It is important to note the extreme sensitivity of the galvanomagnetic properties to the many-valley structure, not merely with respect to magnitudes but particularly with respect to the signs of the mobility variations. A realistic calculation of the galvanomagnetic properties of hot electrons in many-valley semiconductors would necessitate a rather accurate treatment of intervalley repopulation processes and could furnish considerable insight into the details of these processes.

ACKNOWLEDGMENTS

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Mean Free Path of Electrons and Magnetomorphic Effects in Small Single Crystals of Gallium. II*

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The variation of the electrical resistivity of pure gallium has been investigated as a function of specimen size in oriented single crystals for current flow along the A and B axes. An analysis of the data, based on the free-electron model and assuming diffuse scattering at the boundaries, gives $(\rho_0 l_b)_{A\text{ axis}} = 2.30 \times 10^{-11}\ \Omega\ \text{cm}^2$, and $(\rho_0 l_b)_{B\text{ axis}} = 8.17 \times 10^{-12}\ \Omega\ \text{cm}^2$. The longitudinal magnetoresistance was measured for fields up to 1400 G and was found to be dependent upon the dimensions of the wires. The resistivity of the wires was also found to be a function of measuring current, and this variation is attributed to the perturbation of carrier trajectories by the magnetic field of the current.

I. INTRODUCTION

IN a previous paper[1] (hereafter referred to as YC), we have investigated the low-temperature variation of electrical resistivity of 99.9999% pure gallium single crystals as a function of temperature and size for current flow along the C axis. The present paper is an extension of the same work for current flow along the A and B axes. As before, the crystals investigated are in the form of square wires and their dimensions vary from 0.5 mm to 0.1 mm in six steps, which are more or less uniformly spaced as a function of $1/d$, where d is the side of the square. In order to determine the mean free path of the charge carriers for conductivity along these axes, the results are analyzed in exactly the same way as for the C axis. A full discussion of the theory used is given in YC.

II. EXPERIMENTAL DETAILS AND THE PRECISION OF THE MEASUREMENTS

The method of preparing oriented single crystals, the determination of their dimensions in order to calculate the resistivities, the measuring technique employed for the electrical conductivity, and the various corrections applied are discussed in great detail in YC. The crystals were oriented to an accuracy of about 1°. The dimension of the smallest crystals could be ascertained to about 1%. The resistance measurements for the largest B-axis specimens which had the smallest resistance were accurate to about 2% for the lowest temperatures and the smaller specimens could be measured with a much greater precision. Since the greater accuracy of the conductivity measurements for the small crystals was offset by a less accurate determination of their dimensions and vice versa, the result is that for most specimens the accuracy with which the resistivity could be determined is a little better than 1%, except for the two largest specimens in which the errors may be considerably higher. As for the C axis, the resistance of all the crystals was found to be strongly dependent on the measuring current. It was therefore necessary to extrapolate the resistance to zero measuring current. Consequently, an additional error of about 1% could easily be introduced into the resistivities.

* This research was partly supported by the Advanced Research Projects Agency under Contract SD-90, and partly by the National Science Foundation.

[1] M. Yaqub and J. F. Cochran, Phys. Rev. 137, A1182 (1965).

III. RESULTS

The Mean Free Path of Electrons and the Temperature Dependence of the Resistivity

Because of the strong dependence, the resistances from which all the conclusions are drawn are the limiting values defined by

$$R_0=\lim_{I\to 0} V/I,$$

where V is the potential across the specimen in which a current I is flowing.

For each specimen, R_0 was measured at seven different temperatures between 4.2 and 1.2°K and fitted to an expression of the form

$$R_0(T)=R_r+\alpha T^n, \qquad (1)$$

where R_r is the residual resistivity. This was obtained for each specimen as follows. An approximate value of R_r was first determined by extrapolating the R_0-versus-temperature curve to 0°K. Since the temperature-dependent part of the resistivity is expected to obey a power law in temperature, a graph of $\log_{10}[R(T)-R_r]$ against $\log_{10}T$ was then plotted for this value of R_r to see whether the observations obey a simple power law. Any deviations were then corrected by adjusting the value of R_r until a straight line corresponding to a power law was obtained. This is illustrated in Fig. 1, which represents the observations for one of the A-axis crystals. This method could be criticized on the basis of a possible change from one power law to another for the temperature range covered. However, it is difficult to imagine a process which would increase the exponent as the temperature is lowered so that this method certainly eliminated the possibility of selecting too high a value for R_r. Besides, the lower end of the graph is so sensitive to change in the value of R_r that it was possible to obtain an estimate of the uncertainties involved in choosing a particular value.

In Tables I and II we tabulate the results of our observations for the A- and the B-axis crystals, respectively. The values of the resistivities at 77°K for both

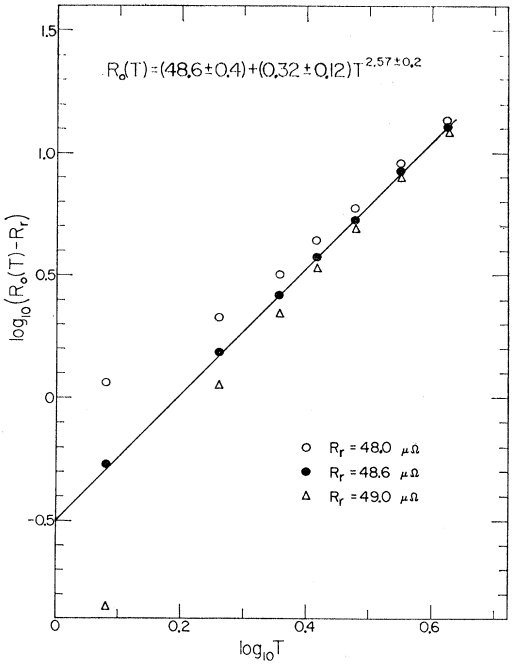

FIG. 1. Temperature variation of the resistance for the A_5 crystal.

these axes do not agree with each other as well as they did for the C axis. No satisfactory explanation for this discrepancy can be found. It is possible that in gallium, which is strongly anisotropic, certain directions are more susceptible to lattice defects than others and this may be the cause for disagreement. Rosenberg,[2] during an investigation of thermal conduction in oriented gallium crystals, has come to the same conclusion. If we assume that our values for the larger crystals are more accurate than those for the smaller ones because of a greater accuracy with which their dimensions can be measured, we obtain the following average values for the resistivities at 77°K:

$$(\rho_{N_2})_{A\text{ axis}}=3.00\times10^{-6}\ \Omega\text{ cm},$$
$$(\rho_{N_2})_{B\text{ axis}}=1.49\times10^{-6}\ \Omega\text{ cm}.$$

TABLE I. Resistivity data for the A-axis crystals.

Specimen	Batch No. from which the crystal was prepared	d at 0°K cm	$1/d$ at 0°K cm^{-1}	ρ at 77°K Ω cm	ρ at 4.2°K Ω cm	ρ_r at 0°K Ω cm	$\alpha\times10^{11}$	n
A_1	2	5.13×10^{-2}	19.49	3.001×10^{-6}	6.45×10^{-10}	3.45×10^{-10}	0.85±0.15	2.47±0.1
A_2	3	3.752×10^{-2}	26.65	3.008×10^{-6}	7.79×10^{-10}	4.46×10^{-10}	1.1 ±0.15	2.36±0.08
A_3	3	2.383×10^{-2}	41.96	3.029×10^{-6}	11.00×10^{-10}	7.24×10^{-10}	1.2 ±0.35	2.40±0.2
A_4	1	1.632×10^{-2}	61.28	2.907×10^{-6}	16.83×10^{-10}	13.56×10^{-10}	0.61±0.06	2.73±0.05
A_5	3	1.229×10^{-2}	81.38	3.062×10^{-6}	20.75×10^{-10}	16.37×10^{-10}	1.1 ±0.4	2.57±0.2
A_6	2	9.35×10^{-3}	106.9	3.10×10^{-6}	27.3×10^{-10}	22.4×10^{-10}	0.99±0.2	2.74±0.08

[2] H. M. Rosenberg, Phil. Trans. Roy. Soc. London **A247**, 441 (1955).

TABLE II. Resistivity data for the B-axis crystals.

Specimen	Batch No. from which the crystal was prepared	d cm at 0°K	$1/d$ cm^{-1} at 0°K	ρ at 77°K Ω cm	ρ at 4.2°K Ω cm	ρ_r at 0°K Ω cm	$\alpha\times10^{10}$	n
B_1	1	5.112×10^{-2}	19.57	1.489×10^{-6}	1.997×10^{-10}	5.95×10^{-11}	0.222±0.016	1.29±0.4
B_2	2	3.795×10^{-2}	26.35	1.476×10^{-6}	2.59×10^{-10}	1.743×10^{-10}	0.115±0.12	1.4 ±0.4
B_3	3	2.167×10^{-2}	46.15	1.495×10^{-6}	4.53×10^{-10}	3.59×10^{-10}	0.027±0.005	2.44±0.14
B_4	1	1.742×10^{-2}	57.42	1.696×10^{-6}	5.27×10^{-10}	4.26×10^{-10}	0.024±0.005	2.68±0.1
B_5	3	1.284×10^{-2}	77.87	1.544×10^{-6}	7.18×10^{-10}	6.05×10^{-10}	0.025±0.005	2.68±0.3
B_6	2	9.795×10^{-3}	102.1	1.488×10^{-6}	8.51×10^{-10}	7.15×10^{-10}	0.049±0.008	2.30±0.1

Corresponding values obtained by Powell[3] are 3.04×10^{-6} and 1.42×10^{-6} Ω cm, respectively. The discrepancy of a few percent between our values and those of Powell can be attributed to the fact that Powell's crystals are not as accurately aligned as ours.

In order to determine the mean free path of the electrons for current flow along these axes, the observations for all temperatures were fitted to the following equation by the method of least squares

$$\rho_d(T)=\rho_b(T)+(A/d)\,, \qquad (2)$$

where $\rho_d(T)$ is the resistivity of the wire at temperature T and $\rho_b(T)$ is the bulk resistivity. A/d, where d is the side of the wire, is a temperature-dependent parameter which takes into account the additional contributions to the resistance due to the diffuse scattering of electrons at the boundaries of the specimens. The results of this computation are plotted in Figs. 2 and 3 for 4.2 and 0°K. In both these figures the deviations from the straight line are larger for 0°K than for 4.2°K and are presumably due to the fact that the resistances which were smaller for the lower temperatures could not be measured with the same degree of precision. An additional cause of error was the fact that at lower temperatures the resistances were more strongly dependent on the measuring current and the extrapolation to zero current could not be achieved with the same accuracy. The values of the slope A_0 for absolute zero calculated by the method of least squares are 2.06×10^{-11} Ω cm^2 for the A axis and 7.33×10^{-12} Ω cm^2 for the B axis. Within the accuracy of the measurements the bulk resistivity is zero for both these axes at absolute zero. According to MacDonald and Sarginson,[4] if the bulk mean free path $l_b\gg d$—the dimension of the square wire—then, on the basis of the free-electron model and diffuse scattering at the walls, Eq. (2) takes the form

$$\rho_d=(\rho_b l_b/d)(1/1.116) \quad \text{for} \quad d/l_b\gg1\,, \qquad (3)$$

which clearly tells us that the bulk free path is much larger than the dimensions of the crystals for electrical conductivity along both these directions. The calculated values of the slope then yield the following values of

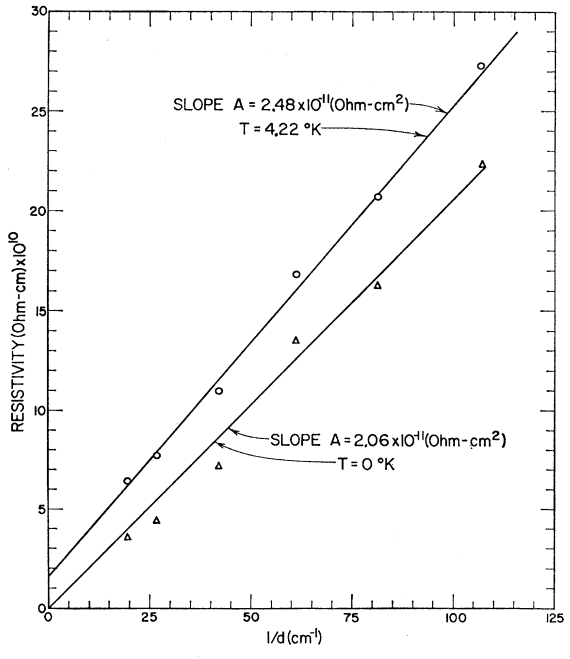

FIG. 2. Resistivity of A-axis crystals as a function of $1/d$, where d is the side of the square wire.

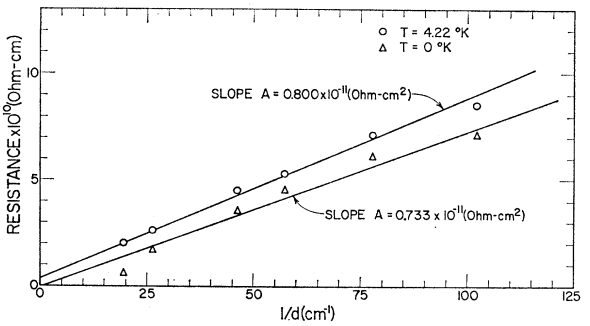

FIG. 3. Resistivity of B-axis crystals as a function of $1/d$, where d is the side of the square wire.

[3] R. W. Powell, Proc. Roy. Soc. (London) **A209**, 525 (1951)

[4] D. K. C. MacDonald and K. Sarginson, Proc. Roy. Soc. (London) **A203**, 223 (1950).

$\rho_b l_b$ for current flow along these axes:

$$(\rho_b l_b)_{A\text{ axis}} = 2.30\times10^{-11}\ \Omega\ \text{cm}^2,$$
$$(\rho_b l_b)_{B\text{ axis}} = 8.17\times10^{-12}\ \Omega\ \text{cm}^2.$$

The same value for the C axis reported in YC is

$$(\rho_b l_b)_{C\text{ axis}} = 8.11\times10^{-11}\ \Omega\ \text{cm}^2.$$

The free electron model with three free electrons per atom suggests the $\rho_b l_b$ should be

$$4.4\times10^{-12}\ \Omega\ \text{cm}^2.$$

This indicates clearly that the Fermi surface of gallium is strongly anisotropic and that only a fraction of the Fermi surface is free for conduction along different axes. This fraction varies from about $\frac{1}{2}$ for the B axis to about 1/20 for the C axis. The ratios of $\rho_b l_b$ for current flow along the A and C axes to that for current flow along the B axis are 2.83 and 9.93, respectively. These values are not very different from the ratios of the resistivities at 20°C obtained by Powell[5]:

$$\rho_A/\rho_B = 2.2, \quad \rho_C/\rho_B = 7.1.$$

Since the resistivity ratios do not depend very strongly upon temperature, the inference can be drawn that the mean free path of the carriers in gallium is nearly isotropic. Hence, even though the scatter in the data is too large to obtain a reliable estimate of the bulk resistivity for A and B crystals, we conclude that the bulk mean free paths for these orientations is comparable with the bulk mean free path for C-axis crystals. In YC we were able to assign a lower limit of 1 cm for the bulk mean free path of C-axis crystals made from the same batches of gallium as were used to prepare the A and B specimens. It is perhaps appropriate to emphasize that the figure of 1 cm obtained for the bulk mean free path was an estimate obtained from the C-axis data using the free electron model for gallium. We can only claim with certainty that the mean free path of the charge carriers is much larger than the dimensions of our crystals. This conclusion is further supported by the observation that the resistivities of these crystals is lowered in the presence of small longitudinal magnetic fields, even at 4.2°K.

On the basis of the free electron model the slope of the ρ_d-versus-$1/d$ curve is an invariant quantity for a particular metal. Experimentally, however, it is always found to be an increasing function of temperature. This increase has been attributed to an enhanced effect of electron-phonon scattering on the conductivity due to boundary scattering[6–9] and is fully discussed in YC. In Tables III and IV, where we have tabulated the calculated values of this slope for different temperatures, we find that it shows a steady increase with temperature for both axes. In order to test how far the enhanced contributions of electron-phonon boundary scattering are responsible for this increase in the slope, we analyze our results on the basis of the explicit mathematical relation, derived by Blatt and Satz,[8] which is given below.

TABLE III. Slope A and slope ϵ for the A-axis crystals.

T°K	Slope A, in Ω cm², obtained from ρ_d versus $1/d$	Slope ϵ obtained from $(10^{10}\rho_d - 0.206/d)$ versus $(1/d)^{2/3}$
4.22	2.48×10^{-11}	0.23
3.56	2.31×10^{-11}	0.15
3.00	2.27×10^{-11}	0.098
2.60	2.24×10^{-11}	0.073
2.25	2.23×10^{-11}	0.053
1.80	2.22×10^{-11}	0.034
0	2.06×10^{-11}	0.00

$$\rho_d(T) = \rho_r + \rho_i(T) + A_0/d + [8\pi f\rho_i(T)]^{1/3} \times[(T/\theta_D)(\rho_b l_b/d)]^{2/3} \quad \text{for} \quad d/l_i \ll 1. \quad (4)$$

In this equation ρ_r is the residual bulk resistivity, $\rho_i(T)$ is the temperature-dependent resistivity of the ideal metal characterized by a mean free path l_i, A_0 is the slope of the resistivity versus $1/d$ straight line at absolute zero, and θ_D is the Debye temperature; f is a fraction which takes into account processes which contribute to the resistance in addition to the normal electron-phonon events, such as umklapp and electron-electron collisions.

From Eq. (4) it is clear that if we plot $\rho_d(T) - A_0/d$ against $(1/d)^{2/3}$, we should obtain a straight line whose slope ϵ is $[8\pi f\rho_i(T)]^{1/3}[T/\theta_D(\rho_b l_b)]^{2/3}$, and whose intercept at the origin of $1/d$ gives the bulk resistivity. Figures 4 and 5 show the results of this calculation for the A axis and the B axis, respectively. It is clear that the data are not accurate enough to be able to say how good the agreement is with the Blatt and Satz theory. If we calculate the slope ϵ from the points on the graph by the method of least squares, we obtain a negative value for the bulk resistivity in each case. Since we

TABLE IV. Slope A and slope ϵ for the B-axis crystals.

T°K	Slope A, in Ω cm², obtained from ρ_d versus $1/d$	Slope ϵ obtained from $(10^{10}\rho_d - 0.0733/d)$ versus $(1/d)^{2/3}$
4.22	8.00×10^{-12}	0.075
3.56	7.74×10^{-12}	0.048
3.00	7.64×10^{-12}	0.031
2.60	7.54×10^{-12}	0.023
2.25	7.50×10^{-12}	0.017
1.80	7.47×10^{-12}	0.011
1.30	7.53×10^{-12}	0.008
0	7.33×10^{-12}	0.000

[5] R. W. Powell, Proc. Roy. Soc. (London) **A209**, 525 (1951).
[6] J. L. Olsen *et al.*, Helv. Phys. Acta **31**, 713 (1958).
[7] B. Luthi and P. Wyder, Helv. Phys. Acta **33**, 667 (1960).
[8] F. J. Blatt and H. G. Satz, Helv. Phys. Acta **33**, 1007 (1960).
[9] M. Ya. Azbel and R. N. Gurzhi, Zh. Eksperim. i Teor. Fiz. **42**, 632 (1962) [English transl.: Soviet Phys.—JETP **15**, 1133 (1962)].

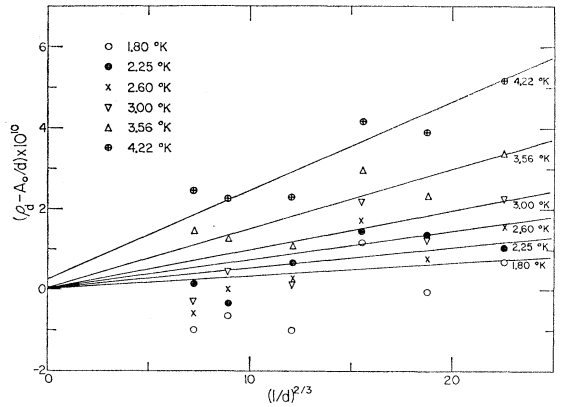

FIG. 4. $[\rho_d(T) - A_0/d]$ against $(1/d)^{2/3}$ for different temperatures for the A-axis crystals.

know that the resistance measurements are less accurate for the larger specimens, we feel somewhat justified in neglecting the values for the larger specimens and forcing the straight lines through zero, thus determining the slope by giving more weight to the points which correspond to the smaller specimens. In this way we arrive at the tabulated values of ϵ given in Tables III and IV. For normal electron-phonon collisions, the Bloch term for the resistivity is theoretically proportional to T^5 and this has been found to be true for most metals. The slope ϵ in this case should be proportional to $T^{2.33}$. A plot of $\log_{10}[100\epsilon]$ gives a straight line (Fig. 6), whose slope is 2.42 for both axes. This implies that for both of these axes the resistivity follows the same temperature dependence and that the normal electron-phonon term is proportional to $T^{5.26}$. Considering the approximate nature of the Blatt and Satz calculations and the almost arbitrary way we determined the values of ϵ, the agreement is more than fair and to some extent justifies our procedure of forcing the curves through zero.

In contrast with the C-axis results, the scatter in the data for the A and B axes is too large to enable us to obtain the temperature variation of the bulk resistivity. However, the temperature dependence with the exception of the two largest B-axis crystals is close to $T^{2.5}$, which points to the conclusion that in gallium of this purity the Bloch term is not very prominent between 4.2 and 1.2°K, and that terms with a lower exponent are contributing to the resistive processes. For most of our specimens the observations can be fitted to an expression of the form

$$\rho(T) = AT^m + BT^n,$$

where m is close to 2 and n is close to 5. For gallium which was at least an order of magnitude less pure, Olsen-Bar, and Powell[10] and Weisberg and Josephs[11] were able to fit their data to a $T^{4.5}$ dependence between 5 and 20°K. Below 5°K they did not find a change of

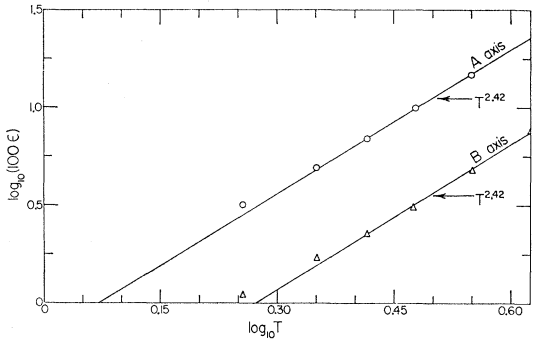

FIG. 6. Temperature variation of the slope ϵ for the two orientations.

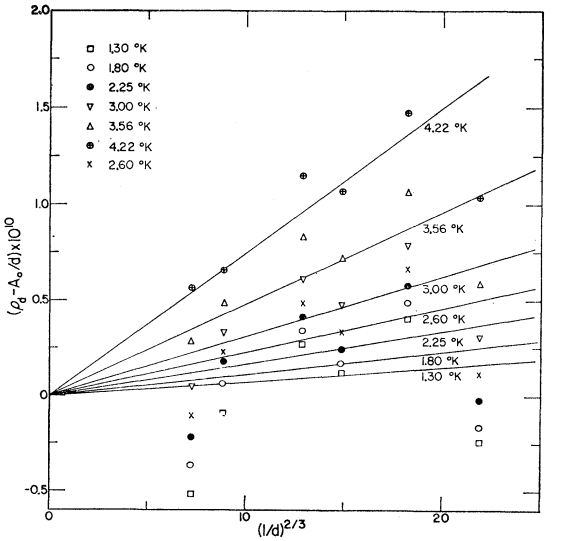

FIG. 5. $[\rho_d(T) - A_0/d]$ against $(1/d)^{2/3}$ for different temperatures for the B-axis crystals.

resistance with temperature because they had already reached the limiting value due to impurities at 4.2°K. Recently, R. Reich[12] has measured the bulk resistivity of pure gallium at low temperatures and has also found that his results can be represented by the following equation:

$$\rho = \rho_0 + AT^2 + bT^5. \quad (5)$$

He attributes the T^2 term to a verification of the so-called "Masharov's Law." Masharov,[13] in a theoretical calculation of the effect of impurities on the ideal resistivity of a metal, claims that the impurities cause the resistivity to vary as T^2 and T^5 due to the perturbation of the lattice modes. According to him for $T/\theta_D \ll 1$

$$\rho = \rho_0 + AT^5 + \beta_1 T^5 + \beta_2 T^2, \quad (6)$$

[10] M. Olsen-Bar and R. W. Powell, Proc. Roy. Soc. (London) **A209**, 542 (1951).
[11] L. R. Weisberg and R. M. Josephs, Phys. Rev. **124**, 36 (1961).
[12] R. Reich, Compt. Rend. **258**, 2814 (1964).
[13] S. I. Masharov, Fiz. Metal. i Metalloved. **13**, 2, 166 (1962) [English transl.: Phys. Metals Metallog. (USSR) **13**, 2, 8 (1962), respectively].

where ρ_0 is the residual resistivity, AT^5 is the Bloch term, β_1 is proportional to A and the concentration of impurities, and β_2 is proportional to impurity concentration only.

For a C-axis crystal of 4 mm^2 cross section Reich obtains the following value for the ideal resistivity:

$$\rho_i(T)=(1.36\times10^{-11}T^2)+1.7\times10^{-13}T^5,$$

which is in rough agreement with our values of the bulk resistivity for the same orientation. Our interpretation of a dominant T^2 term differs from that of Reich in that we attribute it to either umklapp or electron-electron collisions. According to Ziman,[14] a metal in which a considerable part of the Fermi surface is not free is particularly susceptible to umklapp processes. Gallium is just such an anisotropic metal, and the possibility of such processes cannot be ignored. According to Pines,[15] electron-electron collisions should make an appreciable contribution to the resistivity of anisotropic metals having a bulk mean free path of the order of centimeters. Preliminary measurements by Newbower, Neighbor, and Cochran indicate that the resistivity of a 1-cm-diam gallium crystal is still size limited at 1.5°K. This result confirms our earlier conclusion that for our crystals, impurities are not playing an appreciable role in the resistive processes. Thus, it is very likely that the term in the resistivity which varies nearly at T^2 is due to the joint action of umklapp and electron-electron collisions.

Longitudinal Magnetoresistance

It is well known that the resistivity of a conductor whose linear dimensions are less than the bulk electronic mean free path decreases when subjected to a longitudinal magnetic field. This is caused by the lengthening of the effective mean free path due to the spiral motion of the electrons round the magnetic lines of force. An analysis of this phenomenon has been developed by Chambers[16] on the basis of the free electron model. Assuming an isotropic relaxation time and diffuse scattering at the walls, the decrease in resistance depends on two parameters, d/r_0 and d/l_b, where d is the dimensiono f the wire, l_b is the mean free path in the bulk metal, and r_0 is the cyclotron radius. For a given value of d/l_b, provided $d<l_b$, the resistance decreases with increasing field until it approaches the bulk value as d/r_0 tends to infinity. The smaller the ratio d/l_b, the more rapid is the rate of drop in resistance as a function of d/r_0. This gives us a method of determining the electronic mean free path as well as the momentum at the Fermi surface. It need hardly be pointed out that for actual metals the assumption of an isotropic mean path and a free electron model are gross oversimplifications;

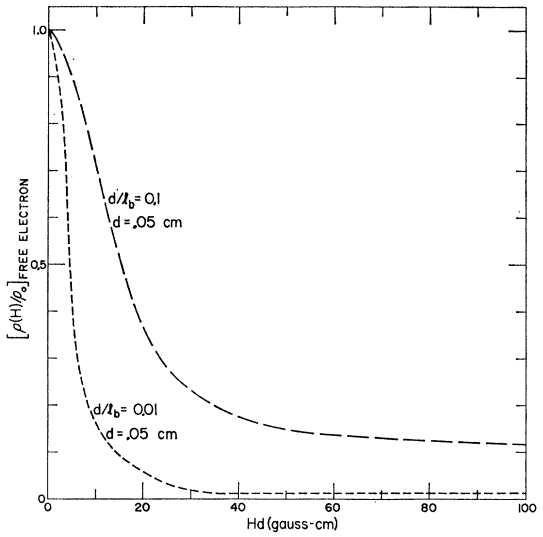

FIG. 7. Longitudinal magnetoresistance for a free-electron model of gallium based on Chambers' calculations.

nevertheless, the experimental results are of the same general form and do indeed provide some information about electronic mean free paths and momenta. According to Chambers' theory, if gallium had a spherical Fermi surface its resistance in a longitudinal magnetic field would behave as shown in Fig. 7, which is a plot of $[\rho_0(H)/\rho_0]$ against Hd, where H is the applied magnetic field, $\rho_0(H)$ is the limiting resistance in a field H, and ρ_0 is the same in zero field, for two values of d/l_b. Since H is proportional to $1/r_0$, our method of presenting the results is equivalent to that of Chambers. Because of the existence of a positive bulk magnetoresistance, the experimental results are somewhat different.[17] They can be seen for our specimens in Figs. 8 and 9 for the A-axis and in Figs. 10 and 11 for the B-axis crystals. For all the crystals the effect of a longitudinal field was measured at 4.2 and 1.2°K up to a maximum of 1400 G. In each case the field was parallel to the axis of the wire to within 2 or 3° and was uniform over the entire length of the specimen to about 1%. For both temperatures every specimen initially shows an increase in resistance which, after attaining a maximum value, begins to fall and ultimately rises again after going through a shallow minimum. The positions of the maximum and minimum for each crystal are obviously a function of the bulk magnetoresistance and the ratio d/l_b. Although Chambers' theory has not been extended to metals with nonspherical Fermi surfaces, it appears that to a first

[14] J. M. Ziman, in *The Fermi Surface*, edited by W. Harrison and M. B. Webb (John Wiley & Sons, Inc., New York, 1960).
[15] D. Pines, *Elementary Excitations in Solids* (W. A. Benjamin, Inc., New York, 1963).
[16] R. G. Chambers, Proc. Roy. Soc. (London) **A202**, 378 (1050).

[17] Kao has recently calculated the dependence of the resistivity of thin plates upon longitudinal magnetic field using Chambers' theory and the free-electron model of a metal [Y. Kao, Phys. Rev. 138, A1412 (1965)]. His calculations show an initial increase of resistivity with magnetic field, followed by a decrease at higher fields. His curves are so similar to our experimental results for the longitudinal magnetoresistivity of small A- and C-axis wires that one wonders if a maximum in the resistance may not be a genuine feature of the free electron theory for wires.

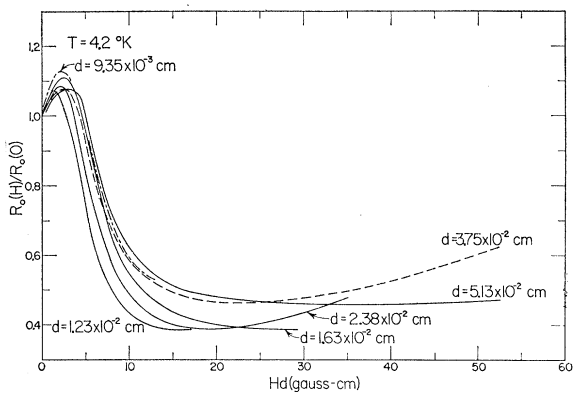

FIG. 8. Normalized resistance as a function of the longitudinal field times the dimensions of the A-axis crystals at 4.2°K.

approximation one can treat the decrease in resistance caused by the lengthening of the mean free path and the increase due to bulk magnetoresistance as independent phenomena and obtain curves similar to those actually observed by multiplying the ratios $[\rho_0(H)/\rho_0]_{\text{free electron}}$ and $[\rho_B(H)/\rho_B]$, where the 2nd factor is the bulk magnetoresistive increase caused by the same field. The results of such a calculation are shown in Fig. 12. The upper curve has been obtained by multiplying the curve of Fig. 7 for $d/l_b=0.1$ by the bulk magnetoresistance of the largest B specimen at 1.2°K. The lower curve of Fig. 12. has been obtained by multiplying the curve of Fig. 7 for $d/l_b=0.01$ by the bulk magnetoresistance of the largest A specimen at 1.2°K. These bulk magnetoresistances were obtained by extrapolating the form of the curves of $R_0(H)/R_0(0)$ versus Hd at large fields to zero field for the largest A and B specimens. The results of this *ad hoc* procedure, Fig. 12, are remarkably similar to the experimental curves corresponding to 1.2°K for the largest A and B crystals.

It is not possible to obtain quantitative values for l_b and the Fermi momentum from the longitudinal magnetoresistance data, but the following qualitative conclusions can be drawn. The smaller magnetomorphic effect exhibited by the B crystals relative to the A crystals, at any temperature, is mainly due to their larger bulk magnetoresistance, although qualitative agreement between the curves of Fig. 12 and the data for the largest A and B crystals at 1.2°K may indicate a longer mean free path for current flow along the A axis than for current flow along the B axis. The existence of a magnetomorphic effect at 4.2°K in the largest wires indicates $l_b \gtrsim 5\times10^{-2}$ cm for both orientations; this is consistent with a mean free path in excess of 1 cm at 1°K because the resistivity of these wires falls approximately as T^2, and the residual resistivity is immeasurably small for both orientations. Finally, the similarity between the curves of Fig. 12 and the experimental data suggests that the momenta of the charge carriers has the same order of magnitude as that calculated for free electrons, 1.75×10^{-19} cgs.

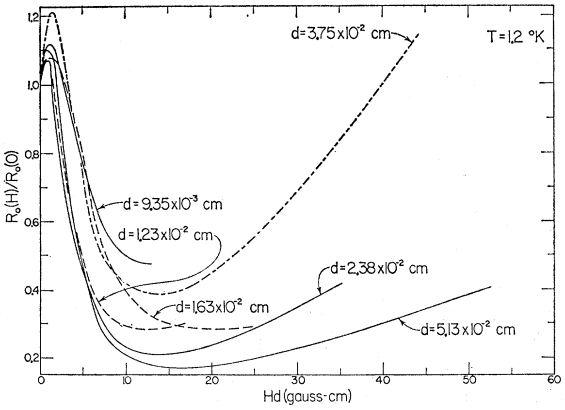

FIG. 9. Normalized resistance as a function of the longitudinal field times the dimensions of the A-axis crystals at 1.2°K.

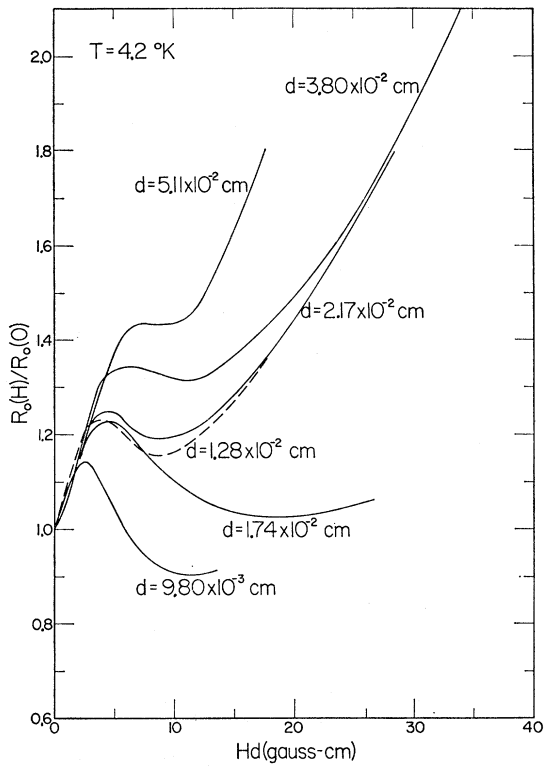

FIG. 10. Normalized resistance as a function of the longitudinal field times the dimensions of the B-axis crystals at 4.2°K.

Current Dependence of the Resistance

As in the case of the C-axis crystals the resistance was found to be a complicated function of the measuring current. It was measured for all the crystals as a function of the current at 4.2 and 1.2°K. The dissipation of

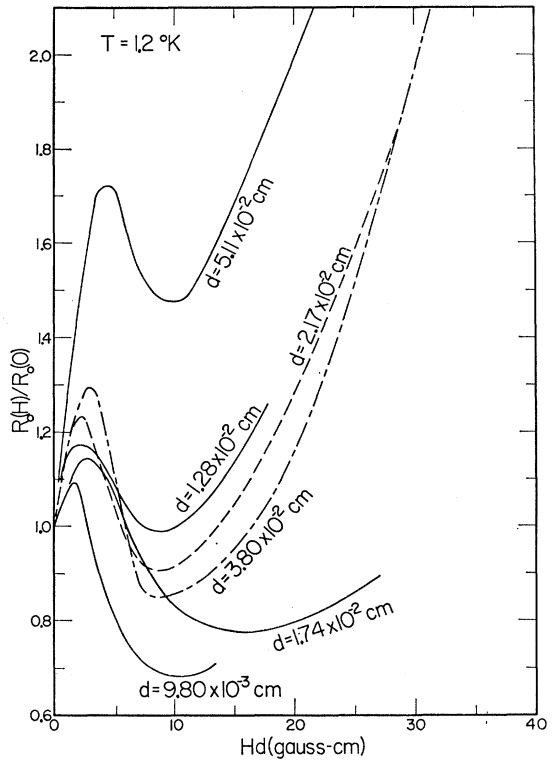

FIG. 11. Normalized resistance as a function of the longitudinal field times the dimensions of the B-axis crystals at 1.2°K.

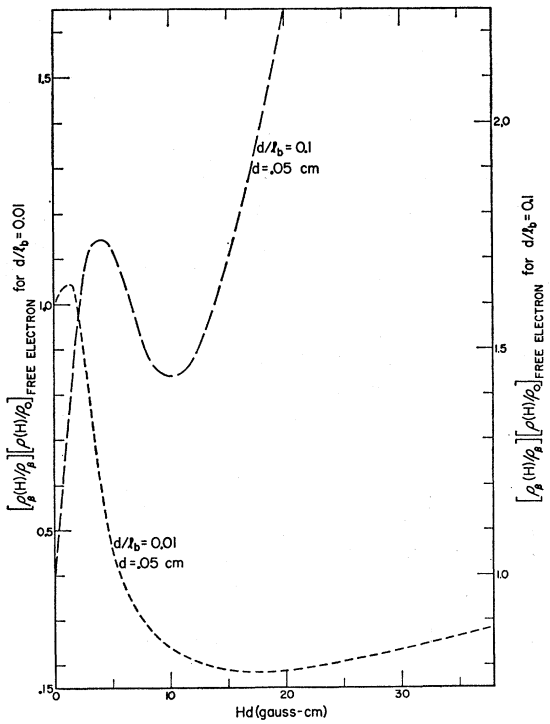

FIG. 12. The product of the longitudinal magnetoresistance for small specimens assuming a free-electron model and the empircal bulk magnetoresistance for two ratios of d/l_b.

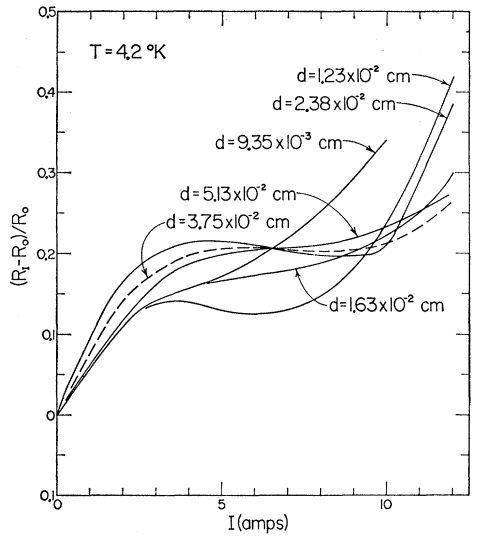

FIG. 13. $\Delta R/R_0$ as a function of the current for the A-axis crystals at 4.2°K.

Joule heat was kept below 0.21 W/cm²; this figure was arrived at empirically and represents the upper limit beyond which the specimens began to rise in temperature. The observations are displayed in Figs. 13 and 14 for the A-axis and in Figs. 15 and 16 for the B-axis crystals. The variation is in general similar for the two axes, although there are considerable differences in detail between different axes as well as between different crystals of the same orientation.

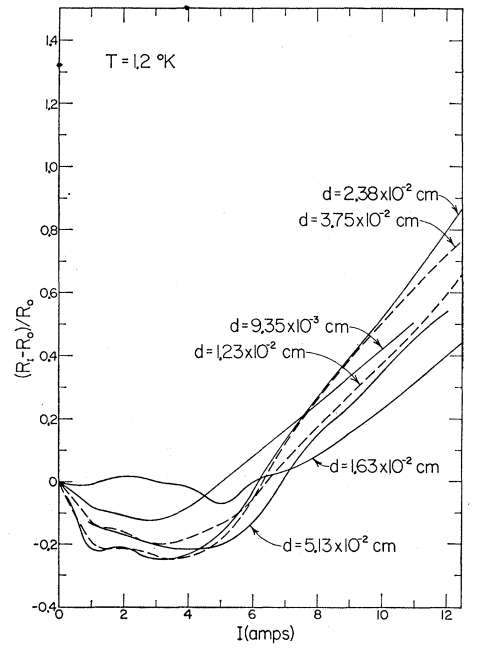

FIG. 14. $\Delta R/R_0$ as a function of the current for the A-axis crystals at 1.2°K.

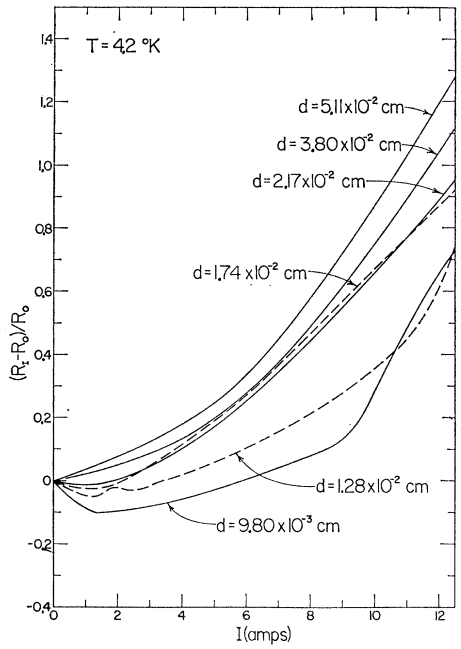

FIG. 15. $\Delta R/R_0$ as a function of the current for the B-axis crystals at 4.2°K.

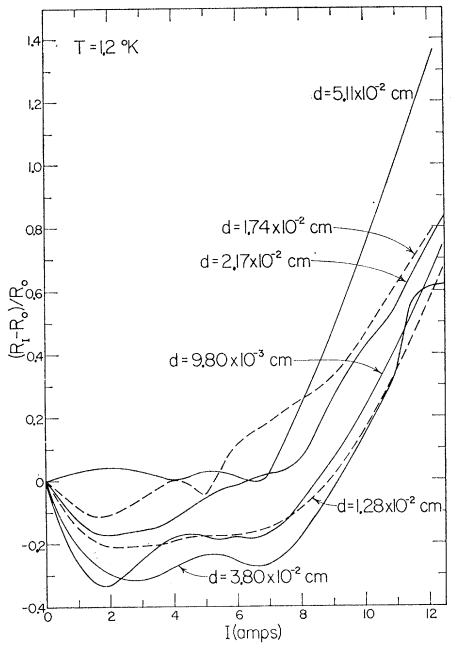

FIG. 16. $\Delta R/R_0$ as a function of the current for the B-axis crystals at 1.2°K.

As explained in YC, these curves can be interpreted in terms of the magnetoresistance effects of the field generated by the measuring current itself. This field performs the double function of increasing the resistance due to the bulk magnetoresistance and of simultaneously decreasing the resistance by trapping the charge carriers so that they are unable to collide with the walls of the specimens. A simple-minded calculation of this effect, based on the free electron model and using the same assumptions used by Chambers[16] in his calculations, was given in YC for cylindrical wires, according to which the resistivity of the wire can be expressed as

$$\bar{\rho}=\rho_d(0)/\{1+[(l_b/2a)]F(\alpha)\}, \quad l_b \ll 2a, \qquad (7)$$

where $\rho_d(0)$ is the limiting resistance of the wire for zero current, $2a$ is the diameter, and $\alpha=eI/5p_fc$. In the last expression I is the current in amperes, p_f is the Fermi momentum, e is the charge, and c is the velocity of light. $F(\alpha)$ is defined as follows:

$$F(\alpha)=\tfrac{1}{64}\alpha(\alpha^2-8\alpha+24) \quad \text{if} \quad 0<\alpha<4$$
$$=[1-(2/\alpha)] \quad \text{if} \quad \alpha>4.$$

According to Eq. (7), wires in which the resistance for small measuring currents is dominated by boundary scattering increase their conductivity with increasing current and ultimately approach their bulk value as $I \to \infty$. The effect is very similar to the case of an externally applied longitudinal magnetic field. The ultimate rise in resistance with current, observed at large currents for every specimen, can be attributed to the bulk magnetoresistance caused by the magnetic field generated by the current itself.

The individual curves of Figs. 13–16 have the same general form as the equivalent curves showing the dependence of resistance on current for the C-axis crystals (see YC). However, they do not display the regular progression with specimen dimensions that was a characteristic feature of the C-axis results. This erratic dependence upon specimen size is probably due to the very rapid variation of the bulk magnetoresistance with the angle between the direction of current flow and the crystal axes. According to Reed and Marcus[18] the magnetoresistance of gallium is quadratic in the field strength for all orientations of current and field except when both the current and the field lie in the AB plane, in which case it saturates. Thus a small misalignment between the crystal axis and wire axis, for a wire axis nominally parallel to A or B, can result in a relatively large change in the average transverse magnetoresistance for fields in the plane perpendicular to the wire axis.

[18] W. A. Reed and J. A. Marcus, Phys. Rev. **126**, 1298 (1962).