We *do not* obtain the results obtained in the static $SU(6)$ model by the inclusion of a "spin-spurion."[17] We also notice that if we include other kinds of $SU(3)$ symmetry breaking terms (e.g., with $W=1$) we get no results apart from Eq. (4). The relation (5) cannot be changed by $\Sigma_\epsilon$-$Y_1^*$ mixing, because both $\Sigma_\epsilon$ and $Y_1^*$ have the same branching ratios for the decays into $\Lambda\pi$ and $\Sigma\pi$.

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# Differential Cross Section for Trident Production

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This paper discusses the differential cross section for pair production by electron scattering off an arbitrary fixed potential (trident). This cross section assumes that the three outcoming electrons' energies are to be measured. A modified version of the Monte Carlo procedures is used to obtain the cross section. Three cases were evaluated on an IBM 7090 computer, to obtain a standard deviation of about 4% to 9%.

THE history of the problem of pair production by electron scattering off a fixed electromagnetic field, succinctly called "trident" production, is a long and varied one.

The first studies of the problem were theoretical. The primary papers were those of Bhabha[1,2] and Racah.[3,4] The Bhabha calculation uses the Weizsäcker-Williams approximation to calculate the *total* cross section for this trident process. This approximation can briefly be described as assuming that the incident particle can be placed in a rest frame by a Lorentz transformation. The nucleus producing the scattering is then treated as though its field of virtual photons were a collection of independent photons. The major use of the Bhabha calculation is with the assumption that the incoming particle is a different particle than the pair-produced electrons. The theoretical analysis of Murota *et al.*[5,6] considered carefully the assumptions of the Weizsäcker-Williams approximation by using Feynman-diagram techniques. They found that the Bhabha formula was valid for trident production under the condition that the initial energy of the electron is greater than 10 GeV. Their analysis took into account two of the eight possible Feynman diagrams (see Figs. 1E and 1G) and gave an estimate of the neglected terms for the total cross section. The Racah calculation was done according to the 1930's version of perturbation theory. That result involved neglecting exchange and considered four of the Feynman diagrams (see Figs. 1A, 1C, 1E, and 1G). These papers constitute the present theoretical analysis of the trident problem for small angles.

There were three distinct groups of experiments. The first group was a series of cloud-chamber experiments essentially culminating with a review article by Crane and Halpern.[7] They analyzed the results of the various cloud-chamber experiments concluding that, for the energy ranges involved (less than 10 MeV), there were no conclusive discrepancies between experiment and theory.

The second group of experiments began after World War II with the advent of nuclear emulsions. There appeared first a series of papers merely giving evidence of the existence of this trident process. Next a series of papers recognized that many of the observed tridents were not the direct result of electron pair production. They were what is usually called pseudotrident, which means that an electron produces a bremsstrahlung which then produces electron pairs. The mean free path of the bremsstrahlung is sufficiently small, so that the fork position of the electron pair and the direct path of the primary electron are not resolved, hence the erroneous assumption that the measurement was a true trident. Because this pseudoprocess demands an extremely large correction to get the "true" trident, there has been a problem as to whether the experimental results of the paper indicating discrepancies are true discrepancies between theory and experiment or a consequence of the experimental method for detection of trident production. The paper of Weil[8] summarizes some of the experimental results of nuclear emulsions

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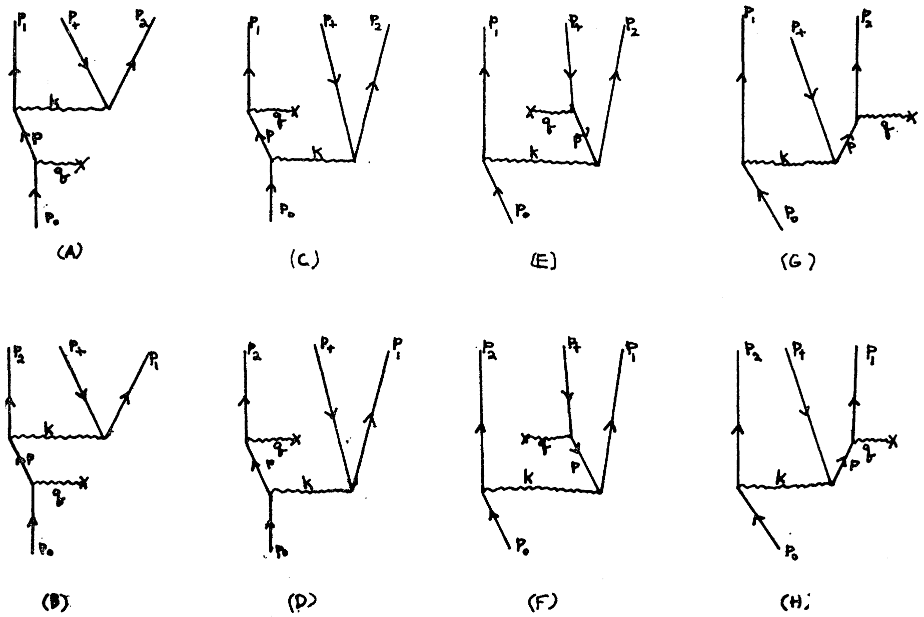

FIG. 1. Feynman diagrams involved in trident production.

and the resulting graphs indicate a discrepancy, after all experimental corrections, of roughly three or four times the theoretical cross section, over an extremely large energy range of 1 to 100 GeV. This comparison is made relative to the Bhabha-Racah theory. Weil comments that experiment and theory will agree if one uses the remark of Stueckelberg.[9] The essence of this is to allow the photons of momentum higher than the rest mass of the electron to contribute to the total cross section in the calculations by the method of the Weizsäcker-Williams approximation. This is not in the spirit of that approximation and makes no theoretical sense although it apparently agrees with the experiments summarized by Weil. The paper by Adita[10] does some very careful investigation of the actual experimental situation of using nuclear emulsions in conjunction with cosmic-ray showers. He essentially concludes that the fluctuation effects, due to the extremely low statistics and large number of corrections for this process, are sufficiently ill defined so that one cannot say there is any true experimental discrepancy with theory. As a result, if such a discrepancy exists, it will not be shown by the above type of experiment.

We are thus led to the third experimental method, viz., using a machine to produce a beam of electrons with a well-defined incident energy. Having this well-defined incident energy avoids one of the major problems of nuclear emulsions, namely, estimating the initial energy of the electron track. There have been to date two such machine experiments.

One is by Camac[11] at roughly 150 MeV. He performs the experiment using a variable thickness copper target and measures the outgoing positrons which have an energy roughly 0.8 to 0.6 of the energy of the incident electron. He shows that the interpolated results are in surprising agreement with the Weizäcker-Williams approximation in spite of the fact that it is not valid for the conditions of this experiment.

The second experiment was done by Criegee[12] at 31.5 MeV. He performed this experiment by measuring the positron distribution, as Camac did for his experiment. The experimental results were found by Criegee to be a factor of one third smaller than those of the Bhabha theory and within the theoretical limits defined by the papers of Murota *et al.* Murota specified these limits by estimating the effects of the neglected Feynman-diagram terms in the trident calculation. (Criegee, incidentally, gave reference to a large number of previous papers on trident production.)

It can now be seen that a better theoretical calculation adaptable to the conditions of the machine experiment and further machine experiments over the high-energy spectrum (1 to 10 GeV) are needed to tell whether a discrepancy between theory and experiment truly exists.

In order to appreciate the need to improve the theory, one must understand the experimental conditions and why the machine procedure is the best way to resolve the question of this alleged discrepancy. In the range of 100 MeV, the trident-production cross section is not significantly large enough to produce good experimental statistics. In the range of 1 GeV and higher for incident energy electrons, the process has a large enough cross section to permit reasonable experimental results. To get a well-defined experiment, a way must be found to

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eliminate all other possible pseudoprocesses which may occur concurrently with the trident process. Essentially, all other multiple-particle processes may be eliminated, with the exception of the pseudo-trident and perhaps Dalitz pairs,[13,14] by simply measuring all three outcoming particles by coincidence techniques. At these extreme energies, one clearly will not try to resolve the angular dependence of the cross section. In fact, this would be unjustified. Since the theoretical calculation performed in this work is only valid to the Born approximation, the angular dependence cannot be considered reliable. The elimination of the pseudotrident can be accomplished by varying the thickness of the target and making use of the fact that the pseudotrident's cross-sectional dependence is proportional to the square of the thickness itself. The question as to the validity of the Born approximation may be resolved by varying the electric charge of the scattering target over a sufficiently large range and extrapolating the result to zero. The question of radiative corrections has not been resolved. Only the results of the paper by Bjorken *et al.*[15] can be used as an estimate of this effect. If the Bjorken calculation is accepted as a reasonable estimate, one would conclude that the radiative correction is, at most, five percent.

The foregoing has defined the essential reasons for this work. That is, the theoretical calculation of the cross section of electron-positron pair production by an electron scattering off a fixed external electromagnetic field. This work provides the means to calculate the differential cross section to sufficient accuracy, so that a test of quantum electrodynamics under this process can be made when its experimental results have been obtained. This calculation is not, however, of the same sensitivity to certain aspects of quantum electrodynamics as the wide-angle experiments suggested by Bjorken.[15,16] The two calculations differ in their respective regions of sensitivity to the field dependence of the electromagnetic field. The wide-angle case is sensitive to field dependence inside the nucleus and this calculation is sensitive to the atomic screening effects. The Born approximation used in the calculation is concluded to be adequate in light of the series of articles by Maximon.[17–19] These papers analyze the pair-production cross section by gamma rays using hydrogenic wave functions. It was concluded in these papers that, as long as the momentum transfer to the nucleus was less than the rest mass of the electron, the Born approximation was a sufficiently valid calculation. The author has found in his calculation that the momentum transfer to the nucleus is less than the rest mass of an electron. As stated, the experimental situation requires that each outgoing particle have a significant percentage of the incident energy. As a result of this situation, the conclusion is quickly reached that all eight Feynman diagrams corresponding to the lowest order for this process are significant (see Fig. 1). Hence begins the tedious task of constructing the differential cross section and devising a method for integrating out the angular dependences by machine (IBM 7090) to obtain the appropriate differential cross section for the experiments, namely

$$d^2\sigma/dE_+dE_2[E_+,E_2,E_0].$$

This is the differential cross section for fixed energy for each of the three exit particles. Because of energy conservation and the assumed character of the electromagnetic potential, the first electron has its energy $E_1$ already uniquely specified.

There were three cases calculated. (See Table I.) These are compared with the Bhabha formulas. The results, found by using for illustration the Yukawa potential, $(q^2+x^2)^{-1}$, with $x^2=(Z/137)^2$, are given in Table II.

As one can see, the exchange terms produce a significant difference between the two results. If one wishes to use a different potential from Yukawa, the thesis has a detailed tabulation of the cross section calculation which contains information allowing determination of cross sections produced by that potential.

There are three separate machine programs that must be used to produce the final integration.

The first is a search program which computes the

TABLE I. Particle energies in units of $10^3$ electron rest masses. Thus, the energy of the incident electron is ≈6 GeV.

| Case | $E_1$ First electron | $E_2$ Second electron | $E_+$ Positron | $E_0$ Incident electron |
|---|---|---|---|---|
| A1 | 3 | 1 | 8 | 12 |
| A4 | 3 | 3 | 6 | 12 |
| A6 | 4 | 4 | 4 | 12 |

TABLE II. Differential cross sections for the 3 cases of Table I, in units of $10^{-37}$ cm² for $Z=1$ and $10^{-33}$ cm² for $Z=80$, according to the author's and Bhabha's calculations.

| | | Author | | Bhabha Non-screening | Bhabha Complete screening |
|---|---|---|---|---|---|
| A4 | $Z=1$ | 0.679 | ±7.0% | 2.06 | 1.33 |
| | $Z=80$ | 0.309 | ±8.4% | 1.32 | 0.55 |
| A1 | $Z=1$ | 0.753 | ±4.1% | 0.435 | 0.282 |
| | $Z=80$ | 0.340 | ±4.4% | 0.279 | 0.117 |
| A6 | $Z=1$ | 0.913 | ±6.8% | 3.470 | 2.240 |
| | $Z=80$ | 0.437 | ±8.1% | 2.220 | 0.930 |

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value of the integrand under the assumption of a Yukawa potential. This program performs the task of searching with the five-dimensional space defined by the above transverse variables for the position of the maximum of the integrand. Also determined within the search is a crude estimate of the rate of change the integrand exhibits around the maximum. The data from this search program is then used as input data for the second program—the hypersearch program.

The purpose of the hypersearch program is to provide more detailed information of the functional dependence of the integrand around the maximum. The hypersearch systematically calculated 256 points within the five-dimensional space. The value of the integrand from these points is then used to construct composite graphs which represent the functional dependence of the integrand for each variable. These composite graphs are then used to provide the necessary input information for the third program—the Monte Carlo program.

This Monte Carlo program is a modification of normal Monte Carlo calculations. By means of the composite graphs an attempt is made to influence the distribution of points within the five-dimensional space used by the Monte Carlo program. This permits reduction of the magnitude of the standard deviation of the quantity to be averaged, namely the integrand.

The programs were tested as follows. The integrand was tested by performing repeated algebraic computations and then a single value of the integrand was calculated by hand and compared with the value found by the machine. Other aspects of the programs were also checked by hand calculations, testing each subroutine. A check was made to see if there was the possibility of overflow. The details of this calculation and the computer programs are on file as the author's Ph.D. thesis at Harvard University (1962).

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# Comparison of Nucleon-Nucleon Scattering Data with the Predictions of $SU(12)_{\mathcal{L}}$[*]

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The form of the nucleon-nucleon scattering amplitude is determined subject to the restriction that baryon-baryon scattering be invariant under $SU(12)_{\mathcal{L}}$ transformations. The results are found to disagree with experiment. The possibility that $SU(12)_{\mathcal{L}}$ may be a "leading approximation" to a true $S$-matrix theory is discussed briefly.

## I. INTRODUCTION

SEVERAL schemes for the calculation of scattering amplitudes have been proposed which are motivated by the desire to extend $SU(6)$ symmetry[1] to states of two or more particles in relative motion.[2,3] We have considered nucleon-nucleon scattering in the particular scheme of Bég and Pais and find disagreement with the experimental data. The calculation involves only two assumptions:

I. The scattering amplitude is invariant under transformations belonging to the group which Bég and Pais have called $SU(12)_{\mathcal{L}}$.

II. Baryon states transform according to the 364-dimensional representation of this group. That is, the baryons are represented by completely symmetric three-index tensors $B_{\lambda\mu\nu}$, where each index runs from 1 to 12.

The disagreement with experiment persists even when assumption I is considerably weakened. We discuss this in Sec. IV.

We feel that nucleon-nucleon scattering provides a particularly good testing ground for this theory for two

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[*] Work performed under the auspices of the U. S. Atomic Energy Commission.

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