Let the trial roots be ordered such that

$$\lambda_i < \lambda_{i+1}.$$

For $\lambda = \lambda_k$, assume values for x_1 and x_{20}. Generate two sequences, using Eq. (7),

$$x_1, x_2, \cdots, x_k$$

and

$$x_{20}, x_{19}, \cdots, x_k;$$

then scale one of the sequences such that the two values of x_k are identical.

Each trial eigenvector is improved by repeated application of the inverse power method, that is, choose $\lambda \neq \lambda_k$ but λ much closer to λ_k than to any other trial eigenvalue and replace the trial eigenvector $\mathbf{x}$ by the solution of

$$(A - \lambda I)\mathbf{z} = \mathbf{x}.$$

The first ten eigenvectors of M_{20} were normalized to unit length. The trial eigenvalues were replaced by

$$\lambda = (\mathbf{x} \cdot A\mathbf{x}).$$

As a final check, the largest observed dot product of two distinct eigenvectors was less than 10^{-14}.

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Electron Capture by High-Energy Deuterons in Gases*

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Measurements of electron-capture cross sections of 12.9- and 21.0-MeV deuterons in He, N_2, and Ar are reported and compared with published theoretical estimates and experimental results for protons with energies $\leqslant 1$ MeV.

I. INTRODUCTION

MANY calculations of cross sections for electron capture by fast protons in gases, especially in atomic hydrogen, have been published. The philosophies of the several approaches are discussed in Refs. 1 through 4 and in the review paper by Bates and McCarroll.[5] There is substantial disagreement as to both the magnitudes and the energy dependence of the published cross sections for energies >1 MeV.

We have measured the capture cross sections for 12.9- and 21.0-MeV deuterons in He, N_2, and Ar.[6] Although these few measurements represent only a beginning, their magnitudes can be checked against the published calculations, and from a comparison with the experimental results of Barnett and Reynolds at proton energies $\leqslant 1$ MeV,[7] inferences can be drawn about the *average* energy dependence for protons in the energy range 1 to 10 MeV.

II. APPARATUS AND PROCEDURE

Deuterons from the Berkeley Hilac were deflected 15 deg, collimated to a diameter of 5 mm, and passed through a differentially pumped gas cell 24 cm long. The neutral particles produced in the gas were detected with a 20-mm-diam solid-state detector, pulse-height analyzed, and counted. Charged particles emerging from the gas cell were deflected into a wide-aperture Faraday cup placed in the analyzing magnetic field to eliminate secondary-electron losses. The currents were recorded with a calibrated integrating electrometer.

At full Hilac beam current it was possible for us to locate the charge-exchange-produced neutral beam by observing it on a phosphor-coated Lucite plate. The Hilac intensity was then reduced and the phosphor replaced by the solid-state detector, which easily contained the entire beam. The energies of the deuterons were determined by the deflection in the bending magnet and by pulse-height analysis with a Li-drifted counter and Al foil degraders. The detector was calibrated with an Am^{241} α source. The uncertainty in the energies is $\pm 1.5\%$.

The pressures in the target were measured with a Schulz-Phelps-type ionization gauge, which was calibrated against a liquid-nitrogen-trapped McLeod gauge. We used the results of Meinke and Reich[8] to correct for the pumping effect of the mercury streaming to the cold trap. The maximum correction was 12%, for the

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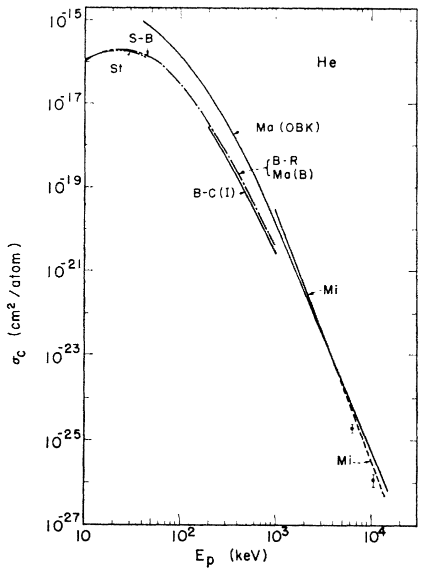

FIG. 1. Electron-capture cross sections per He atom. Experimental: closed circle with error bar, present experiment; St, Stedeford (Ref. 10); S-B, Stier and Barnett (Ref. 11); B-R, Barnett and Reynolds (Ref. 7). B-C(I), Bransden and Cheshire (Ref. 1), impulse approximation; Ma(OBK), Mapleton (Ref. 2), Oppenheimer-Brinkman-Kramers' approximation; Ma(B), Mapleton (Ref. 2), first Born approximation with nucleus-nucleus interaction; Mi, Mittleman (Ref. 3).

case of the 12.9-MeV measurement in Ar. The amount of correction was later verified by cooling the mercury to 0°C. The total uncertainty in the gas-target thicknesses is $\pm 10\%$.

We used previously measured ionization cross sections[9] to correct for the loss of neutrals within the gas cell; these corrections did not exceed 20%. Measurements were made at about six different pressures for each gas.

III. RESULTS AND DISCUSSION

The results of the present experiment are given in Table I, the errors being compounded of those previously mentioned plus internal consistency. Summaries of various experimental[7,10–14] and theoretical[1–4] results are given in Figs. 1 to 3. For discussion purposes, the values from the present measurements are plotted at their equivalent proton energies.

Helium (Fig. 1): The results can be joined to the Barnett and Reynolds' data at $E \leqslant 1$ MeV by a line with an average energy dependence of about $E^{-11/2}$. Our points are approximately a factor of 2 below Mapleton's Oppenheimer-Brinkman-Kramers (OBK) curve. This calculation yields an asymptotic variation of E^{-6} that is not yet reached in the 1- to 10-MeV range. For ease of comparison with experiment, Mittleman has produced a simple approximate expression[15]:

$$\sigma = a^2(2^{18}/5)(1.201)\pi^2 Z E^{-6} n_A(0)[1+O(Z/E)]\ \text{cm}^2,$$

valid in the restricted energy interval

$$10 < E/Z^2 < 42,$$

where a is the Bohr radius, E is measured in units of 25 keV, and $n_A(0)$ is the electron density at the origin in atomic units [$n_A(0)=3.60$ for He]. (The value for helium is from Pekeris.[16]) According to the above criterion Mittleman's calculation (giving essentially the OBK result) is applicable in He for proton energies of 1 to 4 MeV. Extrapolating this E^{-6} curve to our energies (a higher order correction would raise the calculated results slightly), we see that it lies a factor of 2 above our values. Note that this calculated curve is more than a factor of 6 higher than the Barnett and Reynolds' measurements at 1 MeV. The latter are in quite good agreement with an impulse approximation calculation by Bransden and Cheshire[1] (which asymptotically approaches an $E^{-11/2}$ variation) and a Jackson-Schiff type first-Born-approximation calculation[17] by Mapleton.[2] Lower energy measurements by Stedeford[10] and by Stier and Barnett[11] are included in Fig. 1.

Nitrogen (Fig. 2): A straight line joining our points to those of Barnett and Reynolds would have an $E^{-3.8}$ average energy variation. Mapleton's OBK curve[4] lies far below our experimental points. A value obtained by Szostak *et al.*[14] with 4-MeV protons is somewhat below Mapleton's curve. Mittleman's energy criterion indicates that his cross-section formula should be good from approximately 12 to 50 MeV, i.e., slightly higher than our energies. Evaluating $n_A(0)$ for all of the s electrons

TABLE I. Experimental electron-capture cross sections for deuterons (10^{-26} cm²/molecule).

Gas	12.9 MeV	21.0 MeV
He	20 ± 4	1.2 ± 0.4
N_2	$(40\pm8)\times10^2$	$(39\pm8)\times10$
Ar	$(11\pm2)\times10^3$	$(17\pm3)\times10^2$

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[17] This formulation is one in which a proton-nucleus interaction is included. The Brinkman-Kramers' approach is also a first Born approximation, but without a proton-nucleus interaction.

from the calculations of Clementi *et al.*,[18] we obtain $n_A(0) \approx 180$ and hence the line labeled Mi in Fig. 2.

Argon (Fig. 3): A line joining our data to those of Barnett and Reynolds would have an energy variation of roughly $E^{-3.4}$. No calculations are available for this gas. The Mittleman formula should not be applicable below about 90 MeV and presumably would seriously overestimate the cross section at 10 MeV.

IV. REMARKS

Published calculations of electron capture by high-energy protons are not in good agreement and experimental data are almost nonexistent. In the case of helium, the Bransden-Cheshire impulse approximation (which agrees with experimental results to within a factor of 2 in the energy range 0.1 to 1 MeV), if extrapolated with its asymptotic slope ($E^{-11/2}$), passes through our experimental points. The first Born approximation, including nucleus-nucleus interactions, also agrees with measurements, perhaps fortuitously.[1] Numerical values obtained for He from Mittleman's approximate formula agree fairly well with our measurements but not with the Barnett and Reynolds' measurements at 1 MeV. Mapleton's estimate of the first Born approximation for nitrogen lies far below the present measurements, although it is in reasonable agreement with experiments below 1 MeV and at 4 MeV. The Mittleman approximate result for nitrogen, on the other hand, is in fair agreement with our measurements.

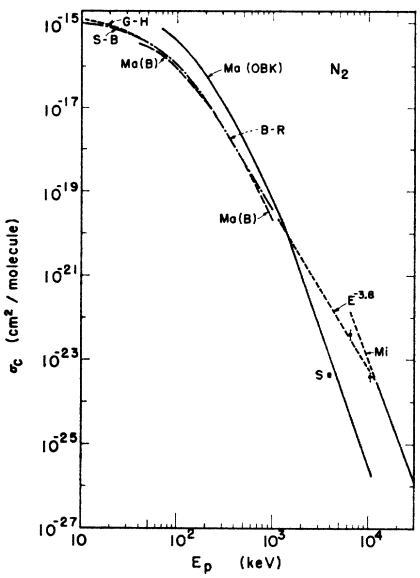

FIG. 2. Electron-capture cross sections per N_2 molecule. Experimental: closed circle with error bar, present experiment; G-H, Gilbody and Hasted (Ref. 12); S-B, Stier and Barnett (Ref. 11); B-R, Barnett and Reynolds (Ref. 7); S. Szostak *et al.*. (Ref. 14). Theoretical: Ma(OBK), Mapleton (Ref. 4), Oppenheimer-Brinkman-Kramers' approximation; Ma(B), Mapleton's estimate (Ref. 4) of first Born approximation with nucleus-nucleus interaction; Mi, Mittleman (Ref. 3). (Cross sections for atomic nitrogen have been multiplied by 2.)

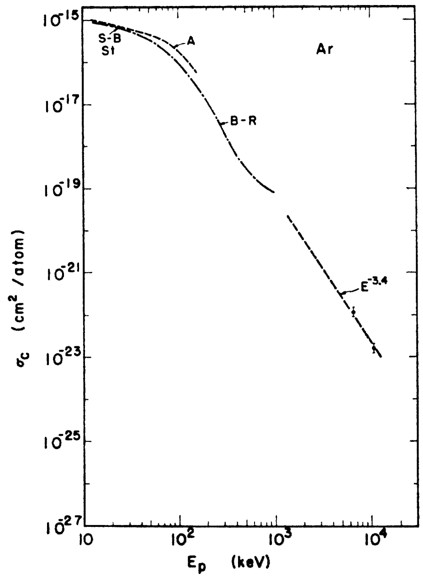

FIG. 3. Electron-capture cross sections per Ar atom. Experimental: closed circle with error bar, present experiment; St, Stedeford (Ref. 10); S-B, Stier and Barnett (Ref. 11); A, Afrosimov *et al.* (Ref. 13); B-R, Barnett and Reynolds (Ref. 7).

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