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Optical Interferometric and Spectroscopic Measurements of Electron Density in a Plasma*

E. A. McLean and S. A. Ramsden†
U. S. Naval Research Laboratory, Washington, D. C.

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A Mach-Zehnder interferometer is used to determine the electron density ($\sim 2\times 10^{17}$ cm^{-3}) behind the reflected shock wave in an electromagnetic, *T*-type shock tube filled with hydrogen to 2 Torr. The results are compared with simultaneous measurements of electron density from Stark broadening of the H_β line and the absolute continuum intensity. Using two-wavelength interferometry, agreement is found to be within the experimental accuracy of about 6%, thus showing the validity of the theory for these three methods of density measurement. Although spectroscopic methods indicate that the plasma is highly ionized ($\sim$90%) so that the refractivity is determined primarily by free electrons, single-wavelength interferometry yields results which are 10–15% too low. This is taken to indicate the presence of a boundary layer of cold, high-density neutral gas near the walls of the shock tube. In addition, it is concluded on the basis of this experimental data that the Griem asymptotic wing formulas for Stark-broadened hydrogen lines is more accurate than the earlier asymptotic wing formulas of Griem, Kolb, and Shen.

INTRODUCTION

THE last few years have seen an increasing use of the techniques of optical interferometry in the determination of plasma densities.[1–6] This has resulted largely from the work of Alpher and White[7] and of Ascoli-Bartoli, De Angelis, and Martellucci,[8] who independently recognized the important role of the free electrons in determining the refractive index of a laboratory plasma, although the first observation of the effect of the free electrons upon an interferogram appears to have been made earlier by Dolgov and Mandelstam.[9] On a per-particle basis, the free electrons can be more than an order of magnitude more effective than atomic or molecular species in determining the refractive index of the plasma. For degrees of ionization greater than a few percent, the electron contribution predominates and the net refractivity becomes negative. The free electrons (refractivity negative and proportional to the square of the wavelength λ) may be distinguished from the atomic and molecular components (refractivity positive and almost independent of λ) by measurements at two wavelengths.

Spectroscopic methods can also be used in the determination of plasma densities. For example, the electron density in a hydrogen plasma can be obtained from a measurement of the half-width of the Stark-broadened lines and from a determination of the absolute continuum intensity. Earlier work by Berg *et al.*,[10] using a *T*-type shock tube and Wiese *et al.*,[11] using a high-current stabilized arc have shown that these two methods yield electron densities which agree to within 5%. However, there are several assumptions implicit in making a comparison between electron densities obtained from the Stark broadening and the continuum theory; e.g., local thermal equilibrium (LTE) is assumed, the optical path length is assumed known so that boundary layers are not considered, and the theoretical contribution of the far wings of spectral lines to the continuum and the lowering of the ionization potential are taken for granted. Thus, the 5% agreement between two measurements could be fortuitous to some extent owing to compensating errors. For these reasons, a third measurement of the electron density which involves few assumptions in analyzing the data would be of interest to evaluate further the precision with which the electron density in dense plasmas ($N_e > 10^{16}$ cm^{-3}) can be established.

In the present experiment, the electron density in a hydrogen plasma produced in a *T*-type shock tube[12,13] was determined by simultaneous measurement of (a) the optical refractivity using a Mach-Zehnder interferometer, (b) the Stark broadening of the H_β line, and (c) the absolute continuum intensity.

The electron temperature is determined from the ratio of the total H_β line intensity to the intensity of the background continuum. This temperature is used in the

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† On leave of absence from the Culham Laboratory, Berkshire, England. Present address: The National Research Council, Ottawa, Canada.

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continuum intensity formula to calculate the electron density. Also, the temperature is used in the Saha equation to calculate the neutral atom density and the degree of ionization within the plasma. Under the conditions of the experiment, the assumption of local thermal equilibrium should be valid; however, this is only of critical importance in the use of the Saha equation.[14]

In our initial work, reported earlier,[15] the electron density behind the incident shock wave[16] was $\sim 5\times 10^{16}$ cm^{-3}. To within the limited accuracy of the measurements (average of $\pm 12\%$), agreement was found between the values of electron density determined from interferometric observations at a single wavelength and from the spectroscopic data. The purpose of the experiments reported in this paper was to achieve better experimental accuracy by (a) working at the higher electron densities ($\sim 2\times 10^{17}$ cm^{-3}) available behind the reflected shock wave,[10,17,18] (b) using two-wavelength interferometry, and (c) using photoelectric techniques to measure the H_β line profile instead of the photographic techniques used earlier.[15]

In the present investigation, it is demonstrated that the three methods of electron-density measurement, i.e., the optical refractivity at two wavelengths, the Stark broadening of H_β, and the absolute continuum intensity, are self-consistent to within the accuracy of the measurements ($\sim 6\%$). On the other hand, in spite of the high degree of ionization within the plasma, the electron densities obtained by single-wavelength interferometry are consistently 10–15% too low. Also, the average density of atoms and molecules obtained from the interferometric data is more than an order of magnitude greater than the neutral density obtained from spectroscopic data. These results are taken to indicate the presence of a high concentration of cold neutral gas in the boundary layers next to the walls of the shock tube.

The interferometric data serve to corroborate the earlier conclusion[10,11] that electron densities can be measured with an accuracy of $\sim\pm 6\%$ and that boundary-layer effects, while they exist, do not obviate the use of spectroscopic techniques. The significance with respect to the line-broadening theory lies in the observation that the electron densities obtained experimentally seem to be more accurate than expected from an analysis of the theoretical approximations involved in the Stark-broadening calculations. Presumably this is due to small compensating errors in the theoretical treatment. Also, this data indicates that the Griem asymptotic wing formulas for Stark-broadened hydrogen lines is more accurate than the earlier Griem, Kolb, Shen formulas.

REFRACTIVITY

The fringe shift s, measured interferometrically for light of wavelength λ, and path length l, is given by

$$s=[\Delta(\mu-1)/\lambda]l,$$

where $\Delta(\mu-1)$ is the phase change in refractivity $(\mu-1)$ due to the plasma. The change in refractivity across the shock is given by

$$\Delta(\mu-1)=(\mu-1)_{\text{gas}} -[(\mu-1)_m+(\mu-1)_a+(\mu-1)_i+(\mu-1)_e]_{\text{plasma}}, \quad (1)$$

where the subscripts "gas" refers to the ambient un-ionized gas ahead of the shock wave,[19] m refers to molecules, a refers to atoms, i refers to ions, e refers to electrons, and "plasma" refers to the heated gas mixture behind the shock wave.

The refractivity of the ambient gas is effectively constant over the visible region of the spectrum and is given by[20]

$$(\mu-1)_{\text{gas}}=5.15\times 10^{-24}N_{m1}, \quad (2)$$

where N_{m1} is the density of molecular hydrogen ahead of the shock wave. [The same equation applies to $(\mu-1)_m$ except N_{m1} is replaced by N_{m2}, which refers to the density of molecular hydrogen behind the shock wave.]

The specific refractivity of atomic hydrogen is obtained from a calculation of the polarizability of Podolsky.[21] Recent experiments by Marlow and Bershader[22] have corroborated these values. Since the specific refractivity varies less than 4% over the wavelength range of interest (4500–5400 Å), a mean value was used, giving

$$(\mu-1)_a=4.35\times 10^{-24}N_a, \quad (3)$$

where N_a is the density of atomic hydrogen behind the shock front.

The phase refractivity of the free electrons is obtained from the well-known dispersion formula used in ionospheric studies.[23] For the conditions of the present experiment, in which the electron density is about 2×10^{17} cm^{-3} and the electron temperature is about 2×10^4 °K, the collision frequency for momentum-transfer colli-

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[16] The term shock wave is used throughout this paper to indicate the luminous front observed. No experiments were done to show that the plasma is only shock-heated gas, and since no comparison is made with shock theory, how the plasma is heated is not our concern here. The only conditions are that the plasma be an essentially homogeneous, optically thin layer.

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[20] C. W. Allen, *Astrophysical Quantities* (The Athlone Press, University of London, 1955), p. 86.

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sions of electrons with heavy particles is much less than the frequency of observation ω, and the equation reduces to the classical dispersion formula for an electron gas, developed by Eccles, Larmor, and Kramers, i.e.,

$$(\mu-1)_e=-\tfrac{1}{2}\omega_p^2/\omega^2=-4.46\times 10^{-14}\lambda^2 N_e\,, \tag{4}$$

where ω_p is the electron-plasma frequency, N_e is the electron density, and λ is the observing wavelength in cm. [The refractivity of the protons is $\sim(m_e/m_a)(\mu-1)_e$, and is completely negligible.]

A value for the electron density, free from any assumption as to the relative contribution of the atomic and molecular components to the refractivity, may be obtained from simultaneous measurements of the fringe shifts s_1 and s_2 at the two wavelengths λ_1 and λ_2, respectively. Because of the large dispersion of the electron contribution and the effectively constant refractivity of the atoms and molecules, the expression relating fringe shift, wavelength of observation, and electron density is given by[7]

$$s_1\lambda_1-s_2\lambda_2=-4.46\times 10^{-14}(\lambda_1^2-\lambda_2^2)\bar{N}_e l\,. \tag{5}$$

Use of Eqs. (1)–(5) allows the determination of the average value for the density of atoms and molecules along the line of sight, as well as the electron density. Thus one can evaluate corrections to the refractivity due to neutral particles in the boundary layer. This, of course, requires the assumption of a single value for the constants in Eqs. (2) and (3); however, since these constants vary by less than 20% for atomic and molecular species, the quantitative conclusions discussed later are not altered significantly.

SPECTROSCOPIC ANALYSIS

For the Stark broadening of H_β, the theoretical profiles calculated by Griem, Kolb, and Shen[24] have been used, interpolating the data according to $N_e^{2/3}$. For the intensity distribution in the wings of the line we have used the asymptotic wing formulas of Griem.[25] (The reason for the choice of these asymptotic wing formulas instead of the earlier asymptotic wing formulas of Griem, Kolb, and Shen[26] will be discussed later in this paper.)

The absolute intensity I of the continuum over a frequency interval $\Delta\nu$ from a hydrogen plasma of thickness l was determined from the usual formula[27]

$$I=\epsilon_{\nu,H}\Delta\nu l[\text{erg cm}^{-2}\text{ sr}^{-1}]\,, \tag{6}$$

where

$$\epsilon_{\nu,H}=6.36\times 10^{-47}[N_e^2/(kT_e)^{1/2}]\exp(-h\nu/kT_e)\,[g_{\text{ff}}\exp(h\nu_g/kT_e)+(2E_{iH}/kT_e)\textstyle\sum(g_{\text{fb}}/n^3)\exp(E_n'/kT_e)]\exp(-\Delta I_i^0/kT_e)\,.$$

Here, T_e is the electron temperature, $h\nu_g$ the energy difference between the last observed discrete level and the theoretical series limit, E_{iH} the ionization energy, g_{ff} and g_{fb} are Gaunt factors for free-free and free-bound transitions,[28] respectively, and the summation is taken over all continua with lower states E_n' which contribute at the frequency ν. The term $\exp(-\Delta I_i^0/kT_e)$ represents a correction for the reduction in the ionization energy at high electron densities,[29] which is of the order $\sim 5\%$.

The ratio of the total intensity of H_β to the intensity of the underlying continuum is independent of the density and is a rapidly varying function of T_e for electron temperatures of the order of a few eV, and thus forms a convenient means of determining the electron temperature in the plasma.[30] Using the Saha equation with a measured value of the electron density, we can then derive the neutral atom density N_a in the plasma and the degree of ionization $N_e/(N_e+N_a)$.

EXPERIMENTAL SYSTEM

Shock Tube

The shock tube, shown schematically in Fig. 1, was made of quartz, 3 cm in diameter and 30 cm long. The tube was provided with a pair of optically flat ($\lambda/10$) glass windows so that approximately one-third of the tube diameter was accessible for observation. The windows were mounted in a Lucite holder which was designed so that two sections of the shock tube could be inserted into the holder in such a way that the cylindrical channel was maintained, apart from a slight step at the windows themselves. To observe the reflected shock wave a reflector was placed about 10 cm from the electrodes. Before each shot the shock tube was evacuated to about 5×10^{-3} Torr and then filled with hydrogen to a pressure of 2 Torr. The shock tube was connected

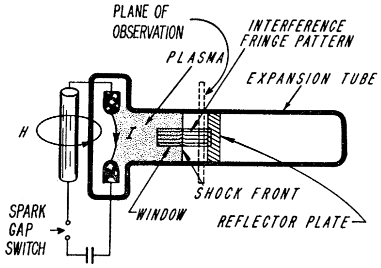

FIG. 1. Cylindrical T-type quartz shock tube used in the experiments.

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[24] H. R. Griem, A. C. Kolb, and K. Y. Shen, Astrophys. J. **135**, 272 (1962).

[25] H. R. Griem, Astrophys. J. **136**, 422 (1962).

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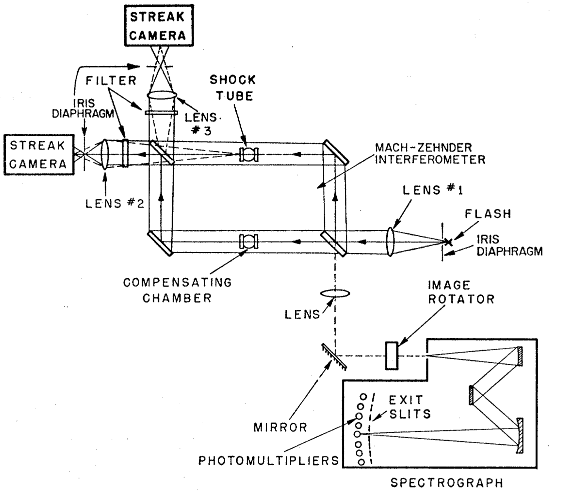

FIG. 2. In the experimental arrangement shown, the interferometer, flash lamp, and streak cameras are used to obtain data on refractivity changes of the plasma in the shock tube. Simultaneous time-resolved measurements of the absolute continuum intensity and the H_β profile for the plasma were obtained with a grating polychromator.

to a 0.65-μF capacitor which was charged to 25 kV. The ring frequency of the discharge was 450 kc/sec.

Interferometry

A Mach-Zehnder interferometer was used to measure the refractivity of the shock-tube plasma. The interferometer was arranged as shown in Fig. 2. A helical flash tube (quartz, 1 mm i.d. and $3\frac{5}{8}$ in. long) filled with 45 Torr xenon and 15 Torr hydrogen was used as the light source for the interferometer. With a 0.25-μF capacitor charged to 7.5 kV, the duration of the light flash was $\sim$8 μsec. A time resolution of 0.1 μsec was obtained by recording the fringes with two continuously recording, high-aperture (f/2.9) streak cameras.[31–33] The fringe system was arranged to be perpendicular to the slits of the cameras and parallel to the direction of propagation of the shock (see Fig. 1). In this way, the fringe shift was recorded as a function of time as the shock passed a particular cross section of the tube (5 to 10 mm ahead of the reflector), and not as a two-dimensional picture at a particular instant, such as was obtained by Alpher and White,[7] for example. The flash lamp was fired ahead of the shock tube in order to observe the undisturbed fringe pattern.

Simultaneous measurements of the fringe shift at two wavelengths were obtained by utilizing the two complementary fringe systems of the Mach-Zehnder interferometer, as shown in Fig. 2. In order to discriminate as much as possible against light from the plasma itself, the observations were made at two wavelengths in the continuum, using interference filters peaked at 5330 and 4545 Å and with half-widths of 140 and 120 Å, respectively. For the measurements involving the reflected shock wave it was necessary to further attenuate the radiation from the plasma by means of diaphragms[33] at the focal points of lenses 2 and 3 in Fig. 2. As illustrated in the diagram, these diaphragms have no effect upon the collimated beams from the flash lamp, but they do reduce the amount of plasma light falling upon the slits of the streak cameras.

Spectroscopy

For each interferogram, simultaneous time-resolved measurements of the absolute continuum intensity and the H_β profile were made using a Czerny-Turner-type grating spectrograph (f/27, 6.8 Å/mm). This spectrograph was used as a polychromator equipped with eight photomultipliers and set to observe the same volume of the shock tube that was observed by the streak cameras. Five of these photomultipliers were used to monitor 3–7 Å increments at points within the H_β profile, a further two were used to monitor 20 Å of the continuum at 5240 and 4575 Å, and the remaining photomultiplier was used to monitor the peak intensity of the H_γ line. This latter photomultiplier channel when used with a Geissler-tube light source served the dual purpose of giving a reference signal for the polychromator and also as a means of obtaining data during the experiment on the relative intensities of the H_γ and H_β lines, which could be used to correct the continuum intensity values (as will be described later). The photomultipliers were calibrated *in situ*[13] using a tungsten ribbon-filament lamp calibrated previously by the National Bureau of Standards. In reducing the measurements to absolute values, corrections were applied for the window transmissions of both the shock tube and the standard lamp.

Five suitable wavelengths for monitoring the H_β profile were chosen as a result of a previous study[17] in which the line was scanned on successive discharges using a grating monochromator. For that study the slight variations in plasma conditions from shot to shot were taken into account by making all the measurements at a predetermined amplitude of the continuum signal recorded simultaneously on the polychromator.[10,18] The profile is shown in Fig. 3. Although corrections were made for self-absorption, the effect on the half-width of the line is small ($\sim$1 %). The curve drawn is a theoretical profile from the work of Griem, Kolb, and Shen,[24] fitted to the experimental data at the peak and the half-width. No correction was needed for either Doppler broadening ($\sim$0.5 Å) of the line or the instrumental width ($\sim$2 Å) of the monochromator.

The half-width of the H_β line measured with the polychromator, and hence the electron density, was determined by iteration. A preliminary value for the

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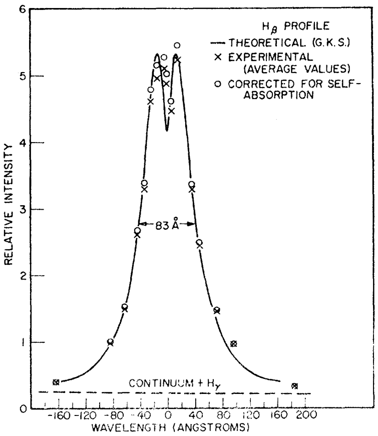

FIG. 3. Relative intensity versus wavelength for the H_β spectral line. The data represent a total of about 100 discharges, and the intensity value shown at each wavelength is an average of at least four measurements.

background intensity was first of all obtained from the measured values of the continuum intensity at 4575 and 5240 Å. This was subtracted from the total line intensity and a preliminary value for the half-width determined. From the theoretical calculations of Griem, Kolb, and Shen,[24] a preliminary value for the electron density was obtained. Then the asymptotic wing formulas calculated by Griem[25] were used and corrections were applied to the measured value of the continuum intensity at 5240 Å for contributions due to the wings of H_β and H_γ. Equation (6) was used to extrapolate the continuum intensity from 5240 Å to the region under the H_β line.[34] The corrected value of the continuum intensity was then used to determine a new value for the half-width of the line and a final value for the electron density.

Because of the possibility of overlapping impurity lines, great care was taken in choosing the wavelength intervals suitable for measurement of the absolute intensity of the continuum. A streak spectrum indicated that the continuum near H_β was free from impurities during the first few microseconds after the shock reflection, i.e., during the time the measurements were to be made, although a number of impurity lines were observed to arise later. However, as only relatively strong lines could be detected in this way, a further check was carried out using two monochromators, one of which was set to view the entire wavelength region in the continuum selected for the measurement while the other was set to scan the same region, 2 Å per shot. No impurity lines were observed within the wavelength interval chosen.

RESULTS

Experimental

A typical streak interferogram is shown in Fig. 4, together with the corresponding oscillograph trace of the continuum signal. The interferogram has been photographically enlarged to give the same time scale as the continuum signal. The incident shock wave, giving a shift of less than one fringe spacing, is seen first in time, followed by the reflected shock wave, which gives a shift of between one and two fringe spacings. Because the continuum intensity is proportional to the square of the electron density, whereas the interferometric fringe shift is only linearly proportional to the electron density, the ratio of the continuum signal for the incident and reflected shock waves is greater than that of the fringe shifts.

Since the three methods used here depend differently on electron-density gradients, it is very important that the plasma be homogeneous along the line of sight. It was found that the *reflected* shock waves met the homogeneity requirement in a fairly high percentage of the shots (~70%)—even when the incident luminous front was nonplanar. Because of the geometry of the window supports, a measurement of the planarity of the luminous front parallel to the optic axis could not be made for each shot; however, the general character of the streak interferogram, which also shows the planarity of the luminous front perpendicular to the optic axis, would usually indicate a plasma of poor homogeneity. If the fringe shift varied by more than 20% across the shock tube window or displayed gross asymmetries, that shot would not be analyzed on grounds of poor homogeneity. The measurements reported here were made at a point in the shock tube 10 mm from the reflector and usually at the time the peak of the reflected shock wave passed the point of observation.

The measured electron densities are summarized in Table I. In column 2 are shown values of the electron density determined from simultaneous measurements of the fringe shifts at 5330 and 4545 Å—analyzed using the two-wavelength formula [Eq. (5)]. The indicated errors are discussed in the following section. In columns 3 and 4 are shown the values of electron density determined from the same fringe-shift measurements at 5330 and 4545 Å but which are individually analyzed using the single-wavelength formula [Eq. (4)]. It is noted that the electron densities in columns 3 and 4 are

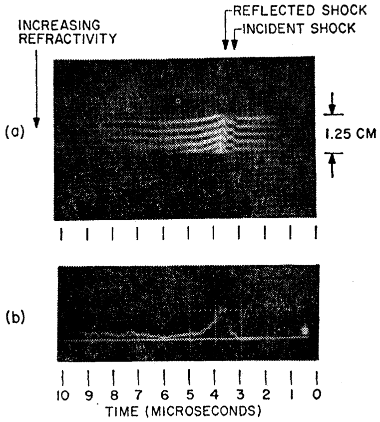

FIG. 4. Typical streak interferogram (a) and corresponding oscillograph trace (b) of the continuum signal.

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[34] If the continuum intensity at 4575 Å is corrected for the relatively large contribution of the overlapping H_β and H_γ lines using the asymptotic formulas, the value of the continuum intensity at 4575 Å falls below the extrapolated intensity based on the continuum intensity at 5240 Å. Thus, it was felt desirable to use the 5240-Å continuum signal instead of the 4575Å continuum signal for the H_β half-width determination and the temperature determination.

TABLE I. Comparison of electron densities measured using spectroscopic and interferometric techniques.

1	2	3	4	5	6	7	8
	Interferometric data $N_e(10^{17}\ \text{cm}^{-3})$				Continuum measurement $N_e(10^{17}\ \text{cm}^{-3})$		
						λ4575	
Run	Two wavelength [Eq. (5)]	λ5330	λ4545	H_β Profile N_e $(10^{17}\ \text{cm}^{-3})$	λ5240	Using Griem wing formula	Using (GKS) wing formula
a	2.24±0.16	1.98	1.89	2.21±0.13	2.24±0.14	2.19±0.26	1.77
b	2.19±0.15	1.81	1.67	2.21±0.13	1.98±0.12	1.86±0.22	1.38
c	2.22±0.16	2.09	2.04	2.18±0.13	2.29±0.14	2.13±0.26	1.62
d	2.76±0.19	2.38	2.24	2.48±0.15	2.54±0.16	2.44±0.29	2.01

consistently lower (average of 12% for column 3 and 17% for column 4) than those in column 2. Owing to the strong wavelength dependence of the electron refractivity, one would expect a larger neutral particle contribution to the refractivity at the lower wavelength than at the higher wavelength, and thus a lower electron density at the lower wavelength.

The electron density determined from the half-width of the H_β line is shown in column 5. Values of the electron density determined from the intensity of the continuum [using Eq. (6)] are shown in columns 6 through 8. As the correction, mentioned earlier, due to the overlapping of the Balmer lines was much greater at 4575 Å (~34%) than at 5240 Å (~9%), the latter values are expected to be the more accurate. At the densities considered here, the correction to the continuum formula for the reduction in the ionization energy is only ~3%. Because of the relatively high temperature (~2 eV) no correction was necessary for the H^- continuum.[35] The most accurate measurements of the electron density are given in columns 2, 5, and 6, where the values agree to within ±6%.

An additional result of this experiment is that the data can be used to test the validity of two asymptotic spectral-line-wing formulas, i.e., the Griem, Kolb, and Shen[26] (GKS) formulas and the later Griem[25] formulas. If we assume the electron densities as given in Table I, columns 2, 5, and 6 are correct, then we can use the electron density as a parameter to evaluate the wing correction used in the continuum measurement of the electron density at 4575 Å, where the wing correction has an appreciable value. (Since the wing correction is small at 5240 Å, no significant difference arises from the two asymptotic wing formulas.) To calculate the electron density, the measured intensity at λ4575 is first corrected for the contribution from both the H_β and H_γ line wings and then substituted in Eq. (6). In column 7 this electron density is listed using the Griem asymptotic-wing formulas and in column 8 the electron density is listed using the earlier (GKS) asymptotic-wing formulas. It is noted that the latter values fall outside of the error bracket for the N_e (interferometer), the N_e (H_β half-width), and the N_e(λ5240 continuum) measurements and thus would be unacceptable, whereas the former values are consistent with these values. This data then would indicate that at an electron density of $2\times10^{17}\ \text{cm}^{-3}$ and 286 Å away from the line center the Griem asymptotic formulas are the more accurate of the two sets of formulas. Also, Weise, Paquette, and Solarski[11] using a high-current, stabilized arc have measured the distant wing intensity to about 300 Å from the line center of the H_β line and they found that at an electron density of $\sim6\times10^{16}\ \text{cm}^{-3}$ the measured wing intensities agreed best with the Griem asymptotic-wing formulas but were always slightly higher. The higher, measured wing intensities in the experiment of Wiese, *et al.* may be caused by systematic errors due to contributions from an additional continuum source and to broadened impurity lines which they suspected might be present in their arc.

The total intensity of H_β was obtained by planimetering the area beneath the profile, assuming the intensity distribution in the wings of the line to follow the asymptotic-wing formulas. From the ratio of the total intensity of the line to the underlying continuum, a value was obtained for the electron temperature in the plasma using the calculations of Griem.[30] These values are listed in Table II, second column. In the third and fourth columns are shown values of the neutral-hydrogen-atom density N_a and the degree of ionization in the plasma, determined by substitution of N_e and T_e into the Saha equation. In the last column is shown the average value for the density N_0 of the atoms and molecules in the line of sight as determined using the interferometric data and Eq. (1). It is noted that the values for N_0 are more than an order of magnitude greater than N_a.

Error Analysis

The errors in the two-wavelength interferometry are primarily associated with the measurement of the

TABLE II. Comparison of neutral particle densities determined using spectroscopic and interferometric techniques.

		Saha equation		Interferometric data
Run	T_e (eV)	Degree of ionization $N_e/(N_e+N_a)$	N_a $(10^{16}\ \text{cm}^{-3})$	N_0 $(10^{17}\ \text{cm}^{-3})$ [Eq. (1)]
a	1.80	0.95	1.27	8.4
b	1.52	0.77	6.59	11.8
c	1.63	0.87	3.24	4.6
d	1.75	0.92	2.08	11.8

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[35] Reference 30, p. 119.

fringe shifts which are estimated to be between 2 and 3%, i.e., about 1/30 of a fringe spacing. Owing to the fact that Eq. (5) represents the difference of the two fringe-shift measurements and that errors of opposite sign greatly augment the net error, the uncertainty in the electron density is 5–10%, which is several times larger than the uncertainty in the fringe shift. (This error enhancement does not occur for single-wavelength interferometry; however, the much more serious systematic error due to the high density of neutral atoms and molecules in the boundaries is then introduced.) Other sources of error in the interferometry, such as the corrections at high plasma density and the corrections due to atoms in excited states, are felt to be small and thus have been ignored. The error in the combined measurement of the atomic and molecular density N_0 depends on the choice of the specific refractivity constants used, so that these values have an estimated error of ±40%.

The uncertainty in the measurement of the H_β half-width has many more facets, since this measurement depends on the calibration, the continuum intensity measurement, the self-absorption of the line, weak overlapping impurity lines (possibly missed in the streak-spectra survey), and the accuracy of the line-broadening theory used to obtain the value of the electron density. Taking the above factors into account, the uncertainty in the H_β half-width is estimated to be 4%, which reflects as a 6% uncertainty in the electron density.

The continuum intensity measurement lends itself more to an accurate estimate of the error. Here the error is primarily due to the uncertainty in the radiation standards used to calibrate the spectrograph and the photomultipliers. From the National Bureau of Standards calibration, the radiance of the tungsten ribbon-filament lamps used in this study is known to about 5%. To this error must be added experimental errors, such as reading errors made in reducing the data from the oscillograms, which will be about 5%, and also the errors made in correcting the data for the overlapping Balmer line intensities. The latter effect would be only 1–2% for the 5240 Å signal, but it could be as high as 10–15% for the 4575 Å signal since the 4575 Å signal includes large intensity contributions from both the H_β and H_γ lines. Therefore, the total error in the intensity measurements for the 5240 Å signal is estimated to be ~12%, which reflects as an error of ±6%.

The uncertainty in the electron temperature, obtained by substituting the errors for the H_β and the continuum intensity in the ratio of H_β to 100 Å continuum, is estimated to be ~6%.

The uncertainty in the degree of ionization in the hot-flow region is estimated to be ~10%, and the uncertainty in the neutral atom density N_a as calculated using the Saha equation, is estimated to be ~50% (mainly due to the 6% temperature uncertainty).

DISCUSSION

Although the spectroscopic measurements indicate a high degree of ionization within the hot-flow region, the values of the electron density determined using single-wavelength interferometry are consistently lower (average of 14%) than the spectroscopic values. Also, the neutral-atom density in the plasma, determined from Saha's equation, is an order of magnitude less than the density of the atoms and molecules determined from the interferometric data. This is taken to indicate the presence of a boundary layer of cold gas along the walls of the tube. Such a layer would not radiate strongly and, consequently, would not be detected by emission spectroscopy.

An estimate of the gas density in the boundary layer can be obtained using the assumption of continuity of pressure, i.e.,

$$N_1kT_1=N_2kT_2,$$

where N_1 and T_1 are the total particle density and temperature in the plasma and N_2 and T_2 are the corresponding values in the boundary layer. For the extreme case of $T_1=20\,000°\text{K}$, $T_2=300°\text{K}$, and $N_1\sim4.6\times10^{17}$ cm^{-3} (90% degree of ionization with $N_e=2.2\times10^{17}$ cm^{-3}) we obtain an average density in the boundary layer of $N_2\sim3\times10^{19}$ cm^{-3}.

The estimate for the boundary-layer thickness Δl (including both sides) may be derived from the relation

$$N_0l=N_2\Delta l+N_n(l-\Delta l),$$

where N_0 is the average density of atoms and molecules determined interferometrically and $N_n=0.1\ N_e$ is the neutral-atom density in the hot-flow region, and $l=3$ cm is the path length between the windows. From the two-wavelength interferometry data, N_0 is $\sim10^{18}$ cm^{-3}. The combined thickness of the boundary layers is then found to be ~1 mm. (It will be several millimeters if T_2 is chosen to have a value several times higher.)

Further evidence of the fact that the boundary-layer thickness must be small would stem from the fact that the electron density measured from the Stark broadening of H_β, which does not depend on the path length $(l-\Delta l)$, gives agreement to about 6% with the electron density measured from the continuum intensity, which depends on $(l-\Delta l)$. Since in the continuum measurement Δl was assumed equal to zero, agreement in the electron density implies that Δl must be small, i.e., the order of a few millimeters. Also, there is no evidence of a thick boundary layer from photographs of the plasma.

Such a high density of particles near the walls would have to be supported by a very high radial-flow rate to the walls. This is, however, not inconsistent with boundary-layer theory and experimental results for shock waves at low pressure.[36,37]

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CONCLUSIONS

The results of this experiment show that the values of the electron density determined by two-wavelength interferometry agree with the values determined from the Stark broadening of H_β and from the absolute continuum intensity at 5240 Å to within the experimental accuracy of about $\pm 6\%$. For the observed electron density of about 2×10^{17} cm^{-3} at temperatures around 20 000°K, this agreement between the three methods of measurement shows the consistency of the theories used. Corrections to the continuum measurements were made for the overlapping Balmer lines using asymptotic-wing formulas. Also, the continuum formula includes the correction for the lowering of the ionization potential due to high-density plasma effects. At these temperatures it is unnecessary to include the contributions due to H^- and H_2^+ continua, so that the continuum measurement of the electron density becomes quite reliable.

These measurements indicate that the calculated half-widths of Griem, Kolb, and Shen for Stark-broadened H_β lines are actually better ($\sim 4\%$) than the theoretical estimates of accuracy ($\sim 10\%$). Also, it can be inferred from this data that at these densities the Griem asymptotic-wing formulas for the Stark-broadened H_β and H_γ lines are more accurate than the earlier Kolb, Griem, and Shen wing formulas.

Concerning the interferometric technique, this study has shown that for the case of shock tubes it is especially important to use the two-wavelength refractivity formula instead of the single-wavelength formula to eliminate the systematic error due to the high neutral-particle density in the boundary layers.

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Measurement of Stark Profiles of Singly Ionized Nitrogen Lines from a T-Tube Plasma*

R. A. Day† and H. R. Griem
University of Maryland, College Park, Maryland

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A hot plasma composed of helium and ionized nitrogen was created by reflecting the shock wave produced in an electromagnetic T tube. The plasma temperature was measured by monitoring three nitrogen-ion lines whose intensities are strong functions of temperature. The profiles of various lines emitted in the plasma were obtained by scanning with a monochromator from shot to shot. The half-width of He I 3889 Å was compared with empirically corrected calculations to determine the electron density. Half-widths of various nitrogen-ion lines were measured from the impact-broadened profiles; their shifts were obtained by comparing these profiles with unshifted lines from a pulsed capillary discharge. The results indicate agreement between experiment and recent Stark-broadening calculations within 20%, except for lines originating from the $4f$ level where Debye screening effects are important. No evidence was found for plasma polarization shifts.

I. INTRODUCTION

THE Stark-broadening theory of isolated lines as first developed for neutral helium[1] has recently been extended to other elements.[2] Several experimental tests[3–9] have been made of the theoretical predictions for neutral atoms, and theory and experiment typically agreed within 20%.[10] However, the only ionic species which has been experimentally tested is helium,[3] and the good agreement found between theory and experi-

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† National Aeronautics and Space Administration Trainee (now at the Harvard College Observatory, Cambridge, Massachusetts). Some of the material in this article is part of a thesis submitted by R. A. Day in partial fulfillment of the requirements for the degree of Doctor of Philosophy at the University of Maryland.

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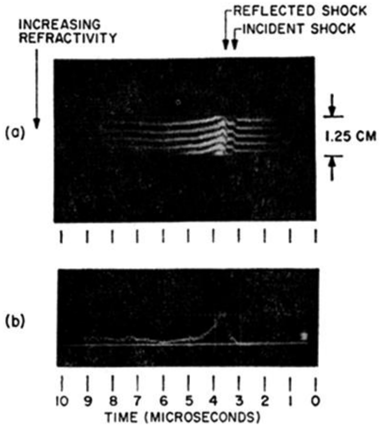

FIG. 4. Typical streak interferogram (a) and corresponding oscillograph trace (b) of the continuum signal.