Transverse Breakdown in a Strong Hall Electric Field

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Impact ionization of electron-hole pairs by hot electrons, which exist when a strong Hall electric field is present, is proposed. Theoretical arguments indicate that the saturated value of the Hall electric field should coincide with the breakdown electric field in a sample whose Hall electric field is shorted. Experiments substantiate this. The existence of electrons and holes in the region of applied field where the Hall electric field saturated was demonstrated by the decrease of resistance between the Hall contacts, as well as by the observation of a strong Suhl effect in a wide sample. The observed decrease in the Hall electric field just above the onset of ionization is related to the contribution of the holes to the Hall field. Thus, the drift-velocity saturation in strong crossed electric and magnetic fields is explained in consistent fashion as due to impact ionization occurring primarily in the $\mathbf{E}\times\mathbf{B}$ direction.

I. INTRODUCTION

AFTER the occurrence of bulk impact ionization from the filled band to the conduction band was observed in InSb,[1,2] considerable interest was focused on plasmas in bulk semiconductors. Further observations by Steele and Glicksman[3] showed the saturation of the electron drift velocity, measured as the product of the Hall coefficient and current density. Recently, the behavior of the carriers in the same material was investigated in more detail to examine the presence of the drift-velocity saturation,[4,5] and to present an explanation for the observed transport properties.

However, the suggestions for the cause of the saturation have not been amplified nor experimentally tested. We propose that the saturation is a result of what we call "transverse breakdown," caused by a nonequilibrium electron distribution containing high-energy electrons, present because of the strong Hall electric field. To our knowledge, this is the first time such a suggestion has been made. Because of the difficulties of presenting a microscopic treatment for the transport properties under strong electric and magnetic fields, we use a phenomenological macroscopic treatment to show that high-energy electrons should be present under these conditions. These electrons have sufficient energy in the "transverse" or Hall-field direction to produce electron-hole pairs at applied electric fields substantially lower than those necessary when such a Hall field is not present.

We present the results of several experiments which are in agreement with this suggestion. In two of these we show that indeed the saturation of the drift velocity occurs only when additional carriers are produced. In another experiment, we use a geometry in which the Hall electric field is shorted out; we see that the applied electric field necessary for impact ionization is much larger than when the Hall field is present, and is, in fact, about the same as the total electric field when a Hall field is present.

Since our experiments show the presence of an electron-hole plasma in that region, it should be pointed out that the current oscillations in electric fields greater than 100 V/cm, reported by H. Ikoma,[6] can come from the plasma instability previously observed in a transverse magnetic field.[7]

II. THEORETICAL DISCUSSION

In the steady state, the drift velocity $\mathbf{v}$ of a single electron is given by the solution of the motional equation (we use mks units throughout):

$$\mathbf{v}=(-\omega_c\tau/B)(\mathbf{E}+\mathbf{v}\times\mathbf{B}),$$

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} -\mu_\perp & \mu_\times & 0 \\ -\mu_\times & -\mu_\perp & 0 \\ 0 & 0 & -\mu_{\parallel} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}, \qquad (1)$$

where

$$\mu_\times=(\omega_c\tau)^2/[(1+\omega_c^2\tau^2)B],$$
$$\mu_\perp=\omega_c\tau/[(1+\omega_c^2\tau^2)B], \qquad (2)$$
$$\mu_{\parallel}=\omega_c\tau/B,$$

ω_c is the electron cyclotron frequency eB/m^*, and τ is the electron scattering time. The z axis is chosen parallel to the magnetic field direction, the x axis parallel to the applied electric field and the y axis will be antiparallel to the Hall electric field, E_H. If we assume that τ is a decreasing function of the electron energy[8] for the region where $\omega_c\tau>1$, then $\mu_\perp$ will increase with electron energy, while $\mu_\times$ will have only a weak dependence on the energy, but will decrease with increasing energy.

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[1] M. Glicksman and M. C. Steele, Phys. Rev. **110**, 1204 (1958).
[2] A. C. Prior, J. Electron. Control **4**, 165 (1958).
[3] M. C. Steele and M. Glicksman, J. Phys. Chem. Solids **8**, 242 (1959).
[4] M. Glicksman and W. A. Hicinbothem, Jr., Phys. Rev. **129**, 1572 (1963).
[5] M. Glicksman and W. A. Hicinbothem, Jr., *Symposium on Plasma Effects in Solids* (Dunod Cie., Paris, 1965), p. 137.
[6] H. Ikoma, J. Phys. Soc. Japan **19**, 419 (1964).
[7] M. Toda, Japan. J. Appl. Phys. **2**, 467 (1963).
[8] This is expected for both acoustic- and optical-mode phonon scattering, which are dominant effects in the experiments performed. As noted by Y. Kanai, J. Phys. Soc. Japan **15**, 830 (1960), and others (Ref. 4), the observed mobility decreases as the electrons are heated, in agreement with this assumption.

These equations hold as long as the assumption that momentum is "relaxed" in every collision holds, so that in the breakdown region we must be careful in applying these equations. By drift velocity we mean the average velocity added in the presence of the applied fields. The time-dependent contributions to the motion will cause the instantaneous velocity of the electrons to oscillate about these average values at the cyclotron frequency. Neglect of this motion may cause difficulties for $\omega_c\tau$ in the vicinity of 1, when the instantaneous velocity may be much larger than the average value. In this case we will overestimate the electric field necessary for breakdown, since we will be using an average electron energy much less than the instantaneous value. Thus, the correction of this error would just reinforce our qualitative demonstration of the lowering of the required applied electric field necessary for breakdown.

A proper theoretical treatment would involve a calculation of the distribution function of the electrons in the presence of the strong electric and magnetic fields, and from this distribution combined with the knowledge of the scattering, a calculation of the conductivity and galvanomagnetic coefficients. A treatment of this kind, valid at fields where impact ionization occurs, is a subject for further research. An approach such as that used by Baraff[9] in the absence of magnetic fields may prove fruitful.

Until such a treatment is made, a proper consideration of the magnetoresistance and its effect on the breakdown is not possible. Thus, although we will see that there is a large increase in the total breakdown field due to an applied magnetic field (see Fig. 6 below) we will assume this as given and not try to explain it theoretically, since we do not yet have an adequate distribution-function treatment available. A treatment using the approximations of equal electron and hole mobilities and densities has recently been formulated[10] but not yet applied to breakdown nor to conditions where those severe restrictions may be relaxed.

A. Boundary Condition: No Hall Current

In the absence of such a theory, we can discuss the situation in physical terms. Equation (1) describes the behavior of the individual electrons in the applied fields. The first case we consider is that for the boundary condition of no current in the y direction. Then, if $\langle\ \rangle$ denotes an appropriate average over the distribution function (and thus over all the electrons)

$$\langle v_y\rangle = 0 = -\langle\mu_\times\rangle E_x - \langle\mu_\perp\rangle E_y\,, \tag{3}$$

$$E_y = -(\langle\mu_\times\rangle/\langle\mu_\perp\rangle)E_x = -\phi_{\rm H}E_x\,, \tag{4}$$

where $\phi_{\rm H}\equiv\langle\mu_\times\rangle/\langle\mu_\perp\rangle$ is the Hall angle. Then

$$\begin{aligned} v_y &= -\mu_\times E_x + \phi_{\rm H}E_x\mu_\perp \\ &= E_x\{-\mu_\times+\phi_{\rm H}\mu_\perp\}\,, \end{aligned} \tag{5}$$

and will be positive or negative, depending on whether or not the ratio of the mobilities is larger or smaller than the Hall angle. Substituting from Eq. (2),

$$v_y = \frac{\omega_c\tau E_x}{B[1+(\omega_c\tau)^2]}\{\phi_{\rm H}-\omega_c\tau\}\,. \tag{6}$$

We have a spread in the transverse distribution of velocities, caused by the dependence of $\omega_c\tau$ on the electron energy. The source of the energy given to these electrons in the transverse direction is, of course, the applied electric field.

It is difficult to estimate the numbers of very energetic electrons in the y direction and hence what electric field would be required to give enough energy in that direction to cause breakdown. The dependence of τ on energy has been discussed in detail by Stratton.[11] He notes that the scattering time is relatively independent of energy for energies less than the optical-phonon energy[12] ($\theta=260°$K; energy thus of 0.022 eV) but becomes much smaller for energies greater than this value. This has a striking effect on the mobility when the lattice temperature is below the optical-phonon temperature, as seen in the strong field dependence observed for $T=77°$K in n-InSb and explained[4] fairly well by Stratton's theory.

The expression for v_y [Eq. (6)] has the following behavior: For very large $\omega_c\tau$, v_y is negative, approaching the value $-E_x/B$ as $\omega_c\tau\to\infty$. It increases monotonically from this value to 0 at $\omega_c\tau=\phi_{\rm H}$ and reaches a maximum positive value of

$$v_y' = \frac{\phi_{\rm H}E_x}{2B}\,\frac{[(1+\phi_{\rm H}^2)^{1/2}-1]\{\phi_{\rm H}-[(1+\phi_{\rm H}^2)^{1/2}-1]/\phi_{\rm H}\}}{1+\phi_{\rm H}^2-(1+\phi_{\rm H}^2)^{1/2}} \tag{7}$$

for

$$\omega_c\tau' = [(1+\phi_{\rm H}^2)^{1/2}-1]/\phi_{\rm H}\,, \tag{8}$$

as $\omega_c\tau$ is decreased from ∞ towards 0. As $\phi_{\rm H}$ becomes very large ($\phi_{\rm H}\gg1$), the maximum value becomes

$$\begin{aligned} v_y' &\approx (E_x/B)\tfrac{1}{2}\phi_{\rm H} \\ &\approx -\tfrac{1}{2}(E_y/B)\quad\text{at}\quad \omega_c\tau'\approx1\,. \end{aligned} \tag{9}$$

Thus, for large Hall angles, those electrons with large $\omega_c\tau$ (larger than the mean) will have velocities in the $-y$ direction up to E_x/B, while the electrons of small $\omega_c\tau$ can have much higher velocities in the $+y$ direction, up to $\frac{1}{2}\phi_{\rm H}(E_x/B)$. Since $\omega_c\tau$ is a function of the final energy of the electrons, this results in the high-

[9] G. Baraff, Phys. Rev. **133**, A26 (1964).
[10] J. Yamashita, Progr. Theoret. Phys. (Kyoto) **33**, 343 (1965).
[11] R. Stratton, Proc. Roy. Soc. (London) **A246**, 406 (1958).
[12] G. Picus, E. Burstein, B. W. Henvis, and M. Hass, J. Phys. Chem. Solids **8**, 282 (1959).

energy electrons (small $\omega_c\tau$) having the highest velocities. This yields a consistent picture of the acceleration process resulting from the application of the crossed E and B fields.

However, in impact ionization, the actual ionization energies are not achieved in one scattering time, so that for the electrons of ionization energy ϵ_i, $\epsilon_i \gg \frac{1}{2}m(\mu E)^2$. Rather, for these electrons, the energy loss per collision has been less than the energy gain, so that over a number of collisions they are able to attain the necessary value of ϵ_i. In order to evaluate the possibility of transverse breakdown, we compare the energy gain of the electrons in the y direction with the case for no applied magnetic field.

When no magnetic field is applied, the same motional equation treatment gives

$$v_x = (\omega_c\tau/B)E_{x,0}$$

where $E_{x,0}$ is the electric field with no applied magnetic field. The maximum value we estimated in the case of a magnetic field was

$$v_y' = \tfrac{1}{2}\phi_{\mathrm{H}}(E_{x,\mathrm{tr}}/B)\,,$$

$E_{x,\mathrm{tr}}$ is the electric field when transverse magnetic fields are applied. Now,

$$\phi_{\mathrm{H}} = \frac{\langle \omega_c^2\tau^2/(1+\omega_c^2\tau^2)\rangle}{\langle \omega_c\tau/(1+\omega_c^2\tau^2)\rangle} \equiv \mu_{\mathrm{H}}B\,,$$

where μ_{H} is the averaged Hall mobility.

Thus, the ratio of electron energy gain in the two cases is

$$\frac{\text{Energy gain in crossed fields}}{\text{Energy gain in electric field alone}} \simeq \frac{v_y' E_{y,\mathrm{tr}}}{v_x E_{x,0}} \simeq \frac{\mu_{\mathrm{H}}^2 B E_{x,\mathrm{tr}}^2}{2(\omega_c\tau/B)E_{x,0}^2}\,. \quad (10)$$

To be meaningful, this ratio should be compared for electrons of the same energy. For example (as we discuss below, in Sec. II C), an important energy for impact ionization will be the optical-phonon energy $k\theta$. At and above this energy τ decreases strongly, so that $\omega_c\tau/B$ is much less than the averaged mobility μ_{H}. Thus, for strong magnetic fields ($\mu_{\mathrm{H}}B>1$) and for equal values of applied electric field, the ratio of the energy gains will be larger than 1. We thus expect that when $\mu_{\mathrm{H}}B\gg 1$, impact ionization due to the electrons moving in the y direction should occur at *comparable* or *lower* electric fields than in the absence of magnetic field. However, the actual value of the fields necessary will depend on the high-energy electron-scattering times. It should also be noted that Eq. (10) presents an upper estimate for this ratio, since the numerator contains v_y', which is achieved by some electrons only if $\omega_c\tau'$ is a possible value for the electron distribution. Since τ has a minimum value at $\epsilon \simeq k\theta$, when B becomes very large, $\omega_c\tau$ has a minimum value which may be larger than $\omega_c\tau'$. In this case, v_y' should be replaced by the appropriate (smaller) value, and we correspondingly expect that at higher magnetic fields, the breakdown field ($\mu_{\mathrm{H}}BE_{x,\mathrm{tr}}$) will increase with magnetic field. This is observed in our experiments. The increase in breakdown field with magnetic field may also be understood in terms of the effect of the magnetic field on the energy gain, i.e., the magnetoresistance. This cannot be evaluated without knowledge of the distribution function.

B. Boundary Condition: No Hall Field

In this case $E_y=0$, and we can write

$$v_x = -\mu_\perp E_x = -\big(\omega_c\tau/(1+\omega_c^2\tau^2)\big)(E_x/B)\,,$$
$$v_y = -\mu_\times E_x = -\big(\omega_c^2\tau^2/(1+\omega_c^2\tau^2)\big)(E_x/B)\,. \quad (11)$$

The situation is actually the same as for the Hall-field case, if we rotate our coordinate system to make the previous x axis coincide with the new direction of current flow. This direction of current flow approaches the new y axis as $\mu_{\mathrm{H}}B$ becomes very much larger than 1. Then electrons traveling in a direction making an angle $90°-\tan^{-1}\phi_{\mathrm{H}}$ with the x axis correspond to our previous y-direction electrons.

We then expect that, for this geometry, the threshold field for breakdown E_x should correspond to the field $E_{y,\mathrm{tr}}=\phi_{\mathrm{H}}E_{x,\mathrm{tr}}$ required when a Hall field is present, since the distributions, forces, etc., are the same. This was observed and is described below.

C. Threshold Electric Field for Breakdown

In the above sections we discussed the energy gain between collisions, noted that it was large and, in fact, expected to be larger than in the absence of a magnetic field, for comparable applied electric fields, and used the results of these discussions to predict the occurrence of "transverse" breakdown. We can carry this further, by estimating the actual required electric field for breakdown from similar arguments and compare this with experiments.

We assume that for the high-energy electrons the scattering is primarily by optical phonons, so that the optical-emission scattering time τ_{op} is set equal to τ, and we will consider the case of no Hall current. Those electrons which will gain enough energy to impact-ionize must gain more energy per collision from the electric field than they lose in a collision, e.g., by emitting an optical phonon of frequency ω_{op}. Then

$$-e\mathbf{v}\cdot\mathbf{E} \geq (\hbar\omega_{\mathrm{op}}/\tau_{\mathrm{op}})\,. \quad (12)$$

Since we consider the case $\phi_{\mathrm{H}}\gg 1$, we can neglect v_xE_x with respect to v_yE_y. Inclusion of this term would lower the threshold field. Substituting for v_y from Eq.

(6), we have

$$e\phi_{\mathrm{H}}\frac{E_x^2}{B}\frac{\omega_c\tau(\phi_{\mathrm{H}}-\omega_c\tau)}{1+\omega_c^2\tau^2}\geq\frac{\hbar\omega_{\mathrm{op}}\omega_c}{\omega_c\tau}. \quad (13)$$

We next solve Eq. (13) for the value of $\omega_c\tau$ which satisfies the equality ($\hbar\omega_{\mathrm{op}}$ in eV):

$$[(\phi_{\mathrm{H}}-\omega_c\tau)\phi_{\mathrm{H}}E_x^2-\hbar\omega_{\mathrm{op}}\omega_cB]\omega_c^2\tau^2-\hbar\omega_{\mathrm{op}}\omega_cB=0. \quad (14)$$

Since we are concerned with the case where the high-energy electrons will be important, we neglect $\omega_c\tau$ with respect to ϕ_{H}. Then

$$\omega_c\tau\simeq[\hbar\omega_{\mathrm{op}}\omega_cB/(\phi_{\mathrm{H}}^2E_x^2-\hbar\omega_{\mathrm{op}}\omega_cB)]^{1/2}. \quad (15)$$

This equation yields a real value for $\omega_c\tau$ only when E_x exceeds a value which we will call the threshold field. It represents the minimum electric field for which the energy gain could satisfy Eq. (13):

$$E_{x,\mathrm{th}}\simeq(\hbar\omega_{\mathrm{op}}\omega_cB)^{1/2}/\phi_{\mathrm{H}}. \quad (16)$$

For indium antimonide, $\hbar\omega_{\mathrm{op}}=0.022$ eV, and

$$\omega_c\simeq2\pi\times2.8\times10^{10}B/(m^*/m_0).$$

An appropriate value for m^*/m_0 is probably in the range[13] 0.018 to 0.030 or larger (corresponding to electron energies of 0.03 to 0.10 eV). Then

$$E_{x,\mathrm{th}}=(3.6-4.6)(B^{1/2}/\phi_{\mathrm{H}})\times10^5\ \mathrm{V/m}. \quad (17)$$

For the higher magnetic fields, where the approximations hold best, Glicksman and Hicinbothem[5] observe the first increase in density at $E_x=1.0\times10^4$ V/m for $B=0.84$ W/m^2, $\phi_{\mathrm{H}}=\mu_{\mathrm{H}}B=22.5$. The calculated value of $E_{x,\mathrm{th}}$ is $(1.45\text{–}1.87)\times10^4$ V/m. If the calculations are modified to include the presence of other scattering mechanisms [i.e., τ_{op} is then larger than τ, so that the right-hand side of Eq. (13) is decreased] the theory will come still closer to the observed threshold field. This good agreement provides further support for the mechanism of transverse breakdown. Of course, when breakdown starts, the Hall field will increase very slowly with current, as the current is increased, because of the usual strong dependence of the generation rate on electric field.[14] This leads to a saturation in the observed Hall field as a function of density.

D. Effect of Holes and Saturation of the Hall Field

The relation between current and electric field is represented by a tensor conductivity, $\mathbf{j}=\boldsymbol{\sigma}\cdot\mathbf{E}$ or

$$\begin{pmatrix}j_x\\j_y\\j_z\end{pmatrix}=\begin{pmatrix}\sigma_\perp & -\sigma_\times & 0\\ \sigma_\times & \sigma_\perp & 0\\ 0 & 0 & \sigma_{\parallel}\end{pmatrix}\begin{pmatrix}E_x\\E_y\\E_z\end{pmatrix}. \quad (18)$$

Values for the conductivity tensor can be calculated by integrating the tensor mobility of Eq. (1) over the electron energy, after multiplying by the appropriate distribution function.

From the experimental condition $j_y=j_z=E_z\equiv0$ and $E_{\mathrm{H}}=-E_y$, we get the Hall electric field,

$$E_{\mathrm{H}}=(\sigma_\times/\sigma_\perp)E_x. \quad (19)$$

If an electron-hole plasma exists, $\sigma_\times$ and $\sigma_\perp$ should be replaced by $\sigma_\times^e-\sigma_\times^h$, $\sigma_\perp^e+\sigma_\perp^h$, respectively, and the Hall electric field will then decrease:

$$E_{\mathrm{H}}=(\sigma_\times^e-\sigma_\times^h)E_x/(\sigma_\perp^e+\sigma_\perp^h), \quad (20)$$

$\sigma_\perp^h$, $\sigma_\times^h$ are the conductivity elements due to the mobile holes. In a strong magnetic field, $\sigma_\times^h$, $\sigma_\perp^h$ are not negligible, because of the decreased values of $\sigma_\times^e$, $\sigma_\perp^e$ caused by the magnetoresistance effect. Thus, E_{H} can decrease due to the existence of holes.

In breakdown it will decrease to a value where the electron-hole generation rate is equal to the recombination rate. If the generation rate increases very steeply with increasing E_{H}, or if the decrease of E_{H} is a strong function of the hole density, E_{H} should saturate at a value slightly higher than the threshold value for pair creation.

III. EXPERIMENTAL DETAILS AND RESULTS

The samples used were n-type InSb in single-crystalline form (unoriented) with electron concentrations of about 9×10^{13} cm^{-3} at 77°K. Sample shapes and dimensions are shown in Fig. 1. Measurement of the current and voltage pulses was made at 77°K with experimental arrangements similar to those used in other experiments.[1,3,5]

In the measurement of the transverse impedance, a

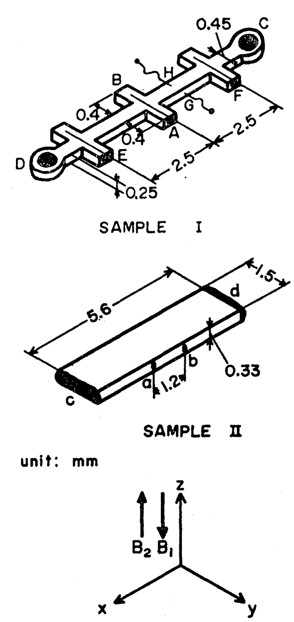

FIG. 1. Indium antimonide sample geometry and dimensions.

[13] B. Lax, J. G. Mavroides, H. J. Zeiger, and R. J. Keyes, Phys. Rev. 122, 31 (1961).
[14] M. Toda, J. Appl. Phys. 36 (1965) (to be published).

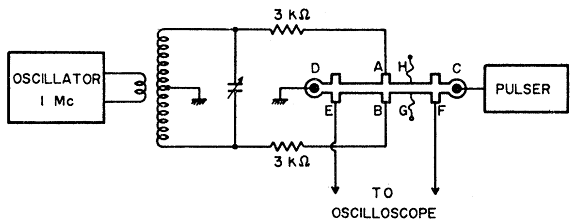

FIG. 2. Circuit used in measuring the impedance during the pulse.

variable-power oscillator, of frequency 2 Mc/sec and maximum output power of 5 W, was used. The circuit is shown in Fig. 2.

A. The Existence of an Electron-Hole Plasma

A continuous 2-Mc/sec sine wave of up to 10-V amplitude was applied to contacts A, B of sample 1, through the high resistances shown in Fig. 2. When high-current pulses were applied to the current contacts C, D, the 2-Mc/sec voltage measured at contacts A, B decreased strongly as the sample voltage, measured on contacts E, F, increased above a threshold value, in a strong magnetic field which was in the z direction in Fig. 1.

The 2-Mc/sec voltage on contacts A, B is proportional to the transverse resistance of the sample. The ratio of the resistance at high electric fields to that at very low applied fields is plotted in Fig. 3 with magnetic field as a parameter. The Hall voltage was measured at contacts G, H, which do not disturb the current-density distribution. The Hall electric fields are shown in Fig. 4. These two figures show that the threshold for the decrease in transverse resistance occurs near the onset of saturation in the Hall electric field. This decrease in resistance confirms the previously observed[5] increase in density and its interpretation as due to the presence of an electron-hole plasma. A frequency of 2 Mc/sec was used for the transverse-resistance measurement to prevent interference from the higher frequency incoherent oscillations present, and also to have a signal frequency below the plasma-generation frequency in order to detect the "dc" properties.

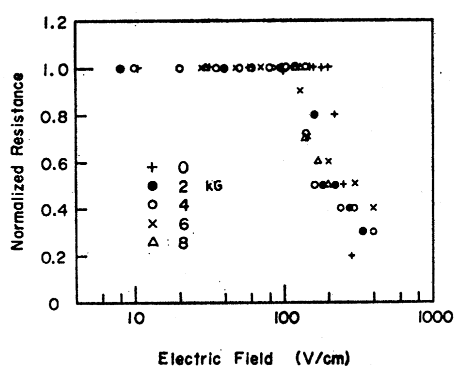

FIG. 3. The resistance of the sample (transverse to the direction of current flow) as a function of applied electric field, for transverse magnetic fields of 0, 2, 4, 6 and 8 kG.

Additional evidence for the existence of electrons and holes in the saturation region of the Hall electric field was obtained using a wider specimen (Sample 2 in Fig. 1) to observe the Suhl effect. The width of this sample (1.5 mm) was much larger than the estimated thickness of less than 0.1 mm for the region of plasma concentration due to the Suhl effect.

The voltage was measured at contacts a, b. When the magnetic field was in the B_1 direction, the plasma is expected to be concentrated on the side near the contacts a, b. No plasma is expected to exist near the current contacts c, d, because the Hall electric field is shorted at these contacts. Therefore, the voltage at a, b is expected to be much lower for the direction B_1 than for B_2 at the same currents if a plasma is present. This effect showed up strongly for a sample with etched surfaces, as can be seen in Fig. 5. When the surfaces were ground, the voltage decrease for the direction B_1 was much suppressed.

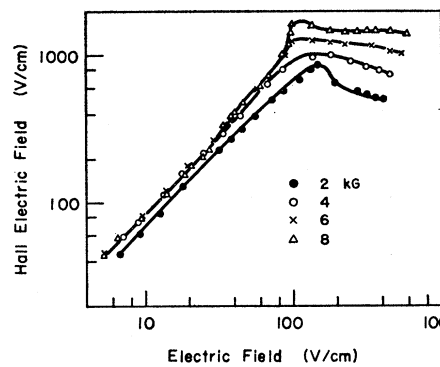

FIG. 4. The measured Hall electric field as a function of applied electric field, for transverse magnetic fields of 2, 4, 6, and 8 kG.

B. Saturated Value of the Hall Electric Field

As described in Sec. II B, the saturated value of the Hall electric field is expected to equal the electric field which sustains breakdown in a sample with infinite dimensions (no Hall field). To test this suggestion, the surfaces of the sides having the Hall contacts G and H in Fig. 1 were plated with copper, and the J-E characteristic between the contacts G and H was measured in a transverse magnetic field. For a sample of this shape, the Hall electric field is shorted by the large current contacts, so that its properties should be close to that of an infinite sample. The result is plotted in Fig. 6. For all values of magnetic field investigated (2000 to 8000 G), the sustaining electric fields agree with the saturated values of the Hall electric fields shown in Fig. 4.

However, the general behavior of the J-E curves of Fig. 6 is much different from that seen in the normal

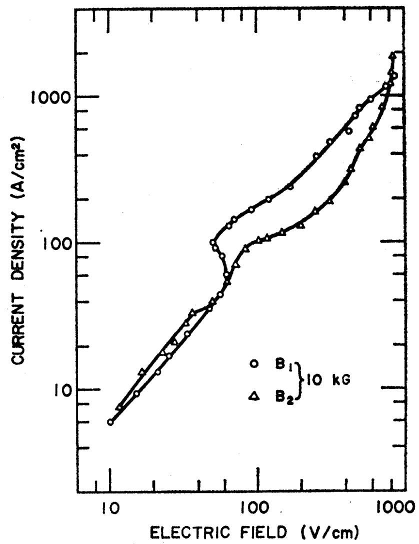

FIG. 5. The current density as a function of "applied" electric field, for sample II in a transverse magnetic field of 8 kG. The direction B_2 drives the carriers away from the side contacts used to measure the electric field, while B_1 drives them to the side where the contacts are located.

geometry. The J-E characteristics for the same sample, taken along its length using the normal method before the plating of the sides, are shown in Fig. 7. For fields up to 100 V/cm the resistance is observed to decrease with increasing electric field, i.e., the curves rise more rapidly than in linear fashion. Since this super-linear property does not appear in the curves of Fig. 6, it must be related to differences in the electron properties caused by the strong Hall field, present in Fig. 7 but not in Fig. 6.

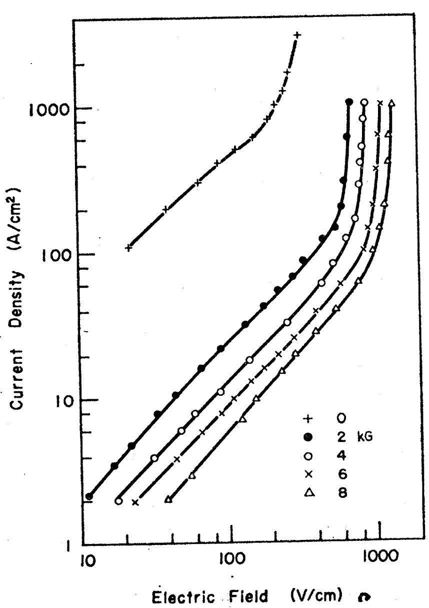

FIG. 6. The current density as a function of applied electric field in an approximately "infinite," or no-Hall-field geometry. Transverse magnetic fields of 0, 2, 4, 6 and 8 kG are applied.

IV. DISCUSSION

In Sec. II C, comparison of a calculated threshold for breakdown with the experiments of Glicksman and Hicinbothem[5] gave fairly good agreement. However, the data presented in this paper would yield values for the threshold field [calculated from Eq. (17)] somewhat higher than is observed, giving poorer agreement than from the previous measurements,[5] because the values of ϕ_H in the experiments reported here were considerably lower than those reported by Glicksman and Hicinbothem.[5] This discrepancy has not been resolved. The bridge geometry used earlier[5] should give a value for ϕ_H closer to that existing throughout the crystal, and thus should provide a better test of the theory.

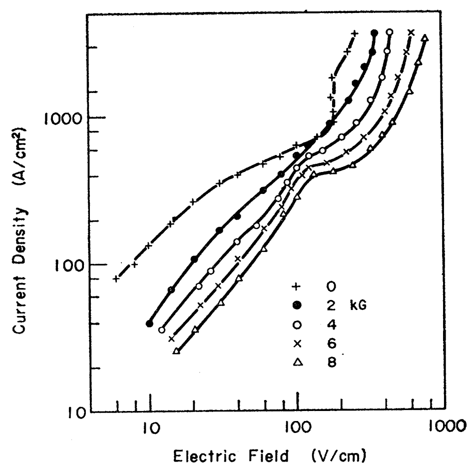

FIG. 7. The current density as a function of applied electric field in a conventional no-Hall-current geometry, for transverse magnetic fields of 0, 2, 4, 6 and 8 kG.

For most of the conditions studied, the magnetic fields are really "intermediate," i.e., ϕ_H is not so large that the longitudinal electric field may be entirely ignored. We expect that as the applied electric field is increased, first "transverse" breakdown will occur. With the Hall field saturated, as the longitudinal field increases, electrons will also gain energy in that direction, so that this field will eventually provide sufficient energy to cause "longitudinal" breakdown. This separation into "longitudinal" and "transverse" is in a sense artificial. It is expected, however, that the distribution function of the electrons will show differences in the two cases, i.e., when the current and electric fields are almost parallel ("longitudinal") and when they are almost perpendicular ("transverse") to each other. Hence, the separation in discussion is warranted.

Until the distribution function under the conditions of strong **E**, strong **B** and impact ionization is better known, discussion of the electron and hole properties in the breakdown region must be based on the experimental observations. The behavior of the Hall voltage[5] "saturation" appears to be fit, qualitatively, by a treatment including the influence of the holes produced in breakdown, as mentioned above, over the range 100–1000 V/cm for the applied longitudinal field. In such a treatment, the electron average mobility was assumed to remain unchanged as the current increased. The quality of the fit indicates that this assumption needs only slight modification.

Thus, we have suggested in this paper that in strong electric and magnetic fields, carriers may have sufficient energy in the $\mathbf{E}\times\mathbf{B}$ direction to cause impact ionization—what we call "transverse breakdown," leading to a "saturation" of the Hall voltage. This suggestion has been reinforced by a simple classical phenomenological calculation showing that, for a crystal in which the electron scattering time decreases with energy, electrons of such large energies can occur at applied electric fields lower than in the absence of a magnetic field. Experiments have been performed which show that breakdown is initiated at the point where the Hall voltage saturation occurs and that this Hall field is equal to the required breakdown field for the same magnetic fields but in a geometry which shorts out the Hall field. These results are in agreement with the predictions of the theory.

A better understanding of the impact ionization process requires calculations of the distribution function and experimental tests of its form. Without such a calculation, we cannot treat the magnetoresistance properly, and thus cannot calculate the change in the total breakdown field due to the magnetic field. It is hoped that the considerations presented here will stimulate further more rigorous theoretical treatments of this problem.

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Electrical Conductivity in Heavily Doped *n*-Type Germanium: Temperature and Stress Dependence*

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Investigations of the piezoresistance of heavily doped *n*-type germanium were made to determine (1) the nature of the carrier scattering mechanisms in degenerate materials and (2) any modification of the conduction-band edge resulting from the large number of impurity states. Stresses large enough to alter the resistivity well beyond the range of linear piezoresistance were applied; saturation of the resistivity at the larger stresses was attained in samples with as many as 10^{19} carriers per cc. Resistivity was measured with extensive parametric variation of dopant, carrier concentration, temperature, and applied stress. The Hall coefficient was also measured as a function of stress for several concentrations. The data are interpreted qualitatively by a four-valley model with a parabolic conduction-band edge, and deviations from this model are discussed. Electron-electron interactions and the temperature dependence of screening contribute significantly to the temperature dependence of the resistivity. The variation of the screening with the relative population of the valley must be considered in interpreting the results of the piezoresistance experiments; in the case of arsenic doping, intervalley scattering is also significant. The mobility anisotropy for screened Coulomb scattering in one valley, as determined from resistivity measurements on antimony-doped samples with large ⟨111⟩ stress, varies with concentration from 5.5 at 1×10^{18} per cc to 3.8 at 1×10^{19} per cc. Evidence for the existence of tail states extending $\approx$0.04 eV below the conduction-band edge is presented.

I. INTRODUCTION

THE discovery of the tunnel diode by Esaki[1] in 1958 has given impetus to both theoretical and experimental investigations of the properties of heavily doped, degenerate semiconductors. Such investigations attempt to answer two questions:

(1) What is the nature of the band structure of semiconductors so heavily doped that the usual simple hydrogenic model for the impurity states no longer holds because of extensive overlap of the ground-state orbitals; and:

(2) What is the nature of the scattering mechanism dominant at such high dopings?

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[1] L. Esaki, Phys. Rev. 109, 603 (1958).