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Angular Distributions for e^--H Scattering near the First Inelastic Threshold*

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Differential cross sections for $e^- - \mathrm{H}$ scattering are presented for several energies above the first inelastic threshold for the $1s \rightarrow 1s$, $1s \rightarrow 2s$, $1s \rightarrow 2p$, and $2s \rightarrow 2s$ transitions. The calculations are based on R matrix elements computed in the $1s$-$2s$-$2p$ close-coupling approximation supplemented by Born-approximation values for the higher angular-momentum states.

I. INTRODUCTION

ALTHOUGH electron-hydrogen scattering was first studied in the 1930's, interest in this reaction has greatly increased in the past few years. The reasons for this great interest are twofold, the improvement in experimental techniques and the full scale theoretical attack allowed upon the problem by the use of large high-speed computers.

A great deal of work is continuing in this field stimulated somewhat by the difficulties being encountered in obtaining a satisfactory theoretical solution for this seemingly simple problem. The sophisticated application of the close-coupling approximation by Burke, Smith[1] and others has not met with unqualified success even though it is perhaps the most reliable theoretical method available at present. Since there exist no experimental data on the angular distribution of inelastically scattered electrons, the results provided by the theoretical calculations give the only available information on these processes. Unfortunately because of this lack of data with which to compare the theoretical results, few angular distributions have been published in the literature, even though for many purposes this information is quite important. For example, Altshuler[2] has demonstrated explicitly the role of the differential cross section for inelastic electron collisions in determining the conductivity of a weakly ionized plasma.

One exception to the paucity of angular-distribution data is the paper of Burke, Schey, and Smith[3] which gives curves for $1s$-$1s$ and $1s$-$2s$ cross sections at incident electron energies of 13.6, 19.6, and 30.6 eV. It is of interest to have these angular distributions at energies nearer the threshold for excitation of the hydrogenic electron to the $2s$ level, and also to have curves for the $1s$-$2p$ transition. Fortunately, Burke *et al.* have presented reactance matrix elements in the $1s$-$2s$-$2p$ close-coupling approximation for six electron energies above threshold. These can be used to compute the desired cross sections.

The purpose of this paper is then, to present the results of differential cross-section calculations for the $1s \rightarrow 1s$, $1s \rightarrow 2s$, $1s \rightarrow 2p$, and $2s \rightarrow 2s$ processes using the reactance matrix elements computed by Burke, Schey, and Smith at six electron energies above the first inelastic threshold. Although there is still considerable controversy over the accuracy of such calculations,[4] they are probably the best possible at the moment. Section II presents the theoretical formulas needed and discusses the method of computation, while Sec. III presents the computed cross sections.

II. CALCULATIONAL METHOD

The cross section for a given reaction can be found from the T matrix which we define in terms of the S

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† Formerly TRW Space Technology Laboratories.

[1] P. G. Burke and K. Smith, Rev. Mod. Phys. 34, 458 (1962). This article contains an extensive listing of earlier references.

[2] S. Altshuler, J. Geophys. Res. 68, 4707 (1963).

[3] P. G. Burke, H. M. Schey, and K. Smith, Phys. Rev. 129, 1258 (1963).

[4] P. G. Burke, Proc. Phys. Soc. (London) 82, 443 (1963).

Table I. The expansion coefficients, $B_L{}^S(1s\rightarrow 1s)$, of Eq. (5) at several energies for elastic e^--H scattering.

	$B_L{}^S(1s\rightarrow 1s)$								
		$k^2=0.81$			$k^2=1.00$			$k^2=1.21$	
L	singlet	triplet	Born	singlet	triplet	Born	singlet	triplet	Born
0	0.4162	1.4923	0.2617	0.3497	1.4654	0.2710	0.3112	1.4132	0.2844
1	−0.0299	1.4758	0.2040	0.1077	1.7574	0.2373	0.1856	1.8884	0.2871
2	0.5683	1.2747	0.0905	0.7407	1.4333	0.1236	0.6808	1.5476	0.1554
3	0.1874	0.6710	0.0286	0.3008	0.9205	0.0513	0.4499	1.0809	0.0801
4	0.1762	0.4093	0.0078	0.2958	0.5497	0.0193	0.3648	0.7181	0.0408
5	0.1426	0.2076	0.0019	0.1927	0.3032	0.0067	0.2339	0.4191	0.0205
6	0.0531	0.1267	0.0004	0.0822	0.1755	0.0022	0.1034	0.2383	0.0101
7	0.0410	0.0362	0.0001	0.0609	0.0553	0.0007	0.0754	0.0834	0.0048
8	0.0146	0.0136	0.0000	0.0235	0.0208	0.0002	0.0398	0.0328	0.0022
9	0.0044	0.0045	0.0000	0.0081	0.0082	0.0001	0.0151	0.0140	0.0010
10	0.0014	0.0014	0.0000	0.0024	0.0025	0.0000	0.0051	0.0049	0.0004
		$k^2=1.44$			$k^2=2.25$			$k^2=4.00$	
L	singlet	triplet	Born	singlet	triplet	Born	singlet	triplet	Born
0	0.2691	1.3594	0.2991	0.2216	1.1930	0.3409	0.2839	0.9659	0.3961
1	0.1753	1.9721	0.3410	0.1587	2.0585	0.5065	0.3853	1.9465	0.7503
2	0.4680	1.6571	0.1972	0.1876	1.8770	0.3706	0.3254	1.9819	0.7071
3	0.4822	1.2118	0.1116	0.2987	1.4721	0.2332	0.2912	1.6427	0.5350
4	0.3717	0.8703	0.0666	0.3259	1.1088	0.1541	0.2956	1.2721	0.3745
5	0.2509	0.6190	0.0411	0.2984	0.8397	0.1125	0.3029	0.9815	0.2640
6	0.1788	0.3909	0.0252	0.2493	0.5927	0.0877	0.2984	0.7634	0.1973
7	0.0849	0.2296	0.0151	0.1413	0.3786	0.0696	0.2699	0.5751	0.1583
8	0.0659	0.0937	0.0088	0.1097	0.2052	0.0546	0.1941	0.4061	0.1338
9	0.0466	0.0423	0.0050	0.0897	0.1199	0.0420	0.1548	0.2777	0.1160
10	0.0237	0.0217	0.0027	0.0662	0.0761	0.0316	0.1303	0.2014	0.1011

matrix by

$$S=1-T=(1+iR)/(1-iR). \qquad (1)$$

We have also introduced the reactance matrix R. The reactance matrix is useful since, in order that S be unitary, R must be real and symmetric. Its matrix elements are convenient to compute and to tabulate in practice. (For the simple case of a central potential the partial-wave expansion gives $R_l=\tan\delta_l$ and thus $S_l=e^{2i\delta_l}$.)

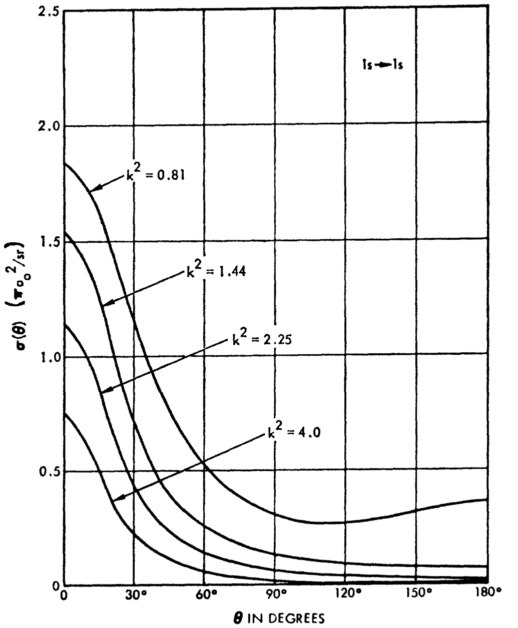

Fig. 1. The differential scattering cross section for elastic e^--H scattering at several energies above the first inelastic threshold.

The T matrix may be computed from the reactance matrix by

$$T=-2iR/(1-iR)=X+iY, \qquad (2)$$

where

$$X=2R/(R+R^{-1}) \text{ and } Y=-2/(R+R^{-1}). \qquad (3)$$

An expression for the differential cross section in terms of the T matrix may be found in the review article by Lane and Thomas.[5] In order to specialize their general formula (Eq. 2.6) to our problem we may note that since the interactions are independent of spin (we assume a Coulomb interaction only) the above matrices are diagonal in the total spin and thus we can consider the singlet and triplet contributions to the cross sections separately. Our notation will thus need a superscript S taking the two values 0 and 1 in the singlet and triplet states, respectively. To further define the particular state of the system with which we are dealing, we will specify the principal quantum number n and orbital angular momentum of the electron in an hydrogenic orbit l_H, the orbital angular momentum of the incident electron l, and J the total orbital angular momentum $J=l+l_H$.

If one then rewrites Eq. (2.6) of Lane and Thomas (note that the channel spin is now l_H) one

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[5] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. 30, 257 (1958).

TABLE II. The expansion coefficients, $B_L{}^S(1s \rightarrow 2s)$, of Eq. (5) at several energies for excitation to the hydrogenic $2s$ level in e^--H scattering.

	$B_L{}^S(1s \rightarrow 2s)$								
		$k^2=0.81$			$k^2=1.00$			$k^2=1.21$	
L	singlet	triplet	Born	singlet	triplet	Born	singlet	triplet	Born
0	0.0936	0.0251	0.0058	0.1857	0.0586	0.0178	0.2159	0.0652	0.0284
1	−0.0566	−0.0179	0.0101	−0.0568	−0.0116	0.0357	−0.0042	0.0167	0.0635
2	0.0548	0.0775	0.0177	0.3951	0.1506	0.0643	0.6528	0.1437	0.1060
3	−0.0251	−0.0163	0.0139	−0.0260	0.0563	0.0664	0.1485	0.1326	0.1242
4	0.1131	0.0490	0.0118	0.2742	0.1061	0.0692	0.4494	0.1417	0.1340
5	−0.0037	−0.0001	0.0065	0.0860	0.0521	0.0612	0.2757	0.1208	0.1305
6	−0.0024	0.0146	0.0028	0.0687	0.0649	0.0473	0.2186	0.1140	0.1171
7	0.0000	0.0033	0.0010	0.0438	0.0361	0.0326	0.1614	0.0919	0.0975
8	−0.0001	0.0005	0.0003	0.0228	0.0228	0.0206	0.1051	0.0774	0.0759
9	−0.0000	0.0001	0.0001	0.0129	0.0117	0.0122	0.0703	0.0536	0.0560
10	0.0000	0.0000	0.0000	0.0072	0.0061	0.0069	0.0473	0.0354	0.0396
		$k^2=1.44$			$k^2=2.25$			$k^2=4.00$	
L	singlet	triplet	Born	singlet	triplet	Born	singlet	triplet	Born
0	0.1807	0.0663	0.0369	0.1404	0.0827	0.0544	0.1414	0.0855	0.0724
1	0.0294	0.0510	0.0881	0.1491	0.1395	0.1449	0.3065	0.2114	0.1990
2	0.5624	0.1433	0.1396	0.3379	0.1772	0.2183	0.4331	0.3024	0.3001
3	0.3036	0.1776	0.1734	0.4094	0.2441	0.2776	0.5077	0.3816	0.3771
4	0.4658	0.1945	0.1925	0.4447	0.3015	0.3189	0.5444	0.4474	0.4312
5	0.3999	0.1837	0.1972	0.4869	0.3505	0.3440	0.5604	0.4912	0.4637
6	0.3609	0.1691	0.1896	0.5155	0.3570	0.3555	0.5728	0.5134	0.4794
7	0.2875	0.1598	0.1722	0.4985	0.3039	0.3548	0.5862	0.5160	0.4818
8	0.2269	0.1384	0.1486	0.4825	0.2939	0.3432	0.5704	0.5026	0.4753
9	0.1680	0.1192	0.1226	0.4427	0.2876	0.3221	0.5710	0.4882	0.4612
10	0.1226	0.0931	0.0971	0.3860	0.2767	0.2941	0.5631	0.4707	0.4414

finds

$$\frac{d\sigma^S}{d\Omega}(nl_{\mathrm{H}} \rightarrow n'l_{\mathrm{H}}') = \frac{(2S+1)}{4(2l_{\mathrm{H}}+1)k_n{}^2\pi} \times \sum_{L=0}^{\infty} B_L{}^S(n'l_{\mathrm{H}}',nl_{\mathrm{H}})P_L(\cos\theta)\,, \quad (4)$$

where

$$B_L{}^S(n'l_{\mathrm{H}}',nl_{\mathrm{H}}) = \frac{(-1)^{l_{\mathrm{H}}'-l_{\mathrm{H}}}}{4} \times \sum_{\substack{l_1,l_2,l_1',l_2'\\ J_1,J_2}} \bar{Z}(l_1J_1l_2J_2,l_{\mathrm{H}}L)\bar{Z}(l_1'J_1l_2'J_2,l_{\mathrm{H}}L) \times T_{n'l_{H}'l_1',nl_Hl_1}{}^{J_1S}T_{n'l_{H}'l_2',nl_Hl_2}{}^{J_2S*} \quad (5)$$

and

$$\bar{Z}(l_1J_1l_2J_2,SL) = [(2l_1+1)(2l_2+1)(2J_1+1)(2J_2+1)]^{1/2} \times (l_1l_200|L0)W(l_1J_1l_2J_2,SL) \quad (6)$$

with $(l_1l_200|L0)$ being the (Condon-Shortley definitions) Clebsch-Gordan coefficient and $W(l_1J_1l_2J_2,SL)$ is the Racah coefficient.[6] $(2S+1)/4$ is the appropriate statistical factor.

The differential cross section is given by Eq. (4) in units of $\pi a_0{}^2$ provided that $k_n{}^2$ is expressed in units of $a_0{}^{-2}$, where a_0 is the usual Bohr radius. This is equivalent to taking the energy units to be rydbergs (13.6 eV) so that $k_1{}^2=0.75$ corresponds to a kinetic energy of 10.2 eV.

Perhaps the relation of the expansion coefficients $B_L{}^S$ to certain properties of the scattering should be pointed

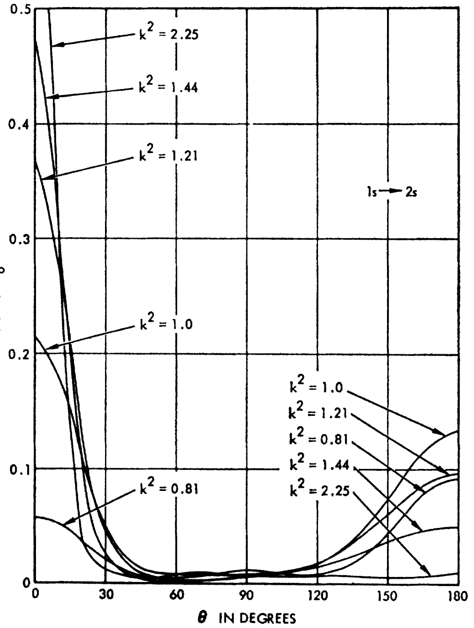

FIG. 2. The differential scattering cross section for excitation of the hydrogenic $2s$ level in e^--H scattering for several energies above threshold.

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[6] See, for example, D. M. Brink and G. R. Satchler, *Angular Momentum* (Clarendon Press, Oxford, 1962).

TABLE III. The expansion coefficients, $B_L{}^S(1s\rightarrow 2p)$, of Eq. (5) at several energies for excitation to the hydrogenic $2p$ level in e^--H scattering.

	$B_L{}^S(1s\rightarrow 2p)$								
	$k^2=0.81$			$k^2=1.00$			$k^2=1.21$		
L	singlet	triplet	Born	singlet	triplet	Born	singlet	triplet	Born
0	0.1812	0.0342	0.0328	0.4479	0.1171	0.1097	0.7510	0.1943	0.1984
1	0.0607	0.0165	0.0365	0.2877	0.1446	0.2061	0.8107	0.3602	0.4436
2	0.2553	0.0368	0.0023	0.7596	0.2087	0.1847	1.3582	0.4290	0.4963
3	−0.0567	0.0048	−0.0124	0.1019	0.1366	0.1124	0.7519	0.4054	0.4383
4	−0.0727	−0.0045	−0.0174	0.1689	0.0659	0.0328	0.8922	0.3087	0.3274
5	−0.0115	0.0002	−0.0108	0.0818	0.0168	−0.0094	0.5470	0.2180	0.2187
6	0.0044	−0.0089	−0.0049	0.0438	0.0252	−0.0221	0.4025	0.1833	0.1369
7	0.0025	−0.0019	−0.0019	0.0261	0.0179	−0.0206	0.2988	0.1554	0.0837
8	0.0004	−0.0006	−0.0007	0.0145	0.0115	−0.0149	0.2014	0.1176	0.0510
9	−0.0000	−0.0001	−0.0002	0.0080	0.0048	−0.0096	0.1400	0.0755	0.0309
10	−0.0000	−0.0001	−0.0001	0.0041	0.0023	−0.0058	0.0927	0.0468	0.0179
	$k^2=1.44$			$k^2=2.25$			$k^2=4.00$		
L	singlet	triplet	Born	singlet	triplet	Born	singlet	triplet	Born
0	0.8999	0.2668	0.2883	0.9560	0.5018	0.5400	0.9806	0.7344	0.7916
1	1.3841	0.5945	0.7088	2.3097	0.5610	1.4822	2.7171	2.0653	2.2503
2	1.7341	0.7444	0.8874	3.0139	1.3000	2.1524	4.0960	3.1628	3.4856
3	1.5784	0.7807	0.9043	3.5233	1.1486	2.5656	5.1569	3.9877	4.4323
4	1.7150	0.7139	0.8214	3.9158	1.7682	2.7777	5.9887	4.5919	5.1259
5	1.3527	0.6262	0.6914	4.0704	1.5250	2.8627	6.5718	4.9695	5.5539
6	1.1342	0.5583	0.5548	4.0303	1.5037	2.7495	6.9329	5.1723	5.7636
7	0.9177	0.4850	0.4323	3.8257	1.5367	2.5708	7.0307	5.1890	5.7397
8	0.7322	0.4091	0.3296	3.5070	1.5691	2.3189	6.8962	5.0552	5.5305
9	0.5475	0.3259	0.2448	3.0668	1.4529	2.0103	6.5099	4.7554	5.1341
10	0.4049	0.2370	0.1741	2.5631	1.2889	1.6692	5.9333	4.3543	4.6265

out. Thus, the total cross section is given by $B_0{}^S$,

$$\sigma_{\text{tot}}{}^S=[(2S+1)/k_n{}^2]B_0{}^S, \tag{7}$$

where we have taken l_H to be zero.

$2S+1$ is the appropriate statistical factor for the singlet and triplet states while one should replace $(2S+1)$ by 4 if one wishes the Born-approximation results. Of course the physical cross section is the sum of the singlet and triplet scattering.

$$\sigma_{\text{tot}}=\sigma_{\text{tot}}{}^{S=0}+\sigma_{\text{tot}}{}^{S=1}. \tag{8}$$

Also the momentum-transfer cross section

$$Q^S=\int d\Omega\sigma^S(\theta)(1-\cos\theta) \tag{9}$$

is given by

$$Q^S=(2S+1)(B_0{}^S-\tfrac{1}{3}B_1{}^S). \tag{10}$$

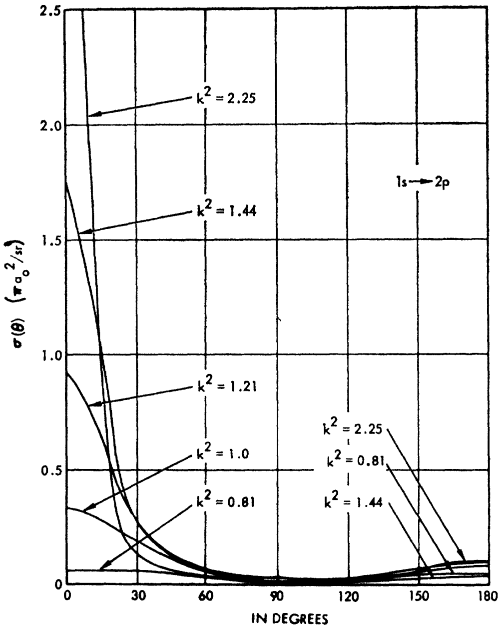

FIG. 3. The differential scattering cross section for excitation of the hydrogenic $2p$ level in e^--H scattering for several energies above threshold.

III. RESULTS

The reactance matrix elements for the coupled $1s$, $2s$, $2p$ states have been computed by Burke *et al.*[3] and presented in tabular form for certain values of l_1 and for energies corresponding to $k_1=0.9$, 1.0, 1.1, 1.2, 1.5, and 2.0 $a_0{}^{-1}$. Using the Burke *et al.* values supplemented by Born-approximation[7] values for higher partial waves, the expression for $B_L{}^S$ in Eq. (5) has been evaluated and the results tabulated in Table I for the $1s\rightarrow 1s$ reaction. For completeness the Born-approximation results are given as well as those for the singlet and triplet couplings. Tables II–IV give the same information for the $1s\rightarrow 2s$, $1s\rightarrow 2p$, and $2s\rightarrow 2s$ reactions, each for the six energies listed above. The resulting differential cross section (in units of $\pi a_0{}^2$) are presented

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[7] M. J. Seaton, Proc. Phys. Soc. (London) **77**, 174 and 184 (1961). J. Lawson, W. Lawson, and M. J. Seaton, *ibid.* **77**, 192 (1961); V. M. Burke and M. J. Seaton, *ibid.* **77**, 199 (1961); Bruce L. Scott, Phys. Rev. **137**, A717 (1965).

TABLE IV. The expansion coefficients, $B_L{}^S(2s \rightarrow 2s)$, of Eq. (5) at several energies for elastic scattering from the excited $2s$ level in e^--H* scattering. The energy k^2 is relative to the ground-state hydrogen atom so that $k^2=0.81$ corresponds to an incident electron energy of 0.06×13.6=0.816 eV.

$B_L{}^S(2s \rightarrow 2s)$

	$k^2=0.81$			$k^2=1.00$			$k^2=1.21$		
L	singlet	triplet	Born	singlet	triplet	Born	singlet	triplet	Born
0	6.44	5.08	7.63	4.76	7.90	6.32	5.06	7.95	5.78
1	16.14	11.31	21.96	11.53	20.38	17.92	12.56	20.83	16.17
2	24.91	20.59	34.62	20.52	29.93	28.20	21.90	30.09	24.88
3	34.46	24.60	44.47	24.90	37.08	36.61	26.96	36.90	31.97
4	40.01	33.52	51.50	31.93	43.69	43.09	34.33	42.50	37.45
5	42.28	35.70	55.82	35.72	48.23	47.53	38.13	46.72	41.26
6	45.12	39.35	57.84	39.15	49.83	50.18	40.75	48.19	43.63
7	46.05	40.51	57.98	41.01	49.14	51.19	42.12	47.62	44.66
8	45.70	40.73	56.60	41.31	47.82	50.84	42.09	45.58	44.57
9	44.11	39.56	53.92	40.55	45.84	49.22	41.06	43.16	43.40
10	41.16	37.63	50.03	38.65	43.65	46.56	39.09	40.90	41.34

	$k^2=1.44$			$k^2=2.25$			$k^2=4.00$		
L	singlet	triplet	Born	singlet	triplet	Born	singlet	triplet	Born
0	5.33	7.94	5.64	7.07	8.39	5.92	8.18	8.82	6.72
1	14.21	20.92	15.70	19.46	22.70	16.62	23.02	24.71	19.25
2	23.08	30.19	23.72	28.69	33.04	24.98	34.89	37.37	29.77
3	29.43	36.69	29.96	35.43	39.92	30.87	43.61	46.62	37.75
4	35.69	41.78	34.64	40.20	44.40	34.73	49.55	52.84	43.27
5	39.02	45.18	37.77	42.95	46.92	36.85	53.05	56.41	46.54
6	40.95	46.76	39.62	43.94	48.03	37.63	54.53	57.84	47.93
7	41.50	46.30	40.32	43.42	47.79	37.29	54.25	57.51	47.74
8	41.14	44.62	40.10	41.79	45.99	36.17	52.62	55.95	46.31
9	39.77	41.93	39.03	39.43	42.93	34.45	49.76	52.89	43.85
10	37.79	39.17	37.30	36.76	39.44	32.38	45.99	48.77	40.62

in Figs. 1–4 as computed using Eq. (4) and the $B_L{}^S$ in Tables I–IV. In all graphs l values up to 13 have been used, resulting in L up to 26. We have only presented $L \leqslant 10$ in the tables for the sake of conserving space although for angles in the forward direction $L > 10$ contributes significant amounts to the differential cross section.

IV. DISCUSSION

Figure 1 contains the elastic differential cross section and is essentially the same as that presented in Ref. 3 except that the lower incident energies are given here. The cross sections vary smoothly with energy and angle and have no distinguishing features.

Figure 2 contains the differential cross section for excitation of the hydrogen atom to the $2s$ level. There are two features of interest. First, at low energies there is considerable peaking in the backward directions; indeed the cross section is larger at 180° than at 0° for $k^2=0.81$. Second, a glance at Table II will show that the series giving the cross section is very slowly converging for high energies so that we must include very large L values in order to obtain the cross section at 0° correctly. This explains why the cross section at 0° presented here for $k^2=2.25$ is substantially larger than that of Burke, Schey, and Smith.[3] They had not included enough L values to obtain substantial convergence of the expansion. At larger angles the agreement is satisfactory since the higher $P_L(x)$ introduce many sign changes which greatly enhance the convergence of the expansion.

It is of interest to consider the convergence of the expansion of Eq. (4) in somewhat greater detail for this case. Table V gives the coefficients $B_L{}^S(1s \rightarrow 2s)$

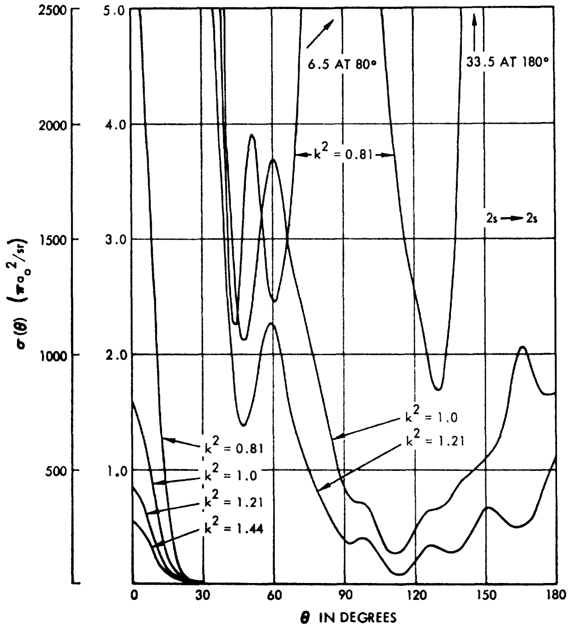

FIG. 4. The differential scattering cross section for elastic e^--H* scattering from the hydrogenic $2s$ level at several energies. (Note that the total energy k^2 is relative to the ground-state hydrogen atom so that $k^2=0.81$ corresponds to an incident electron energy of 0.06×13.6=0.816 eV.)

TABLE V. Values of the expansion coefficients $B_L{}^S(1s \to 2s)$ at $k^2=2.25$ for large values of L. See the text for a discussion of these numbers.

	$B_L{}^S(1s \to 2s)$		
		$k^2=2.25$	
L	singlet	triplet	Born
12	0.2788	0.2115	0.2271
14	0.1760	0.1453	0.1519
16	0.1000	0.0837	0.0926
18	0.0537	0.0499	0.0516
20	0.0246	0.0246	0.0246
22	0.0101	0.0101	0.0101
24	0.0033	0.0033	0.0033
26	0.0001	0.0001	0.0001

at $k^2=2.25$ for L values up to 26. Two points, applicable to all of the tabulated numbers, should be noted. First, for $L \geqslant 20$ (at $k^2=2.25$), the singlet, triplet, and Born results are all equal. The reason for this is that results for the R-matrix elements were presented by Burke, Schey, and Smith only for l up to 6 at this energy so that for $7 \leqslant l \leqslant 13$ only Born-approximation values are available. Since values of l greater than 13 have not been used in this work, it follows that $l_1=6$ and $l_2=13$ can couple to $L=19$ at the outside. This means that for $L=20$ only angular-momentum states for which Born values must be used can contribute. Thus the singlet, triplet, and Born results, as computed here, must be identical.

The second remark is that the convergence indicated by the numbers in Table V is somewhat misleading in that the numbers presented are, at most, lower bounds. This occurs because one cannot include all contributions to a particular L value because of the cutoff in l at $l=13$. For example, at $L=26$ only $l_1=13$ and $l_2=13$ have been taken into account even though there are substantial contributions from $l_1=14$, $l_2=12$, $l_1=15$, $l_2=11$, etc. At lower L values the effects of these neglected terms will of course be much less, but in any case the effect is that of greatly increasing the apparent convergence of the series. Thus one must realize that at the higher energies the true cross section in the forward direction is larger than that presented.

Although the results of Figs. 1 and 2 have been recorded before (at different energies), it does not appear that any information on the differential cross section for the $1s$-$2p$ and $2s$-$2s$ transitions has been published. Figure 3 gives this information for the $1s$-$2p$ transition. Of interest here is the nearly symmetric cross section at low energy ($k^2=0.81$) changing into an extremely peaked one at higher energies.

The $2s$-$2s$ curve is of substantial interest and is given in Fig. 4. Of necessity the figure has been broken into two regions of different scale, one for forward angles and another for larger angles because of the extreme peaking in the forward direction especially at the lower energies. (One should recall that k^2 is the total energy of the e^--H system referred to a zero of a ground-state hydrogen atom with the electron at rest so that $k^2=0.81$ for the $2s$-$2s$ reaction corresponds to an incident electron of $k^2=0.81-0.75=0.06$ or 0.8 eV.) The enormous magnitude of the $2s$-$2s$ cross section is substantially due to the $1/k_n{}^2$ factor in Eq. (4). This cross section also has a substantial amount of structure which is not seen in any of the other curves.[8]

For all of these reactions there are some convergence difficulties at the higher energies, but for the $2s \to 2s$ transition these difficulties appear even at $k^2=0.81$ as may be inferred from Table IV. For this reason the $2s$-$2s$ cross sections are substantially less accurate than the others but still are probably accurate to 10–15%.

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[8] This is not to say that there is not structure in the other curves, but that the structure is so small as to be invisible on the curves, while for $2s \to 2s$ the large magnitude of the cross sections renders the structure plainly visible.