CONCLUSIONS

The results of this experiment show that the values of the electron density determined by two-wavelength interferometry agree with the values determined from the Stark broadening of H_β and from the absolute continuum intensity at 5240 Å to within the experimental accuracy of about $\pm 6\%$. For the observed electron density of about 2×10^{17} cm^{-3} at temperatures around 20 000°K, this agreement between the three methods of measurement shows the consistency of the theories used. Corrections to the continuum measurements were made for the overlapping Balmer lines using asymptotic-wing formulas. Also, the continuum formula includes the correction for the lowering of the ionization potential due to high-density plasma effects. At these temperatures it is unnecessary to include the contributions due to H^- and H_2^+ continua, so that the continuum measurement of the electron density becomes quite reliable.

These measurements indicate that the calculated half-widths of Griem, Kolb, and Shen for Stark-broadened H_β lines are actually better ($\sim 4\%$) than the theoretical estimates of accuracy ($\sim 10\%$). Also, it can be inferred from this data that at these densities the Griem asymptotic-wing formulas for the Stark-broadened H_β and H_γ lines are more accurate than the earlier Kolb, Griem, and Shen wing formulas.

Concerning the interferometric technique, this study has shown that for the case of shock tubes it is especially important to use the two-wavelength refractivity formula instead of the single-wavelength formula to eliminate the systematic error due to the high neutral-particle density in the boundary layers.

ACKNOWLEDGMENTS

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Measurement of Stark Profiles of Singly Ionized Nitrogen Lines from a T-Tube Plasma*

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A hot plasma composed of helium and ionized nitrogen was created by reflecting the shock wave produced in an electromagnetic T tube. The plasma temperature was measured by monitoring three nitrogen-ion lines whose intensities are strong functions of temperature. The profiles of various lines emitted in the plasma were obtained by scanning with a monochromator from shot to shot. The half-width of He I 3889 Å was compared with empirically corrected calculations to determine the electron density. Half-widths of various nitrogen-ion lines were measured from the impact-broadened profiles; their shifts were obtained by comparing these profiles with unshifted lines from a pulsed capillary discharge. The results indicate agreement between experiment and recent Stark-broadening calculations within 20%, except for lines originating from the 4*f* level where Debye screening effects are important. No evidence was found for plasma polarization shifts.

I. INTRODUCTION

THE Stark-broadening theory of isolated lines as first developed for neutral helium[1] has recently been extended to other elements.[2] Several experimental tests[3–9] have been made of the theoretical predictions for neutral atoms, and theory and experiment typically agreed within 20%.[10] However, the only ionic species which has been experimentally tested is helium,[3] and the good agreement found between theory and experi-

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[2] H. R. Griem, Phys. Rev. **128**, 515 (1962); U. S. Naval Research Laboratory Report No. NRL 6084, 1964 (unpublished), see also Ref. 13.

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[10] W. L. Wiese, in *Plasma Diagnostics*, edited by R. H. Huddlestone and S. L. Leonard (Academic Press Inc., New York, to be published), Chap. 6; see also Ref. 13.

ment there cannot be extrapolated to other ions since the helium-ion lines are superpositions of various overlapping lines (because of the hydrogenic degeneracy). Thus, an experimental test of the Stark-broadening parameters for nonhydrogenic ions was undertaken. (The principal additional approximation in the theory is the use of straight-line classical paths for the perturbers instead of hyperbolic orbits.) Nitrogen was chosen as an example because its unperturbed wave functions are reasonably well known and because it is experimentally easily handled. By mixing the nitrogen with helium the shock-tube reproducibility was improved, and the helium lines could be used as an electron-density probe.

II. EXPERIMENTAL APPARATUS

The use of the electromagnetic T tube for scanning line profiles from shot to shot has been described before.[3] Employed in the present experiment was a Pyrex T tube of 1.6-cm inside diameter (Fig. 1); the shock wave was allowed to run into an aluminum block with a reflector, and the plasma behind the reflected wave was then observed through several 0.5-mm diameter holes. With He:N mixtures of 9.5:1 and 1:1, and initial pressures between 1 and 2 Torr, temperatures of about 20 000°K and electron densities between 10^{17} and 6×10^{17} cm^{-3} were produced at a repetition rate of about one shot per minute.

The light from the shock tube was observed by two photomultiplier-monochromator systems whose outputs were displayed on a dual-beam oscilloscope and recorded on film for measurement. One low-resolution monochromator always monitored the same region of the spectrum which contained three nitrogen-ion lines. The signal from this monochromator thus acted as a standard reference signal, giving a check on the shot-to-shot reproducibility. Also, since the intensity of the nitrogen-ion lines is a strong function of temperature, an absolute intensity calibration of the monochromator-photomultiplier combination with the aid of the carbon arc as an intensity standard[11] gave a means of measuring the temperature.

The other monochromator was a 50-cm Ebert-Fastie instrument with a measured resolution of 0.23 Å in first order. This monochromator was used to scan the various spectral lines emitted by the plasma; a modification to the drive mechanism of the monochromator permitted scans to be made in 0.1-Å steps.

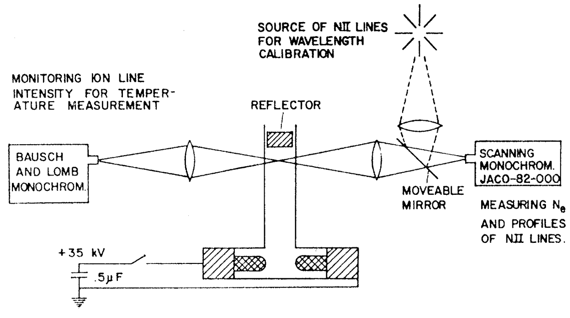

Fig. 1. Schematic diagram of experiment.

By means of a movable mirror, the scanning monochromator could also view the light emitted by a pulsed capillary discharge, which was operated with nitrogen at a pressure of 50 mTorr. The nitrogen-ion lines emitted by the discharge were shown to be unshifted with respect to adjacent iron lines. Therefore the spectral lines emitted by the source could serve as undisplaced reference lines.

III. TEMPERATURE AND ELECTRON DENSITY MEASUREMENTS

The dependence of the intensity of the nitrogen-ion lines on temperature was calculated from

$$I(\mathrm{N}^+)=\frac{4\pi^2}{\lambda^3}\frac{\hbar e^2}{m_e}\frac{g_m}{Z^+}f_{nm}lN(\mathrm{N}^+)\exp(-E_n/kT),\quad(1)$$

and $N(\mathrm{N}^+)$ was calculated from the coupled Saha equations

$$\frac{N(\mathrm{N}^+)N_e}{N(\mathrm{N}^0)}=\frac{2Z(\mathrm{N}^+)}{Z(\mathrm{N}^0)}\left(\frac{m_ekT}{2\pi\hbar^2}\right)^{3/2}\exp\left(-\frac{E_\infty(\mathrm{N}^0)-\Delta E_\infty{}^0}{kT}\right),\quad(2)$$

$$\frac{N(\mathrm{He}^+)\mathrm{N}_e}{N(\mathrm{He}^0)}=\frac{2Z(\mathrm{He}^+)}{Z(\mathrm{He}^0)}\left(\frac{m_ekT}{2\pi\hbar^2}\right)^{3/2}\times\exp\left(-\frac{E_\infty(\mathrm{He}^0)-\Delta E_\infty{}^0}{kT}\right),\quad(3)$$

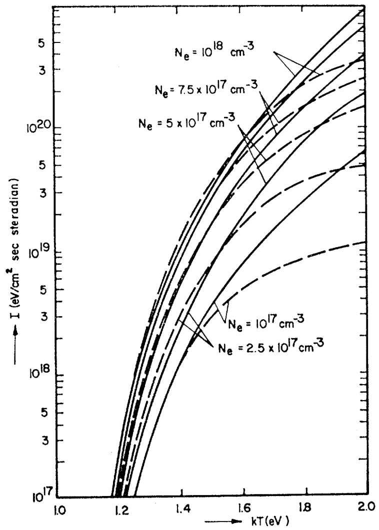

Fig. 2. Intensity of (combined) N II 5676, 5679, 5686-Å lines versus temperature for various electron densities. Gas mixture of He:N is 1:1 (solid curves) or 9.5:1 (dashed curves). The length of the radiating layer is 1.6 cm.

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[11] J. Euler, Ann. Physik 11, 203 (1953).

TABLE I. Experimental and theoretical (full) line widths $2w$ and shifts d (in Å units).

Expt[a]	λ[Å]	Transition	$2w_{\text{exptl}}$	$2w_{\text{th}}$[b]	$2w_{\text{th}}'$[c]	d_{exptl}	d_{th}[b]	d_{th}'[c]	d_{pol}[d]
A	5495	$3p\,^3D–3d\,^3P^0$	1.2 ±0.15	1.09		0.5 ±0.15	0.45		−1.6
	5405	$3s\,^3P^0–3p\,^3S$	1.0 ±0.15	0.85		0.5 ±0.15	0.43		−0.7
	4614	$3s\,^3P^0–3p\,^3D$	0.9 ±0.15	0.81		0.5 ±0.15	0.45		−0.7
	3006	$3p\,^1P–4s\,^1P^0$	1.65±0.15	1.69		1.1 ±0.15	1.05		−0.9
B	3838	$3p\,^3P–4s\,^3P^0$	1.9 ±0.2	1.68		0.9 ±0.15	1.02		−0.6
	3006	$3p\,^1P–4s\,^1P^0$	0.9 ±0.15	0.92		0.6 ±0.15	0.57		−0.5
	4553	$3d\,^1F^0–4f\,^3G$	1.3 ±0.15	3.50	6.1	0.15±0.15	−1.27	−1.1	−1.5
C	4026	$3d\,^3F^0–4f\,^1G$	2.15±0.35	2.52	3.1	−0.1 ±0.35	−0.40	−0.3	−0.6
	4530	$3d\,^1F^0–4f\,^1G$	2.5 ±0.25	3.12	3.5	−0.2 ±0.25	−0.75	−0.6	−0.8
	4553	$3d\,^1F^0–4f\,^3G$	0.75±0.2	1.72	3.9	−0.1 ±0.2	−0.62	−0.6	−0.7

[a] Experiment A: $T=18\,800\pm940°\text{K}$; $N_e=6.3\pm0.9\times10^{17}\ \text{cm}^{-3}$. Experiment B: $T=19\,600\pm980°\text{K}$; $N_e=3.3\pm0.5\times10^{17}\ \text{cm}^{-3}$. Experiment C: $T=19\,100\pm950°\text{K}$; $N_e=1.56\pm0.23\times10^{17}\ \text{cm}^{-3}$.

[b] Calculated as for helium, i.e., neglecting Debye shielding of perturbing electrons.

[c] Estimated analogous to the hydrogen calculations, i.e., including shielding, but taking the limit of zero splitting and neglecting all but the closest level.

[d] Plasma polarization shift according to Refs. 3 and 13.

the quasineutrality condition and the mixing ratio

$$M\equiv\frac{N(\text{N}^+)+N(\text{N}^0)}{N(\text{He}^+)+N(\text{He}^0)}. \tag{4}$$

Here, $I(\text{N}^+)$ is the intensity of the nitrogen-ion line, E_n the energy of the upper state of the line, g_m the statistical weight of the lower state, and f_{nm} the absorption oscillator strength of the line (measured by Mastrup and Wiese[12]). Further, $N(\text{N}^+)$ and $N(\text{He}^+)$ are the nitrogen- and helium-ion densities and $N(\text{N}^0)$ and $N(\text{He}^0)$ the corresponding neutral densities, N_e is the electron density, the Z's are the partition functions of the various species, and E_∞ is the ionization energy of nitrogen or helium as indicated. The lowering of the ionization energy, $\Delta E_\infty{}^z$, is given by[13] $\Delta E_\infty{}^z=(z+1)e^2/\rho_D$, where ρ_D is the Debye length and z is the charge on the atom ($z=0$) or ion (in our case $z=1$). The results of these calculations are given in Fig. 2. An absolute intensity calibration of the detecting apparatus will now yield the temperature. Estimated errors of 40% in the knowledge of the intensity (including theoretical errors due to deviations from local thermal equilibrium) give an error in temperature of only 5%; this suffices for measurements of Stark-broadening parameters which are not very temperature-sensitive.[1,2,13]

The measurement of the electron density of the plasma was made by comparing the experimental Stark width of He I 3889 Å to empirically corrected (by minus 10%[14]) theoretical values[13] of the width. The rather high continuum under the He I 3889-Å line (and nitrogen-ion lines as well) could be evaluated by using the theoretical wavelength dependence of the intensity,[1,13] i.e., $I\approx I_{\max}w^2/(w^2+\Delta\lambda^2)$. Here w is the half-width at half-intensity of the line (determined from the measured intensity assuming a reasonable continuum level) and $\Delta\lambda$ the wavelength distance measured from the line center. Then, picking two points on the line wings, $\Delta\lambda_1$ and $\Delta\lambda_2$, having total intensities (line plus continuum) of I_1 and I_2 associated with them, an improved continuum intensity is calculated from

$$I_{\text{cont}}=\frac{(w^2+\Delta\lambda_1{}^2)I_1-(w^2+\Delta\lambda_2{}^2)I_2}{\Delta\lambda_1{}^2-\Delta\lambda_2{}^2}. \tag{5}$$

With this continuum intensity, a new value for w could be measured and the process be repeated. Iteration never changed I_{cont} more than a few percent.

IV. MEASUREMENT OF SHIFTS AND HALF-WIDTHS OF NITROGEN-ION LINES

Various lines emitted by the plasma were scanned from shot to shot, the intensity of each point being recorded at least twice. For points near the center of the spectral line, at least three shots were made, so that the statistical error of each point became less than 10%. Near the center of the undisplaced line, the pulsed discharge was carefully monitored in between shots of the T tube, so that the position of the undisplaced ion line was known to ±0.1 Å. The intensity of the spectral line was plotted versus wavelength, the continuum was calculated using Eq. (5), and the half-width and the shift of the spectral line were measured. This was done for all of the lines scanned in an experimental run, i.e., normally for at least three nitrogen-ion lines and the He I 3889-Å line. The results of three of the best runs are presented in Table I.

The errors of the measured widths were estimated to ±10% except at small line widths where the coarseness of the wavelength scan limited measurements to ±0.15 Å. Also indicated in Table I are estimated errors in measured shifts, which are made up of the ±0.1-Å error in the location of the reference line and a similar but independent error in reducing the line shift from the measured profile. (In experiment C the coarseness of the scan increased this error slightly.)

Not listed in Table I are errors in calculated widths and shifts due to uncertainties in plasma density and temperature. As mentioned before, temperature errors

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[12] F. Mastrup and W. Wiese, Z. Astrophys. 44, 259 (1958).

[13] H. R. Griem, *Plasma Spectroscopy* (McGraw-Hill Book Company, Inc., New York, 1964).

[14] R. P. H. Lincke, dissertation, University of Maryland, 1964 (unpublished); Air Force Cambridge Research Laboratory Report No. AFCRL-64-960, 1964 (unpublished).

are expected to be of the order of 5%. Because Stark-broadening parameters are rather insensitive to temperature, this source of error may be neglected. Much more important for Stark broadening is the electron density measurement where an estimated uncertainty of 10% in the empirically corrected theoretical Stark width parameter and a 10% experimental error in the measurement of the width of He I 3889 Å combine to an estimated uncertainty of 15% in the electron density and therefore also in the calculated widths and shifts of the N II lines. (Added to this should be any errors from various approximations made in the theory.)

V. DISCUSSION OF RESULTS

The results, shown in Table I, are in about 20% agreement with theory both for the shifts and widths, except for lines originating from the $4f$ level of N II. A typical trace of an N II line, λ3006 Å, given in Fig. 3 shows the almost Lorentzian profile typical of impact broadening even though the line is strongly shifted relative to its width. (For this line, quasistatic, i.e., ionic, effects contribute less than 10% to the width and and shift.[13])

The lines originating in the $4f$ level of N II are to some extent similar to hydrogen lines (where the levels are completely degenerate), since the perturbing $4d$ levels are only 200 to 400 cm^{-1} away. Since the plasma frequency corresponds to wave numbers of 150 and 100 cm^{-1} in experimental runs B and C, the fields of the perturbing electrons should, to some degree, be considered as Debye-shielded.[1] This causes both the shift and the width of the lines to be overestimated by calculations neglecting the shielding.[1,2,13] An upper limit to the width of the spectral line can be calculated from the formula for the hydrogenic broadening[13]; if this width is quite close to the width predicted by the nonhydrogenic theory, then the experimental width is expected to be smaller. The effect of Debye screening on the shifts is much more drastic. According to the estimates in Ref. 13, a quantity $b(\omega_{\alpha\alpha'}/\omega_p)$ must be subtracted from $b(z_{\alpha\alpha'}^{\min})$, which together with the dipole matrix element squared determines the shift of level α due to the perturbing level α'. Here b is a slowly varying function ($b=\pi/2$ at $z=0$ and $b\approx 1$ at $z=1$), and $z_{\alpha\alpha'}^{\min}$ is defined as $z_{\alpha\alpha'}^{\min}\equiv\omega_{\alpha\alpha'}\rho_{\min}/v$, $\rho_{\min}$ being an impact parameter at which second-order perturbation theory becomes invalid. The frequencies $\omega_{\alpha\alpha'}$ and ω_p denote the splitting between levels α and α' and the plasma frequency, respectively. As they are about equal to each other in this example, the contribution of the $4d$ levels to the shifts is reduced by perhaps a factor of 2. However, because of their larger separation, the $5d$ and $5g$ level contributions, which are somewhat less important but act in the opposite direction, are not affected by the shielding. The reduction in the total shift is therefore certainly much larger and would bring theory and experiment into even closer agreement.

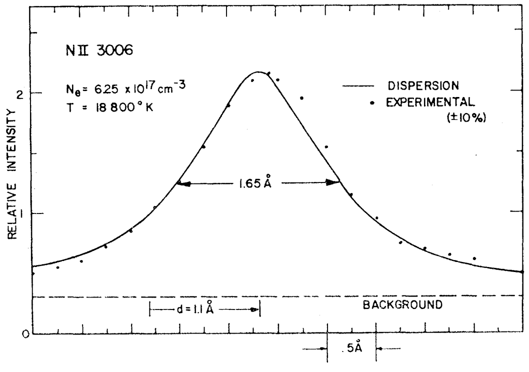

FIG. 3. Profile of typical nitrogen-ion line (λ3006 Å) fitted to a dispersion profile having the same center, width, and maximum intensity.

The above discussion accounts well for the behavior of λ4026 and 4530 Å; additional confidence in these experimental and theoretical widths can be gained from the fact that their widths are the same within the experimental error when the proper wavelength scaling with a λ^2 factor is used. However, the experimental width of λ4553 Å is in strong disagreement with theory, which is probably due to a superimposed Si III line at 4552.65 Å. (If account were taken of the well-known breakdown of L-S coupling[15] in the theory, then the relevant perturbing levels would be still closer to the $4f\,^3G$ level, and an even stronger disagreement with theory would result, since a larger theoretical width would be predicted.)

VI. SUMMARY

To summarize, in this experiment the over-all validity of the Stark-broadening parameters as calculated in Refs. 1, 2, and 13 is established also for ion lines provided that the frequency separation of the perturbing levels from the level of interest is much greater than the plasma frequency ($\omega_{\alpha\alpha'}\gg\omega_p$). When $\omega_{\alpha\alpha'}\approx\omega_p$, then the widths of two such lines are somewhat reduced from theoretical predictions (about 20%), whereas their shifts are much smaller than calculated. Results for the line shifts further indicate that the plasma polarization shift seems to be greatly overestimated by the simple semiclassical estimate given in Refs. 3 and 13. Such small shifts are consistent with quantum-theoretical estimates[16] in which the perturbing electrons are assumed to be in highly excited bound states.

There are some other lines not listed in Table I which were studied under optically thick conditions. Widths and shifts of these lines, namely λ3437, λ3994, λ4447, and λ5710 Å, were adjudged to agree with the theory within 20% if allowance were made for reabsorption near the line centers. For diagnostic purposes the line λ3006 Å is recommended, which is quite bright, reasonably wide, and located in an isolated portion of the spectrum. This line was studied under various experimental conditions, always yielding good agreement between theory and experiment.

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[15] K. B. S. Eriksson, Phys. Rev. **102**, 102 (1956).

[16] H. R. Griem, Proceedings of the Seventh International Conference on Phenomena in Ionized Gases, Belgrade, 1965 (to be published).