value of the integrand under the assumption of a Yukawa potential. This program performs the task of searching with the five-dimensional space defined by the above transverse variables for the position of the maximum of the integrand. Also determined within the search is a crude estimate of the rate of change the integrand exhibits around the maximum. The data from this search program is then used as input data for the second program—the hypersearch program.

The purpose of the hypersearch program is to provide more detailed information of the functional dependence of the integrand around the maximum. The hypersearch systematically calculated 256 points within the five-dimensional space. The value of the integrand from these points is then used to construct composite graphs which represent the functional dependence of the integrand for each variable. These composite graphs are then used to provide the necessary input information for the third program—the Monte Carlo program.

This Monte Carlo program is a modification of normal Monte Carlo calculations. By means of the composite graphs an attempt is made to influence the distribution of points within the five-dimensional space used by the Monte Carlo program. This permits reduction of the magnitude of the standard deviation of the quantity to be averaged, namely the integrand.

The programs were tested as follows. The integrand was tested by performing repeated algebraic computations and then a single value of the integrand was calculated by hand and compared with the value found by the machine. Other aspects of the programs were also checked by hand calculations, testing each subroutine. A check was made to see if there was the possibility of overflow. The details of this calculation and the computer programs are on file as the author's Ph.D. thesis at Harvard University (1962).

ACKNOWLEDGMENTS

This work was carried out with the help of a number of individuals and organizations. First has been the continuing, understanding assistance of the author's thesis adviser, Professor W. H. Furry. Programming assistance was given by Dr. J. Averell, Dr. A. Brenner, Janice MacDonald, Judy Spall, and Stephen Russell. The major portion of the computing was done at the M.I.T. Computation Center and the remainder at the Harvard Computation Center. Funds for the Harvard computations were provided by the Harvard Physics Department. An industrial fellowship from the Raytheon Corporation is gratefully acknowledged.

PHYSICAL REVIEW VOLUME 140, NUMBER 4B 22 NOVEMBER 1965

Comparison of Nucleon-Nucleon Scattering Data with the Predictions of $SU(12)_{\mathfrak{L}}$*

P. B. Kantor, T. K. Kuo, Ronald F. Peierls, and T. L. Trueman
Brookhaven National Laboratory, Upton, New York
(Received 7 July 1965)

The form of the nucleon-nucleon scattering amplitude is determined subject to the restriction that baryon-baryon scattering be invariant under $SU(12)_{\mathfrak{L}}$ transformations. The results are found to disagree with experiment. The possibility that $SU(12)_{\mathfrak{L}}$ may be a "leading approximation" to a true S-matrix theory is discussed briefly.

I. INTRODUCTION

SEVERAL schemes for the calculation of scattering amplitudes have been proposed which are motivated by the desire to extend $SU(6)$ symmetry[1] to states of two or more particles in relative motion.[2,3] We have considered nucleon-nucleon scattering in the particular scheme of Bég and Pais and find disagreement with the experimental data. The calculation involves only two assumptions:

I. The scattering amplitude is invariant under transformations belonging to the group which Bég and Pais have called $SU(12)_{\mathfrak{L}}$.

II. Baryon states transform according to the 364-dimensional representation of this group. That is, the baryons are represented by completely symmetric three-index tensors $B_{\lambda\mu\nu}$, where each index runs from 1 to 12.

The disagreement with experiment persists even when assumption I is considerably weakened. We discuss this in Sec. IV.

We feel that nucleon-nucleon scattering provides a particularly good testing ground for this theory for two

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964); B. Sakita, Phys. Rev. **136**, B1756 (1964).

[2] M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 267 (1964), and earlier papers by the same authors. We follow the notation of this paper.

[3] See, for example, A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965); K. Bardakci, J. M Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters **14**, 48 (1965); P. Roman and J. J. Aghassi, Phys. Letters **14**, 68 (1965); B. Sakita and K. C. Wali, Phys. Rev. Letters **14**,404 (1965); W. Rühl, Phys. Letters **14**, 346 (1965).

reasons.[3a] First, there is a wealth of accurate data available. Second, the proton and neutron masses are essentially the same. This is to be contrasted with other experimental tests of higher symmetries where large mass splittings make it difficult to decide at what energies to compare different reactions.

II. THE $SU(12)_{\mathfrak{L}}$-INVARIANT SCATTERING AMPLITUDE

The various observables which we shall discuss are conveniently expressed in terms of the center-of-mass scattering amplitude. We shall therefore proceed in two steps: First, we determine the $SU(12)_{\mathfrak{L}}$-invariant scattering amplitude in a form invariant under homogeneous Lorentz transformations; second, we evaluate this amplitude in the center-of-mass frame in the usual (two-component-spinor) form.

Since the profusion of indices may obscure the argument let us consider first the case of two spin $\frac{1}{2}$ particles with no internal quantum numbers. In particular, let the initial particles have momenta and spin components p_1i_1, p_2i_2 and the final particles $p_1'i_1'$; $p_2'i_2'$. The most general Lorentz-invariant scattering amplitude T_{fi} can be expressed in terms of the four-component Dirac spinors $u(p,i)$:

$$T_{fi}=\langle p_1'i_1'p_2'i_2'|T|p_1i_1p_2i_2\rangle$$
$$=\bar{u}_{\alpha_1'}(p_1',i_1')\bar{u}_{\alpha_2'}(p_2',i_2')\mathfrak{M}_{\alpha_1'\alpha_2';\alpha_2\alpha_1}\times(p_1'p_2',p_1p_2)u_{\alpha_1}(p_1i_1)u_{\alpha_2}(p_2i_2)\,. \quad (1)$$

In particular, assuming the usual space-time invariance and the Pauli principle, $\mathfrak{M}$ contains five independent amplitudes which may be chosen in a variety of ways.

In the center-of-mass frame, we can express the four-momenta as follows:

$$\begin{aligned} p_1&=(\mathbf{k},E)\,, & p_2&=(-\mathbf{k},E)\,,\\ p_1'&=(\mathbf{k}',E)\,, & p_2'&=(-\mathbf{k}',E)\,,\\ E&=(k^2+m^2)^{\frac{1}{2}}, & \cos\theta&=\mathbf{k}\cdot\mathbf{k}'/k^2. \end{aligned} \quad (2)$$

To reduce T_{fi} to an expression involving two-component spinors we recall that the index i specifies the component of spin along a particular direction in the rest frame of each particle. In particular, we choose to reach the rest frame by a pure velocity transformation in the direction of motion (the "boost" or "acceleration" convention). If, in its rest frame, the particle is described by the Pauli spinor $\chi(i)$, then it follows that the Dirac spinor is given by

$$u_\alpha(p,i)=\sum_{a=1}^{2}D_{\alpha a}(p)\chi_a(i)\,. \quad (3)$$

[3a] There are, of course, other theories which add some dynamical assumptions to I and for which this might not be a good testing ground. For example, the assumed dominance of the nearest poles in the crossed channel would, on account of the small mass of the pion, lead to symmetry violation for nucleon-nucleon scattering. This point of view has been put to us by Professor R. Oehme.

In this expression D is chosen so that the matrix in Dirac spinor space representing the Lorentz transformation mentioned above has the form:

$$\tfrac{1}{2}\sqrt{2}\begin{bmatrix} D_{11} & D_{12} & 0 & 0\\ D_{21} & D_{22} & 0 & 0\\ 0 & 0 & D_{31} & D_{32}\\ 0 & 0 & D_{41} & D_{42}\end{bmatrix}. \quad (4)$$

The explicit form of the $D_{\alpha a}(p)$ is given by

$$D(p)=\frac{1}{[2m(p_0+m)]^{\frac{1}{2}}}\begin{pmatrix}(p_0+m)\mathbf{1}+\boldsymbol{\sigma}\cdot\mathbf{p}\\(p_0+m)\mathbf{1}-\boldsymbol{\sigma}\cdot\mathbf{p}\end{pmatrix}, \quad (5)$$

where $\mathbf{1}$ and $\boldsymbol{\sigma}$ are the 2×2 unit and Pauli matrices. Inserting the expression Eq. (3) everywhere into Eq. (1) we have

$$T_{fi}=\chi_{a_1'}{}^{\dagger}(i_1')\chi_{a_2'}{}^{\dagger}(i_2')M_{a_1'a_2';a_2a_1}(\mathbf{k}',\mathbf{k})\times\chi_{a_1}(i_1)\chi_{a_2}(i_2)\,, \quad (6a)$$

$$M_{a_1'a_2';\,a_1a_2}=(D^{\dagger}(p_1')\gamma_4)_{a_1'\alpha_1'}(D^{\dagger}(p_2')\gamma_4)_{a'_2\alpha_2'}\times\mathfrak{M}_{\alpha_1'\alpha_2';\,\alpha_1\alpha_2}D(p_1)_{\alpha_1a_1}D(p_2)_{\alpha_2a_2}. \quad (6b)$$

Of course, M still has five independent amplitudes and they are chosen as follows[4]:

$$M(\mathbf{k}',\mathbf{k})=BP_S+C(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{n}+N\boldsymbol{\sigma}_1\cdot\mathbf{n}\boldsymbol{\sigma}_2\cdot\mathbf{n}P_T+\tfrac{1}{2}G(\boldsymbol{\sigma}_1\cdot\mathbf{K}\boldsymbol{\sigma}_2\cdot\mathbf{K}+\boldsymbol{\sigma}_1\cdot\mathbf{P}\boldsymbol{\sigma}_2\cdot\mathbf{P})P_T+\tfrac{1}{2}H\times(\boldsymbol{\sigma}_1\cdot\mathbf{K}\boldsymbol{\sigma}_2\cdot\mathbf{K}-\boldsymbol{\sigma}_1\cdot\mathbf{P}\boldsymbol{\sigma}_2\cdot\mathbf{P})P_T. \quad (7)$$

The symbols P_S and P_T denote the spin singlet and triplet projection operators; $\mathbf{n}$, $\mathbf{K}$, $\mathbf{P}$ are unit vectors in the directions $\mathbf{k}\times\mathbf{k}'$, $\mathbf{k}-\mathbf{k}'$, and $\mathbf{k}+\mathbf{k}'$, respectively. The Wolfenstein parameters B, C, N, G, and H are functions of E and θ only.

We can now easily discuss the $SU(12)_{\mathfrak{L}}$ symmetric case. The spinors $u_\alpha(p,i)$ are replaced by "spinor-tensors" $B_{\mu_1\mu_2\mu_3}(\mathbf{p},i,I)$, where I denotes the $SU(3)$ quantum numbers. Each index μ runs from 1 to 12 and denotes a pair (αA), $A=1,2,3$, $\alpha=1,\cdots 4$. The fact that α and A are paired together represents the mixing of spin and $SU(3)$ symmetries. In place of Eq. (1) we now have

$$T_{fi}=\langle p_1'i_1'I_1'p_2'i_2'I_2'|T|p_1i_1I_1p_2i_2I_2\rangle$$
$$=\bar{B}_{\mu_1'\mu_2'\mu_3'}(p_1'i_1'I_1')\bar{B}_{\mu'_4\mu_5'\mu_6'}(p_2'i_2'I_2')\times\mathfrak{M}_{\mu_1'\cdots\mu_6';\,\mu_1\cdots\mu_6}(p_1'p_2';p_1p_2)\times B_{\mu_1\mu_2\mu_3}(p_1i_1I_1)B_{\mu_4\mu_5\mu_6}(p_2i_2I_2)\,. \quad (8)$$

In addition to invariance under homogeneous Lorentz transformations we now require invariance under all the transformations in $SU(12)_{\mathfrak{L}}$. This means precisely that the tensor components of $\mathfrak{M}$ are given by Kronecker deltas:

$$\mathfrak{M}_{\mu_1'\cdots\mu_6';\,\mu_1\cdots\mu_6}=\sum_P g_P(s,t,u)\prod_{i=1}^{6}\delta_{\mu_i'\mu(i)}. \quad (9)$$

[4] M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Ann. Rev. Nucl. Sci. **10**, 296 (1960). (Note particularly pp. 296–301.)

The sum runs over all permutations of the numbers 1 to 6, and $s,t,u,$ are the usual invariants $s=-(p_1+p_2)^2$, $t=-(p_1-p_1')^2$, $u=-(p_1-p_2')^2$. Because each tensor B is completely symmetric, when Eq. (9) is inserted into Eq. (8) only four linearly independent combinations of the 6! terms occur. Thus we have

$$\begin{aligned}T_{fi}=f_0(s,t,u)\bar{B}(2')_{\mu_1\mu_2\mu_3}B(2)_{\mu_1\mu_2\mu_3}&\\ \times\bar{B}(1')_{\mu_1'\mu_2'\mu_3'}B(1)_{\mu_1'\mu_2'\mu_3'}&\\ +f_2(s,t,u)\bar{B}(2')_{\mu_1\mu_2\mu_3'}B(2)_{\mu_1\mu_2\mu_3}&\\ \times\bar{B}(1')_{\mu_1'\mu_2'\mu_3}B(1)_{\mu_1'\mu_2'\mu_3'}&\\ +f_1(s,t,u)\bar{B}(2')_{\mu_1\mu_2'\mu_3'}B(2)_{\mu_1\mu_2\mu_3}&\\ \times\bar{B}(1')_{\mu_1'\mu_2\mu_3}B(1)_{\mu_1'\mu_2'\mu_3'}&\\ +f_3(s,t,u)\bar{B}(2')_{\mu_1'\mu_2'\mu_3'}B(2)_{\mu_1\mu_2\mu_3}&\\ \times\bar{B}(1')_{\mu_1\mu_2\mu_3}B(1)_{\mu_1'\mu_2'\mu_3'}.&\quad(10)\end{aligned}$$

We have denoted $B(p_1',i_1',I_1')$ by $B(1')$, etc. The generalized Pauli principle requires T_{fi} to be odd under the exchange of $1'$ and $2'$, which implies that

$$f_0(s,t,u)=-f_3(s,u,t)\,,$$

and

$$f_1(s,t,u)=-f_2(s,u,t)\,. \qquad (11)$$

In order to determine the Wolfenstein parameters, we express T_{fi} in the form of Eq. (6a). This requires the analog of Eq. (3):

$$B(p,i,I)_{\alpha_1A_1\alpha_2A_2\alpha_3A_3}=\sum_{a_1a_2a_3}D_{\alpha_1a_1}(p)D_{\alpha_2a_2}(p)\times D_{\alpha_3a_3}(p)B(iI)_{a_1A_1a_2A_2a_3A_3}. \qquad (12)$$

The 56-component baryon tensor $B(iI)$ describing a particle at rest is related to the spinor $\chi(i)$ by Eq. (3) of Bég and Pais.[2] The explicit reduction to the form Eq. (7) is best done in two steps. Inserting Eq. (12) into Eq. (10) and using the center-of-mass variables Eq. (2), we perform all the indicated α summations. Each sum contributes a factor $d_{a'a}(\mathbf{k'k})$ or $d_{a'a}(-\mathbf{k}',\mathbf{k})$:

$$\begin{aligned}d_{a'a}(\mathbf{k}',\mathbf{k})&=\textstyle\sum_\alpha\,(D^\dagger(k')\gamma_4)_{a'\alpha}D(k)_{\alpha a}\\ &=[1/m(E+m)][(E+m)^2\\ &\qquad-\mathbf{k}\cdot\mathbf{k}'+i\boldsymbol{\sigma}\cdot\mathbf{k}\times\mathbf{k}']_{a'a}. \qquad (13)\end{aligned}$$

If the quantization direction is taken as $\mathbf{k}\times\mathbf{k}'$, the d's

TABLE I. pp Wolfstein parameters.

$$\begin{aligned}B&=64[f_0|\alpha|^6-f_3|\beta|^6+(1/81)f_2|\alpha|^2(14\,\mathrm{Re}\alpha^2\bar{\beta}^2-17|\alpha|^2|\beta|^2)\\ &\qquad-(1/81)f_1|\beta|^2(14\,\mathrm{Re}\alpha^2\bar{\beta}^2-17|\alpha|^2|\beta|^2)].\\ C&=32i[f_0|\alpha|^4\,\mathrm{Im}\alpha^2+f_3|\beta|^4\,\mathrm{Im}\beta^2\\ &\qquad+(1/81)f_2(10\,\mathrm{Re}\alpha^2\bar{\beta}^2\,\mathrm{Im}\alpha^2+21|\alpha|^4\,\mathrm{Im}\beta^2)\\ &\qquad+(1/81)f_1(10\,\mathrm{Re}\alpha^2\bar{\beta}^2\,\mathrm{Im}\beta^2+21|\beta|^4\,\mathrm{Im}\alpha^2)].\\ N&=64[f_0|\alpha|^4\,\mathrm{Re}\alpha^2+f_3|\beta|^4\,\mathrm{Re}\beta^2\\ &\qquad+(1/81)f_2(10\,\mathrm{Re}\alpha^2\bar{\beta}^2\,\mathrm{Re}\alpha^2+21|\alpha|^4\,\mathrm{Re}\beta^2)\\ &\qquad+(1/81)f_1(10\,\mathrm{Re}\alpha^2\bar{\beta}^2\,\mathrm{Re}\beta^2+21|\beta|^4\,\mathrm{Re}\alpha^2)].\\ G-N&=64[f_0|\alpha|^6+f_3|\beta|^6+(1/81)f_2|\alpha|^2(14\,\mathrm{Re}\alpha^2\bar{\beta}^2+17|\alpha|^2|\beta|^2)\\ &\qquad+(1/81)f_1|\beta|^2(14\,\mathrm{Re}\alpha^2\bar{\beta}^2+17|\alpha|^2|\beta|^2)].\\ H&=0.\end{aligned}$$

TABLE II. np Wolfenstein parameters.

$$\begin{aligned}B&=64[f_0|\alpha|^6+(1/81)f_2|\alpha|^2(22\,\mathrm{Re}\alpha^2\bar{\beta}^2+8|\alpha|^2|\beta|^2)\\ &\qquad+(1/81)f_1|\beta|^2(8\,\mathrm{Re}\alpha^2\bar{\beta}^2+25|\alpha|^2|\beta|^2)]\\ C&=32i[f_0|\alpha|^4\,\mathrm{Im}\alpha^2+(1/81)f_2(8\,\mathrm{Re}\alpha^2\bar{\beta}^2\,\mathrm{Im}\alpha^2+6|\alpha|^4\,\mathrm{Im}\beta^2)\\ &\qquad+(1/81)f_1(2\,\mathrm{Re}\alpha^2\bar{\beta}^2\,\mathrm{Im}\beta^2+15|\beta|^4\,\mathrm{Im}\alpha^2)].\\ N&=64[f_0|\alpha|^4\,\mathrm{Re}\alpha^2+(1/81)f_2(8\,\mathrm{Re})\alpha^2\bar{\beta}^2\,\mathrm{Re}\alpha^2+6|\alpha|^4\,\mathrm{Re}\beta^2)\\ &\qquad+(1/81)f_1(2\,\mathrm{Re}\alpha^2\bar{\beta}^2\,\mathrm{Re}\beta^2+15|\beta|^4\,\mathrm{Re}\alpha^2)].\\ G-N&=64[f_0|\alpha|^6+(1/81)f_2|\alpha|^2(22\,\mathrm{Re}\alpha^2\bar{\beta}^2-8|\alpha|^2|\beta|^2)\\ &\qquad+(1/81)f_1|\beta|^2(-8\,\mathrm{Re}\alpha^2\bar{\beta}^2+25|\alpha|^2|\beta|^2)]\\ H&=0\end{aligned}$$

are diagonal, with matrix elements proportional to α, $\bar{\alpha}$, β, $\bar{\beta}$:

$$\begin{aligned}\alpha&=[1/2m(E+m)][(E+m)^2-k^2e^{-i\theta}]\,,\\ \beta&=[1/2m(E+m)][(E+m)^2+k^2e^{-i\theta}]\,.\end{aligned} \qquad (14)$$

The second step is to consider specific spin transitions such as $(++)\rightarrow(+-)$, $(++)\rightarrow(++)$, etc., computing the matrix elements both in Eq. (7) and in the partially reduced form of Eq. (10), thus obtaining simple linear equations for the Wolfenstein parameters. The results are shown in Tables I and II. The striking fact that $H=0$, on which most of our conclusions are based, is seen at once by considering the $(++)\rightarrow(--)$ matrix element of T, which vanishes. The origins of this result can be seen by considering the explicit form of the tensor $B(iI)_{a_1A_1a_2A_2a_3A_3}$. If, for the moment, we let each a_i take the values $\pm\frac{1}{2}$ in the natural way, we find that the elements of B vanish unless $a_1+a_2+a_3=i$. Since the matrices $d_{a'a}$ are diagonal it follows that

$$\sum_{i=1}^{6}a_i'=\sum_{i=1}^{6}a_i\,,$$

and thus that the total spin component normal to the scattering plane is conserved. This result is true for all scatterings described by the amplitude Eq. (10) and is not restricted to nucleon-nucleon scattering.

III. RELATIONS BETWEEN OBSERVABLES

We have seen that an $SU(12)_{\mathfrak{L}}$ symmetric baryon-baryon scattering amplitude has only four independent complex amplitudes at every energy and angle except 90° in the center-of-mass system, where there are two. Even though we are considering only nucleon-nucleon scattering, this is a sharp reduction from the 10 complex amplitudes of the ordinary charge-independent theory. Nonetheless, a perfectly straightforward test would require more independent measurements at each of several energies and angles than are now available experimentally. [The unitarity restriction usually used to reduce the number of independent experiments required for a phase shift analysis is irrelevant here, since it is known[5] that this $SU(12)_{\mathfrak{L}}$ prescription does not

[5] M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 509 (1965).

lead to amplitudes satisfying unitarity.] However, there are a number of definite predictions which follow from the vanishing of H, and the Pauli principle.

Because $H=0$ the following constraints apply to the pp and np triple scattering parameters[6] at all energies and angles.

$$R-A'=0, \tag{15}$$

whence

$$R'+A=0 \tag{16}$$

$$|C_{KP}|\leq\cos\alpha_L. \tag{17}$$

The angle between the two final nucleons in the lab frame is denoted by α_L.

For pp scattering the Pauli principle leads to the following additional relations at 90° c.m.:

$$A(90°)-R(90°)\cot\theta_L=0, \tag{18}$$

$$2[1-D(90°)]-[1-C_{NN}(90°)]=0, \tag{19}$$

$$D(90°)\geq 0. \tag{20}$$

The laboratory scattering angle is denoted by θ_L.

We also obtain relations between pp and np cross sections:

(a) the following inequality on the differential cross sections at 90°, valid to order $[k/(E+m)]^8$,

$$\left(\frac{d\sigma(pp)}{d\Omega}\right)_{90°}\geq\left(\frac{d\sigma(pn)}{d\Omega}\right)_{90°}; \tag{21}$$

(b) a relation between the triplet and singlet scattering lengths

$$a_s=a_t=128(f_0-(1/27)f_2). \tag{22}$$

It should be noted that this is really a prediction of $SU(6)$ and is independent of the manner in which the theory is extended to moving particles. Note also that, at rest, there are only two independent amplitudes since Eq. (11) becomes

$$f_0=-f_3,\quad f_1=-f_2. \tag{23}$$

IV. COMPARISON WITH EXPERIMENTAL DATA

A. Scattering Lengths

The experimental values of the np scattering lengths are[7]:

$$a_s=-23.680\pm0.028\text{ F},\qquad a_t=5.399\pm0.011\text{ F}. \tag{24}$$

The interpretation of this discrepancy in the np system presents some difficulties. If the $SU(12)_{\mathfrak{L}}$ symmetry is regarded as a purely phenomenological theory describing scattering amplitudes, this is strong counter evidence. On the other hand, it might be argued that the scattering lengths are so strongly dependent on the locations of the nearby bound and virtual states that this large difference between them is a reflection of a small breaking of the symmetry. Detailed analysis of this question leads to many deep questions "which we couldn't answer if we could think of them."

B. Differential Cross Section at 90°

Experimentally[7]

$$\left(\frac{d\sigma(np)}{d\Omega}\right)_{90°}>\left(\frac{d\sigma(pp)}{d\Omega}\right)_{90°} \tag{25}$$

for energies below about 75 MeV. However, the fractional difference is not always large, and in some regions a small symmetry-breaking term might account for the discrepancy.

TABLE III. The correlation coefficient C_{KP} and its predicted upper bound.

T_{lab} (MeV)	$\theta_{\text{c.m.}}$ (deg)	Value[a]	Upper bound
52.3	90°	0.10±0.14	0.014
315	45°	0.74±0.51[b]	0.039
382	90°	0.63±0.10[c]	0.086
380	30°	0.12±0.10	0.047
400	90°	0.32±0.09	0.095
450	90°	0.37±0.14	0.104
660	90°	0.22±0.18	0.147

[a] Data taken from a reference in Ref. 7.
[b] Yu. M. Kazarinov *et al.*, Zh. Eksperim. i Teor. Fiz. **47**, 848 (1964) [English transl.: Soviet Phys.—JETP **20**, 565 (1965)].
[c] J. V. Allaby, A. Ashmore, A. N. Diddens, J. Eades, G. B. Huxtable, and K. Skarsvåg, Proc. Phys. Soc. (London) **77**, 234 (1961).

TABLE IV. The prediction $R'=-A$.

T_{lab} (MeV)	$\theta_{\text{c.m.}}$	R'	T_{lab} (MeV)	$\theta_{\text{c.m.}}$	$-A$
140[a]	31.4°	0.625±0.062	139	31.1°	0.368±0.032
	41.7°	0.548±0.062		41.4°	0.344±0.031
	52.0°	0.470±0.069		51.7°	0.311±0.035
	61.8°	0.343±0.058		61.9°	0.231±0.046
	72.1°	0.466±0.095		72.0°	0.189±0.056
	82.2°	0.190±0.177		82.1°	0.099±0.079
213[b]	30°	0.491±0.025	213	30°	0.400±0.019
	40°	0.390±0.024		40°	0.317±0.019
	50°	0.177±0.022		50°	0.205±0.021
	60°	0.120±0.025		60°	0.102±0.025
	70°	−0.277±0.045		70°	0.012±0.036
	80°	−0.208±0.068		80°	0.090±0.046
	90°	−0.340±0.104		90°	0.180±0.077

[a] O. N. Jarvis *et al.*, Nucl. Phys. **50**, 529 (1964).
[b] K. Gotow and F. Lobkowicz, Phys. Rev. **136**, B1345 (1964).

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[6] There is no good single reference for these parameters. A,R,A', and R' are essentially polarization rotation coefficients. They are defined by Wolfenstein [Ann. Rev. Nucl. Sci. **6**, 43 (1954)]. The nonrelativistic forms are correctly tabulated in Ref. 4. The relativistic corrections were first discussed by Stapp [Phys. Rev. **103**, 425 (1956)]. A readable account is given by Roth [Princeton Technical Report #33, 1964 (unpublished)], and the results for A,R,A',R', and C_{KP} are correctly given in a note by Sprung [Phys. Rev. **121**, 925 (1961)].

[7] R. Wilson, *The Nucleon-Nucleon Interaction* (Interscience Publishers, Inc., New York, 1963).

TABLE V. The prediction $R=A'$.

T_{lab} (MeV)	$\theta_{c.m.}$	R	A'
430[a]	30°	0.06±0.11	0.47±0.20
	45°	0.40±0.11	0.06±0.11
	60°	0.43±0.08	0.06±0.09
	75°	0.47±0.07	0.22±0.08
	90°	0.47±0.05	0.36±0.07
	105°	0.35±0.11	0.01±0.11
	120°	0.34±0.18	0.08±0.04

[a] R. F. Roth, Princeton Technical Report #33, 1964 (unpublished report).

TABLE VI. The prediction $2(1-D)=1-C_{NN}$ and $A=R\cot\theta_L$ for $\theta_{c.m.}=90°$.

T_{lab} (MeV)	$2(1-D)$	T_{lab} (MeV)	$1-C_{NN}$
310	1.16±0.12[d]	310	$0.16_{-0.10}^{+0.22}$[b]
430[c]	0.66±0.20	425[a]	0.34±0.15
660	0.14±0.34	650	0.07±0.20
T_{lab} (MeV)	A	$R\cot\theta_L$	
213	−0.180±0.077	0.234±0.055	
430	0.27 ±0.07	0.52 ±0.05	

[a] Data averaged over $T=400$ and 450 MeV.
[b] I. M. Vasilevski *et al.*, Zh. Eksperim i Teor. Fiz. 39, 889 (1960) [English transl.:—JETP 12, 616 (1961)].
[c] R. F. Roth (see footnote, Table V).
[d] Extrapolation of data in Wilson (see Ref. 7).

C. Triple-Scattering Parameters

The *pp* data relevant to the four results given in Sec. III are listed in Tables III–VI.

It is important to note that these predictions are quite stable under the introduction of a small symmetry-breaking interaction. This is because each quantity predicted to be zero is, in fact, of the form

$$\mathrm{Re}H^*A/(|H|^2+\sum|A|^2)\,, \qquad (26)$$

where the A's are appropriate linear combinations of the other Wolfenstein parameters. Clearly if H is changed from zero to some value small compared to A, the resulting change in this fraction will be even smaller. Hence we feel that the comparisons in Tables III–VI test the suggestion that $SU(12)_{\mathfrak{L}}$ symmetry provides a phenomonological leading approximation to some true theory of strong interactions.

V. DISCUSSION

It is possible that careful study of these data will provide some clue to where the symmetry is good and why it fails in other regions, but we do not find any suggestive regularity.

Other results dealing with polarization have been obtained for meson-baryon scattering by Blankenbecler, Goldberger, Johnson, and Treiman.[8] However, Chang and Shpiz[9] have pointed out that that result depends upon the assignment of the mesons to the 143-dimensional representation of $SU(12)_{\mathfrak{L}}$, an assignment which is not necessary for the successful predictions of the scheme. The present results are clearly independent of the assignment of the mesons. The equality of the scattering lengths is mentioned by Akyeampong *et al.*[10] in a discussion of nucleon-nucleon scattering and the $\tilde{U}(12)$ symmetry of Salam *et al.*[3]

Bég and Pais have argued that their scheme for the relativistic completion of $SU(6)$ should be regarded as a procedure for calculating effective matrix elements. Because of the nonunitary nature of the scheme it can at best be expected to give a good approximation to the true matrix elements. We have found that the approximation is poor in the case of nucleon-nucleon scattering and we expect that the predictions of this scheme will not, in general, be reliable.

Note added in proof. A number of authors have proposed schemes for the relativistic extension of $SU(6)$ which would impose on the scattering amplitude requirements less restrictive than assumption I. We call attention to the fact that *all of the relations between observables* given in Sec. III, with the possible exception of Eq. (21), *are obtained from any of the following schemes:* $U(6,6)$ broken by kinetic energy spurions,[11] $M(12)$ broken by kinetons,[12] $\tilde{U}(12)\otimes T(143)$,[13] $I\ \tilde{U}(12)$,[14] and $U(3)\times U(3)$.[15] We would like to thank Dr. S. Meshkov and Professor R. Oehme for discussion of this point.

ACKNOWLEDGMENTS

We are particularly indebted to Dr. Bég and Dr. Pais for helpful and stimulating discussions of their work. We would like also to thank Dr Weneser, Dr. Wick, and Dr. Yao for helpful comments.

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