdot\mathbf{p}'/\mathbf{p}^2=(t-u)/(t+u)]$.

We obtain the following general relations[17]:

$$\alpha=[E^2+m^2-3(E^2-m^2)\cos\theta]F_1+2[m(2E+m)-(E^2-m^2)\cos\theta+(E-m)^2\cos^2\theta]F_2-2(2E^2+m^2)F_3 +2[E(E+2m)+(E-m)^2\cos\theta^2]F_4+(E^2-m^2)(1+\cos\theta)F_5; \quad (22)$$

$$\beta=-[E^2+m^2+(E^2-m^2)\cos\theta]F_1+2[m(2E-m)+(E^2-m^2)\cos\theta+(E-m)^2\cos^2\theta]F_2+2[2E^2+m^2 -2(E^2-m^2)\cos\theta]F_3+2[E(2m-E)+(E-m)^2\cos^2\theta]F_4-(E^2-m^2)(1+\cos\theta)F_5; \quad (23)$$

$$\gamma/\sin\theta=(E^2-m^2)(F_1+F_3)-2(E-m)^2\cos\theta(F_2+F_4); \quad (24)$$

$$\delta=-[E^2+m^2+(E^2-m^2)\cos\theta]F_1+2[m^2+2(E^2-m^2)\cos\theta]F_2+2[-E^2+4m^2+(E^2-m^2)\cos\theta]F_3 +2[E^2-(E^2-m^2)\cos\theta]F_4+(E^2-m^2)(-3+\cos\theta)F_5; \quad (25)$$

$$\epsilon=-[E^2+m^2+(E^2-m^2)\cos\theta]F_1+2m^2F_2+2[5E^2-2m^2+(E^2-m^2)\cos\theta]F_3 +2[E^2+(E^2-m^2)\cos\theta]F_4+(E^2-m^2)(1+\cos\theta)F_5. \quad (26)$$

These connections between the F_i and the $\alpha, \beta, \gamma, \delta, \epsilon$ may prove valuable in their own right, without reference to $\tilde{U}(12)$.

Without further ado we shall state the physical observables in terms of the nonrelativistic parameters[6,15]:

(a) The differential cross section for an initially unpolarized beam is

$$d\sigma/d\Omega=[128\pi m^2 s]^{-1}[|\alpha|^2+|\beta|^2 +2|\gamma|^2+|\delta|^2+|\epsilon|^2]. \quad (27)$$

(b) The polarization $\langle P\rangle$ of the final baryon (perpendicular to the scattering plane) is proportional to $\mathrm{Im}[\gamma^*(\alpha+\beta)]$. From (22), (23) and (24) we get

$$\langle P\rangle\propto -8(E^2-m^2)[2mE-(E-m)^2\cos^2\theta] \times\sin\theta\,\mathrm{Im}[(F_1+F_3)^*(F_2+F_4)] \sim\mathbf{p}^2\sin\theta\,\mathrm{Im}[(F_1+F_3)^*(F_2+F_4)] \quad (28)$$

near threshold.

(c) The spin-singlet and spin-triplet scattering lengths (obtained at threshold), a_S and a_T are given by

$$\alpha(0)=m^2(3a_T+a_S),\quad \beta(0)=m^2(a_T-a_S).$$

Hence,

$$a_T=2(F_2+F_4)_{s\rightarrow 4m^2},\quad a_S=2(F_1-3F_3)_{s\rightarrow 4m^2}. \quad (29)$$

[15] R. Wilson, *The Nucleon-Nucleon Interaction* (Interscience Publishers, New York, 1963).

[16] M. L. Goldberger, Y. Nambu, and R. Oehme, Ann. Phys. 2, 226 (1957).

[17] A. O. Barut and M. Samiullah, Phys. Rev. **135**, 1356 (1964) have evaluated corresponding expressions in the Weyl representation. They are related to our nonrelativistic amplitudes by a similarity transformation.

Note that the Pauli principle forbids triplet–singlet transitions for each of the $SU(3)$ amplitudes.

(d) The optical theorem relates the total cross section to forward scattering as follows:

$$\sigma^{\text{tot}}(s)=\operatorname{Im}\alpha(s,0)/4m^2[s(s-4m^2)]^{1/2}, \qquad (30)$$

where

$$\alpha(s,0)=\tfrac{1}{2}s(2F_4+F_5-F_1-2F_3)+2m^2(2F_1+3F_2-F_3+F_4-F_5). \qquad (31)$$

(e) Connections with other experimental observables may be obtained from standard tables. For example the correlation parameter C_{qP} is proportional to $\operatorname{Im}[\gamma^*\times(\delta-\epsilon)]$ and so on.

Finally we make some observations on the question of mass differences. Throughout the whole of the previous work we have assumed the baryon octet degenerate at a common mass m. At high energies this is probably a good approximation, but at low energies where we shall compare scattering lengths, polarizations, etc., this will certainly not do. There is to date no well-defined prescription for taking exact account of mass corrections within any symmetry scheme, but a plausible method is to make comparisons of amplitudes at the same Q values after correcting for phase space and flux factors.[18] We shall follow this approach and thereby need modify the external momentum factors. Consider typically the forward-scattering case where we meet the factor $s(s-4m^2)$ in the degenerate situation $m_1=m_2=m_3=m_4=m$. If $m_1=m_2$ and $m_3=m_4$ we make the modification $s(s-4m^2)\to s[(s-4m_1^2)(s-4m_3^2)]^{1/2}$, whereas if $m_1=m_3$ and $m_2=m_4$ we make the modification

$$s(s-4m^2)\to[s-(m_1-m_2)^2][s-(m_1+m_2)^2].$$

Similar replacements may be envisaged for the various factors occurring in Eqs. (9)–(14), and (19), and will be used particularly for $p\bar{p}\to N\bar{N}$ reactions. Although they correctly incorporate threshold effects, these prescriptions at best can only be a rough way of handling mass corrections. However, any more sophisticated treatment must make the theoretical analysis many orders of magnitude more difficult and really quite unnecessary, bearing in mind that $\tilde{U}(12)$ is by its very nature an approximate theory.

5. COMPARISON WITH pN SCATTERING EXPERIMENTS

In the direct channel, apart from the extremely well-studied nucleon-nucleon problem, there is a scarcity of data all of which is confined to Yp scattering as Ξ processes are entirely out of reach at the present time. Nevertheless we shall concentrate on the seven processes $pp\to pp$, $pn\to pn$, $p\Lambda\to p\Lambda$, $p\Sigma^\pm\to p\Sigma^\pm$, $p\Lambda\to p\Sigma^0$, and $p\Xi^-\to p\Xi^-$ since the last is physically attainable in the crossed channel $p\bar{p}\to N\bar{N}$. The analysis will be confined to comparisons of scattering lengths, polarizations and total cross sections obtained from forward scattering.

The F_i for each of these processes is collected in Tables III and IV, while in Table V we have for convenience listed the combinations $(F_2+F_4)_{s=4m^2}$, $(F_1-3F_3)_{s=4m^2}$, and

$$[\tfrac{1}{2}s(-F_1-2F_3+2F_4+F_5)+2m^2(2F_1+3F_2-F_3+F_4-F_5)]_{t=0},$$

which have physical relevance. Theoretical predictions are then easily read off. Notice that in threshold limit $\mathcal{A}(s,t,u)\to\mathcal{A}'(s,t,u)\to\mathcal{A}(4m^2,0,0)$ but that otherwise $\mathcal{A}$, $\mathcal{A}'$, $\mathcal{B}$, $\mathcal{B}'$ remain distinct amplitudes in general. However if we neglect all but s waves (i.e., neglect the t, u dependence of these amplitudes) $\mathcal{A}=\mathcal{A}'$ and $\mathcal{B}=\mathcal{B}'$ and it is this special situation that has been studied for $SU(6)$ by Barger and Rubin.[19] We shall make no such restriction unless we have no other option,

TABLE III. The $F_i^{pp,np,\Xi p}$ as deduced from $\tilde{U}(12)$; $x=s/4m^2$, $y=t/4m^2$, $z=u/4m^2$.

F_i \ Process	$pp\to pp$	$pn\to pn$	$p\Xi^-\to p\Xi^-$
F_1	$((9/2)\mathcal{A}-(8/9)\mathcal{B})(1-y)^2+(1/18)\times\mathcal{B}[5(1+y)(x-z)+2(1-y)]+(y\leftrightarrow z)$	$((9/2)\mathcal{A}-(1/9)\mathcal{B})(1-y)^2+(1/18)\times\mathcal{B}[4(1+y)(x-z)+7(1-y)]-(1/18)\times\mathcal{B}'[14(1-z)^2+5(1-z)-(x-y)(1+z)]$	$((9/2)\mathcal{A}+(7/18)\mathcal{B})(1-y)^2+(1/18)\times\mathcal{B}[4(1+y)(x-z)-2(1-y)]$
F_2	$((9/4)\mathcal{A}+(7/9)\mathcal{B})(1-y)^2+(1/36)\times\mathcal{B}[(7+3y)(x-z)-4(1-y)]-(y\leftrightarrow z)$	$((9/4)\mathcal{A}+(17/36)\mathcal{B})(1-y)^2+(1/36)\times\mathcal{B}[(11-3y)(x-z)-14(1-y)]-(1/36)\times\mathcal{B}'[11(1-z)^2+10(1-z)-(x-y)(4-6z)]$	$((9/4)\mathcal{A}-(1/36)\mathcal{B})(1-y)^2+(1/36)\times\mathcal{B}[(2+6y)(x-z)+4(1-y)]$
F_3	$-(1/9)\mathcal{B}(1-y)+(y\leftrightarrow z)$	$-(7/18)\mathcal{B}(1-y)+(5/18)\mathcal{B}'(1-z)$	$(1/9)\mathcal{B}(1-y)$
F_4	$((9/4)\mathcal{A}+(2/3)\mathcal{B})(1-y)^2+(1/36)\times\mathcal{B}[(3+7y)(x-z)+4(1-y)]-(y\leftrightarrow z)$	$((9/4)\mathcal{A}+(1/12)\mathcal{B})(1-y)^2+(1/36)\times\mathcal{B}[(-3+11y)(x-z)+14(1-y)]-(1/36)\times\mathcal{B}'[21(1-z)^2-10(1-z)+(x-y)(6-4z)]$	$((9/4)\mathcal{A}+(1/12)\mathcal{B})(1-y)^2+(1/36)\times\mathcal{B}[(6+2y)(x-z)-4(1-y)]$
F_5	$(1/9)\mathcal{B}(1-y)(3+2y-2z)+(y\leftrightarrow z)$	$(7/18)\mathcal{B}(1-y)(3+2y-2z)-(5/18)\mathcal{B}'(1-z)(3+2z-2y)$	$-(1/9)\mathcal{B}(1-y)(3+2y-2z)$

[18] S. Meshkov G. A. Snow, and G. B. Yodh, Phys. Rev. Letters **13**, 213 (1964).
[19] V. Barger and H. Rubin, University of Wisconsin report (unpublished).

TABLE IV. The $F_i^{p\Lambda,p\Sigma\pm,p\Lambda'p\Sigma}$ as deduced from $\tilde{U}(12)$; $x=s/4m^2$, $y=t/4m^2$, $z=u/4m^2$.

F_i \ Process	$p\Lambda\to p\Lambda$	$\sqrt{3}(p\Lambda\to p\Sigma^0)$	$p\Sigma^+\to p\Sigma^+$	$p\Sigma^-\to p\Sigma^-$
F_1	$(9/2)\mathcal{A}(1-y)^2+(1/8)\times\mathcal{B}[2(1+y)(x-z)+(1-y)]-(3/4)\mathcal{B}'(1-z)^2$	$(1/24)\mathcal{B}[2(1+y)(x-z)-7(1-y)-4(1-y)^2]+(1/4)\mathcal{B}'[(1-z)-(1-z)^2]$	$((9/2)\mathcal{A}-(23/18)\mathcal{B})(1-y)^2+(1/18)\mathcal{B}[(1+y)(x-z)+4(1-y)]+(1/18)\times\mathcal{B}'[4(1+z)(x-y)-2(1-z)+7(1-z)^2]$	$((9/2)\mathcal{A}-(1/18)\mathcal{B})\times(1-y)^2+(1/36)\times\mathcal{B}[4(1+y)(x-z)+7(1-y)]$
F_2	$((9/4)\mathcal{A}+(5/16)\mathcal{B})(1-y)^2+(1/16)\mathcal{B}[(3+y)(x-z)-2(1-y)]-(3/8)\mathcal{B}'(1-z)^2$	$(1/48)\mathcal{B}[2(1+y)(x-z)+14(1-y)+(1-y)^2]-(1/8)\mathcal{B}'[(1-z)(x-y)-2(1-z)+2(1-z)^2]$	$((9/4)\mathcal{A}+(29/36)\mathcal{B})(1-y)^2+(1/36)\mathcal{B}[(5-3y)(x-z)-8(1-y)]-(1/36)\times\mathcal{B}'[(2+6z)(x-y)+4(1-z)-(1-z)^2]$	$((9/4)\mathcal{A}+(17/72)\mathcal{B})\times(1-y)^2+(1/72)\times\mathcal{B}[(11-3y)(x-z)-14(1-y)]$
F_3	$-(1/8)\mathcal{B}(1-y)$	$(7/24)\mathcal{B}(1-y)-(1/4)\mathcal{B}'(1-z)$	$-(2/9)\mathcal{B}(1-y)+(1/9)\mathcal{B}'(1-z)$	$-(7/36)\mathcal{B}(1-y)$
F_4	$((9/4)\mathcal{A}+(3/16)\mathcal{B})\times(1-y)^2+(1/16)\times\mathcal{B}[(1+3y)(x-z)+2(1-y)]-(3/8)\times\mathcal{B}'(1-z)^2$	$(1/48)\mathcal{B}[(9-5y)(x-z)-14(1-y)+15(1-y)^2]+(1/8)\mathcal{B}'[(1-z)(x-y)-2(1-z)]$	$((9/4)\mathcal{A}+(7/12)\mathcal{B})(1-y)^2+(1/36)\mathcal{B}[(-3+5y)(x-z)+8(1-y)]-(1/36)\times\mathcal{B}'[(6+2z)(x-y)-4(1-z)+3(1-z)^2]$	$((9/4)\mathcal{A}+(1/24)\mathcal{B})\times(1-y)^2+(1/72)\times\mathcal{B}[(-3+11y)\times(x-z)+14(1-y)]$
F_5	$(1/8)\mathcal{B}(1-y)\times(3+2y-2z)$	$-(7/24)\mathcal{B}(1-y)\times(3+2y-2z)+(1/4)\times\mathcal{B}'(1-z)(3+2z-2y)$	$(2/9)\mathcal{B}(1-y)(3+2y-2z)-(1/9)\mathcal{B}'(1-z)(3+2z-2y)$	$(7/36)\mathcal{B}(1-y)\times(3+2y-2z)$

TABLE V. Scattering lengths and forward scattering amplitudes for various $pN\to pN$ processes; $x\equiv s/4m^2$.

Reaction	$a_S=2(F_1-3F_3)_{x=1}$	$a_T=2(F_2+F_4)_{x=1}$	$\alpha(s,0)/2m^2=x(-F_1-2F_3+2F_4+F_5)+(2F_1+3F_2-F_3+F_4-F_5)$
$pp\to pp$	$18\mathcal{A}-(2/3)\mathcal{B}$	0	$36\mathcal{A}+10x\mathcal{B}-(32/3)x^2\mathcal{B}-18x^3\mathcal{A}'$
$pn\to pn$	$9\mathcal{A}-(1/3)\mathcal{B}$	$9\mathcal{A}-(1/3)\mathcal{B}$	$36\mathcal{A}+8x\mathcal{B}-(28/3)x^2\mathcal{B}'$
$p\Lambda\to p\Lambda$	$9\mathcal{A}$	$9\mathcal{A}$	$36\mathcal{A}+6x\mathcal{B}-6x^2\mathcal{B}'$
$p\Sigma^+\to p\Sigma^+$	$9\mathcal{A}-(1/3)\mathcal{B}$	$9\mathcal{A}+(7/3)\mathcal{B}$	$36\mathcal{A}+8x\mathcal{B}-(4/3)x^2\mathcal{B}'$
$p\Sigma^-\to p\Sigma^-$	$9\mathcal{A}+(5/3)\mathcal{B}$	$9\mathcal{A}+(7/9)\mathcal{B}$	$36\mathcal{A}+4x\mathcal{B}$
$p\Xi^-\to p\Xi^-$	$9\mathcal{A}+(1/3)\mathcal{B}$	$9\mathcal{A}+(5/9)\mathcal{B}$	$36\mathcal{A}+2x\mathcal{B}$
$p\Lambda\to p\Sigma^0$	$-\mathcal{B}/\sqrt{3}$	$\mathcal{B}/3\sqrt{3}$	...

such as when we consider low-energy $p\bar{p}\to N\bar{N}$ cross sections.

(a) Firstly, regarding the question of polarization which is given by the interference of (F_1+F_3) and (F_2+F_4), we immediately conclude from Tables III and IV that the outgoing proton may well be polarized for the reactions considered. To illustrate, take proton-proton scattering where

$$(F_1+F_3)^{(pp)}=[(9/2)\mathcal{A}-(8/9)\mathcal{B}](1-y)^2+(5/18)\mathcal{B}(1+y)(x-z)+(y\leftrightarrow z),$$

$$(F_2+F_4)^{(pp)}=[(9/2)\mathcal{A}+(13/9)\mathcal{B}](1-y)^2+(5/18)\mathcal{B}(1+y)(x-z)-(y\leftrightarrow z).$$

The interference takes place between amplitudes of opposite symmetry types $\mathcal{A}\pm\mathcal{A}'$, $\mathcal{B}\pm\mathcal{B}'$ and is obviously nonzero. Similarly for each of the reactions in Tables III and IV there is never a single amplitude and polarization of the final proton is quite feasible within $\tilde{U}(12)$. Evidently there exist processes which we have not considered such as $p\Xi^-\to n\Xi^0$ which are predicted to give vanishing polarizations on the basis of $\tilde{U}(12)$ because they involve just one amplitude. However, once irregular couplings are admitted into the theory, such conclusions will no longer apply.[4]

(b) The six independent scattering lengths of $SU(3)$ are reduced to just two by $\tilde{U}(12)$. Thus in addition to $SU(3)$ predictions such as $a_S^{\Sigma^+p}=a_S^{np}$ we have the relations (see Table V)

$$a_S^{np}=a_T^{np},\qquad a_S^{\Lambda p}=a_T^{\Lambda p},$$
$$8a_S^{\Lambda p}=a_T^{\Sigma^+p}+7a_T^{np},\quad 6a_S^{\Lambda p}=a_S^{\Sigma^-p}+5a_S^{np}. \tag{32}$$

The only experimental evidence regarding these lengths[20] after eliminating possible Coulomb effects is as follows:

$$a_S^{np}=-23.4\text{ F},\qquad a_T^{np}=5.4\text{ F},$$
$$a_S^{\Lambda p}=3\pm1\text{ F},\qquad a_T^{\Lambda p}=?, \tag{33}$$
$$-3\lesssim a_T^{\Sigma^+p}\lesssim1.7\text{ F},\qquad 4F\gtrsim a_S^{\Sigma^+p}\gtrsim30\text{ F}.$$

[20] A. C. Melissinos *et al.*, Phys. Rev. Letters **14**, 604 (1965). H. G. Dosch *et al.*, Phys. Letters **14**, 162 (1965).

The very first theoretical conclusion $a_S{}^{np}=a_T{}^{np}$ is so badly violated that comparison with $a^{\Sigma^+p}-8a_s{}^{\Lambda p}$ in (32) is rendered meaningless. Most likely the discrepancy with $\tilde{U}(12)$ lies in the great sensitivity of the results to the position of the deuteron poles (real and virtual) near threshold. Such excuses have already been invoked to account for deviations from $SU(3)$ symmetry[21] and therefore the lack of equality of singlet and triplet nucleon-nucleon scattering lengths could well represent only small $\tilde{U}(12)$ symmetry breakdown. We must mention that kineton terms from inhomogeneous $\tilde{U}(12)$ reduce to regular couplings and cannot affect this argument.

(c) In view of these remarks it is desirable to make comparisons with experiment at somewhat higher energies. Unfortunately, except for nucleon-nucleon scattering, this is not available. All Yp scattering data are known[22] for laboratory momenta of only about 150 MeV/c:

$$\sigma^{\text{tot}}(\Lambda p)=125\pm35 \text{ mb},$$
$$\sigma^{\text{tot}}(\Sigma^+p)=185\pm55 \text{ mb},$$
$$\sigma^{\text{tot}}(\Sigma^-p)=230\pm50 \text{ mb},$$

and at these same energies,[6,15]

$$\sigma^{\text{tot}}(pp)\approx500\pm50 \text{ mb},$$
$$\sigma^{\text{tot}}(pn)\approx900\pm100 \text{ mb}.$$

On the theoretical side, making use of (30) and (31), we get, apart from the standard $SU(3)$ relations such as $\sigma_{\text{tot}}(\Xi^0p)=\sigma_{\text{tot}}(\Sigma^-p)$, the extra predictions (unchanged even with irregular couplings)

$$\sigma^{\text{tot}}(\Sigma^+p)-3\sigma^{\text{tot}}(\Sigma^-p)=4\sigma^{\text{tot}}(np)-6\sigma^{\text{tot}}(\Lambda p),$$
$$3\sigma^{\text{tot}}(\Sigma^+p)-8\sigma^{\text{tot}}(\Sigma^-p)+4\sigma^{\text{tot}}(\Xi^-p)=3\sigma^{\text{tot}}(np)-4\sigma^{\text{tot}}(\Lambda p). \quad (34)$$

The fact that the first relation is violated at these low energies cannot again be regarded as a serious objection; $\sigma(np)$ shows very large variations in this region and, quite apart from that, Coulomb forces will muddy the strong interaction amplitudes. Until data is available at considerably higher momenta, (34) will not prove a useful test of $\tilde{U}(12)$.

(d) Dashen and Gell-Mann[7] have shown that for (coplanar) nucleon-nucleon scattering the invariance under $U(3)\otimes U(3)$, which is the invariance group left with insertion of all irregular couplings of inhomogeneous $\tilde{U}(12)$, leads to $C_{pq}=0$. We are easily able to reproduce this result and extend it to all NN scattering processes using Tables III and IV, by noting that $\delta-\epsilon=0$. *This is the single constraint on the F_i imposed by $\tilde{U}(12)$ invariance.* This condition also implies $R'=-A$, $R=A'$ (see Ref. 15 for the definitions of these parameters). The pp scattering experiments at about 400 MeV show $C_{pq}\approx0.4$, which is in disagreement with $\tilde{U}(12)$.

6. COMPARISON WITH $p\bar{p}\rightarrow N\bar{N}$ EXPERIMENTS

By applying crossing symmetry to the direct channel, $p(1)+N(2)\rightarrow p(3)+N(4)$ we discover the amplitude for $p(1)+\bar{p}(2)\rightarrow \bar{N}(3)+N(4)$. This is expressed by the substitution law $p_2\leftrightarrow -p_3$, $N(p_2)\leftrightarrow \bar{N}(-p_3)$ in Eqs. (2) and (3). At the same time $\mathcal{A}(s,t,u)\rightarrow \bar{\mathcal{A}}(s,t,u)=\mathcal{A}(t,s,u)$, etc. We could also apply crossing to obtain the processes $p\bar{N}\rightarrow p\bar{N}$ but these reactions do not hold much physical interest, and we shall address ourselves only to $p\bar{p}\rightarrow N\bar{N}$ for which there is considerable data, especially in the form of total cross sections.

One immediate consequence of $\tilde{U}(12)$ invariance is the vanishing of all inelastic (absorptive) cross sections $p\bar{p}\rightarrow N\bar{N}$ at threshold.[23] The only surviving amplitude is then $\mathcal{A}'J_B{}^B(p_4,p_1)J_C{}^C(p_3,p_2)$. It is interesting that this symmetry scheme automatically leads to the dominance of $p\bar{p}$ elastic scattering without recourse to physical arguments. On the experimental side it is known that $\sigma(p\bar{p})$ and $\sigma(n\bar{n})$ have values of about 100 mb and 20 mb very near threshold, while the data[24] on $Y\bar{Y}$ production at 3 GeV/c $\bar{p}$ laboratory momenta show $\sigma(Y\bar{Y})\approx50$ μb at this energy $\sigma(p\bar{p})\approx20$ mb and $\sigma(n\bar{n})\lesssim1$ mb. Thus the $\tilde{U}(12)$ prediction is in fair agreement with experiment.

To make more detailed comparisons with $\tilde{U}(12)$ we must calculate the $\bar{F}_i$ *above* threshold; for $p\bar{p}\rightarrow Y\bar{Y}$ we cannot use the Johnson-Treiman[2] device to circumvent Eq. (27). However, though differential cross sections[24] are known for these processes we will limit ourselves to comparing total cross sections and for that purpose we make the S-wave approximation (supposedly we are near threshold) of discarding all angular dependence $(t-u)$ and retaining those terms to first order in $(s-4m^2)$. $d\sigma/d\Omega$ often show a strong peaking in the forward direction indicating a peripheral mechanism with K^* exchange and invalidating this approach, but as far as *rough* comparisons are concerned our approximation is permissible. In this limit the crossed non-

[21] H. Harari, in Proceedings of Seminar on High-Energy Physics and Elementary Particles, International Centre for Theoretical Physics, Trieste, 1965 (I.A.E.A., Vienna, to be published). We are also indebted to Professor H. P. Stapp for further discussions on this point.

[22] B. Sechi-Zorn *et al.*, Phys. Rev. Letters **13**, 282 (1964). G. Alexander *et al.*, *ibid.* **13**, 484 (1964). H. G. Dosch *et al.*, Phys. Letters **14**, 162 (1965).

[23] D. A. Akyeampong, R. Delbourgo and Fayyazuddin, Intertional Centre for Theoretical Physics Report No. IC/65/50 (unpublished).

[24] B. Musgrave *et al.*, Nuovo Cimento **35**, 735 (1965).

relativistic amplitudes (22)–(26) become

$$(\bar{\alpha}-\bar{\beta})/4m^2=\tfrac{1}{2}(x-1)[-2\bar{F}_1-3\bar{F}_2+5\bar{F}_3-\bar{F}_4+\bar{F}_5],$$
$$(\bar{\alpha}+\bar{\beta})/4m^2=[1+\tfrac{3}{2}(x-1)][-\bar{F}_1+\bar{F}_2-\bar{F}_3+\bar{F}_4],$$
$$\bar{\gamma}/4m^2=\tfrac{1}{2}i(x-1)^{1/2}[\bar{F}_1+2\bar{F}_2+\bar{F}_3+2\bar{F}_4]-2(x-1)(\bar{F}_2+\bar{F}_4),$$
$$(\bar{\delta}+\bar{\epsilon})/4m^2=(-\bar{F}_1+2\bar{F}_2+2\bar{F}_3+\bar{F}_5)+\tfrac{1}{2}(x-1)[-\bar{F}_1+3\bar{F}_2+\bar{F}_3-\bar{F}_4+2\bar{F}_5],$$
$$\bar{\delta}-\bar{\epsilon}\equiv 0,$$

with $x-1=s/4m^2-1$ as the expansion parameter. We then obtain the following theoretical leading behaviors of the cross sections from (27) valid for $(x-1)\ll 1$:

$$\begin{aligned}
\sigma(p\bar{p}) &\to 81|\mathcal{A}'|^2/x,\\
\sigma(n\bar{n}) &\to 25(x-1)|\mathcal{B}'|^2/4x,\\
\sigma(\Lambda\bar{\Lambda}) &\to 81(x-1)|\mathcal{B}'|^2/16x,\\
\sigma(\Sigma^0\bar{\Lambda}) &\to 3(x-1)|\mathcal{B}'|^2/16x,\\
\sigma(\Sigma^+\bar{\Sigma}^+) &\to (x-1)|\mathcal{B}'|^2/4x,\\
\sigma(\Sigma^-\bar{\Sigma}^-) &\to 49(x-1)^4|\mathcal{B}|^2/648x,\\
\sigma(\Xi^-\bar{\Xi}^-) &\to 2(x-1)^4|\mathcal{B}|^2/81x.
\end{aligned}\tag{35}$$

Experimentally $p\bar{p}$ annihilations at 3 GeV/c lab momentum have yielded[24]

$$\begin{aligned}
\sigma(p\bar{p}) &= 21\text{ mb},\\
\sigma(n\bar{n}) &= 0+1.3\text{ mb},\\
\sigma(\Lambda\bar{\Lambda}) &= 117\pm 18\ \mu\text{b},\\
\sigma(\Sigma^0\bar{\Lambda}) &= 51\pm 8\ \mu\text{b},\\
\sigma(\Sigma^+\bar{\Sigma}^+) &= 36\pm 16\ \mu\text{b},\\
\sigma(\Sigma^-\bar{\Sigma}^-) &= 10\pm 4\ \mu\text{b},\\
\sigma(\Xi^-\bar{\Xi}^-) &= 2\pm 1\ \mu\text{b}.
\end{aligned}\tag{36}$$

A first crude comparison of (35) and (36) is achieved by taking a common mass ≈ 1.1 GeV for the baryon octet (when $x-1\approx 0.6$). This gives the rough theoretical estimates,

$$\sigma(n\bar{n}):\sigma(\Lambda\bar{\Lambda}):\sigma(\Sigma^0\bar{\Lambda}):\sigma(\Sigma^+\bar{\Sigma}^+)\approx 125:100:4:5,$$
$$\sigma(\Sigma^-\bar{\Sigma}^-):\sigma(\Xi^-\bar{\Xi}^-)\approx 3:1;$$

and in the event that the s dependence of the amplitudes is neglected so that $\mathcal{B}=\mathcal{B}'$,

$$\sigma(\Sigma^-\bar{\Sigma}^-):\sigma(\Sigma^+\bar{\Sigma}^+)\approx 1:6.$$

A more "correct" computation of the theoretical ratios (not entirely in accordance with the standard rules because the Q values of the processes are slightly different) must take account of the phase-space factors in (35) and the different masses involved. The prescriptions of Sec. 4 give the corrected $\tilde{U}(12)$ amplitudes to be $\sigma(n\bar{n}):\sigma(\Lambda\bar{\Lambda}):\sigma(\Sigma^0\bar{\Lambda}):\sigma(\Sigma^+\bar{\Sigma}^+)=130:80:3:4$, $\sigma(\Sigma^-\bar{\Sigma}^-):\sigma(\Xi^-\bar{\Xi}^-)\approx 25:1$, and $\sigma(\Sigma^-\bar{\Sigma}^-):(\Sigma^+\bar{\Sigma}^+)\approx 1:6$, as before, if $\mathcal{B}\approx\mathcal{B}'$.

(a) Now in view of the approximations made to derive (35), the quantitative results cannot be wholly reliable, particularly as $(x-1)_{n\bar{n}}\approx 1.1$ and $(x-1)_{\Lambda\bar{\Lambda}}\approx 0.75$ are hardly small parameters. Moreover, the introduction of irregular couplings will in general radically change the answers although not in the approximation which gave (35) when we extrapolated from the forward direction. Therefore the most we can hope is for qualitative successes of $\tilde{U}(12)$; these we do obtain except for the over estimate of $\sigma(\Lambda\bar{\Lambda})$. It is a curious feature that in contrast to $SU(6)$, $\tilde{U}(12)$ is fairly satisfactory in the annihilation channels because it somehow incorporates absorption effects. In the direct scattering channel it seems to fail at low energies [just as with $SU(6)$]; it is conceivable that this blame is carried by bound-state effects.

(b) Our comments about polarizations in Sec. 5(a) carry over unmodified to the crossed channel, viz., for the processes considered it will in general exist.

While this work was in progress we received a number of communications[25] on the same subject, mainly in the context of $SU(6)$. We have not suppressed parts of our work, since it has unified all these approaches for the various channels in a single relativistic framework (Tables I and II) and yielded new results for $p\bar{p}\to N\bar{N}$:

Note added in proof. It has been recently demonstrated by Ruegg[26] that, contrary to Ref. 13, the statement in the first paragraph of Sec. 4 [as well as that following Eq. (29)], there exist not only the 30 S-diagonal "classical" amplitudes considered above, but two additional amplitudes, $A^{FD}\pm A^{DF}$, that describe triplet-singlet transitions in the octet channel ($pY\to pY$ processes) and which we have not evaluated. The essential results of this paper, and in particular those relating to S waves, are completely unaffected by the presence of the two new amplitudes.

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[26] H. Ruegg, CERN Report No. 65/1334/5—TH 601 (unpublished).