Magnetic-Field Dependence of the Rf Skin Depth of Gallium*

J. F. COCHRAN AND C. A. SHIFFMAN
Department of Physics and The Center for Materials Science and Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 27 May 1965)

The variation with magnetic field of the surface reactance of gallium single crystals at 2 Mc/sec has been investigated at helium temperatures and for fields up to 1500 G. The changes observed in the surface reactance depend in a very complicated way upon magnetic field, and correspond to variations of nearly 100% in the skin depth. The field dependence of the skin depth was found to depend upon specimen orientation, the mean free path of the charge carriers, and rf magnetic field amplitude. A number of small anomalies in the surface impedance have been identified as the rf size effect first reported by Gantmacher for tin.

I. INTRODUCTION

WHEN a metal is placed in a high-frequency electric field, the amplitude of the field which penetrates the surface falls to zero in a short distance called the "skin depth." For the case of a plane slab of metal at room temperature and in a parallel field, the amplitude decreases exponentially with distance according to the relation

$$E=E(0)e^{-z/2\delta}e^{-i(z/2\delta-\omega t)},$$

where

$$\delta=\tfrac{1}{2}(c^2/2\pi\sigma\omega)^{1/2}. \qquad (1)$$

This classical result is valid so long as $\delta \gg l$, where l is the mean free path of the charge carriers in the metal. Mean free paths of the carriers in very pure metals become comparable to 1 mm at liquid-helium temperatures, and for such materials the above inequality does not obtain even for frequencies in the kilocycle range. For the case $\delta < l$, it is necessary to use the theory of the "anomalous skin effect."[1] This theory predicts a nonexponential penetration of the rf fields into the metal, falling to small values in a distance much larger than the classical skin depth. Table I compares the classical and anomalous skin depth calculations for gallium at low temperatures and a frequency of 1 Mc/sec. In the regime of the anomalous skin effect the skin depth can be defined as that distance into the specimen which the rf fields would have to penetrate without attenuation in order that the flux per unit length be equal to the actual flux contained in the metal:

$$\delta=\mathrm{Re}\left\{\frac{1}{H(0)}\int_0^\infty H(z)dz\right\}.$$

For free electrons which are diffusely scattered from the specimen surface, it is given by[1]

$$\delta^2=(c^2/8\pi\omega\sigma)(2.8\delta/l)^{-1}. \qquad (2)$$

This expression has the same form as (1) but the conductivity of the metal has been reduced by a factor proportional to δ/l. The metal behaves as if only those electrons which can complete a mean free path in the skin layer contribute to the surface currents which shield the interior of the metal from the applied fields.[1] The number of "effective" electrons is inversely proportional to the mean free path, and δ becomes independent of l, since the ratio σ/l ($=ne^2/p_F$ for free electrons) is a constant characteristic of a given metal.

If a static magnetic field is applied parallel to the surfaces of an infinite plane slab of thickness d, two lengths are introduced into the problem in addition to δ and l. These are R_0, the cyclotron radius corresponding to a Fermi surface extremum ($=p_Fc/eH_0$ for free electrons), and the thickness itself. d enters the problem because charge carriers moving parallel to the surface are deflected out of the skin depth and can contribute to the current density in the skin layer on the opposite side of the specimen if the mean free path is sufficiently long. Consider explicitly the case in which the rf magnetic field is parallel to the static magnetic field, and l, d, $R_0 \gg \delta$, with $l \geq d$. At frequencies of the order of 1 Mc/sec one has a relatively simple situation because the charge carriers are, on the average, scattered in a time which is short compared to the period of the oscillations, i.e., $\omega\tau \ll 1$. The rf time dependence is therefore unimportant, and serves only to localize the fields within a distance of order δ of the specimen surface. Theory has divided the problem of the magnetic field

TABLE I. The characteristic resistivities of gallium for current flow along its three principle directions, and the classical and anomalous skin depths at 1 Mc/sec calculated from them.

Direction of current flow	ρl[a] Ω cm^2	σ/l esu	Classical skin depth at 10^6 cps and $l=1$ mm, Eq. (1) (cm)	Reactive anomalous skin depth at 10^6 cps. $\delta=(c^2/4\pi\omega)X$[b] (cm)
A	2.3×10^{-11}	3.9×10^{22}	3.8×10^{-6}	3.8×10^{-4}
B	0.82×10^{-11}	11×10^{22}	2.3×10^{-6}	2.6×10^{-4}
C	8.1×10^{-11}	1.11×10^{22}	7.2×10^{-6}	5.7×10^{-4}

[a] Characteristic resistances for current flow along the A, B axes were obtained from unpublished size effect data of Cochran and Yaqub; that for current flow along the C axis has been taken from Yaqub and Cochran (Ref. 6).
[b] Calculated from the formula for free electrons (Ref. 1) assuming diffuse scattering of the charge carriers at the specimen surfaces. $\delta^3=c^2/8\pi\omega(2.8\sigma/l)$.

* This research was supported by the Advanced Research Projects Agency under contract SD-90.

[1] E. H. Sondheimer, *Advances in Physics* (Francis & Taylor, Ltd., London, 1952), Vol. 1, p. 1.

dependence of the skin depth for this geometry, and for $\omega\tau \ll 1$, into three characteristic regions:

(a) $2(R_0\delta_0)^{1/2} \gg l$. The quantity on the left side of the inequality is the length of the chord subtended at a distance $\delta_0/2$ from the circumference of a circle with radius R_0. The inequality is therefore the statement that the magnetic field is too small to bend a charge carrier out of the skin layer within one mean free path, if it starts out $\delta_0/2$ from the surface with a velocity in the plane of the slab. According to Azbel' and Kaner[2] the change in penetration depth with magnetic field for a free-electron metal is given by

$$\Delta\delta = (4.85 \times 10^{-3}) l^4 / R_0^2 \delta_0 . \tag{3}$$

Note that the skin depth is expected to increase as H_0^2, and that the magnitude of the change in the skin depth is expected to be very dependent upon mean free path in contrast with δ_0, the skin depth in zero magnetic field. For gallium with a mean free path of 1 mm and $\delta_0 = 4 \times 10^{-4}$ cm (c.f. Table I) we find that R_0 must be larger than 6 cm, and hence the field must be less than 1.7 G in order to observe this effect. In the calculation of the magnetic field corresponding to a given cyclotron radius we have used the expression for a free-electron gas having an electron density appropriate to gallium,

$$R_0 = 10.9 / H_0 . \tag{4}$$

(b) $R_0 < d$, $\ll l$, *but* $R_0 \gg \delta$. A charge carrier which begins a mean free path within δ of the surface can return to the skin layer a number of times before being scattered. Consequently, it can sample the surface electric field a number of times, and thus contribute more to the surface current density than a carrier which is scattered after having made only one pass. According to Kaner[3] this mechanism causes the skin depth to decrease with increasing magnetic field. For free electrons and a magnetic field which lies within the angle $\varphi_0 = \delta_0 / l$ of the plane of the specimen, Kaner gives

$$\delta/\delta_0 = 1.2 (2R_0/l)^{1/3} . \tag{5}$$

Taking $l = 1$ mm for gallium, condition (b) is satisfied for fields much larger than 100 G. Equation (5) is expected to be valid, however, only if the field lies within $\sim\frac{1}{2}°$ of the plane of the specimen. This is a stringent condition which it is difficult to achieve in practice. Kaner has shown that the effect of tilting the field is to change the exponent in Eq. (5) from $\frac{1}{3}$ to $\frac{1}{4}$, and to multiply by a constant factor and by a function of the tilt angle φ. One might regard the difference between the two power laws as too slight to be decided empirically. It turns out, however, that the value of the constant multiplier involved depends (in part) on whether $R_0 \gg l^2\varphi^2/\delta_0$ or $R_0 \ll l^2\varphi^2/\delta_0$. Therefore, one must anticipate that varying the field at fixed φ may give rise to a nonmonotonic variation in δ, despite the apparent simple-power-law dependence given in Eq. (5).

(c) $R_0 \ll \delta$. A charge carrier which begins its mean path in the skin layer must remain in the skin layer. The classical expression (1) becomes valid, where the conductivity to be used is the bulk conductivity for the field H_0. The penetration depth must eventually increase for large enough fields since a magnetic field causes the bulk conductivity to decrease.

Superposed on the above gross variations of the skin depth with magnetic field there are small anomalies. When a charge-carrier orbit corresponding to a Fermi surface extremum can just fit between the surfaces of the slab, current is carried from one side of the specimen to the other in the correct phase for enhancement of the screening of the rf field. This effect was first observed by Gantmacher[4] in tin. Gantmacher resonances provide a caliper measure of the Fermi surface in a direction perpendicular to both magnetic field and the normal to the slab. For a resonance occuring at the field H_0, the projection of the orbit in k space along the above direction is given by

$$\Delta k = (eH_0/\hbar c) d = 1.52 \times 10^7 (H_0 d) \mathrm{cm}^{-1} . \tag{6}$$

Kaner has calculated both the form and magnitude of the dependence of δ on magnetic field at resonance. He finds that the strength and width of the resonance are given by

$$\Delta\delta/\delta \propto (\delta/d)^{1/2} ,$$
$$\Delta H / H_0 \propto \delta / d . \tag{7}$$

The experiments described in this paper are a survey of the magnetic-field dependence of the skin depth in gallium single crystals. It will be seen that the results of the experiments are in general agreement with the behavior sketched above, although a detailed comparison with theory is not possible without an explicit knowledge of the carrier mean free path l and the skin depth in zero magnetic field δ_0.

II. EXPERIMENTAL DETAILS

Surface Impedance

The geometry used in these experiments is shown in Fig. 1. The specimen, a slab d centimeters thick and having a total surface area of A centimeters^2 was inserted into an oscillator tank coil having n turns per centimeter. The tank coil was approximately twice as long as the specimen in order to obtain a uniform rf field over the surfaces of the slab. We calculate that the field did not vary by more than 3% over the volume of the specimen. Flux contained in the specimen contributes to the inductance of the coil, and changes in inductance are proportional to changes in the surface reactance component X_{yy}, where

$$Z_{yy} = R_{yy} + iX_{yy} = -(4\pi/c) E_y(0) / H_x(0) .$$

[2] M. Ya. Azbel' and E. A. Kaner, J. Phys. Chem. Solids **6**, 113 (1958).

[3] E. A. Kaner, Zh. Eksperim. i Teor. Fiz. **44**, 1036 (1963) [English transl.: Soviet Phys.—JETP **17**, 700 (1963)].

[4] V. F. Gantmacher, Zh. Eksperim. i Teor. Fiz. **44**, 811 (1963) [English transl.: Soviet Phys.—JETP **17**, 549 (1963)].

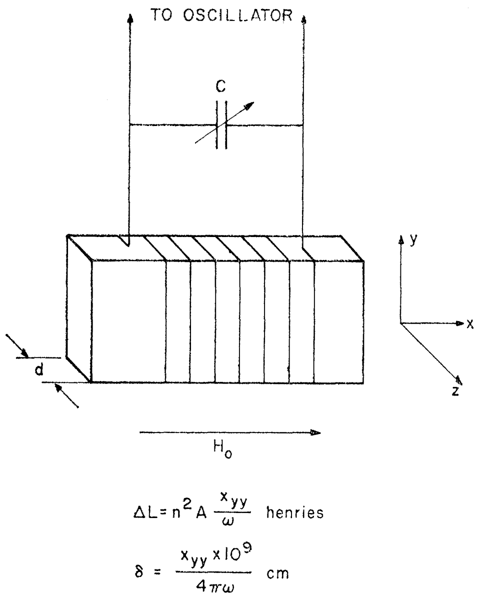


FIG. 1. The geometry used to investigate the magnetic field dependence of the surface impedance of gallium. The surface impedance X_{yy} is in ohms.

Referring to Fig. 1, $E_y(0)$ and Z_{yy} are the components along y of the electric field at the surface of the specimen and the surface impedance, respectively. $H_x(0)$ is the rf magnetic field amplitude at the surface of the slab. The skin depth can be expressed in terms of the surface impedance by

$$\delta \equiv \tfrac{1}{2}\mathrm{Re}\left\{\frac{1}{H_x(0)}\int_0^d H_x(z)dz\right\} = \frac{c^2 X_{yy}}{4\pi\omega}\text{centimeters}. \quad (8)$$

X_{yy} is in electrostatic units; $\omega = 2\pi f$ is the circular frequency. The inductance of the tank coil is increased by the finite penetration of the rf fields into the metal,

$$\Delta L = n^2 A X_{yy}/\omega,$$

where A is the surface area of both sides of the specimen. For a loosely fitting coil changes in oscillator frequency are proportional to ΔL, and hence to changes in skin depth,

$$\Delta\delta = (L/2\pi n^2 A)(\Delta f/f)\times 10^3 \text{ centimeters}, \quad (9)$$

where the coil inductance in zero field is L μh.

Apparatus

The oscillator consisted of a wide-band amplifier, having a gain of 100, in which a fraction of the output signal was fed back to the input through a resistor in series with the tank circuit. The feedback signal was taken from a voltage divider, and fed from a germanium diode limiting circuit across the amplifier output, so that the voltage across the tank coil could be varied. Tank voltages ranging from $\frac{1}{10}$ to 35 V in amplitude were obtained, corresponding to rf magnetic field amplitudes of 0.03 to 10 G at the specimens. The resonant frequency of the tank circuit was adjusted to 2.00 Mc/sec by means of a variable air capacitor.

The tank coil and specimen were mounted in a conventional helium cryostat with provision for varying the temperature between 1.5 and 4.2°K. The static magnetic field, H_0, and the low-frequency sweep field, were generated by means of solenoids mounted in the helium bath and fixed with respect to the tank coil so that these fields were parallel within $\frac{1}{2}°$ to the specimen surfaces. A copper solenoid was used for measurements with magnetic fields less than 100 G, and a superconducting niobium solenoid was used to obtain fields up to a maximum of 1500 G. The earth's magnetic field was compensated to 0.01 G at the specimen by means of Helmholtz pairs.

Changes in the oscillator frequency were obtained by recording changes in the output voltage of a ratio detector-limiter combination[5] scavenged from an ordinary FM receiver and retuned to 2 Mc/sec. In order to obtain absolute changes in skin depth it was necessary to measure the inductance of the tank coil plus specimen combination at zero field. This inductance was obtained from the variation of oscillator frequency with tank capacitance. A 5–10% correction was necessary because of transmission line effects in the 1-m long coaxial cable which joined the tank coil in the liquid helium to the capacitor at room temperature.

The derivative of skin depth with respect to magnetic field $d\delta/dH_0$ was obtained by applying a small ac magnetic field parallel to H_0. The resulting signal from the ratio detector at the sweep frequency f_s was measured with a lock-in amplifier. Most derivative measurements were made using $f_s = 35$ cps, although 75 cps was also used. No peculiar behavior due to the finite penetration of the sweep field into the specimens was observed for frequencies less than 500 cps, but 1500 cps could definitely not be used to obtain meaningful data. The sensitivity of the apparatus was such that changes of 10^{-7} cm/G in $(d\delta/dH_0)$ were detectable using a 3-sec time constant and a sweep amplitude of $\frac{1}{10}$ G.

TABLE II. Properties of the crystals used in this work. The crystals are $\frac{1}{2}$ in. square wafers of thickness d. The letter in the specimen label indicates the gallium crystal axis which is normal to the plane of the specimen.

Specimen	d[a] (10^{-2} cm)	$1/d$ (cm^{-1})	Total surface area, A (cm^2)
A_{13}	9.04	11.1	3.08
A_{15}	3.94	25.4	3.16
A_{14}	1.75	57.1	3.10
B_{06}	4.06	24.6	3.06
C_{05}	4.06	24.6	3.10

[a] Average thickness calculated from the weight and area of the specimens using a density [T. W. Richards and S. Boyer, J. Am. Chem. Soc. **43**, 274 (1921)] $\rho = 5.907$ g/cc.

[5] S. Seely, *Electron Tube Circuits* (McGraw-Hill Book Company, Inc., New York, 1958), 2nd ed.

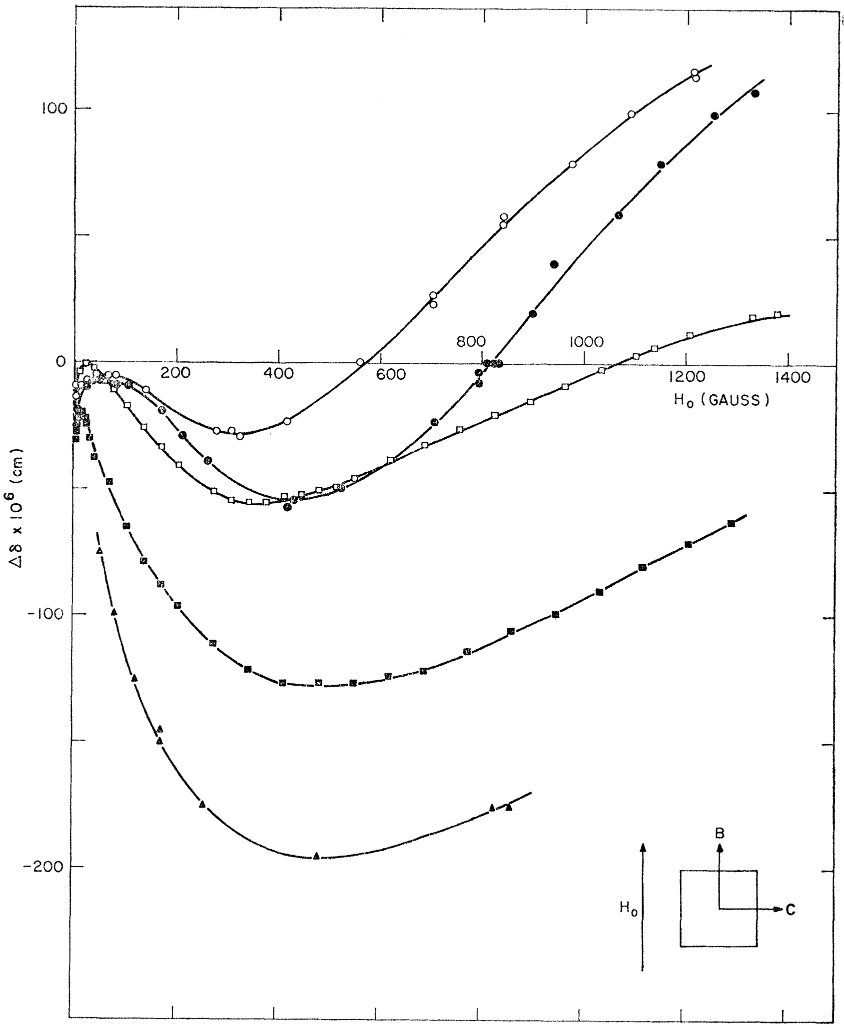


FIG. 2. The change in rf skin depth as a function of magnetic field for three A crystals with the field parallel to the B axis. Open symbols, $T=4.2$°K. Closed symbols, $T=1.6$°K. ○, ●—0.90 mm thick. ▲—0.39 mm thick. □, ■—0.18 mm thick.

Specimens

The properties of the five gallium crystals used in this investigation are summarized in Table II. These crystals, $\frac{1}{2}$-in. square wafers, were grown in Lucite molds, following the prescription of Yaqub and Cochran[6] and from gallium of comparable purity. The surfaces of the crystals were mirror smooth, apart from a few small flaws. The crystallographic axes were parallel to the specimen edges to within 1° of arc.

III. RESULTS AND DISCUSSION

The Magnetic-Field Dependence of the Skin Depth

The gross magnetic-field dependence of the gallium skin depth is shown in Figs. 2, 3, and 4. The data has been replotted on an expanded scale in Fig. 5 in order to demonstrate the structure of the curves at small fields. The most remarkable features of these curves are the nonmonotonic dependence of δ on field and the magnitude of the changes observed. Referring to Table I it will be seen that the changes in the skin depth can be very nearly as large as the skin depths calculated for zero field. Data for the three A crystals, Fig. 2 and 3, show that the magnetic field dependence of δ is not related to specimen thickness in any simple way, but is very sensitive to mean free path. Not only are the observed variations temperature-dependent, but they also differ wildly from specimen to specimen. In fact, difference in behavior within the group of A crystals are nearly as great as difference in behavior for the different orientations, shown in Fig. 4. All orientations have in common a rapid decrease of skin depth with field at low fields with a minimum at approximately 5 G (Fig. 5) followed by an increase, then a decrease again to a second minimum which occurs between 200 and 1200 G depending upon orientation. The strength of the intervening maximum shows some correlation with orientation of the crystal. In particular, if the field is normal to the A axis the maximum is not strong enough to reach positive values of $\Delta\delta$.

[6] M. Yaqub and J. F. Cochran, Phys. Rev. 137, A1182 (1965).

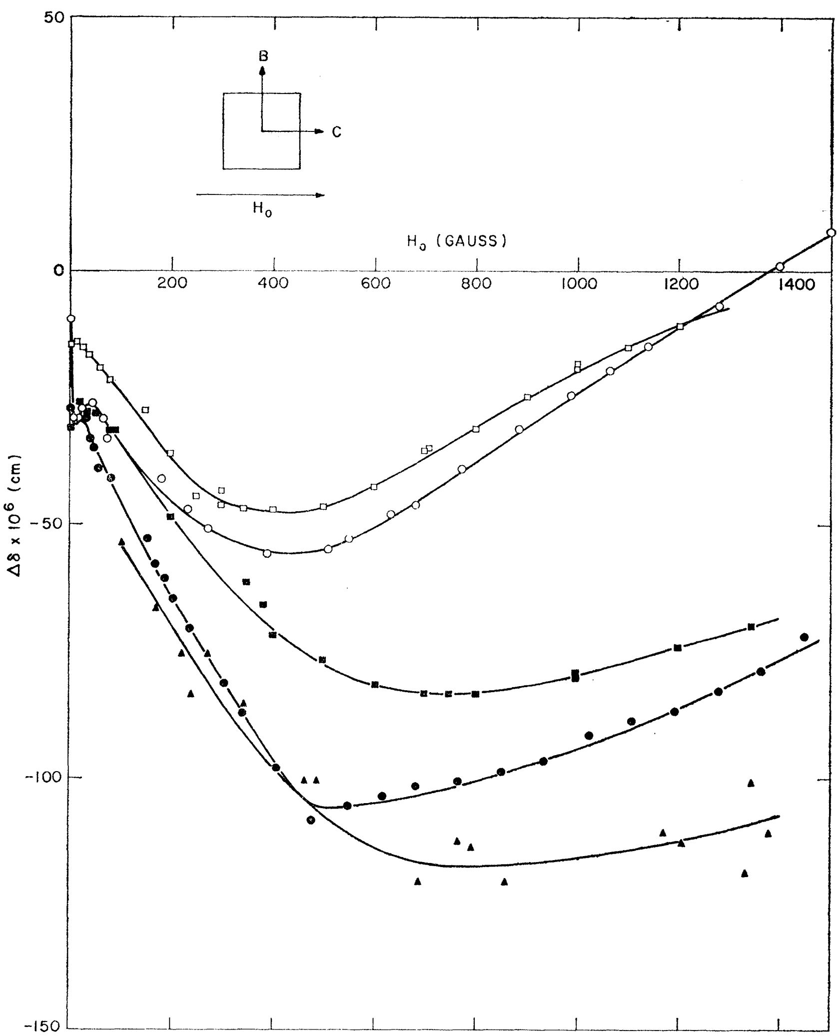


FIG. 3. The change in rf skin depth as a function of magnetic field for three A crystals with the field parallel to the C axis. Open symbols, $T=4.2$°K. Closed symbols, $T=1.6$°K. ○, ●—0.90 mm thick. ▲—0.39 mm thick. □, ■—0.18 mm thick.

These maxima suggest that the effect of tilting the field which was discussed in the introduction may be operative. It seems unlikely, however, that the tilt effect could cause the maxima for two reasons. The first is that δ exceeds δ_0 in some orientations, contrary to the theoretical prediction. Secondly, although we allow a tilt of 10^{-2} rad in our apparatus, this figure is only an estimate based on the clearance left in the specimen holder. Thus the actual angle would vary from day to day. Despite this we observe basically the same behavior for a given specimen when measured on different occasions. We think, therefore, that it is unlikely that the dependence of the skin depth upon field, as illustrated in Figs. 2–5, can be ascribed to the misalignment of field and specimen, but a clarification of this point must await further investigation.

TABLE III. The bulk magnetoresistance at 12 kG and 4.2°K for various directions of magnetic field and current flow for gallium. Data taken from W. A. Reed and J. A. Marcus [Phys. Rev. **126**, 1298 (1962)].

Direction of normal to specimen plane	Direction of the magnetic field	Direction of current flow	$\Delta\rho/\rho \times 10^{-4}$	Character of the field variation of $\Delta\rho/\rho$
A	B	C	2	quadratic
A	C	B	26	quadratic
B	A	C	4	quadratic
B	C	A	10	quadratic
C	A	B	~ 0	saturates
C	B	A	~ 0	saturates

There appears to be no correlation between the magnitude of $\Delta\delta/\delta_0$ at 1500 G for the various orientations and the magnetoresistance of the bulk metal, Table III. It would be of interest to know the behavior of δ at larger fields, but we were limited to 1500 G.

Two orientations, A with H_0 parallel to B, and B with H_0 parallel to A, exhibited an initial increase of skin depth with magnetic field. It is tempting to identify this increase with that predicted by Azbel' and Kaner,[2] Eq. (3). A quantitative comparison of the data with their theory is not possible since l, δ_0, R_0 are all unknown. The increase in skin depth observed is consistent

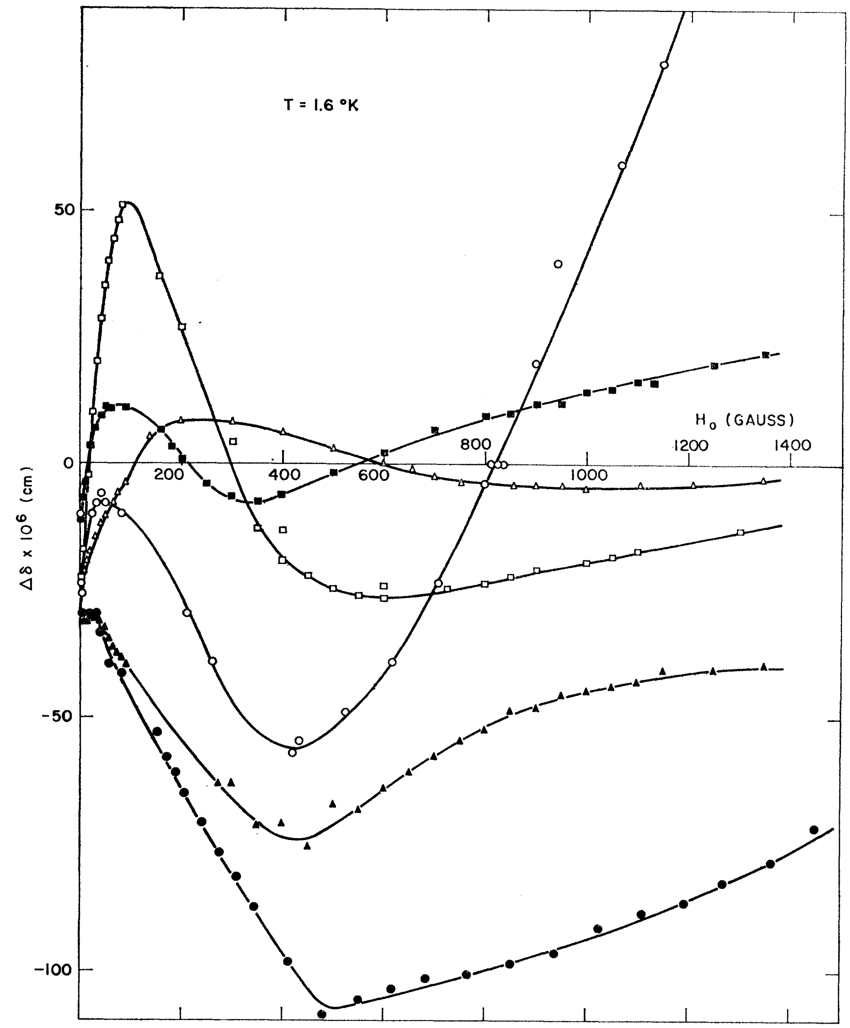


FIG. 4. The change in rf skin depth as a function of magnetic field at 1.6°K for A, B, and C crystals. ○—A_{13}, H_0 parallel to B. ●—A_{13}, H_0 parallel to C. □—B_{06}, H_0 parallel to A. ■—B_{06}, H_0 parallel to C. △—C_{05}, H_0 parallel to A. ▲—c_{05}, H_0 parallel to B.

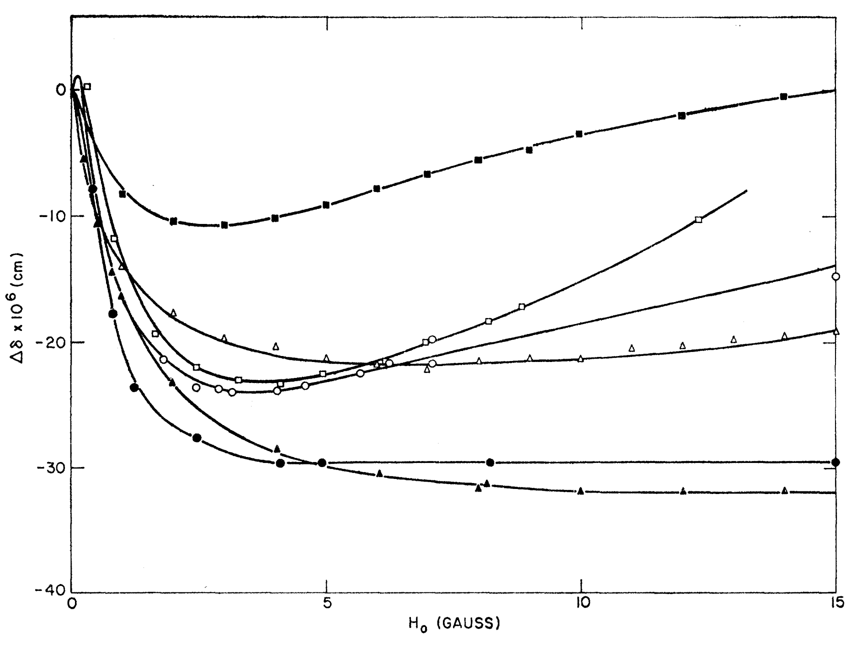


FIG. 5. The variation of rf penetration depth with magnetic field at 1.6°K for A, B, and C crystals. ○—A_{13}, H_0 parallel to B. ●—A_{13}, H_0 parallel to C. □—B_{06}, H_0 parallel to A. ■—B_{06}, H_0 parallel to C. △—C_{05}, H_0 parallel to A. ▲—C_{05}, H_0 parallel to B.

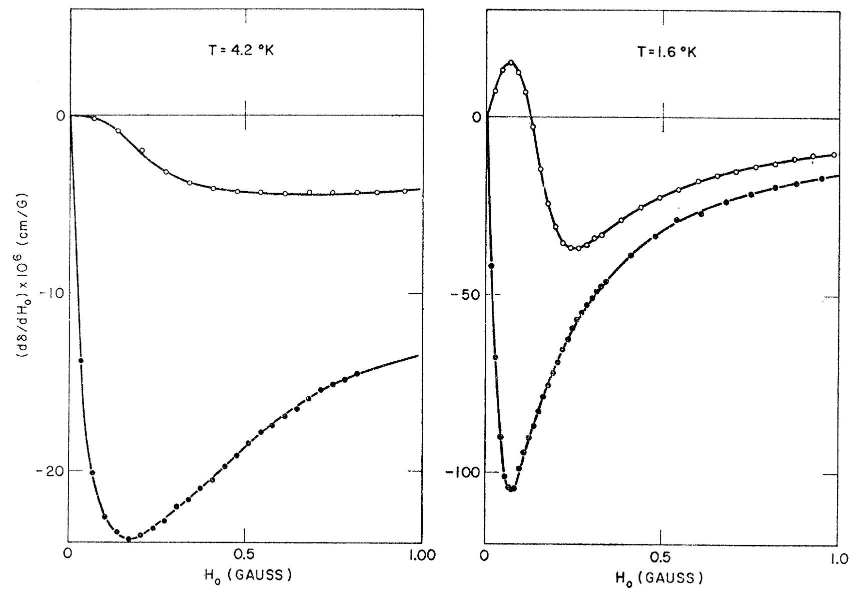


FIG. 6. The variation of the derivative of the rf penetration depth with magnetic field for specimen A_{13}. ○—H_0 parallel to B. ●—H_0 parallel to C.

with $l \sim \frac{1}{2}$ mm, if $\delta_0 \sim 5 \times 10^{-4}$ cm and $R_0 \sim 0.2/H_0$ centimeters as obtained from Gantmacher resonance data to be presented below. It is interesting that Gantmacher and Sharvin[7] have reported a magnetic-field-dependent skin depth for tin which also showed an initial increase with field followed by a rapid decrease to a minimum at approximately 40 G. This suggests that variations of δ of the type shown in Figs. 2–5 may be a general property of conduction in metals at high frequencies.

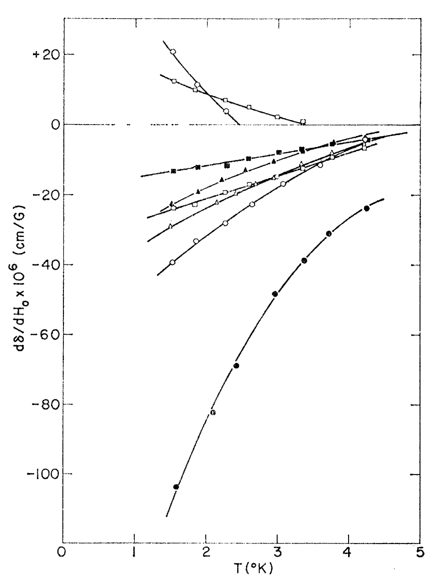


FIG. 7. The temperature dependence of the magnitude of the low field extrema in the derivative of rf skin depth with magnetic field for A, B, and C crystals. ○—A_{13}, H_0 parallel to B. ●—A_{13}, H_0 parallel to C. □—B_{06}, H_0 parallel to A. ■—B_{06}, H_0 parallel to C. △—C_{05}, H_0 parallel to A. ▲—C_{05}, H_0 parallel to B.

[7] V. F. Gantmacher and Yu. V. Sharvin, Zh. Eksperim. i Teor. Fiz. **39**, 512 (1960) [English transl.: Soviet Phys.—JETP **12**, 358 (1961)].

Variation of the Skin Depth for Small Fields

The dependence of δ upon H_0 for small fields can best be displayed through the derivative, $d\delta/dH_0$. The two types of low-field behavior which we have observed in gallium are shown in Fig. 6. That for A_{13}, H_0 parallel to B, is typical for the two orientations which displayed an initial increase of skin depth with field, namely H_0 normal to A and parallel to B, and the reverse. The behavior of A_{13}, H_0 parallel to C, is typical for the remaining orientations, which had no initial increase of δ with H_0. The positive maxima were found to be far more sensitive to the effects of accidental strain than were the negative peaks, particularly at 4.2°K. In some specimens, for instance, the skin-depth derivative was negative but approached the field axis with zero slope. Annealing at room temperature for several days restored the positive peak in those cases. In "unstrained" specimens the amplitude of the positive peak was very dependent on the temperature, but the field at which it occurred was not. Between 1.5 and 4.2°K, the amplitude might change by as much as a factor of 30, for example, while the position changed by at most 10%. By way of contrast both the amplitude and position of the negative maxima were temperature-dependent as illustrated in Figs. 7 and 8.

The correlation between negative peak amplitude and peak field is demonstrated in Fig. 8. Peak amplitude varied nearly as the inverse of the square of peak position for all but the C crystals, in which it varied more nearly as the inverse of the peak position. This observation is the only feature of our results which is correlated in a simple way with the bulk magneto-

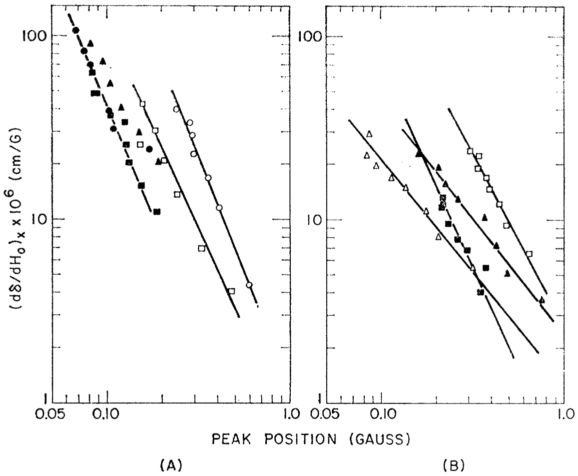


FIG. 8. The relation between position and amplitude for the low-field negative peak in the derivative of rf skin depth. (A) Three A-axis crystals. Open symbols, H_0 parallel to B, closed symbols, H_0 parallel to C. ○, ●—A_{13}. □, ■—A_{14}. ▲—A_{15}. (B) B and C axis crystals. □—B_{06}, H_0 parallel to A. ■—B_{06}, H_0 parallel to C. △—C_{05}, H_0 parallel to A. ▲—C_{05}, H_0 parallel to B.

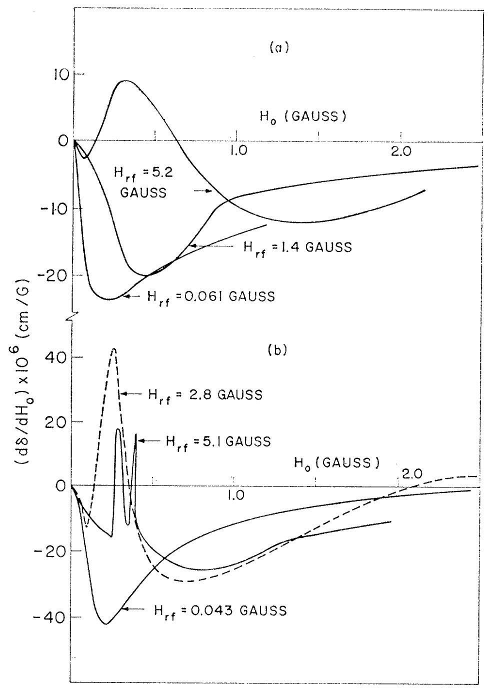


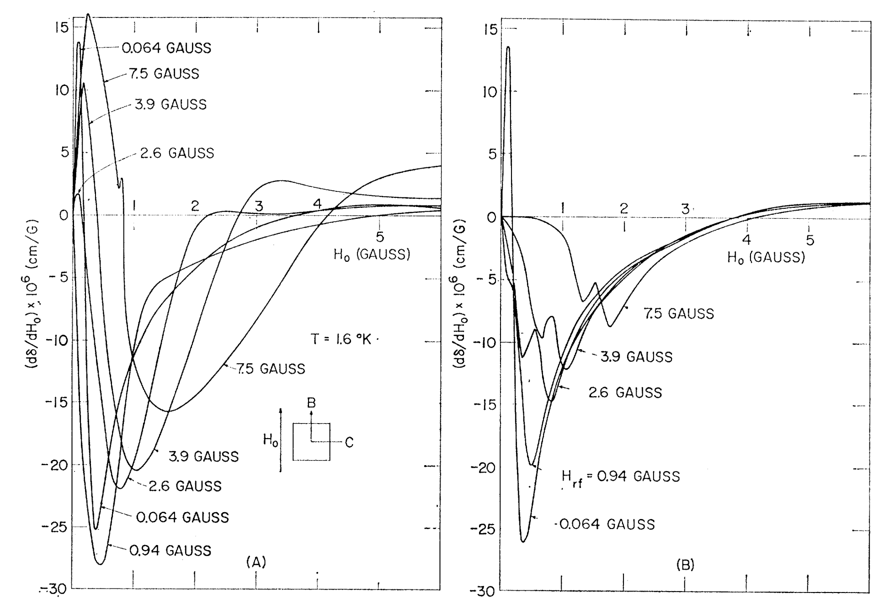
FIG. 9. Typical examples of the effect of large rf field amplitudes on the $(d\delta/dH_0)$ versus H_0 curves: (a) Specimen A_{15} with H_0 parallel to the C axis, at 4.2°K; (b) Specimen B_{05} with H_0 parallel to the A axis, at 1.6°K.

resistivity of gallium, Table III. This correlation may be spurious; in any case, we do not understand its significance. Comparison of the low-field data with Table III also shows that the occurrence of a positive peak is restricted to those orientations in which the magnetoresistance is small but still quadratic in the field. The gap between "small" and "large" dependences on field, is not so great however, as to allow a qualitative differentiation.

Rf Magnetic Field Amplitude Dependence

The form of the magnetic field dependence of δ is sensitive to the amplitude of the rf magnetic field, as is illustrated in Figs. 9 and 10(a). The data presented above were all obtained using an rf magnetic field amplitude of $\frac{1}{10}$ G for which the rf amplitude effect is negligible. Increasing the rf magnetic field at first reduces the amplitude of the derivative peaks and shifts their positions to higher magnetic fields—this behavior has been observed for all crystal orientations.

FIG. 10. (A) The variation of the derivative of the rf skin depth with dc magnetic field for specimen A_{13}, illustrating the effect of increasing rf magnetic field amplitude. (B) Calculated from Eq. (13) with $H'_{rf}=0.2H_{rf}$.



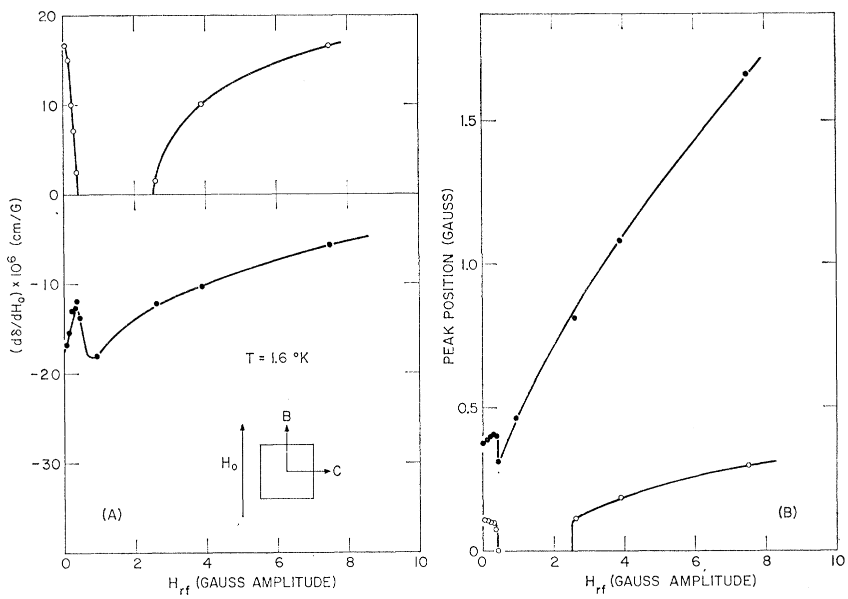


FIG. 11. The dependence of the extrema in the low-field derivative of the skin depth upon rf magnetic field for specimen A_{13} at 1.6°K. ○—positive peak. ●—negative peak. (A) The variation of the extrema amplitudes. (B) The variation of the magnetic field at which the extrema occur.

The effect has been studied for large rf fields in the A crystals for which a further increase of rf amplitude causes peaks to disappear and reappear as shown in Fig. 11. Similar, but less pronounced, distortions were observed at 4.2°K.

The question immediately arises, to what extent does this dependence on the rf field strength reflect simply the time average of δ over one period of the rf field? The average is defined by

$$\delta \propto \frac{1}{\tau}\int_0^\tau \delta(H_0+H'_{\text{rf}}\sin\omega t)dt, \tag{10}$$

where $\delta(x)$ is the limiting form of the dependence of δ on H_0 at low rf fields, and where H'_{rf} is an effective rf field amplitude. The physical idea underlying (10) is that at very low fields and low frequencies the skin depth will appear to depend only on the instantaneous value of the *total* field averaged over the surface layer. This is based on the observation that the radius of curvature of the carrier orbits at these low fields is so large that the field distribution in the interior of the metal can be ignored as a first approximation, and on the fact that the radio-frequency field varies very little during the passage of a carrier through the skin layer. The effective rf field amplitude H'_{rf} appearing in (10) is taken to mean a suitable average of the existing distribution over the skin depth, but no attempt has been made to evaluate it. Instead, it is treated as an empirical parameter. What gives credence to this simple model is the observation that for all specimens the negative extrema in $d\delta/dH_0$ show an orderly progression to higher fields as H_{rf} increases, provided H_{rf} is greater than about twice the value of H_0 at which the peak is observed in the limit $H_{\text{rf}}\to 0$. In fact, for a given orientation the position of the peak is very close to linear in H_{rf}, which indicates that the effective field H'_{rf} is a constant factor times the rf field at the surface, for a given orientation and temperature. Equation (10) has been used to predict the variation of $d\delta/dH_0$ for a number of specimen orientations. The general result is that the model reproduces many of the features of the observations, but certainly not all. As an example, we show in Fig. 10(b) the calculated curves which should be compared with the observations given in Fig. 10(a). The calculations were based on the data at $H_{\text{rf}}=0.06$ G, with the assumption $H'_{\text{rf}}/H_{\text{rf}}=0.2$. There was no adjustment of the amplitude of the results.

A significant area of disagreement is the failure of the

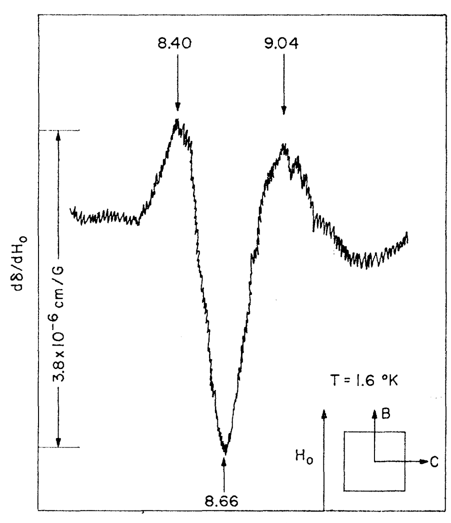


FIG. 12. The Gantmacher resonance observed at 8.7 G in specimen A_{15}, H_0 parallel to the B axis.

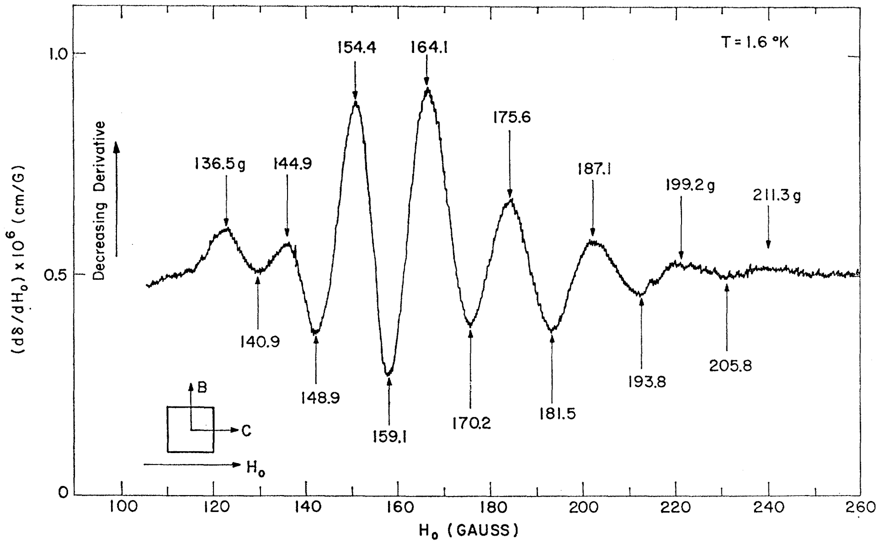


FIG. 13. A Gantmacher resonance complex observed in specimen A_{14} at 1.6°K, H_0 parallel to the C axis.

model to reproduce the behavior at low dc fields and high rf fields. Equation (10) fails to give a low-field positive peak at high rf fields, and it introduces a spurious wiggle in the negative peak. We recall here the earlier remarks on the orientation, temperature, and strain dependences of the positive peaks and suggest that they arise either from different parts of the Fermi surface from those responsible for the negative peaks, or from a different mechanism.

Gantmacher Resonances

Gantmacher resonances were observed in all but the B crystals.[8] The dimensions of the corresponding Fermi surface extrema have been listed in Table IV. These dimensions are consistent with the Fermi surface model for gallium proposed by Wood.[9] Wood's model predicts a profusion of extrema ranging from 0.01×10^8 cm^{-1} to 0.7×10^8 cm^{-1} for all directions of the magnetic field, so approximate agreement with the data is not very significant. A serious investigation of his model would require the complete mapping of the shapes of the various extremal orbits.[10]

The resonance line shapes observed varied from the simple form predicted by Kaner,[3] Fig. 12, to large complexes such as that shown in Fig. 13. Whether such complexes are simply groups of isolated resonances or whether they represent a basically more complicated type of resonance cannot be decided at this stage. However, the first possibility seems less likely, since the relative amplitudes of the peaks were the same at 4.2 and 1.6°K, whereas, for recognizably distinct resonances this was not the case (for example, see Fig. 16). It should be noted that the amplitudes of these resonances are so small as to preclude their direct observation on the scale shown in Figs. 2–5. Only in the case of the 20.8- and 25.6-G resonances observed in A_{14}, H_0 parallel to B, were the variations in δ large enough to be directly observable. Moreover, the positions of the resonances are very insensitive to rf field amplitude, although the strength of the resonances decreases as the rf field is increased. The data reported in this section were all obtained using $H_{\rm rf}\approx0.06$-G amplitude.

TABLE IV. Fermi surface dimensions as deduced from Gantmacher resonances. $\Delta k=1.52\times10^7(H_0d)$cm^{-1}.

Direction of H_0	Direction along which Δk is measured	Δk (10^8 cm^{-1})	Remarks
A	B	0.32 to 0.50	A complex. Prominent peaks at $\Delta k=0.42\times10^8$ cm^{-1}.
B	A	0.34	
		0.38	
		0.44	
B	C	0.053	
		0.062	
		0.067	
C	B	0.032	
		0.072	
		0.09 to 0.15	A complex. Prominent peaks at $\Delta k=0.13\times10^8$ cm^{-1}.
		0.19	
		0.27	
		0.37 to 0.53	A complex. Prominent peaks at $\Delta k=0.4\times10^8$ cm^{-1}.

[8] This does not imply that there are no Gantmacher resonances observable in B crystals, but only that they were not as strong as those which were observed in A and C crystals.

[9] J. H. Wood, MIT Solid State and Molecular Theory Group, Quarterly Progress Report No. 55, 1965 (unpublished).

[10] While this work was in progress D. M. Sparlin and D. S. Schreiber [*Low Temperature Physics, Proceedings of LT9,* edited by J. G. Daunt *et al.* (Plenum Press, New York, 1965)] reported measurements of singularities in the resistive part of the surface impedance of gallium which they interpret as Gantmacher resonances. The resonances reported in this paper occur at lower fields than do theirs, after correcting for differences in specimen thickness.

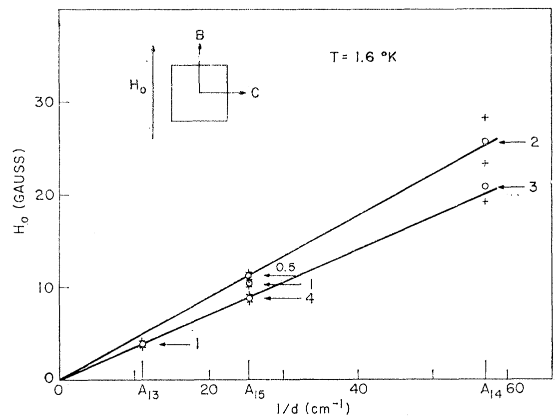


FIG. 14. Gantmacher resonances observed in A crystals at 1.6°K with the magnetic field parallel to the B axis. ○—negative peaks. +—positive peaks. The amplitudes of the strongest peaks are shown in units of 10^{-6} cm/G.

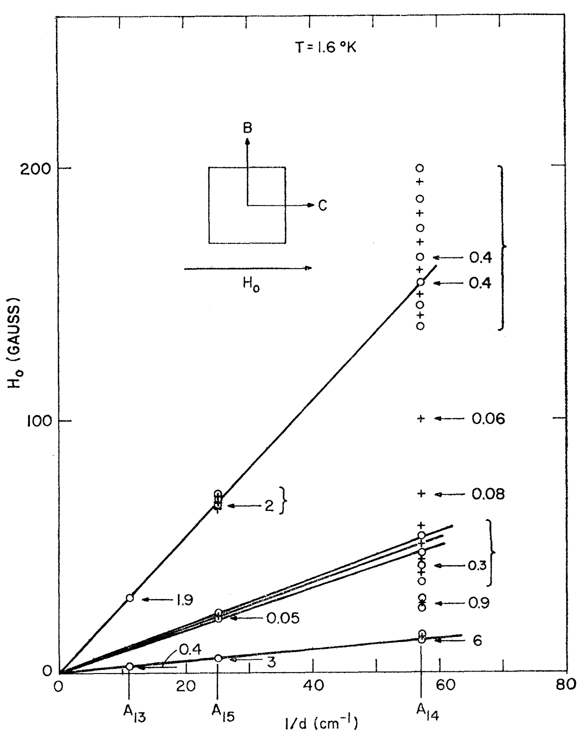


FIG. 15. Gantmacher resonances observed in A crystals at 1.6°K with the magnetic field parallel to the C axis. ○—negative peaks. +—positive peaks. The amplitudes of the strongest peaks are shown in units of 10^{-6} cm/G. The brackets indicate groups of peaks which appear to form a single complex.

The thickness dependence of Gantmacher resonance fields was measured for the A crystals, as shown in Figs. 14 and 15. The fields at which Gantmacher peaks occurred were found to be proportional to $1/d$, as predicted by Eq. (6).

The temperature dependence of the amplitudes of the 20.8- and 25.6-G Gantmacher peaks, $A_{14}H_0$ parallel to B, was measured and the results are shown in Fig. 16. Kaner[3] does not explicitly calculate the temperature dependence of the Gantmacher resonance amplitude for free electrons. However, he does show that the conductivity depends upon electron mean free path through the factor

$$\sigma \propto \{1-\exp[-2\pi(i\omega/\Omega+R_0/l)]\}^{-1}. \tag{11}$$

(Ω is the cyclotron frequency.) The strength of the

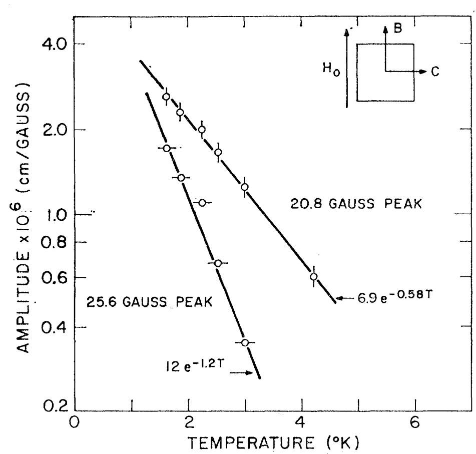


FIG. 16. Temperature dependence of the amplitudes of the 20.8- and 25.6-G Gantmacher resonance peaks observed in A_{14}, H_0 parallel to the B axis. ⌽—20.8 G peak. -○-—25.6 G peak.

Gantmacher peaks depends in a complicated way upon conductivity, but the exponential term $e^{-2\pi R_0/l}$ must appear as one of the factors which determines the amplitude of the resonance. It is for this reason that the data shown in Fig. 16 have been fitted to exponential functions. The data are neither precise enough nor do they extend over a sufficiently large range of temperatures to attach much significance to the temperature dependence of the exponent. It is unlikely that Gantmacher resonances will be a useful tool for studying the mean free path of the carriers in a metal because of the complicated dependence of their amplitude on l.