Decays of Positive-Parity Baryon Resonances in a Broken $\tilde{U}(12)$

H. Harari
Israel Atomic Energy Commission, Soreq Nuclear Research Center, Yavne, Israel

AND

D. Horn
Department of Physics, Tel-Aviv University, Tel-Aviv, Israel

AND

M. Kugler and H. J. Lipkin
The Weizmann Institute of Science, Rehovoth, Israel

AND

S. Meshkov*
National Bureau of Standards, Washington, D. C.

(Received 25 June 1965)

The simplest representations of $SU(6)$ and $\tilde{U}(12)$ which are capable of describing the low-lying positive-parity baryon resonances are the three-quark representations. All particle states in these multiplets are eigenstates of W spin and $SU(6)_W$, the subgroup of $\tilde{U}(12)$ and $U(6)\times U(6)$ which remains invariant for two-body decay processes, even when kinetic-energy terms and derivative couplings are included in the calculations. It is shown that $SU(6)_W$ leads to various selection rules and to definite branching ratios for the decays of these resonances. The branching ratios for the decays of the spin-$\frac{3}{2}$, $SU(3)$-octet resonances in the **70** representation of $SU(6)$ and $SU(6)_W$ are calculated. An attempt is made to include $SU(3)$ symmetry-breaking effects in a calculation of the decay rates of the $\frac{3}{2}^+$ decuplet within the **56** of $SU(6)_W$.

ONE of the most attractive features of the static $SU(6)$ theory[1] is the complete classification of low-lying mesonic and baryonic states into the **35** and **56** representations. A tentative assignment of some higher baryon resonances to the **70** was suggested,[2] while the higher meson resonances are, presumably, in the **189** or **405**. It is, however, obvious that any "static" classification cannot be complete, because it involves only properties such as spin, parity, and mass values. The relative decay rates of the various states remain beyond any analysis which is based on $SU(6)$ since most of the decays are forbidden in this limit. This difficulty is *not* removed when we consider the same problem within the framework of the $\tilde{U}(12)$ theory,[3,4] for even then, in the case of complete symmetry, almost all decay modes of higher baryon and meson resonances are forbidden.[5] This is not surprising at all, as it is easy to see that the inclusion of symmetry-breaking kinetic-energy terms or "momentum spurions" is inevitable in this theory, and that these lead to nonvanishing matrix elements for the relevant decays. The explicit calculation may proceed by assuming symmetry breaking to a certain order in the momentum spurions, hoping that the contributions of higher orders are comparatively small.[6] Alternatively, one may take into account symmetry breaking to all orders, thus completely violating $\tilde{U}(12)$ but preserving its subsymmetry[7] $SU(6)_W$ under which all collinear processes are supposed to be invariant even in the presence of an arbitrary number of symmetry-breaking momentum spurions.[8] We prefer the second possibility because at present we have no reason to believe that the first orders in the symmetry-breaking interaction are really dominant. The $SU(6)_W$ subgroup of $\tilde{U}(12)$ provides us with a useful tool for calculating branching ratios between the two-body decays of the eigenstates of $\mathbf{W}^2$. It should, however, be noted that the states of $\tilde{U}(12)$ multiplets appearing in systems of two quarks and two antiquarks or four quarks and one antiquark are, in general, mixtures of W-spin eigenstates. Consequently, the only decays that are calculable are those of the positive-

* Supported in part by the U. S. Office of Naval Research, Contract No. NA-onr-11-65.

[1] B. Sakita, Phys. Rev. **136**, B1756 (1964); F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964); A. Pais, *ibid.* **13**, 175 (1964).

[2] I. P. Gyuk and S. F. Tuan, Phys. Rev. Letters **14**, 121 (1965). See also A. Pais, Ref. 1.

[3] A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965); M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 267 (1965); B. Sakita and K. C. Wali, *ibid.* **14**, 404 (1965).

[4] For attempts of classifying meson and baryon resonances in $\tilde{U}(12)$, see R. Delbourgo, M. A. Rashid, and J. Strathdee, Phys. Rev. Letters **14**, 719 (1965); H. Harari, D. Horn, M. Kugler, H. J. Lipkin, and S. Meshkov, Phys. Rev. **140**, B431 (1965).

[5] H. Harari, Phys. Rev. Letters **14**, 1100 (1965).

[6] It should be mentioned that most of the results which have been obtained for the case of first order symmetry breaking, such as the $\bar{p}p$ annihilation at rest into two mesons, are valid to all orders in the symmetry breaking, because they involve collinear processes.

[7] H. J. Lipkin and S. Meshkov, Phys. Rev. Letters **14**, 670 (1965).

[8] To be exact, symmetry-breaking kinetic-energy corrections may be imposed on all external lines of collinear processes, and on some internal lines of the appropriate diagrams, without violating $SU(6)_W$. For more complicated diagrams one might introduce momentum spurions whose components in other directions do not vanish.

parity baryon resonances of the $3q$ system. As the **20** of $SU(6)_W$ cannot decay into **35**+**56**, and the **56** is already completely filled, we are left only with the **70** at our disposal.[9] Note that the **70** of $SU(6)_W$ is also a **70** of the ordinary static $SU(6)$ and is described in $\tilde{U}(12)$ by the **572** multiplet. The $SU(3)\times SU(2)$ content of the **70** is

$$\mathbf{70}=(\mathbf{8},\mathbf{4})+(\mathbf{8},\mathbf{2})+(\mathbf{10},\mathbf{2})+(\mathbf{1},\mathbf{2}). \qquad (1)$$

The only candidates for the (**8**,**4**) representation are $Y_1^*(1660)$ and $\Xi^*(1810)$. Their parities are, however, not yet established and the possibility of negative parity is not ruled out. The possible $N^*(1480)$ with $J^P=\frac{1}{2}^+$ may be a candidate for the (**8**,**2**). It is hard to say more before the existence of the $N\eta$, $\Lambda\eta$ and such resonances is verified. Using the Clebsch-Gordan coefficients[10] for the decay $\mathbf{70}\to\mathbf{56}\times\mathbf{35}$ in $SU(6)_W$ we may express all two-body decays of the components of the **70** in terms of one parameter. As the (**8**,**4**) seems to be the most interesting[11] at present, we list here all branching ratios for the decays of its components into a pseudoscalar meson and a $\frac{1}{2}^+$ or $\frac{3}{2}^+$ baryon of the **56**. Denoting the decaying particles by N_ϵ, Σ_ϵ, Λ_ϵ, and Ξ_ϵ, we get[12]

$$\Gamma(N_\epsilon\to\Lambda K)=\Gamma(\Lambda_\epsilon\to N\bar{K})=\Gamma(\Xi_\epsilon\to\Xi\eta)=0, \qquad (2a)$$

$$\begin{aligned}\bar{\Gamma}(N_\epsilon\to N^*\pi)&=4\bar{\Gamma}(N_\epsilon\to Y^*K)=20\bar{\Gamma}(N_\epsilon\to N\pi)=5\bar{\Gamma}(N_\epsilon\to\Sigma K)=20\bar{\Gamma}(N_\epsilon\to N\eta)\\ &=4\bar{\Gamma}(\Lambda_\epsilon\to\Xi^*K)=(8/3)\bar{\Gamma}(\Lambda_\epsilon\to Y^*\pi)=20\bar{\Gamma}(\Lambda_\epsilon\to\Xi K)=40\bar{\Gamma}(\Lambda_\epsilon\to\Lambda\eta)=(40/3)\bar{\Gamma}(\Lambda_\epsilon\to\Sigma\pi)\\ &=3\bar{\Gamma}(\Sigma_\epsilon\to N^*\bar{K})=12\bar{\Gamma}(\Sigma_\epsilon\to\Xi^*K)=12\bar{\Gamma}(\Sigma_\epsilon\to Y^*\pi)=8\bar{\Gamma}(\Sigma_\epsilon\to Y^*\eta)=15\bar{\Gamma}(\Sigma_\epsilon\to N\bar{K})\\ &=60\bar{\Gamma}(\Sigma_\epsilon\to\Sigma\pi)=40\bar{\Gamma}(\Sigma_\epsilon\to\Lambda\pi)=40\bar{\Gamma}(\Sigma_\epsilon\to\Sigma\eta)=60\bar{\Gamma}(\Sigma_\epsilon\to\Xi K)=4\bar{\Gamma}(\Xi_\epsilon\to\Omega K)\\ &=8\bar{\Gamma}(\Xi_\epsilon\to\Xi^*\pi)=8\bar{\Gamma}(\Xi_\epsilon\to\Xi^*\eta)=8\bar{\Gamma}(\Xi_\epsilon\to Y^*\bar{K})=40\bar{\Gamma}(\Xi_\epsilon\to\Sigma\bar{K})\\ &=40\bar{\Gamma}(\Xi_\epsilon\to\Lambda\bar{K})=10\bar{\Gamma}(\Xi_\epsilon\to\Xi\pi).\end{aligned} \qquad (2b)$$

$\bar{\Gamma}$ is the partial width for the appropriate decay, "corrected" by a phase-space factor.

Some of these processes may be forbidden by energy conservation. For all other processes, phase-space factors should be included while an actual comparison with experiment is made. We notice that some processes which are allowed by $SU(3)$ are forbidden in $SU(6)_W$.[13] The decay $\Lambda_\epsilon\to\bar{K}N$ is one example. Thus, Λ_ϵ will not be found as a peak in the $\bar{K}N$ cross section. Moreover, its production in the reactions $\pi^-+p\to\Lambda_\epsilon+K^0$, $K^-+p\to\Lambda_\epsilon+\pi^0$ will not proceed through one-particle exchange mechanisms, since both the $K\Lambda_\epsilon\bar{p}$ vertices are forbidden.

In general, we have to include the contribution of the $SU(3)$ symmetry breaking interaction in all calculations based on higher symmetry groups. It is, however, premature to do it for the decays of the **70**; the $SU(6)_W$ predictions should be regarded merely as a guide in the classification of the particles within this representation. On the other hand, we may include $SU(3)$ symmetry breaking terms while calculating the decay modes of the $\frac{3}{2}^+$ decuplet of the **56**. Exact $SU(3)$ predicts

$$(N^*|N\pi)=\sqrt{2}(Y_1^*|\Lambda\pi)=\sqrt{3}(Y_1^*|\Sigma\pi)=\sqrt{2}(\Xi^*|\Xi\pi). \qquad (3)$$

This is in clear contradiction with the data,[14] because $(Y_1^*|\Sigma\pi)$ seems to be consistent with zero while at least $(N^*|N\pi)$ and $(Y_1^*|\Lambda\pi)$ are much larger. Including an interaction which transforms like the $I=Y=0$ member of an octet, one gets[15]

$$2(N^*|N\pi)+3\sqrt{2}(Y^*|\Lambda\pi)-3(Y^*|\Sigma\pi)+2\sqrt{2}(\Xi^*|\Xi\pi)=0. \qquad (4)$$

This seems to be satisfied by the data. If we now assume that the $SU(3)$ symmetry breaking term is a W-spin scalar of a **35** of $SU(6)_W$ [an assumption which is consistent both with $\tilde{U}(4)$ conservation and the inclusion of an arbitrary number of momentum spurions], we get one more relation. However, this is the one which contradicts the experimental data, namely[16]

$$\sqrt{2}(Y_1^*|\Lambda\pi)=\sqrt{3}(Y_1^*|\Sigma\pi). \qquad (5)$$

[9] Note that our **70** has nothing to do with that of Gyuk and Tuan, as they discussed only negative parity states.

[10] J. C. Carter, J. J. Coyne, and S. Meshkov, Phys. Rev. Letters **14**, 523 (1965); and erratum **14**, 850 (1965); C. L. Cook and G. Murtaza (to be published).

[11] The apparent absence of S-wave resonances both in the meson-meson and the baryon-meson systems might be explained by the following qualitative argument: If p-wave resonances are 100-MeV wide (e.g., the ρ and N^*), one might expect S-wave resonances with comparable coupling constants to be several hundreds of MeV wide and thus escape detection, unless some selection rule is dictating a much smaller value for the matrix element.

[12] *Note added in proof.* The couplings in Eq. (2a) also vanish in $S\tilde{U}(12)$, since $SU(6)_W$ is a subgroup of $S\tilde{U}(12)$. Thus our results (2a) are also obtained from Eq. (5) of R. Delbourgo and M. A. Rashid, Proc. Roy. Soc. (London) **A286**, 412 (1965) and are unaffected by the inclusion of momentum spurions to all orders in $S\tilde{U}(12)$. We wish to thank R. Delbourgo for pointing out that all momentum spurion couplings reduce to the simple $S\tilde{U}(12)$ form for this case.

[13] The selection rule which forbids most of these decays is the conservation of $SU(4)_I\times SU(2)_S$ within $SU(6)_W$.

[14] *Note added in proof.* Recent work (Armenteros *et al.*, Cern unpublished report, 1965) indicates that $(Y_1^*|\Sigma\pi)$ may be greater than the previously accepted value. If this newer value is, in fact, consistent with Eq. (3), then discussions of symmetry breaking become irrelevant for this case.

[15] V. Gupta and V. Singh, Phys. Rev. **135**, B1443 (1964); M. Konuma and K. Tomozawa, Phys. Letters **10**, 347 (1964); C. Becchi, F. Eberle, and G. Morpurgo, Phys. Rev. **136**, B808 (1964).

[16] Notice that a $W=0$, $SU(3)$-octet spurion in the **35** is a singlet of $SU(4)_I\times SU(2)_S$ within $SU(6)_W$; hence, any relation which is obtained by the use of this subsymmetry is not changed by introducing an arbitrary number of such spurions.

We *do not* obtain the results obtained in the static $SU(6)$ model by the inclusion of a "spin-spurion."[17] We also notice that if we include other kinds of $SU(3)$ symmetry breaking terms (e.g., with $W=1$) we get no results apart from Eq. (4). The relation (5) cannot be changed by Σ_ϵ-Y_1^* mixing, because both Σ_ϵ and Y_1^* have the same branching ratios for the decays into $\Lambda\pi$ and $\Sigma\pi$.

[17] R. Ferrari and M. Konuma, Phys. Rev. Letters **14**, 378 (1965); A. de Alfaro and K. Tomozawa, Phys. Rev. **138**, B1193 (1965).

PHYSICAL REVIEW VOLUME 140, NUMBER 4B 22 NOVEMBER 1965

Differential Cross Section for Trident Production

Eric G. Johnson, Jr.*
Harvard University, Cambridge, Massachusetts
(Received 14 December 1964; revised manuscript received 21 April 1965)

This paper discusses the differential cross section for pair production by electron scattering off an arbitrary fixed potential (trident). This cross section assumes that the three outcoming electrons' energies are to be measured. A modified version of the Monte Carlo procedures is used to obtain the cross section. Three cases were evaluated on an IBM 7090 computer, to obtain a standard deviation of about 4% to 9%.

THE history of the problem of pair production by electron scattering off a fixed electromagnetic field, succinctly called "trident" production, is a long and varied one.

The first studies of the problem were theoretical. The primary papers were those of Bhabha[1,2] and Racah.[3,4] The Bhabha calculation uses the Weizsäcker-Williams approximation to calculate the *total* cross section for this trident process. This approximation can briefly be described as assuming that the incident particle can be placed in a rest frame by a Lorentz transformation. The nucleus producing the scattering is then treated as though its field of virtual photons were a collection of independent photons. The major use of the Bhabha calculation is with the assumption that the incoming particle is a different particle than the pair-produced electrons. The theoretical analysis of Murota *et al.*[5,6] considered carefully the assumptions of the Weizsäcker-Williams approximation by using Feynman-diagram techniques. They found that the Bhabha formula was valid for trident production under the condition that the initial energy of the electron is greater than 10 GeV. Their analysis took into account two of the eight possible Feynman diagrams (see Figs. 1E and 1G) and gave an estimate of the neglected terms for the total cross section. The Racah calculation was done according to the 1930's version of perturbation theory. That result involved neglecting exchange and considered four of the Feynman diagrams (see Figs. 1A, 1C, 1E, and 1G). These papers constitute the present theoretical analysis of the trident problem for small angles.

There were three distinct groups of experiments. The first group was a series of cloud-chamber experiments essentially culminating with a review article by Crane and Halpern.[7] They analyzed the results of the various cloud-chamber experiments concluding that, for the energy ranges involved (less than 10 MeV), there were no conclusive discrepancies between experiment and theory.

The second group of experiments began after World War II with the advent of nuclear emulsions. There appeared first a series of papers merely giving evidence of the existence of this trident process. Next a series of papers recognized that many of the observed tridents were not the direct result of electron pair production. They were what is usually called pseudotrident, which means that an electron produces a bremsstrahlung which then produces electron pairs. The mean free path of the bremsstrahlung is sufficiently small, so that the fork position of the electron pair and the direct path of the primary electron are not resolved, hence the erroneous assumption that the measurement was a true trident. Because this pseudoprocess demands an extremely large correction to get the "true" trident, there has been a problem as to whether the experimental results of the paper indicating discrepancies are true discrepancies between theory and experiment or a consequence of the experimental method for detection of trident production. The paper of Weil[8] summarizes some of the experimental results of nuclear emulsions

* Present address: Boulder Laboratories, National Bureau of Standards, Boulder, Colorado.

[1] H. Bhabha, Proc. Cambridge Phil. Soc. **31**, 394 (1935).

[2] H. Bhabha, Proc. Roy. Soc. (London) **A152**, 559 (1935).

[3] G. Racah, Nuovo Cimento 4, 66 (1936).

[4] G. Racah, Nuovo Cimento 4, 112 (1937).

[5] T. Murota, A. Veda, and H. Tanaka, Progr. Theoret. Phys. (Kyoto) **16**, 482 (1956).

[6] T. Murota and A. Veda, Progr. Theoret. Phys. (Kyoto) **16**, 497 (1956).

[7] H. Crane and J. Halpern, Phys. Rev. **55**, 838 (1938).

[8] R. Wel, Helv. Phys. Acta **31**, 641 (1958).