Microwave Scattering Due to Acoustic-Ion-Plasma-Wave Instability*

V. ARUNASALAM† AND SANBORN C. BROWN
Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts

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An analysis is presented of an experimental study of microwave scattering from nonequilibrium plasmas in which the electrons had a large steady drift velocity v_d with respect to the ions. According to theory, such plasmas must support "relatively undamped" acoustic-ion-plasma waves due to a two-stream instability type of mechanism. Measurements of the frequency-power spectrum of the scattered signal show agreement with theory. In particular, as predicted, the measured scattering cross sections were large in comparison with the Thomson cross section. The effects of collisions predicted by theory did not show up in the experimental results. As expected from theoretical considerations of nonlinear effects (such as the nonlinear coupling between the plasma modes of different wave numbers), the relative drift velocity and the electron-ion temperature ratio tend toward a saturation value as the plasma current is increased. The experimental results indicate that the nonlinear effects are relatively weak.

I. INTRODUCTION

THE dispersion relation of a two-component plasma will, in general, have two roots corresponding to two longitudinal normal modes. This is analogous to the vibrational modes of a diatomic lattice.[1] One of these longitudinal normal modes corresponds to the high-frequency optical mode (the electron plasma oscillations) in which the electrons and ions vibrate out of phase so that their center of gravity is approximately at rest, and the other corresponds to the low-frequency acoustic mode (the ion plasma oscillations) in which the electrons and ions vibrate in phase with approximately the same amplitude. For a plasma whose electron temperature T_- is less than the ion temperature T_+, one can show[2] that the low-frequency acoustic mode is highly damped and this collective mode cannot be distinguished from those modes associated with the motion of individual ions. For a plasma whose $T_- \gg T_+$, the low-frequency acoustic mode has a relatively long lifetime and thus becomes a well-defined excitation. In this paper we shall only consider the latter case.

If one produces a nonequilibrium plasma (one whose electrons and ions do not have a Maxwellian distribution of velocities) in which the electrons have a steady drift velocity $\mathbf{v}_d$ with respect to the ions, the distribution functions for the component of velocity $\mathbf{v}$ along $\mathbf{v}_d$ take the forms

$$F_-(v) = (1/(2\pi)^{1/2} v_-) \exp[-(v-v_d)^2/2v_-^2] \quad (1a)$$

and

$$F_+(v) = (1/(2\pi)^{1/2} v_+) \exp[-v^2/2v_+^2]. \quad (1b)$$

Here $v_\mp = (KT_\mp/m_\mp)^{1/2}$ are the average speeds of electrons (of mass m_- and temperaure T_-) and ions (of mass m_+ and temperature T_+), respectively, and K is Boltzmann's constant. Such a situation will arise in an ordinary dc discharge. In this case the dc voltage applied across the cathode and anode of a discharge tube produces a steady dc electric field $\mathbf{E}$ across the plasma column. Under the action of such a dc electric field $\mathbf{E}$ the electrons and ions of the plasma will show a steady drift velocity $\mathbf{v}_{d\pm} = \mu_\mp \mathbf{E}$, where $\mu_\mp$ are the mobilities of electrons and ions, respectively. In a coordinate system in which the ions have zero drift velocity, the electrons will have a steady drift velocity $\mathbf{v}_d = \mathbf{v}_{d-} - \mathbf{v}_{d+} \approx \mathbf{v}_{d-}$, where $m_+ \gg m_-$, $\mu_+ \ll \mu_-$, and therefore $v_{d-} \gg v_{d+}$. In such a coordinate system, the electron and ion velocity distribution functions will take the forms given by Eqs. (1a) and (1b), respectively.

It is clear from Eq. (1a) that $F_-(v)$ increases as v increases for electron velocities $v \leq v_d$. If this drift velocity v_d is large enough that the phase velocity v_p of any one of the longitudinal normal modes mentioned earlier satisfies the inequality $v_p = \omega/k \leq v_d$, it is possible for the waves corresponding to that normal mode to grow in amplitude. Such a growth in amplitude of the plasma wave is a consequence of the transfer of the "extra" kinetic energy of those electrons whose velocities lie in the neighborhood of the phase velocity of the plasma wave to that plasma wave under consideration. This transfer of energy from electrons to the plasma wave occurs in the following way. Electrons with velocities slightly greater than $v_p = \omega/k$ will be trapped in the potential trough of the plasma wave and decrease their average velocity to v_p, transferring their "extra" kinetic energy to the plasma wave. Electrons with velocities slightly less than v_p will be accelerated by this trapping mechanism and increase their average velocity to v_p by absorbing the energy from the plasma wave. Therefore, if $F_-(v)$ increases with increasing v, it is possible that the plasma wave will grow in amplitude, since there will be more fast particles to transfer energy to the plasma

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† Now at Princeton University, Plasma Physics Laboratory, James Forrestal Research Center, Princeton, New Jersey.

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wave than slow particles to absorb energy from it. If, furthermore, the wave frequency is sufficiently large compared with the frequency appropriate to the damping mechanism (that is, the collision frequency) it can be shown[2] that these plasma waves obtain a net exponential growth rate. Thus, for sufficiently large drift velocities and sufficiently low collision frequencies, it is possible to have plasma-wave instabilities. That is, the plasma becomes unstable against a growing wave of plasma oscillations, corresponding to a coherent excitation of the plasma oscillations by the drifting electrons (the familiar two-stream instability). Here, we shall consider only the acoustic-ion-plasma-wave instabilities.

Recently, several authors[2–9] have investigated the collective behavior in nonequilibrium plasmas. In particular, they have investigated the behavior of the acoustic-ion-plasma waves in nonequilibrium plasmas whose electrons had a steady drift velocity v_d with respect to the ions. They find that the energy associated with these acoustic ion plasma waves should increase steadily from its thermal equilibrium value of KT_- toward extremely large values if one increases this relative drift velocity v_d from the value zero toward a value equal to the phase velocity v_{p+} of these acoustic-ion-plasma waves. When $v_d \approx v_{p+}$ the plasma should undergo a transition from a stable to an unstable region with respect to these acoustic-ion-plasma waves. That is, they show that the density-density correlation function corresponding to the collective part of the electron density fluctuations should exhibit a large increase (in a form somewhat analogous to the critical fluctuations in the vicinity of a liquid-gas phase transition) as the plasma approaches from a region of stability to a critical point ($v_d \approx v_{p+}$) corresponding to the onset of the acoustic-ion-plasma-wave instabilities. Therefore, as the plasma approaches this critical condition for the onset of such instabilities, the cross sections for the scattering of electromagnetic waves from the "relatively undamped" acoustic-ion-plasma waves should exhibit a large increase in a form somewhat analogous to the critical opalescence for a liquid-gas phase transition. With such large enhancement in the scattering cross sections it is possible to carry out experiments on the scattering of microwaves from the density fluctuations associated with the onset of such acoustic-ion-plasma-wave instabilities. This paper reports the results of such an experiment on microwave scattering from laboratory nonequilibrium hydrogen and helium plasmas. When electromagnetic waves are scattered from acoustic-ion-plasma waves, the difference between the frequency of the scattered radiation and the frequency of the incident radiation is equal to the frequency of the acoustic-ion-plasma wave responsible for the scattering, and the scattering cross section is proportional to the potential energy of the acoustic-ion-plasma wave responsible for the scattering.

II. THEORY OF INSTABILITIES

The basic quantity for the description of the dynamics of the collective behavior in plasmas is the retarded, frequency and wave-vector dependent, longitudinal dielectric coefficient $\epsilon[\mathbf{k},\omega]$. By using the distribution functions given by Eqs. (1a) and (1b) in the linearized self-consistent Boltzmann-Poisson equations, Ichimaru and others[2–4,8,9] have derived an expression for $\epsilon[\mathbf{k},\omega]$. By letting $\mathrm{Re}\,\epsilon[\mathbf{k},\omega]=0$, one gets the plasma dispersion relation. For the low-frequency acoustic-ion-plasma oscillations this procedure gives[2]

$$(k_-^2/k^2)=[(\omega_+^2/\omega_k^2)-1]=[(f_+^2/f^2)-1], \tag{2}$$

where

$$k_-^2=(4\pi n_0e^2/KT_-), \tag{3a}$$

$$\omega_+^2=(2\pi f_+)^2=(4\pi n_0e^2/m_+), \tag{3b}$$

and $\omega_k=2\pi f$ is the frequency of the acoustic-ion-plasma wave of wave number k, and n_0 is the average number density of electrons or ions.

By letting $\mathrm{Im}\,\epsilon[\mathbf{k},\omega]=0$, one can get the boundary between the growing and damped plasma waves. For the low-frequency acoustic-ion-plasma oscillations this procedure gives[2–4] the equation of this boundary as

$$v_d(k)=v_{d_1}(k)+v_{d_2}(k), \tag{4}$$

where

$$\left(\frac{v_{d_1}(k)}{v_-}\right)\approx\left(\frac{m_-}{m_+}\right)^{1/2}[1+(k^2/k_-^2)]^{-1/2} \times\left\{1+\left(\frac{m_+}{m_-}\right)^{1/2}\left(\frac{T_-}{T_+}\right)^{3/2}\exp\left[-\frac{1}{2}\left(\frac{T_-/T_+}{1+k^2/k_-^2}+3\right)\right]\right\} \tag{5a}$$

is the boundary when collisions are neglected, and

$$\left(\frac{v_{d_2}(k)}{v_-}\right)\approx\left(\frac{2}{\pi}\right)^{1/2}\frac{1}{\omega_+\tau_+}\frac{[1+(k^2/k_-^2)]^{3/2}}{k/k_-} \quad \text{for}\quad \omega_+\tau_+\gg 1 \tag{5b}$$

represents the effect of collisions, and τ_+ is the relaxation time of the ions. Here $v_d(k)$ represents the drift velocity at which the acoustic-ion-plasma wave of wave number k is neither damped nor growing. Since, in an ion oscillations, the electrons and ions vibrate with almost equal amplitude, the wave momentum is essentially deter-

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mined by the ion vibrational motion. Therefore, the effect of electron collisions can be neglected in comparison with that of the ions as long as $(m_-/m_+)<(\tau_-/\tau_+)$, where τ_- is the relaxation time of the electrons.

It is convenient to represent this boundary between the growing and damped acoustic-ion-plasma waves by a curve $v_d(k)$ in the v_d-k plane. Figures 1 and 2 show the shape of this boundary curve for a hydrogen plasma as given by Eqs. (4), (5a), and (5b). The region to the left of these threshold curves corresponds to damped acoustic-ion-plasma waves [that is, $\text{Im}\,\epsilon(\mathbf{k},\omega)>0$], and that to the right, to unstable growing acoustic-ion-plasma waves [that is, $\text{Im}\,\epsilon(\mathbf{k},\omega)<0$]. With the aid of Eqs. (4), (5a), and (5b) one can define a critical point k_c as that wave vector for which, with increasing v_d, the ion sound wave first becomes unstable. Let $v_c=v_d(k_c)$ be the associated minimum drift velocity for instability. That is, $v_d=v_c$ is the condition of "marginal stability." Furthermore, by comparing the curves of Fig. 1 with the corresponding curves of Fig. 2, it can be seen that the effect of collisions is to damp out the low wave number oscillations and thus to make the critical wave number k_c nonzero. It is also seen from Fig. 1 that the boundary curves exhibit a slight folding back toward the k axis for higher values of the temperature ratio T_-/T_+. That is, for higher values of the temperature ratio T_-/T_+, there exists some $k>0$ such that $v_d(k>0)<v_d(k=0)$. Let $(T_-/T_+)_c$ be the critical temperature ratio above which this folding back occurs. [See Eqs. (12a) and (12b).] That is, for $(T_-/T_+)>(T_-/T_+)_c$, the critical wave number k_c is nonzero.

III. THEORY OF SCATTERING

In general, the scattering of electromagnetic waves from electron density fluctuations in a plasma is determined by the dynamic form factor $S(\mathbf{k},\omega)$ which is the statistical average of the space-time Fourier transform of the density-density correlation function [see Eq. (7)].

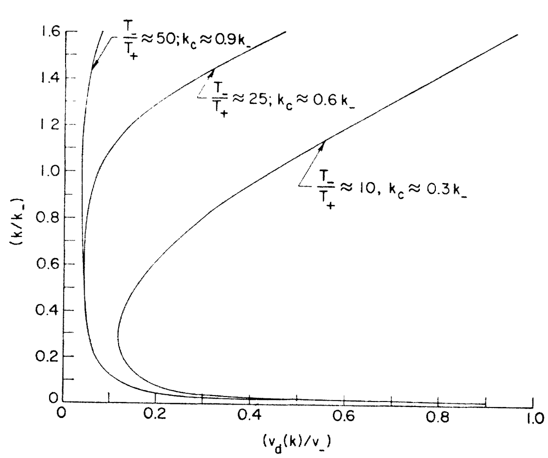

FIG. 1. Boundary between growing and damped waves for a hydrogen plasma when $\omega_+\tau_+=\infty$.

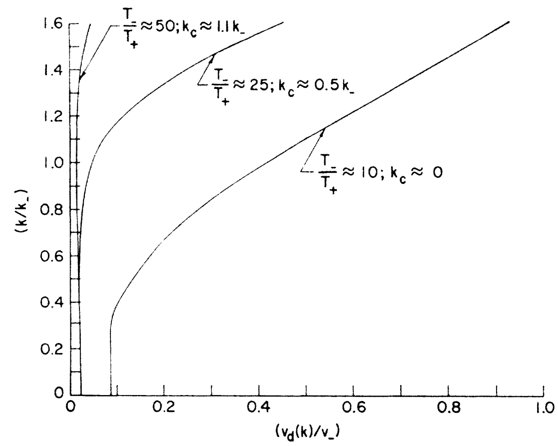

FIG. 2. Boundary between growing and damped waves for a hydrogen plasma when $\omega_+\tau_+=100$.

The differential cross section $(d^2\sigma/d\Omega d\omega)$ for the transfer of momentum $\hbar\mathbf{k}$ (corresponding to scattering into a solid angle $d\Omega$) and energy $\hbar\omega$ from an electromagnetic wave to the electrons in a plasma is given by[3,4]

$$(d^2\sigma/d\Omega d\omega)=(e^2/m_-c^2)^2(1-\tfrac{1}{2}\sin^2\theta)S(\mathbf{k},\omega)\,, \qquad (6)$$

where θ is the angle between the incident and scattered waves. $S(\mathbf{k},\omega)$ is given by

$$S(\mathbf{k},\omega)=\frac{L^3}{2\pi}\int_{L^3}d\mathbf{r}\int_{-\infty}^{\infty}dt\langle n(\mathbf{r}',t')n(\mathbf{r}+\mathbf{r}',t+t')\rangle \times\exp[-i(\mathbf{k}\cdot\mathbf{r}-\omega t)]\,, \qquad (7)$$

where $n(\mathbf{r},t)$ denotes the electron number density of N electrons in a box of volume L^3, and the angular brackets refer to statistical average over the electron states. The measurement of the intensity of radiation scattered into a given angle yields directly the structure factor in momentum space $S(\mathbf{k})$, given by

$$S(\mathbf{k})=\frac{1}{N}\int_{-\infty}^{\infty}d\omega S(\mathbf{k},\omega)\,. \qquad (8a)$$

The measurement of the intensity of radiation scattered into a given frequency yields directly the structure factor in energy space $S(\omega)$,

$$S(\omega)=\frac{1}{N}\int_{-\infty}^{\infty}d\mathbf{k}S(\mathbf{k},\omega)\,. \qquad (8b)$$

If one uses a detector that has a bandpass filter that passes frequencies in the range $(\omega-\frac{1}{2}\Delta\omega;\omega+\frac{1}{2}\Delta\omega)$ for the measurements of the intensity of the scattered radiation, it is convenient to define an "average" cross section $\sigma(\omega)$ as

$$\sigma(\omega)\Delta\omega=\int_{\omega-\frac{1}{2}\Delta\omega}^{\omega+\frac{1}{2}\Delta\omega}\left(\frac{d\sigma}{d\omega}\right)d\omega\,. \qquad (9)$$

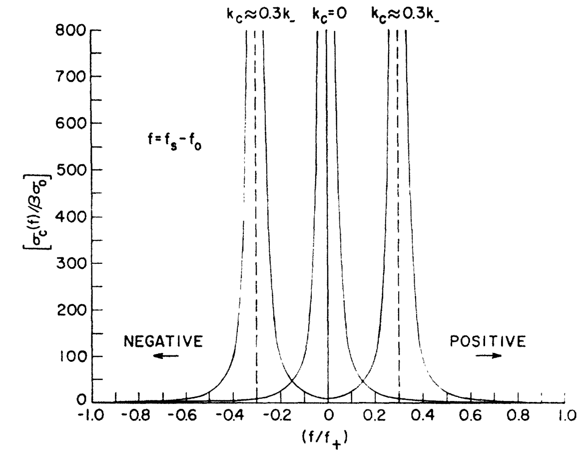

FIG. 3. Scattering cross section $\sigma_c(f)$ as a function of the wave frequency f for a plasma at "marginal stability" with respect to acoustic-ion-plasma waves.

The scattered power is then given by

$$P_s(\omega)\Delta\omega = n_0\sigma(\omega)\Delta\omega \text{ (volume of scatterer)} \text{ (incident power /area of incidence)}. \quad (10)$$

Using the conventional "dressed-particle" method, Ichimaru and others[3,4] have shown that near the onset of the acoustic-ion-plasma-wave instability, the structure factor $S(\mathbf{k})$ in momentum space is given by

$$S_c(\mathbf{k}) \approx \frac{1}{2}\left[\frac{(m_-/m_+)^{1/2}}{\{(v_c - v_d)/v_-\} + (v_1/v_-)(k^2/k_-^2)}\right] \quad (11)$$

for $k^2 < k_-^2$ and $k_c \approx 0$. The critical wave number k_c is zero only when the effects of collisions can be neglected and when $(T_-/T_+) \lesssim (T_-/T_+)_c$. For sufficiently large temperature ratios $[10 \leq (T_-/T_+) \leq (T_-/T_+)_c]$,

$$(v_1/v_-) \approx \tfrac{1}{2}[\{(T_-/T_+)^{5/2} \exp[-\tfrac{1}{2}\{(T_-/T_+)+3\}]\} - (m_-/m_+)^{1/2}]. \quad (12a)$$

The critical temperature ratio $(T_-/T_+)_c$ above which the boundary curve folds back toward the k axis is the solution of the equation

$$v_1 \approx 0 \quad \text{for} \quad (T_-/T_+) = (T_-/T_+)_c. \quad (12b)$$

In these results, it has been assumed that the direction of the wave vector $\mathbf{k}$ is parallel to that of the drift velocity $\mathbf{v}_d$.

Let us now examine the structure factor that is appropriate to the condition of "marginal stability." By letting $v_d = v_c$ in Eq. (11), we get the structure factor in momentum space $S_c(\mathbf{k})$ for "marginal stability":

$$S_c(\mathbf{k}) \approx \tfrac{1}{2}[(m_-/m_+)^{1/2}/(v_1/v_-)](k_-^2/k^2) \text{ for } k^2 < k_-^2. \quad (13)$$

By using Eq. (2) in Eq. (13), we get the structure factor in frequency space $S_c(f) = S_c(\omega_k/2\pi)$ for "marginal stability":

$$S_c(f) \approx \beta[(f_+^2/f^2) - 1] \quad \text{for} \quad f \lesssim f_+, \quad (14)$$

where

$$\beta \approx \tfrac{1}{2}[(m_-/m_+)^{1/2}/(v_1/v_-)]. \quad (15)$$

From Eqs. (6), (8b), (8), and (14), we get the "average" cross section $\sigma_c(f)$ at "marginal stability":

$$[\sigma_c(f)/\sigma_0] \approx \beta[(f_+^2/f^2) - 1] \quad \text{for} \quad f \lesssim f_+, \quad (16)$$

where

$$\sigma_0 = (e^2/m_-c^2)^2 \int d\Omega(1 - \tfrac{1}{2}\sin^2\theta) = (8\pi/3)(e^2/m_-c^2)^2$$

is the free-electron Thomson cross section.

Equation (16) has the following physical meaning. Consider an electromagnetic wave of frequency f_0 incident on a nonequilibrium plasma whose electrons have a steady drift velocity v_d with respect to the ions, and let f_+ be the ion plasma frequency appropriate to this plasma. Let this $v_d = v_c$, the minimum drift velocity for the acoustic-ion-plasma-wave instability (that is, at the condition of "marginal stability"), and let the frequency of the scattered radiation $f_s = f_0 \pm f$. Then the "average" cross section $\sigma_c(f)$ of Eq. (16) represents the fraction of the incident electromagnetic wave energy that is scattered by a single plasma electron. Stated differently, $\sigma_c(f)$ of Eq. (16) represents the "average" cross section per electron for the transfer of an energy proportional to $2\pi\hbar f = hf$ from the incident electromagnetic wave to the "relatively undamped" acoustic-ion-plasma wave of frequency f.

When the temperature ratio $(T_-/T_+) > (T_-/T_+)_c$, that is, when the boundary curve $\mathbf{v}_d(k) - \mathbf{k}$ folds back toward the k axis, or when the effect of collisons becomes appreciable, the critical fluctuations occur in the vicinity of a nonzero value of k_c. In such cases one can show[3] that "marginal stability" the structure factor in momentum space $S_c(k) \propto [k_-^2/(k-k_c)^2]$.

Figure 3 shows the approximate shape of the scattering cross section as a function of the frequency f of the acoustic-ion-plasma wave responsible for the scattering. Figure 4 shows the dependence of the factor β on the temperature ratio (T_-/T_+). As indicated in this figure, for a hydrogen plasma $(T_-/T_+)_c \approx 19.6$, and for a helium plasma $(T_-/T_+)_c \approx 21.5$. It is seen that as (T_-/T_+) tends toward the value $(T_-/T_+)_c$, the factor β tends toward extremely large values. For a cylindrical plasma column of diameter d and length L, the volume of scatterer $\approx L\pi d^2/4$ and the area of incidence $\approx Ld$, we get the scattered power from Eqs. (10) and (16):

$$P_s(f) \approx (\tfrac{1}{4}\beta\sigma_0\pi d P_i n_0)[(f_+^2/f^2) - 1] \quad \text{for} \quad f \lesssim f_+, \quad (17)$$

where P_i is the incident power. It is seen from this equation that for such a scattering, a plot of the scattered power $P_s(f)$ as a function of $1/f^2=1/(f_s-f_0)^2$ should yield a straight line whose

$$\text{slope}\approx(\tfrac{1}{4}\beta\sigma_0\pi dP_i n_0 f_+^2) \tag{18a}$$

and

$$\left|\frac{\text{slope}}{\text{intercept on } P_s \text{ axis}}\right|=f_+^2. \tag{18b}$$

Thus, from such a straight-line graph one can obtain the value of f_+ (and consequently the value of the particle density $n_0\approx n_-\approx n_+$) and the value of the factor β. Furthermore, for a cylindrical plasma column of diameter d, the total discharge current J is given by

$$(J/\tfrac{1}{4}\pi d^2)\approx n_0 e v_d. \tag{19}$$

By using Eq. (19) and the relation $f_+^2=(n_0e^2/\pi m_+)$ in Eq. (17), we get

$$P_s(f)\approx\left(\frac{4}{\pi^2}\frac{\beta\sigma_0 P_i}{m_+d^3v_d^2}\right)\left(\frac{J^2}{f^2}\right) \quad \text{for} \quad f^2\ll f_+^2. \tag{20}$$

It is seen from this equation that if d and v_d remain constant independently of J, the scattered power should be proportional to (J^2/f^2) for $f^2\ll f_+^2$.

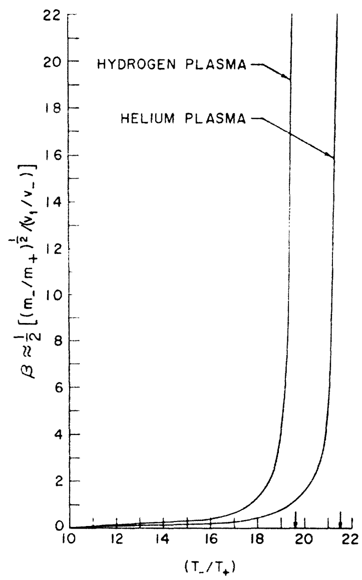

FIG. 4. The approximation [Eq. (15)] to the factor β of Eq. (14) as a function of electron-ion temperature ratio.

IV. EXPERIMENT

The experimental arrangement is illustrated in Fig. 5. The length of the discharge tube was ~15 cm and the diameter was ~2 cm. The distance between the two probes on either side of the scattering volume was ~6.5 cm. The cathode was an oxide-coated tungsten spiral whose axis was along the axis of the discharge tube. The shield around the cathode was a cylindrical can of molybdenum tied to one of the cathode leads and insulated from the other cathode lead by an insulation bead. The face of the water-cooled anode was of stainless steel and the rest was of copper. The electromagnet was used for applying a uniform magnetic field parallel to the axis of the discharge tube.

The maximum discharge current used for the measurements was 1 A. Readings were taken for currents J in steps of $\frac{1}{10}$ A. The gas was either hydrogen or helium at pressures in the range of 60–510 μ Hg. The voltage across the two probes was measured for each value of the current J. This voltage varied slightly as the discharge current was increased from 0.1 to 0.4 A, but remained constant as the discharge current was increased from 0.4 to 1.0 A. This seemed to indicate that the values of E/p (and consequently those of v_d and T_-/T_+), tend to saturate for higher values of the discharge current J. Here, p is the pressure, and E is the steady dc electric field in the plasma.

Incident X-band microwave power was supplied by a Varian X-13 klystron, and the incident frequency was varied in the range of 9255±200 Mc/sec by steps of approximately 5 Mc/sec for scanning the scattered spectrum. With the aid of the directional coupler, wavemeter, crystal, and meter, the frequency of the incident power was read directly in Mc/sec and the incident power was kept constant as the frequency spectrum was scanned. The detector line was tuned to a fixed frequency of $9255\pm\frac{3}{4}$ Mc/sec. The incident power of 2.5 mW was modulated in the line by the ferrite switch at 100 cps and the detector was synchronized to this modulation frequency. In this way the detector was made not to respond at all for direct microwave emission,[10,11] from the plasma such as Bremsstrahlung, cyclotron radiation, and any other nonthermal radiations. For the experimental conditions given in Table I, the amplitude-modulated incident power of 2.5 mW can at most give rise to 0.01% modulation in the electron temperature T_-. This, in turn, can at most give rise to 0.01% modulation of the direct microwave emission from the plasma. This corresponds to less than 1% of the minimum detectable signal. Thus, our detector would respond only to the scattered radiations arising from the incident power from the klystron. With the aid of the three resonant cavities, each of $Q\approx2300$, we were able to avoid any direct pickup as the incident frequency was brought to 5 Mc/sec on either side of the detector frequency. The horn-to-horn spacing was ~3.2 cm, and the transmission loss caused by this spacing was approximately 7 dB. These horns were arranged so that the electric vectors and the propagation vectors of both the incident and detector lines were at right angles to the axis of the discharge tube (and therefore at right angles to the uniform B field).

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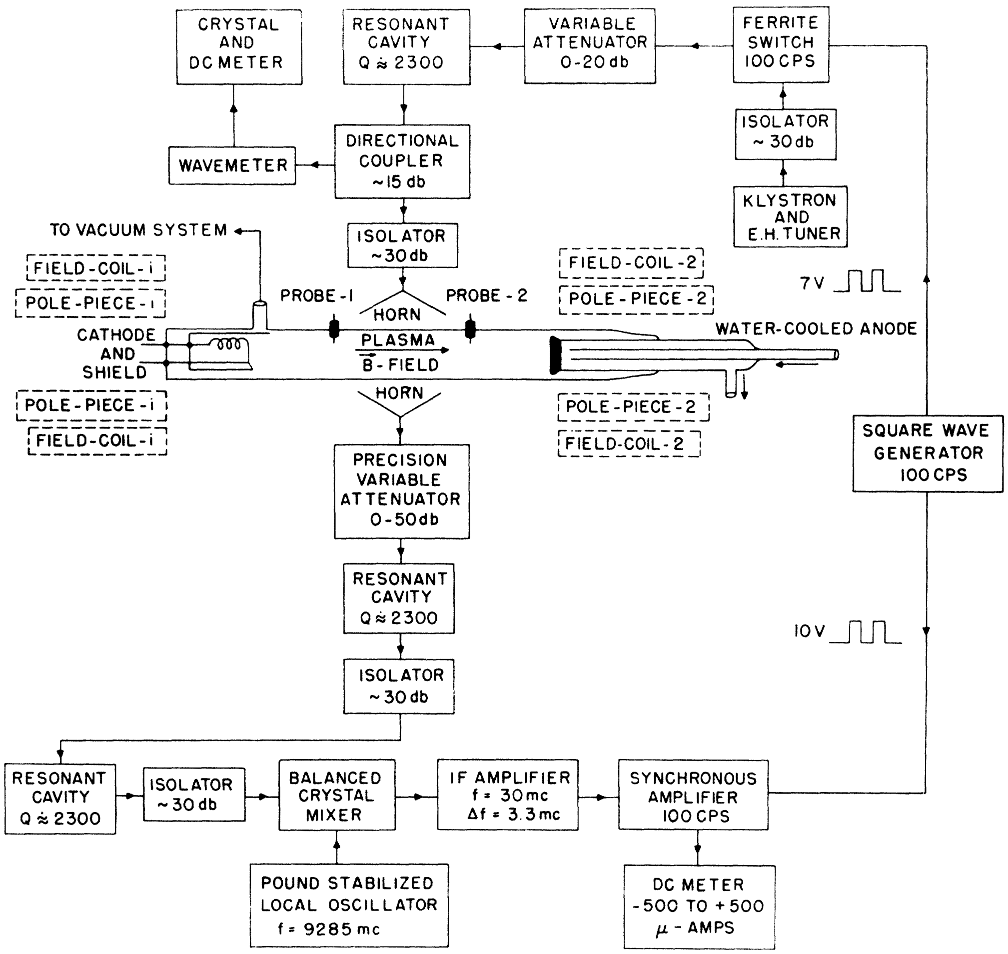

FIG. 5. Block diagram of the experimental setup.

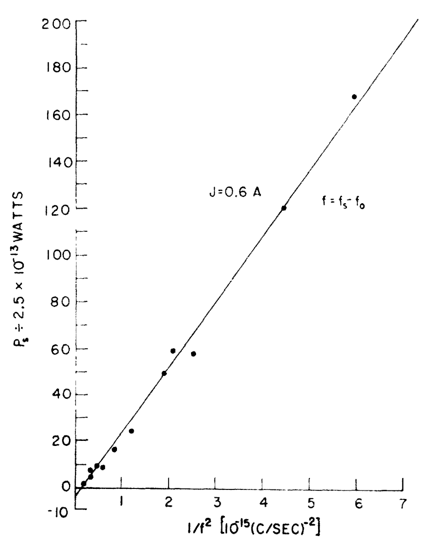

FIG. 6. Scattered power P_s as a function of f^{-2}, where f is the acoustic-ion-plasma-wave frequency.

This uniform magnetic field B serves, first, to overcome the difficulties associated with the diffusion losses in the low-pressure high-current dc discharges and, second, to overcome the difficulties associated with the momentum-conservation requirements for the scattering process under study. Since, the acoustic-ion-plasma oscillations have their wave vectors $\mathbf{k}$ in the direction of the drift velocity $\mathbf{v}_d$ and consequently along the $\mathbf{B}$ field, and since the frequencies of the incident microwaves (f_0 in the range 9255±200 Mc/sec) are much higher than the electron cyclotron frequencies appropriate to the B fields used in our experiments, the uniform B field cannot have played any significant role in our experiments.

The principle of operation of the detector is as follows. The radiation scattered from the plasma is collected by the receiver horn and passed down the detector line. The two resonant cavities in the detector line allow only those radiations in the frequency interval 9255 Mc/sec plus or minus approximately 1 Mc/sec to reach the balanced crystal mixer. At the balanced crystal mixer

TABLE I. The range of experimental conditions.

Gas	H_2	H_2	He	He
Pressure p (mm Hg)	0.060	0.080	0.150	0.510
Electric field E for 0.8 A $\leqslant J \leqslant$ 1.0 A (V/cm)	1.00	4.00	1.80	4.00
E/p for 0.8 A $\leqslant J \leqslant$ 1.0 A (V/cm-mm Hg)	16.7	50.0	12.0	7.8
$\Delta_1(E/p)$ (V/cm-mm Hg)[a]	4.0	10.0	2.0	2.0
$\Delta_2(E/p)$ (V/cm-mm Hg)[b]	$\cdots$	4.0	0.5	0.8
Magnetic field B (G)	2290	2290	2780	2780
Diameter of plasma column d (cm)	0.50	0.50	1.0	2.0
v_d (cm/sec)	6.40×10^6	1.80×10^7	9.40×10^6	6.10×10^6
v_- (cm/sec)	9.55×10^7	1.45×10^8	1.72×10^8	1.62×10^8
(v_d/v_-)	0.067	0.124	0.055	0.038
Ion plasma frequency f_+ (Mc/sec)	$110\sqrt{J}$[c]	$140\sqrt{J}$	$50\sqrt{J}$	$27\sqrt{J}$
$\omega_+\tau_+=2\pi f_+\tau_+$	$365\sqrt{J}$	$355\sqrt{J}$	$66\sqrt{J}$	$11\sqrt{J}$
Particle density $n_0\approx n_-\approx n_+$ (per cm³)	$2.75\times10^{11}\,J$	$4.50\times10^{11}\,J$	$2.30\times10^{11}\,J$	$6.70\times10^{10}\,J$
β	1470	228	460	1010

[a] $\Delta_1(E/p)=[(E/p)_{J=0.4\,\mathrm{amp}}-(E/p)_{J=0.1\,\mathrm{amp}}]$.
[b] $\Delta_2(E/p)=[(E/p)_{J=1.0\,\mathrm{amp}}-(E/p)_{J=0.4\,\mathrm{amp}}]$.
[c] J is the total discharge current in amperes.

this scattered radiation was mixed with a fixed amount of the local oscillator signal of frequency 9285±0.1 Mc/sec. The crystal mixer sends out two signals at frequencies 9285–9255 Mc/sec to the intermediate-frequency amplifier. The intermediate-frequency amplifier picks out only the signal at the frequency (9285 −9255) Mc/sec = 30 Mc/sec, and amplifies it approximately 100 dB. This signal was then amplified, integrated, and compared with a standard reference signal by the synchronous amplifier and the dc meter.

V. RESULTS

Typical experimental data are shown in Figs. 6 and 7. The conditions for the experimental runs are given in Table I. In Fig. 7 we have plotted the scattering cross section $\sigma_c(f)$ as a function of (f/f_+) as given by the theoretical Eq. (16), with our experimental measurements of $\sigma_c(f)$ fitted to the theoretical curve at one point only. For any particular experimental run, data were taken at discharge currents J, from 0.1 to 1.0 A in steps of $\frac{1}{10}$ A, and we used the relations $f_+\sim\sqrt{J}$ and $\sigma_c(f)\sim P_s(f)/J$ in plotting Fig. 7. These two relations are valid when the relative drift velocity and the cross-sectional area of the cylindrical plasma column are independent of the discharge current J. Table I illustrates the range of parameters varied experimentally. The average electric field E is the voltage across the two probes (see Fig. 5) divided by the distance between them. The average diameters of the plasma column given in this table are approximate values of the "visual diameter" as measured by a centimeter scale. From the measured values of E/p, the values of v_d, v_-, and τ_+ were calculated from available data.[12] The values of f_+ and β were determined from the straight-line graphs as illustrated in Fig. 6 and explained in Sec. III. The particle density $n_0\approx n_-\approx n_+$ was calculated from the known values of $f_+=(n_0e^2/\pi m_+)^{1/2}$. These values of the particle density were checked by determining the plasma "cut-off" condition of the extraordinary wave.[13] Since our measurements lack the experimental determination of the ion temperature T_+, it was not possible to make a direct comparison between the experimental values of β and that predicted by the theoretical Eq. (15). Some of our measurements showed departures from the relation $P_s(f)\sim J^2$, but all of them showed close agreement with the relation $P_s(f)\sim(1/f^2)=1/(f_s-f_0)^2$. Here, f_s is the frequency of the scattered radiation and f_0 is that of the incident radiation. This departure is due to the slight dependence of d (and presumably of v_d) on the discharge current J.

The results of all of our measurements satisfy Eq. (16). We recall from our discussions in Sec. III that Eq. (16) is valid if and only if, first, the conditions are such that the critical wave number k_c is zero, second, the

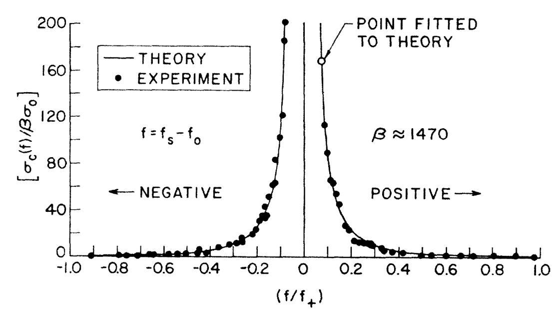

FIG. 7. Scattering cross section $\sigma_c(f)$ as a function of the acoustic-ion-plasma-wave frequency f.

[12] S. C. Brown, *Basic Data of Plasma Physics* (John Wiley & Sons, Inc., New York, 1959), p. 336.

[13] W. P. Allis, S. J. Buchsbaum, and A. Bers, *Waves in Anisotropic Plasmas* (The M.I.T. Press, Cambridge, Massachusetts), p. 280.

conditions are appropriate to that of "marginal stability," and third, the amplitudes of the "relatively undamped" acoustic-ion-plasma waves are sufficiently small so that one could neglect all nonlinear effects. In the light of our experimental measurements, let us discuss these conditions of validity of Eq. (16).

We have shown in Sec. II that one of the necessary conditions for the critical wave number k_c to be zero is that the effects of collisions must be negligibly small. Indeed, it is seen from Eqs. (4), (5a), and (5b) that for k_c to be exactly zero, it is necessary to have $\omega_+\tau_+=2\pi f_+\tau_+=\infty$. But, it is seen from Table I for our experimental measurements $11\sqrt{J}\lesssim\omega_+\tau_+=2\pi f_+\tau_+\leq 365\sqrt{J}$, where J is the discharge current in amperes. Therefore, the critical wave number k_c must have been significantly different from zero for some of our experimental measurements. It is seen (from Fig. 2) that for $\omega_+\tau_+\lesssim 100$, $k_c\gtrsim 0.3k_-$ according to the theory presented in Sec. II. For such nonzero values of k_c, the analysis of Sec. III predicts a structure factor of the type shown in curve b of Fig. 3. Our measurements of this structure factor have been made for the range $0.06\lesssim f/f_+\approx k/k_-\lesssim 1$, and we should have been able to see such effects (arising from the nonzero value of k_c) if k_c was greater than $0.05k_-$. (The range $f/f_+\lesssim 0.05$ is not accessible for making measurements with our equipment due to large direct pickup signals.) It would, therefore, appear that the experiments on helium [for which $\omega_+\tau_+<100$ and consequently $(k_c/k_-)>0.3$ according to the theory presented in Sec. II] should have yielded a "double-peak" structure factor [of the type shown in curve b of Fig. 3], but instead we found that even these measurements gave a "single-peak" structure factor [of the type shown in curve a of Fig. 3; that is, they also satisfied Eq. (16)] which is appropriate to the case $k_c\approx 0$. It is somewhat surprising that in our measurements collisions did not seem to play a major role (in making k_c nonzero) in contradiction to the theory.

VI. NONLINEAR EFFECTS

We have seen that a linearized solution of the self-consistent Boltzmann-Poisson equations yields exponentially growing acoustic-ion-plasma waves for a nonequilibrium plasma whose electrons have a sufficiently large steady drift velocity v_d with respect to the ions. The question arises as to the behavior of the system once the threshold for the growth of instabilities is reached. Once the instability sets in, the amplitude of the plasma oscillations becomes sufficiently large for nonlinear effects (of which relatively little is known) to begin to play a role. In order to get a physical feeling for the ultimate behavior of these waves in the nonlinear region, it is convenient to divide[14,15] the nonlinear effects into two groups, one of which arises as a consequence of those nonlinear terms which when combined with the linear terms gives rise to the "nonlinear dispersion relation," and the other arise as a consequence of the nonlinear coupling between plasma modes of different wave numbers. The nonlinear effects of the first type will[14,15] tend to produce a flattening in the distribution function $F_-(v)$ in the heighborhood of $v\approx v_{p+}$ [where, v_{p+} is the phase velocity of the acoustic-ion-plasma waves] caused by a "diffusion" in velocity space. But the nonlinear effects of the second type, that is, the nonlinear mode-coupling mechanism will[2] cause the modes of lower wave numbers to decay into modes of a higher wave number. The higher wave-number modes decay in turn into still higher wave-number oscillations until those oscillations whose wave number is approximately equal to the reciprocal of the Debye-length decay into a fine-grained random or "thermal" motion. Therefore, the temperature of the electrons and ions will increase because of this nonlinear mode-coupling decay. Since in an ion oscillation, the electrons and ions vibrate with almost equal amplitude, such a decay of the plasma waves into random thermal motion will result in a comparatively larger increase in the ion temperature and a relatively smaller increase in the electron temperature. This is due to the fact that since $m_+\gg m_-$, for the same vibrational velocity, the energy carried by the ions is very much larger than that carried by the electrons. This would imply that this nonlinear mode-coupling decay will tend to reduce the value of (T_-/T_+). But the usual Joule heating tends to favor a large value of (T_-/T_+). It would therefore appear that (T_-/T_+) will tend to saturate at some equilibrium value. Furthermore, since these plasma oscillations grow by absorbing their necessary energy from the directed particle drift motion, the relative drift velocity v_d will tend to saturate at some equilibrium value. The weaker the nonlinear mode couplings, the larger is the portion of the available drift energy going into the plasma oscillations. Therefore, if this nonlinear mode-coupling mechanism is relatively weak, the level of plasma oscillations at quasi-steady state will be very large. This would imply that for weak nonlinear effects, the saturation values of v_d and (T_-/T_+) must be such that they tend to favor a large value of the factor β. That is, v_d should tend to saturate near v_c and (T_-/T_+) should tend to saturate near $(T_-/T_+)_c$.

By comparing the values of $\Delta_2(E/p)$ with the corresponding values of E/p and $\Delta_1(E/p)$, as given in Table I, we find that the values of E/p remain sensibly constant (independent of J) for discharge currents in the range 0.4 to 1.0 A. Therefore, we conclude that the relative drift velocity v_d and the temperature ratio (T_-/T_+) tend to saturate in our experiments for the higher values of the discharge currents. We also note from Table I that the measured values of β for all our experiments lie in the range $228\lesssim\beta\lesssim 1470$. Thus, the saturation values of

[14] W. E. Drummond and D. Pines, Nucl. Fusion, 1962 Suppl., Part 3, pp. 1049–1057.

[15] E. Frieman and P. Rutherford, Ann. Phys 28, 134 (1964).

E/p (and consequently those of v_d and T_-/T_+) are such that they tend to favor a large value of the factor β. Therefore, we are further led to conclude that the nonlinear effects are weak. (A very large value of the factor β implies that the level of acoustic-ion-plasma oscillations which exists at steady state is very large and this in turn implies that the nonlinear effects are weak.) Such a saturation value of E/p (and consequently those of v_d and T_-/T_+) may have resulted directly from electron-neutral collisions rather than from the decay of these growing plasma oscillations to thermal motions due to nonlinear mode couplings as discussed above. But, as may be seen from Table I, such a saturation of E/p occurred for all measurements in the pressure range 0.060 mm Hg$\lesssim p \lesssim$0.510 mm Hg. Therefore, it would appear that (since, even the low-pressure measurements showed such a saturation) the nonlinear mode-coupling mechanism is at least partly responsible for producing such saturation values of E/p (and consequently those of v_d and T_-/T_+).

VII. CONCLUSION

Thus our scattering experiment yields the following physical information about the collective behavior of a current-carrying plasma in which the particle distribution functions are non-Maxwellian [that is, of a nonequilibrium plasma]. First, our measurements show that the cross sections for the scattering of electromagnetic waves from the density fluctuations of a nonequilibrium plasma are very much larger than the Thomson cross section σ_0. Since these measurements satisfy Eqs. (16) to (20), we conclude that this scattering is from the "relatively undamped" acoustic-ion-plasma waves. Second, our measurements of the structure factor show that the critical wave number k_c is zero even when $\omega_+\tau_+$ is sufficiently small [that is, even when $\omega_+\tau_+ < 100$], and this is in disagreement with the theory. Third, since the measured values of the factor β are very large, we conclude that the nonlinear effects are relatively weak. Fourth, since the measured values of E/p remained sensibly constant for higher values of the discharge current J, we conclude that v_d and T_-/T_+ tend to saturate, but the saturation value of v_d seems slightly larger than the value v_c of the theoretical prediction for saturation. This may imply that the efficiency of the energy transfer from the growing acoustic-ion-plasma oscillations to the "thermal" motions caused by decay by nonlinear mode-coupling mechanism is not high. This is consistent with our earlier conclusion that the nonlinear effects are relatively weak.

Finally, since $[\sigma_c(f)/\sigma_0]\approx[E_f/KT_-]$, where E_f is the energy density associated with the acoustic-ion-plasma wave of frequency f, we conclude that the energy distribution among the "relatively undamped" acoustic-ion-plasma waves of the current-carrying plasmas used in our measurements is given by

$$E_f\approx(KT_-)\beta[(f_+^2/f^2)-1] \quad \text{for} \quad f\lesssim f_+.$$

This is an important piece of experimental information that one should bear in mind if one wishes to formulate a nonlinear theoretical analysis of this problem.

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