THE VELOCITY OF SOUND AND THE RATIO OF THE SPECIFIC HEATS FOR AIR.

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SYNOPSIS.—The ratio of the specific heats for air obtained from the author's determination of the velocity of sound does not seem to be as high as that obtained by different observers using the Lummer-Pringhselm method. This was thought at first to be due to experimental errors in the determination of the velocity of sound. A redetermination gives, however, a value very close to the previously determined value.

The cause of the discrepancy is found, however, to be due to the method of obtaining the velocity of sound for dry ir at 0° C. from observations made in air containing moisture. The usual transformation leaves out the term $\sqrt{\gamma_0/\gamma_m}$ where γ_0 is for dry air at 0° C. and γ_m is for air containing a certain per cent. of moisture. The term is small but not negligible. It can be calculated with sufficient accuracy.

The corrected values of the velocity of sound can then be used to determine γ for dry air at 0° C. The results obtained are very satisfactory.

SOME years ago I made a determination of the velocity of sound by a new method which seemed to give very accurate results.[1] At that time I was not interested in the ratio of the specific heats, and, hence, made no effort to calculate its value. The value of the velocity obtained at that time, however, has since been used by Moody[2] in his paper on the ratio of the specific heats; but he failed to correct for the fact that the ideal gas laws were used in its calculation. Even if this correction be applied the value of γ thus obtained is still lower than the value obtained by different observers using the Lummer and Pringsheim method. This is due to the fact that I failed to apply a correction which is necessary in obtaining the velocity of sound for dry air at 0° C. from observation made in moist air.

Now both of these corrections can be calculated, and, hence, it is of interest to see what value of γ is obtained by applying them. This paper contains an application of these corrections and also the account of a new determination of the velocity of sound. This determination was made two years ago and before I saw what was the explanation for the difference in value between γ as obtained by my velocity of sound and that obtained by the Lummer-Pringsheim method. I may forecast

[1] PHYS. REV., Vol. XX., No. 2, Feb., 1905.

[2] PHYS. REV., Vol. 34, No. 4, p. 275, 1912.

the results of the experiment by saying that the value of the velocity obtained is practically the same as that obtained in 1904.

1. Velocity of Sound.

The method used was the same as that used in my previous determination. The same apparatus was used which had been carefully preserved at Ryerson Physical Laboratory, University of Chicago, where the experiment was done. The method was of an interference type. Two coaxial paraboloids—5 ft. in diameter—had each a telephone transmitter. In one paraboloid the transmitter was placed in the focus, and in the other it was placed near the focus. Each transmitter was in series with a battery and one primary of an induction coil containing two primaries and one secondary. A telephone receiver was in series with the secondary of the induction coil. A source of sound was placed in the focus of the paraboloid, having a transmitter near its focus. This paraboloid was always kept stationary. If the other was moved, always keeping it coaxial with the stationary one, the sound in the receiver of the telephone passed through maximum and minimum values. By this means it was possible to measure the wave-length of the particular note used. And, as a single setting could be made with considerable accuracy, and, as it was possible to move the one paraboloid through a distance of 80–100 feet, it was possible to obtain the wave-length with an accuracy of about one tenth of one per cent., as was shown in my previous paper.

A whistle blown with a constant air pressure was used as a source of sound. This, although presenting difficulties, was the most constant source of frequency, with sufficient intensity, available. To insure constancy when observations were made, the whistle was kept in unison with a tuning fork of known frequency.

The experiment was performed indoors in a basement hall, and, hence, there were no wind disturbances. The temperature was determined by eleven thermometers spaced along the distance over which the observations were made. The dew point was determined with an Alluard hygrometer.

In my previous experiment I used only one frequency, owing to the fact that I had only one tuning fork of sufficiently high pitch. In this experiment, however, I had at my disposal a set of standard tuning forks. These forks, with the exception of the 4,096 d.v., had been kept in good condition. The one marked 4,096 d.v. had apparently been hit and dented with a sharp metallic hammer. On comparing the forks among themselves it was found that it was impossible to get beats due to combination tones, excepting in the case of the 4,096 fork, which was found to have a frequency of 4,099.4 relative to the 2,048 d.v. fork. It was later found, however, that this fork could not be used as its frequency was too great.

In order to be sure that the absolute pitches of the forks were as marked, three of them were tested in the following manner. First, a fork with a frequency of about 100 was compared with a standard clock; and, second, the forks to be tested were compared with the standardized tuning fork.

In order to get the frequency of this intermediate fork, a stylus was attached to one prong so that it could make a trace on a smoked drum. By this means the period was found, (1) by direct comparison with the clock, and, (2) by comparison with a pendulum which had been compared with the clock. As the comparison took a great deal of time, the tuning fork was kept in vibration by means of an electromagnet between its prongs. This electromagnet was operated once a second from the clock circuit. This method resulted in an occasional phase change for which, however, correction was made. The results of five determinations are given in the following table:

TABLE I.

Method.	Time, Seconds.	t.	N_t.	N_{21}.
1. Clock	120	23° C.	99.817	99.839
2. "	120	21° C.	99.848	99.848
3. "	120	21° C.	99.812	99.812
4. Pendulum	106	32.2° C.	99.815	99.839
5. "	80	23.2° C.	99.800	99.824
			Mean	99.833

The probable error of the mean is .004.

To find the frequency of the forks to be directly used in the experiment, a high-frequency generator was used. On its shaft was fitted a switch which operated once for every revolution of the generator. This actuated an electromagnet which recorded on a revolving drum with the trace of the standardized fork alongside of it. The idea entertained at first was to get the note produced by a telephone receiver in series with the generator, in unison with that of the fork whose pitch was to be determined, and, then, calculate its pitch from the speed of the generator as obtained from the tracings on the drum. This did not succeed, as it was found to be impossible to keep the generator speed sufficiently constant to keep the two notes in unison. It was found, however, that it was possible to keep the generator-telephone note out of unison with the tuning fork just enough to enable the number of beats in a certain time to be counted. Then, if the precaution was taken to be sure which frequency was the higher, it was possible to determine the relative frequencies. Table II. will make this clearer.

Table II.

Revolutions of Generator from Drum Record.	Corresponding Beats.	No. of Revolutions to Make Unison.	No. of Waves of Standard Fork.	Waves per Revolution.
279	30	278.34	869.0	3.1221
257	20	256.34	800.0	3.1212
214	20	213.34	665.4	3.1190
300	20	299.34	934.1	3.1205
219	20	218.34	680.97	3.1188
246	20	245.34	765.65	3.1207
			Mean..........	3.1204

In this case the generator-telephone note was higher than that of the tuning fork, and the rotor of the generator had 30 teeth. These results were obtained with the fork marked 960 d.v. It will be noticed that the number of beats is always the same. By the use of a switch in the circuit which recorded the revolutions of the generator it was possible to begin the record at any instant. Starting with the close of the switch 20 beats were counted and then the switch was opened.

Two other such determinations as given in Table II. gave means as follows:

$$\text{Means}_2 = 3.1199,$$
$$\text{Means}_3 = 3.1208.$$

The mean of these three means is 3.1204. The frequency of the 960 d.v. tuning fork, therefore, at 21° C. was:

$$N = \frac{30 \times 99.833}{3.1204} = 959.84.$$

As a check on these results the frequencies of the 1,024 d.v. and the 2,048 d.v. forks were determined in the same manner. The three means obtained for the 1,024 d.v. fork were:

Mean_1..........2.9250
Mean_2..........2.9247
Mean_3..........2.9253

Mean.......... 2.9250.

The frequency of the 1024 d.v. fork was, therefore, at 21° C.:

$$N = \frac{30 \times 99.833}{2.9250} = 1023.93.$$

As only a single set of observations was made for the 2048 d.v. fork I include it in Table III. For this case the rotor of the generator contained 60 teeth.

TABLE III.

Revolutions of Generator Taken from Drum Record.	Corresponding Beats.	Number of Revolutions to Make Unison.	Number of Waves of Standard Fork.	Waves Per Revolution.
313	0	313.0	915.7	2.9256
457	50	456.17	1334.1	2.9246
491	30	491.5	1438.0	2.9257
213	20	212.67	621.8	2.9238
306	30	305.5	893.5	2.9247
667	20	666.67	1949.9	2.9248
442	50	441.17	1290.0	2.9240
			Mean..........	2.9247

Hence the frequency of the 2,048 d.v. fork was at 21° C.:

$$N = \frac{60 \times 99.833}{2.9247} = 2048.02.$$

These results show that the 960 d.v. fork was in error about .02 per cent.; the 1,024 d.v. fork about .01 per cent.; and the 2,048 d.v. fork was about correct. As the previous experiments had shown that the forks were consistent among themselves it was assumed from these results that all the forks were accurate at 20° C. It should be observed that all through this paper the following formula has been used:

$$N_t = N_0(1 - .00011t),$$

where N_t is the frequency of a fork at $t°$ C. and N_0 is its frequency at 0° C.

With the frequencies known and the wave-length determined as indicated above, it was easy to get the corresponding velocity. The following table gives the results of my observations on page 78.

It will be noticed that only two results are given for the 3,072 frequency. No attempt was made to use it on August 11, and it was impossible to get any results with it on August 21. This was due to the fact that the reflected wave caused a change in the frequency of the whistle just as soon as the one paraboloid was moved. This caused trouble for all frequencies but the effect increased with the frequency. The trouble can be obviated by adjusting the air pressure supplied to the whistle. This is a long and tedious process and was not followed out on August 21. It was this reflected wave which made it impossible to use the 4,096 frequency.

An examination of the table reveals the fact that the mean velocity

Date.	Number of Waves.	Distance in Inches.	Tempera-ture °C.	Dew Point °C.	Pressure Mm.	λ_t in Inches.	N_{20}.	V_t in Inches.	V_0 (Meters.)
Aug. 11	90	957.0	22.1	14	746.9	10.633	1,280	13,610	331.43
" 15	91	968.4	22.1	16	746.6	10.642	"	13,621	331.69
" 19	91	968.1	22.1	16.5	743.4	10.638	"	13,617	331.43
" 21	91	969.1	22.4	18.5	741.8	10.649	"	13,631	331.51
								Mean .	331.51
Aug. 11	108	957.5	22.1	14	746.9	8.866	1,536	13,618	331.62
" 15	109	964.9	22.1	16	746.6	8.8521	"	13,597	331.11
" 19	109	965.5	22.1	16.5	743.4	8.8582	"	13,606	331.16
" 21	109	966.4	22.4	18.5	741.8	8.8660	"	13,618	331.19
								Mean .	331.27
Aug. 11	144	956.3	22.1	14	746.9	6.6406	2,048	13,600	331.19
" 15	146	970.9	22.1	16	746.6	6.6499	"	13,619	331.64
" 19	145	963.5	22.1	16.5	743.4	6.6448	"	13,609	331.23
" 21	146	971.0	22.4	18.5	641.8	6.6507	"	13,621	331.26
								Mean	331.33
Aug. 15	219	970.0	22.1	16	746.6	4.4292	3,072	13,606	331.33
" 19	218	967.0	22.1	16.5	743.4	4.4358	3,072	13,627	331.68
								Mean .	331.51

for a frequency of 1,280 is somewhat higher than the mean in the cases of the 1,536 and 2,048 frequencies. Frequencies of 960 and 1,024 were also tried, but it was found that they gave still higher values. This agrees with what was found in the previous experiment and appears to be related to the relative size of reflector and wave-length. Therefore, the results with the 1,280 frequency have not been used, although their addition does not affect the results seriously. The mean value of the velocity for the 1,536, 2,048 and 3,072 frequencies is 331.34 m. per second. This has to be corrected for the change in pitch due to the fact that the tuning forks were used at about 22.2° C., whereas they were correct at 20° C. The corrected value is, therefore, 331.26 m. per second, with a probable error of .05. This value is very close to the value obtained in my previous experiment, viz., 331.29. When, however, the correction given in the next paragraph is applied the agreement is not so close.

In calculating the results of Table IV. the relation

$$V_0 = V_m\sqrt{\frac{\rho_m}{\rho_0}}$$

was used where V_0 is the velocity in dry air at 0° C., V_m is the velocity as determined by the experiment, ρ_m is the density of the air at the time of the experiment and ρ_0 is the density of the air at 0° C. and at a pressure equal to that at the time of the experiment. Now this formula assumes that the ratio of the specific heats for dry air is the same as that for moist air, which is not correct. The correct transformation should be obtained in this manner:

$$V_0 = \sqrt{\frac{p\gamma_0}{\rho_0}} \quad \text{and} \quad V_m = \sqrt{\frac{p\gamma_m}{\rho_m}},$$

where γ_m is the ratio of the specific heats for air at the time of the experiment and γ_0 is the ratio for dry air at 0° C.

Hence

$$V_0 = V_m\sqrt{\frac{\rho_m}{\rho_0}} \cdot \sqrt{\frac{\gamma_0}{\gamma_m}},$$

i. e., the results given in Table IV. should be multiplied by $\sqrt{\gamma_0/\gamma_m}$ in order to give an accurate result. As γ_0/γ_m is nearly equal to one we can calculate its value with sufficient exactness.

In order to do that consider a unit mass of a mixture of three gases—mono-, di- and triatomic. The three gases referred to are argon, oxygen and nitrogen as one gas, and water vapor.

Then $C_v = C_{v_1}m_1 + C_{v_2}m_2 + C_{v_3}m_3$, where the C_v's are the specific heats at constant volume of the mixture and individual gases respectively and m_1, m_2 and m_3 are the masses of the ingredients.

But $C_p - C_v = R$ for the mixture

$$\therefore \quad \gamma - 1 = \frac{R}{C_v}.$$

Where γ is the ratio of the specific heats for the mixture

$$\therefore \quad \gamma - 1 = \frac{R}{C_{v_1}m_1 + C_{v_2}m_2 + C_{v_3}m_3}$$

$$= \frac{R}{\frac{R_1}{\gamma_1 - 1}m_1 + \frac{R_2}{\gamma_2 - 1}m_2 + \frac{R_3}{\gamma_3 - 1}m_3},$$

where γ_1, γ_2 and γ_3 are the ratios of the specific heats for the three gases and R_1, R_2 and R_3 are their gas constants;

$$\therefore \quad \gamma - 1 = \frac{1}{\frac{1}{\gamma_1 - 1} \cdot \frac{m_1R_1}{R} + \frac{1}{\gamma_2 - 1} \cdot \frac{m_2R_2}{R} + \frac{1}{\gamma_3 - 1} \cdot \frac{m_3R_3}{R}}.$$

But

$$\frac{m_1R_1}{R} = \frac{p_1}{p}, \quad \frac{m_2R_2}{R} = \frac{p_2}{p} \quad \text{and} \quad \frac{m_3R_3}{R} = \frac{p_3}{p},$$

where p, p_1, p_2 and p_3 are the pressures of the mixture and the component gases respectively.

Therefore

$$\gamma = 1 + \frac{1}{\frac{1}{\gamma_1 - 1}\cdot\frac{p_1}{p} + \frac{1}{\gamma_2 - 1}\frac{p_2}{p} + \frac{1}{\gamma_3 - 1}\frac{p_3}{p}}.$$

It will be sufficiently accurate to use the theoretical values of γ_1, γ_2 and γ_3, *i. e.*, $= \gamma_1 = 1\frac{2}{3}$, $\gamma_2 = 1.4$ and $\gamma_3 = 1\frac{1}{3}$;

$$\therefore \quad \gamma = 1 + \frac{1}{3/2(p_1/p) + 2.5(p_2/p) + 3(p_3/p)}.$$

The per cent. of the gases at the time of my experiment were .95 for argon, 1.85 for water vapor and 97.2 for oxygen and nitrogen.

$$\therefore \quad \gamma_m = 1 + \frac{1}{3/2 \times .0095 + 2.5 \times .972 + 3 \times .0185},$$

where γ_m is the ratio of the specific heats for air with 1.85 per cent. of water vapor in it.

$$\therefore \quad \gamma_m = 1.4000.$$

If $p_3 = 0$, *i. e.*, if the air is dry

$$\gamma_0 = 1 + \frac{1}{3/2 \times .0095 + 2.5 \times .9905}$$

$$= 1.4015.$$

Hence

$$\sqrt{\frac{\gamma_0}{\gamma_m}} = \sqrt{\frac{1.4015}{1.4000}} = 1.00053.$$

Hence the corrected velocity of sound for dry air at 0° C. as obtained from these experiments is

$$= 331.26 \times 1.00053 = 331.44 \text{ m.p.s.}$$

It is of interest to correct my previous value of the velocity of sound. The value obtained was 331.29 m. per second. Now the average value of the water vapor for that experiment was 1.03 per cent.

Hence

$$\gamma_m = 1 + \frac{1}{3/2 \times .0095 + 2.5 \times .9802 + 3 \times .0103}$$

$$= 1.4007;$$

$$\therefore \quad \sqrt{\frac{\gamma_0}{\gamma_m}} = \sqrt{\frac{1.4015}{1.4007}} = 1.000285;$$

$$V_0 = 331.29 \times 1.000285$$

$$= 331.38 \text{ m. per second.}$$

If we give these two values of the velocity of sound equal weights we get for the mean:

$$V_0 = 331.41,$$

which appears to me cannot have an error of more than 2 or 3 centimeters.

2. The Ratio of the Specific Heats.

To calculate the ratio of the specific heats it is necessary to apply a correction to the formula $v = \sqrt{p\gamma/\rho}$ in order to take into account the deviation of air from the ideal gas law. This correction may be obtained in the following manner: An infinitely small adiabatic change may be expressed by the following equation—

$$C_p\,dT_p + C_v\,dT_v = 0,$$

where dT_p and dT_v are the changes in temperature when the pressure and volume are constant respectively. Their values can be obtained from the ideal gas law: $pv = RT$ and hence

$$Cp \cdot pdv + C_v vdp = 0.$$

In this equation C_p and C_v are the specific heats for an ideal gas. If, however, we determine the dT's from Van der Wall's equation then C_p and C_v will be the specific heats of the gas under consideration in so far as Van der Wall's equation represents the characteristics of that gas. Thus:

$$\left(p + \frac{a}{v^2}2\right)(v - b) = RT;$$

hence

$$dT_p = \left[\frac{p - \frac{a}{v^2} + \frac{2ab}{v^3}}{R}\right] dv$$

and

$$dT_v = \frac{v - b}{R} dp.$$

$$\therefore \quad C_p \frac{p - \frac{a}{v^2} + \frac{2ab}{v^3}}{R} dv + C_v \frac{v - b}{R} dp = 0.$$

$$\therefore \quad \gamma \left[\frac{p - \frac{a}{v^2 p} + \frac{2ab}{v^3 p}}{1 - \frac{b}{v}}\right] \times pdv + vdp = 0.$$

Now the term in the brackets is small and hence may be considered as a constant.

$\therefore \quad pv^{\gamma_1}$ = a constant is the equation of the adiabatic for this gas, where

$$\gamma_1 = \gamma \left[\frac{p - \frac{a}{v^2 p} + \frac{2ab}{v^3 p}}{1 - \frac{b}{v}} \right].$$

As the term $2ab/v^3p$ is negligible in comparison with the rest of the expression under the brackets we get

$$\gamma_1 = \gamma \left[\frac{1 - \frac{a}{v^2 p}}{1 - \frac{b}{v}} \right].$$

Using the critical constants for air, viz., $T_c = -140°$ C. and $P_c = 39$ atmospheres we get for the expression in the brackets at 0° C. and 76 cm. pressure the fraction .99896.

$$\therefore \quad \gamma = \gamma_1 \times 1.00104.$$

Now γ_1 is obtained from the velocity of sound thus:

$$\gamma_1 = \frac{Vo^2 \rho_0}{P_{760}} = \frac{331.44^2 \times .0012928}{760 \times 13.5955 \times 980.616}$$

$$= 1.4017.$$

$$\therefore \quad \gamma = 1.4017 \times 1.00104$$

$$= 1.4031.$$

This is the value from the velocity of sound reported in this paper.

The value of γ obtained by using my previous value of V_0 as corrected in this paper, viz., 331.38 is

$$\gamma = 1.00104 \times \frac{331.38^2 \times .0012928}{760 \times 13.5955 \times 980.616}$$

$$= 1.00104 \times 1.4012$$

$$= 1.4026.$$

This value differs considerably from the value Moody obtained from my velocity of sound.

These two values of γ, 1.4031 and 1.4026, agree very well with the values obtained in recent years by the Lummer & Pringsheim method. This is shown in the following table, part of which is taken from Miss Shield's paper on the ratio of the specific heats of hydrogen.[1]

[1] Phys. Rev., Ser. 2, Vol. X., No. 5, p. 525, 1917.

TABLE V.

Observer.	Method.	
Lummer & Pringsheim	L. & P.	1.4025
Moody	"	1.4003
Partington	"	1.4034
Shields	"	1.4029
Hebb (1904)	Velocity of Sound	1.4026
Hebb (1919)	"	1.4031

The average of the two values of γ presented in this paper is 1.40285 and as I am of the opinion that the average of the two velocities of sound presented is accurate to 2 or 3 parts in 33,000, I am also of the opinion that this value of γ must be accurate to 2 or 3 parts in 14,000. This value, 1.40285, is almost identical with what one would expect from a theoretical standpoint. In the section on the velocity of sound it was shown that a gas consisting of .95 per cent. of argon and 99.05 per cent. of oxygen and nitrogen would have a ratio of the specific heats equal to 1.4015 considered as an ideal gas. If there be added to this the correction due to its deviation from an ideal gas, viz., .0014, there results the value 1.4029.

I wish to thank the staff of Ryerson Physical Laboratory for showing me every courtesy during the time I spent at the laboratory doing the experimental part of this work.

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