centroid of the coordinates of the π (electron) track to the point of intersection of the tracks.

B. Causes of "Wild" Events

The values of x and σ for the four wild events excluded from the final sample of events are italicized in Table IV. Possible causes for these figures are

(i) that the first grain of one of the secondary tracks was formed so close to the K^+ ending as to be indistinguishable from the true end of the track,

(ii) that the electron or π^+ suffers a single scatter near the K^+ ending; if this took place before the first grain of the track were formed, it could not be detected.

The cause (i) does not seem to be possible as the grain diameter is only $\sim 0.3\,\mu$. The second possibility appears the more reasonable especially when the other events are considered in which only one of the electron tracks gave a wild result while the other was well behaved.

PHYSICAL REVIEW VOLUME 139, NUMBER 4B 23 AUGUST 1965

Direct and Y_1^*-Resonant Production of Λ^0 and π^- and Σ-Λ^0 Conversion Following $\bar{K}$-Meson Nuclear Absorption. Final-State Interactions in the $\bar{K}+{}^4\mathrm{He} \rightarrow \pi^- + \Lambda^0 + {}^3\mathrm{He}$ Reaction

P. SAID AND J. SAWICKI

Laboratoire Joliot-Curie de Physique Nucléaire, Orsay (Seine et Oise) France

(Received 7 April 1965)

In the reaction of $\bar{K}$-meson nuclear absorption, three fundamental processes are considered: a direct (nonresonant) $\bar{K}+N \rightarrow \pi^- + \Lambda^0$ reaction, the same reaction with the Y_1^* resonance formation in the intermediate state, and the reaction with a Σ production in the first stage and its subsequent conversion into a Λ^0 in a successive collision with another nucleon. The "zero-range" impulseapproximation is assumed. The initial $\bar{K}$ state is an nS or an mP Bohr mesoatomic orbit. Several forms of the Λ^0-nucleus final-state interaction are considered. For the case of the $\bar{K}+{}^4\mathrm{He} \rightarrow \pi^- + \Lambda^0 + {}^3\mathrm{He}$ reaction the recoiling-^{3}He momentum distribution, the pion momentum distribution, and the $\pi^- + \Lambda^0$-^{3}He angular distribution are analyzed. It turns out that in order to explain the ^{3}He momentum distribution no elastic scattering distortion of the Λ^0-^{3}He wave is sufficient, and one has to introduce the Σ-Λ^0 conversion amplitude, which is most important at high ^{3}He-momenta, and which also improves our pion momentum distribution.

1. INTRODUCTION

IN a previous paper by one of us[1] the reaction $\bar{K}+{}^4\mathrm{He} \rightarrow \pi^- + \Lambda^0 + {}^3\mathrm{He}$ has been studied. Such a reaction is important for the study of the mechanism of the absorption of $\bar{K}$ mesons in nuclei and for the study of final-state interactions, especially of the outgoing hyperon-nucleus interaction. In I the formation of the Y_1^* resonance in the intermediate state is assumed to be the most important absorption mechanism, in contrast to some previous publications listed in that reference. This assumption helps us to understand fairly well the position of the maximum of $R_3(p_3)$, the distribution of the reaction rate as a function of the momentum p_3 of the recoiling ^{3}He nucleus. Another distribution function $R_\pi(p_\pi)$ of the reaction rate as a function of the outgoing pion momentum p_π is rather model-insensitive.

In explaining the function $R_3(p_3)$, however, neither the far-off tail (large p_3) of the experimental histogram[2] nor even the region of less small p_3 can be understood well. It was speculated in I that the discrepancies mentioned could be accounted for by introducing a really correct nuclear wave function (form factor) and/or by a proper treatment of the final-state interaction (the elastic-scattering Λ-^{3}He distortion). Indeed, neither of these two points has been satisfactorily treated in I. It will be seen from the present analysis, however, that neither of these two speculations of I and of other papers is correct, i.e., that neither of these two effects alone nor both of them combined suffice to explain the observed $R_3(p_3)$.

It turns out, indeed, that it is necessary to consider the Σ-Λ conversion amplitude as an extra component, and actually as a combined effect of the three generally interfering components: (1) a direct (nonresonant) amplitude, (2) a Y_1^* resonance amplitude, and (3) a direct Σ-production t-matrix element with the (second-stage) Σ-Λ conversion amplitude (or, equivalently, an

[1] J. Sawicki, Nuovo Cimento **33**, 361 (1964) referred to hereafter as I.

[2] Helium Bubble Chamber Collaboration Group, Nuovo Cimento **20**, 724 (1961); J. Auman, *et al., Proceedings of the 1962 Annual International Conference on High-Energy Nuclear Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 330; cf., also *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester*, edited by E. C. G. Sudarshan, J. H. Tincot, and A. C. Melissions (Interscience Publishers, Inc., New York, 1960), p. 426.

amplitude with the "inelastic Σ distortion" related to a complex Σ-^{3}He phase shift). Already in the best known reaction, $\bar{K}+D \rightarrow \pi^- + \Lambda + p$, Dahl *et al.*[3] have conjectured from the analysis of their data that the presence of three such components is most likely. On the other hand, Condo and Hill[4] find that the Σ-Λ conversion process, though important for heavy nuclei, is of little importance for light ones (of the order of 10% or so).

The essential basis of the calculation mentioned is the impulse approximation, i.e., the use of single t-matrix elements as transition operators in the reaction amplitudes corresponding to $\bar{K}$ interactions with only one nucleon at a time. Of the multiple interaction effects we shall consider only the Σ-Λ conversion. The impulse model itself has been criticized by Chand[5] and others with the suggestion that it is probably worse in the case of ^{4}He than in the considered case of deuterium. The "multiple scattering" corrections to the $\bar{K}$ absorption in D in flight considered in Ref. 5 represent the $\bar{K}$-N pseudopotential matrix elements for one of the nucleons, containing the part due to elastic distortion of the other nucleon by the same $\bar{K}$ meson. They appear to be important at appreciable relative $\bar{K}$-N momenta in the case of deuterium (this effect may be quite sensitive to the magnitudes of the real and the imaginary parts of the effective $\bar{K}$-N scattering length). On the other hand, a very crude estimate based on the numerical values of the complex $\bar{K}$-N scattering lengths available in the literature (cf. Ref. 6) seems to indicate that the "multiple scattering" correction in question should be rather negligible in general. [*Note added in proof.* Such corrections or an equivalent nuclear potential distortion of the $\bar{K}$ mesoatomic wave function (for nS, $n\neq 1$, mP orbits) could result only in a small modification for our particular reaction amplitude due to the extremely small kinetic energy of the $\bar{K}$. This situation is in contrast to that described in Ref. 53.] At very small $\bar{K}$-N relative momenta, the absorption cross section dominates very strongly over the elastic-scattering cross section and, in our case of a $\bar{K}$ mesoatomic orbit, the relevant $\bar{K}$-^{4}He relative momenta are very small indeed. However, in view of the result of Ref. 5 the problem is still open and requires a detailed numerical analysis. It may be that the apparent discrepancy is due to the poorness of the approximation in the estimate of the "multiple-scattering" correction in Ref. 5.

This question is indirectly related to the problem of the "multiple" (two-nucleon) $\bar{K}$ absorption without final pion production. This effect is small ($\sim$1%) for deuterium, but for ^{4}He the ratio of this double absorption rate to the rate of the ordinary single-nucleon absorption (with a final pion) is about 16% according, e.g., to the review article by Fowler.[7] According, e.g., to Condo *et al.*,[8] this ratio is about 17% for light nuclei such as C, N, or O. The same authors claim that this ratio is even considerably higher for heavier nuclei (Ag,Br) where $\bar{K}$ capture from low Bohr orbits ($1S$,$2P$, $3D$) predominates. This latter result seems to be difficult to reconcile with the peripheral model (alpha clusters in the surface) of Jones,[9] Wilkinson,[10] or Rook,[11] for the interior absorption model of McCarthy and Prowse.[12] Other surface mechanisms have also been considered (cf. Fowler and Crossland[13] or Biswas[14]).

In such "multiple-capture" events, at least one free nucleon unaccompanied by any final pion is observed with a final nucleus. The ^{4}He data which we discuss below do not contain any admixture of such multiple events. In this sense we do not have to worry about them directly. However, a high percentage rate of such events in the $\bar{K}$ absorption in ^{4}He would seem to indicate the possible importance of either many-body-force or higher cluster-term corrections even to our single-nucleon-absorption reaction amplitude with the ^{3}He (ground) final state. On the other hand, in terms of the rather short kaon-nucleon force range, which is considerably smaller than the average nucleon-nucleon separation in the nucleus, large multiple-scattering corrections would, in general, appear hard to understand at small $\bar{K}$ energies. In the following we employ the impulse approximation plus the Σ-Λ conversion correction term.

In place of the rather vague definition in I of the initial $\bar{K}$-meson state (appropriate rather to the case of an nS orbit of the corresponding mesonic atom; this is practically equivalent to a plane wave in our approximation), we consider here specific bound states of the kaon in definite Bohr orbits. It turns out (cf. our detailed calculations and discussion in subsequent sections) that it is sufficient to consider only the two

[3] O. Dahl, N. Horwitz, D. H. Miller, J. J. Murray, and S. G. White, Phys. Rev. Letters **6**, 142 (1961); cf., also *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester*, edited by E. C. G. Sudarshan, J. H. Tincot, and A. C. Melissions (Interscience Publishers, Inc., New York, 1960), p. 415; and for the Σ-Λ conversion cf., also: Helium Bubble Chamber Collaboration Group, Nuovo Cimento **20**, 423 (1961); E. H. S. Burhop, D. H. Davis, and J. Zakrzewski, Progr. Nucl. Phys. **9**, 157 (1964).

[4] G. T. Condo and R. D. Hill, Phys. Letters **13**, 271 (1964).

[5] R. Chand, Nuovo Cimento **31**, 1013 (1964); Ann. Phys. (N.Y.) **22**, 438 (1963); R. Chand and R. H. Dalitz, *ibid.* **20**, 1 (1962); A. K. Bhatia and J. Sucher, Phys. Rev. **132**, 855 (1963); T. B. Day, G. A. Snow and J. Sucher, *ibid.* **119**, 1100 (1960).

[6] R. Chand, Nuovo Cimento **34**, 1796 (1964).

[7] G. N. Fowler, Nucl. Phys. **57**, 100 (1964); A. K. Common and K. Higgins, *ibid.* **60**, 465 (1964).

[8] G. T. Condo and R. D. Hill, Phys. Rev. **129**, 388 (1963); G. T. Condo, R. D. Hill, and A. D. Martin, *ibid.* **133**, A1280 (1964); cf., also Y. Eisenberg, M. Friedmann, G. Alexander, and D. Kessler, Nuovo Cimento **22**, 1 (1961).

[9] P. Jones, Phil. Mag. **3**, 33 (1958), cf., also G. B. Chadwick *et al.*, *ibid.* **3**, 1193 (1958).

[10] D. H. Wilkinson, Phil. Mag. **4**, 215 (1959); *Proceedings of the Rutherford Jubilee International Conference, 1961*, edited by J. B. Birks (Academic Press Inc., New York, 1961), p. 339.

[11] J. R. Rook, Nucl. Phys. **39**, 479 (1962).

[12] I. E. Mc Carthy and D. J. Prowse, Nucl. Phys. **17**, 96 (1960).

[13] G. N. Fowler and A. D. Crossland, Nucl. Phys. **42**, 229 (1963).

[14] N. N. Biswas, Nuovo Cimento **22**, 654 (1961).

most important types of Bohr orbits: nS and mP (n and m general). In fact, the interesting angular and momentum distributions obtained are almost independent of n and m, and thus it is sufficient to calculate explicitly only the basic cases $1S$ and $2P$. Actually, the $\bar{K}$ radial wave functions $R_{nl}(r)$ are given essentially by the leading term $\bar{N}_{nl}r^l$, and only the lowest l's are expected to be important. For example,

$$R_{10}(r)=\bar{N}_{10}\exp(-Zr/a_{\text{Bo}})\cong\bar{N}_{10}=2(Z/a_{\text{Bo}})^{3/2}=\text{const},$$

$$R_{21}(r)=\bar{N}_{21}r\exp(-Zr/2a_{\text{Bo}})\cong\bar{N}_{21}r=(Z/\sqrt{3}a_{\text{Bo}})\times(Z/2a_{\text{Bo}})^{3/2}r,$$

$$R_{20}(r)=\bar{N}_{20}[1-(Zr/2a_{\text{Bo}})]\exp(-Zr/2a_{\text{Bo}})\cong\bar{N}_{20}=2(Z/2a_{\text{Bo}})^{3/2}=\text{const},$$

etc.

This is so, because the Bohr radius of the kaon, $a_{\text{Bo}}=\hbar^2/m_Kc^2$, is many times greater than the nuclear radius. Consequently, we can approximate the $\bar{K}$ wave function by its amplitude near the origin. For an nS orbit this gives

$$\phi_K(\mathbf{r})\cong N_{nS}=\pi^{-\frac{1}{2}}(Z/na_{\text{Bo}})^{3/2},$$

and for an mP orbit:

$$\phi_{\bar{K}}(\mathbf{r})\cong N_{mP}(\mathbf{e}\cdot\mathbf{r})=\left(\frac{m^2-1}{3\pi}\right)^{1/2}\left(\frac{Z}{ma_{\text{Bo}}}\right)^{5/2}(\mathbf{e}\cdot\mathbf{r}).$$

Unfortunately, the general question of the critical Bohr orbit involved is still open in spite of a vast amount of literature. This literature abounds in contradictions. In accordance with Day[15] it is generally believed that the meson is initially captured from the continuum into a bound orbit with the principal quantum number $n\approx30$, ejecting one of the atomic electrons. Day and others[15,16] have made theoretical estimates of the magnitudes of several effects which might be of importance in determining the history of a typical $\bar{K}$ meson during its cascade. It is just the relative importance of these competing radiative processes and of the direct nuclear capture which decides about the Bohr orbits relevant in our calculation, i.e., we have to multiply our probability of the direct nuclear capture from a given Bohr orbit by the relative-population probability for this orbit in order to see which orbits contribute most to our total nuclear-capture rate. For example, although the probability of direct nuclear capture from the orbit nS is much larger (by a factor $\sim10^4$) than that from the orbit nP ($m=n$; cf., Day[15]) the strongly competing radiative and other processes depopulating any given $\bar{K}$-mesonic state can change considerably the actual relative importance of the states in question. Of these one should consider the ordinary Auger transitions, the "external" Auger effect, the "polarization capture," the direct X radiation and, finally, the "molecular" Stark effect. For high n it is just this Stark effect which seems to Day[15] to be important in preventing the meson from cascading down through the lower P states from whence they might be absorbed; thus direct nuclear capture from the nS orbits would be most important. This conclusion is repeated by Day and Snow[16] for the case of ^{4}He (the same authors[17] find the $2P$ orbit to be the most important for the case of deuterium; cf., also the discussions by Karplus and Rodberg[18] and by Kotani and Ross[19]). Day, Snow, and Sucher[20] suggest a predominant capture of pions and kaons from S states, the $2P$ state being suppressed by the molecular Stark effect. On the other hand, Fetkovich and Pewitt[21] claim to have disproved the Stark-effect argument of Day, and have suggested that probably about 95% or more of the $\bar{K}$ in ^{4}He undergo nuclear absorption from P states. This in turn has been contradicted by Condo.[22] According to Condo and Hill[5] even the $3D$ Bohr orbit may be important around $A\sim14$ (C, N, O), while states such as $5G$ may be important for $A\sim100$ (cf., also Condo, Hill, and Martin[8]). Adair[23] assumes $3D$ in the case of ^{12}C. Rook[24] argues that higher l states might be important for complex nuclei. A similar rather controversial suggestion concerning higher l Bohr orbits is proposed by Eisenberg and Kessler.[25] A decisive experiment on K-mesonic x rays, giving information on the populations in the cascades, is being prepared by Schluter.[26] A quantitative analysis of the initial relative populations of the K-mesonic Bohr orbits in K atomic capture is given by Baker, Jr.[27]

Another novel improvement on the crude calculation of I is our present treatment of the recoil effect of the final nucleus.

The nuclear wave function of the relative motion of the absorbing nucleon and the center of mass of ^{3}He is taken empirically, i.e., the nuclear form factor (the Fourier transform with respect to the n-^{3}He relative coordinates) is taken with parameters fitting the ^{4}He(p,$2p$) momentum distribution. We have taken the corresponding data from Jacob.[28] The analytic form of

[15] T. B. Day, Nuovo Cimento **18**, 381 (1960).

[16] T. B. Day and G. A. Snow, Phys. Rev. Letters **5**, 112 (1960); cf. also G. A. Snow, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester*, edited by E. C. G. Sudarshan, J. H. Tincot, and A. C. Melissions (Interscience Publishers, Inc., New York, 1960), p. 407.

[17] T. B. Day and G. A. Snow, Phys. Rev. Letters **2**, 59 (1959).

[18] R. Karplus and L. S. Rodberg, Phys. Rev. **115**, 1058 (1959).

[19] T. Kotani and M. Ross, Nuovo Cimento **14**, 1282 (1959); cf., also A. Fuji and R. E. Marshak, *ibid.* **8**, 643 (1958).

[20] T. B. Day, G. A. Snow and J. Sucher, Phys. Rev. Letters **3**, 61 (1959); Phys. Rev. **118**, 864 (1960); R. K. Adair, Phys. Rev. Letters **3**, 438 (1959).

[21] J. G. Fetkovich and E. G. Pewitt, Phys. Rev. Letters **11**, 290 (1963); and private communication from Dr. J. B. Kopelman; cf., also J. E. Russell, Proc. Phys. Soc. (London) **A85**, 245 (1965).

[22] G. T. Condo, Phys. Letters **9**, 65 (1963).

[23] R. K. Adair, Phys. Letters **6**, 86 (1963).

[24] J. R. Rook, Nucl. Phys. **43**, 363 (1963).

[25] Y. Eisenberg and D. Kessler, Phys. Rev. **130**, 2352 (1963).

[26] R. A. Schluter (to be published); J. B. Kopelman (private communication).

[27] G. A. Baker, Jr., Phys. Rev. **117**, 1130 (1960).

[28] G. Jacob, *Proceedings of the International Conference on Nuclear Structure, 1960*, edited by D. A. Bromley and E. W. Vogt, (University of Toronto Press, Toronto, 1960), p. 432.

the corresponding (S-state) wave function is that of a Hulthén function with adjusted parameters.

The distortion of the final-state wave function of the Λ-^{3}He relative motion is represented for the S waves by a scattered spherical outgoing wave minus an exponentially decaying wave, giving the correct asymptotic behavior both at infinity and at the origin. The corresponding phase shift is considered both as calculated from appropriate Λ-nuclear potentials and as an adjustable parameter; the exponent-function parameter is also adjustable within certain limits ($+\infty$ and $\sim$nuclear radius).

2. CALCULATION

A. Λ Production without the Σ-Λ Conversion

As in I, but with the assumption of a definite initial $\bar{K}$ meson (Bohr orbit) wave function and with the inclusion of the recoil effect of the residual nucleus, we find, in the impulse approximation, the following expression for the reaction matrix element:

$$M\equiv(2\pi)^4\delta(E_i-E_f-E_{\pi}{}^0)\times\int d\mathbf{p}d\mathbf{p}_n d\mathbf{p}_K\delta(\mathbf{P}_i-\mathbf{p}_K+\mathbf{p}-\mathbf{P}_f-\mathbf{p}_n)\times\int d\xi\,\phi_f{}^*\left(\frac{m_\Lambda}{M_3+m_\Lambda}\mathbf{P}_f-\mathbf{p};\xi\right)\times\phi_4(\mathbf{p}_n-\tfrac{1}{4}(\mathbf{P}_i-\mathbf{p}_K);\xi)\phi_K\left(\mathbf{p}_K-\frac{m_K}{M_4+m_K}\mathbf{P}_i\right)\times\langle\mathbf{p}\mathbf{p}_\pi|t|\mathbf{p}_n\mathbf{p}_K\rangle, \quad (1)$$

where E_i, E_f, $E_\pi{}^0$ and $\mathbf{P}_i$, $\mathbf{P}_f$, $\mathbf{p}_\pi$ are the initial total energies and momenta of the ^{4}He $\bar{K}$-mesonic atom, the $\Lambda+{}^3$He system, and the pion, respectively (only $E_\pi{}^0$ is relativistic); $\mathbf{P}_f=\mathbf{p}_3+\mathbf{p}_\Lambda=$ sum of the final ^{3}He and the Λ momenta; $\mathbf{p}$, $\mathbf{p}_n$, and $\mathbf{p}_K$ are the virtual (Fourier) momenta of the Λ, of the nucleon (the absorbing neutron), and of the $\bar{K}$, respectively; the Λ-^{3}He, the ^{4}He nuclear, and the $\bar{K}$ form factors are

$$\phi_f(\boldsymbol{\lambda};\xi)=(2\pi)^{-3}\int d\mathbf{r}\,e^{i\boldsymbol{\lambda}\cdot\mathbf{r}}\Psi_f(\mathbf{r};\xi),$$

$$\phi_4(\boldsymbol{\kappa};\xi)=(2\pi)^{-3}\int d\mathbf{u}\,e^{-i\boldsymbol{\kappa}\cdot\mathbf{u}}\Psi_4(\mathbf{u};\xi),$$

and

$$\phi_K(\boldsymbol{\nu})=(2\pi)^{-3}\int d\mathbf{v}\,e^{-i\boldsymbol{\nu}\cdot\mathbf{v}}\phi_K(\mathbf{v}),$$

where $\mathbf{r}=\mathbf{r}_\Lambda-\mathbf{r}_3$ is the distance vector from the ^{3}He c.m. to the Λ; $\mathbf{u}=\mathbf{r}_n-\mathbf{r}_3$ that from the ^{3}He to the nucleon n, and $\mathbf{v}=\mathbf{r}_{K4}=\mathbf{r}_K-\mathbf{R}_4$ that from the ^{4}He c.m. to the $\bar{K}$. In the above the initial K-mesonic atom wave function is taken to be a simple product of the nuclear ^{4}He wave function and the Bohr orbit ϕ_K; the symbol ξ denotes the intrinsic coordinates of the ^{3}He system. As in I we have neglected here any final-state interaction of the outgoing pion with the other particles.

The t-matrix element of the elementary process can, in view of total momentum conservation, be written as

$$\langle\mathbf{p}\mathbf{p}_\pi|t|\mathbf{p}_n\mathbf{p}_K\rangle=(2\pi)^3\delta(\mathbf{p}_n+\mathbf{p}_K-\mathbf{p}-\mathbf{p}_\pi)\langle\mathbf{q}_1|t|\mathbf{q}_0\rangle, \quad (2)$$

where

$$\mathbf{q}_0=(M\mathbf{p}_K-m_K\mathbf{p}_n)(M+m_K)^{-1},$$
$$\mathbf{q}=(m_\Lambda\mathbf{p}_\pi-m_\pi\mathbf{p})(m_\Lambda+m_\pi)^{-1}$$

are the initial- and final-state relative momenta. To simplify the kinematics we consider only the case (the only important one from the point of view of the bubble-chamber data) of capture at rest, i.e., we set $\mathbf{P}_i=0$.

On performing the $\mathbf{p}_n$ integration we obtain

$$M=(2\pi)^7\delta(E_i-E_f-E_\pi{}^0)\delta(\mathbf{p}_\pi+\mathbf{P}_f)\times\int d\mathbf{p}_K d\mathbf{p}\int d\xi\phi_f{}^*\left(\frac{m_\Lambda}{M_3+m_\Lambda}\mathbf{P}_f-\mathbf{p};\xi\right)\times\phi_4(\mathbf{p}-\mathbf{P}_f-\tfrac{3}{4}\mathbf{p}_K;\xi)\phi_K(\mathbf{p}_K)\langle\mathbf{q}|t|\mathbf{q}_0\rangle. \quad (3)$$

We introduce now a very important approximation by replacing the t-matrix element by a corresponding one where the final state is taken on the momentum shell of the final physical Λ and π, i.e., in the matrix element we replace $\mathbf{p}$ by $\mathbf{p}_\Lambda$, the final state Λ momentum. This is analytically equivalent to the Dalitz-Downs[29] "zero-range" approximation [cf., Eq. (D2) of Ref. 29 or cf., Karplus and Rodberg[18]] if one works in the configuration space. In other terms, a similar result could be obtained automatically, as in I, if one considers the final-state Λ-^{3}He distortion of Ψ_f as a small perturbation, i.e., the following result is exact if Ψ_f is replaced by a plane wave of the Λ-^{3}He relative motion.

Upon this replacement $\mathbf{q}$ becomes $\mathbf{q}_1=(m_\Lambda\mathbf{p}_\pi-m_\pi\mathbf{p}_\Lambda)\times(m_\Lambda+m_\pi)^{-1}$; then $\langle\mathbf{q}_1|t|\mathbf{q}_0\rangle$ does not depend on $\mathbf{p}$, and we can readily perform the $\mathbf{p}$ integration to get

$$M=(2\pi)^7\delta(T_3+T_\Lambda+E_\pi{}^0-Q)\delta(\mathbf{p}_\pi+\mathbf{P}_f)\times\int d\mathbf{p}_K\phi_K(\mathbf{p}_K)\mathfrak{F}_{(3,n,\Lambda)}\left(\frac{M_3}{M_3+m_\Lambda}\mathbf{P}_f+\tfrac{3}{4}\mathbf{p}_K\right)\times\langle\mathbf{q}_1|t|\mathbf{q}_0\rangle\equiv(2\pi)^7\delta(en)\delta(\mathbf{p}_\pi+\mathbf{P}_f)\bar{M}, \quad (4)$$

where in the energy-conservation δ function $T_3=p_3{}^2/2M_3$ and $T_\Lambda=p_\Lambda{}^2/2m_\Lambda$ stand for the ^{3}He and the Λ kinetic energies, and $Q\equiv M_4-M_3+m_K-m_\Lambda$ (here the notation for the masses is self-explanatory; the units are $c=1$). The $\bar{K}$ binding energy of our Bohr orbit is neglected as small in Q. The one common ^{3}He-neutron

[29] R. H. Dalitz and B. W. Downs, Phys. Rev. **111**, 967 (1958), cf. their Eq. (D2).

form factor is

$$\mathfrak{F}_{(3,n,\Lambda)}(\boldsymbol{\omega})=(2\pi)^{-3}\int d\mathbf{u}\exp(i\boldsymbol{\omega}\cdot\mathbf{u})\times\int d\xi\,\Psi_f^*(\mathbf{u};\xi)\Psi_4(\mathbf{u};\xi). \quad (5)$$

In view of momentum conservation,

$$\mathbf{q}_0=\mathbf{p}_K+\frac{m_K}{M+m_K}\mathbf{p}_3,$$

and

$$\mathbf{q}_1=\mathbf{p}_\pi+\frac{m_\pi}{m_\Lambda+m_\pi}\mathbf{p}_3.$$

If we take now ϕ_K to be an nS Bohr orbit, $\phi_K(p_K)\cong N_{nS}\delta(\mathbf{p}_K)$ to a good approximation from the point of view of our calculation. [*Note added in proof.* In the extreme limit of $\langle t\rangle$ independent of $\mathbf{p}_K$ our theory of Eq. (4) reduces to that of Refs. 18 and 19.] In the case of an mP orbit $\phi_K(\mathbf{p}_K)\cong N_{mP}i(\mathbf{e}\cdot\nabla_{p_K})\delta(\mathbf{p}_K)$ in the same sense.

For the predominant nS capture, we finally obtain

$$\bar{M}_{nS}=N_{nS}\mathfrak{F}_{(3,n,\Lambda)}((M_3/M_3+m_\Lambda)\mathbf{P}_f)\times\langle\mathbf{q}_1|t|\mathbf{q}_0\rangle|_{p_K=0}, \quad (6)$$

and for the mP case

$$\bar{M}_{mP}=N_{mP}\{\mathfrak{F}_{(3,n,\Lambda)}((M_3/M_3+m_\Lambda)\mathbf{P}_f)\times i(\mathbf{e}\cdot\nabla_{q_0}\langle\mathbf{q}_1|t|\mathbf{q}_0\rangle)|_{p_K=0}+i\tfrac{3}{4}[\mathbf{e}\cdot\nabla_\omega\mathfrak{F}_{(3,n,\Lambda)}(\boldsymbol{\omega})]_{\omega=(M_3/M_3+m_\Lambda)\mathbf{P}_f}\times\langle\mathbf{q}_0|t|\mathbf{q}_0\rangle|_{p_K=0}\}, \quad (7)$$

where the last term of Eq. (7) is connected with the recoil effect of the final ^{3}He nucleus (the term $\propto+\frac{3}{4}\mathbf{p}_K$ in the argument of $\mathfrak{F}$). In the next section we shall compare the numerical results for the two Bohr orbits. Any other higher l Bohr orbits appear to be relatively unimportant in the case of ^{4}He.

As for the t matrix, it has been shown in I that the most important component of the vertex function of the elementary $\bar{K}+n\rightarrow\pi^-+\Lambda^0$ process is a resonant term corresponding to the Y_1^* with energy $M_1=1385$ MeV and width $\frac{1}{2}\Gamma_1\cong 20$ MeV. A Breit-Wigner t-matrix element can in this case shift the peak of the final ^{3}He-momentum distribution $R_3(p_3)$ to a position of p_3 around 250 MeV/c. The Y_1^* resonance is well known as a $P_{3/2}$ state, and can, therefore, be represented by

$$\langle\mathbf{q}_1|t|\mathbf{q}_0\rangle_{\text{res}}=a_r\frac{\mathbf{q}_0\cdot\mathbf{q}_1+\frac{1}{2}i\boldsymbol{\sigma}_\Lambda\cdot(\mathbf{q}_0\times\mathbf{q}_1)}{s^{1/2}-M_1^*+\frac{1}{2}i\Gamma_1} \quad (8)$$

where $s^{1/2}=(m_\pi^2+q_1^2)^{1/2}+m_\Lambda+q_1^2/2m_\Lambda$ is the total energy in the $\Lambda+\pi$ c.m. system.

Unfortunately, the existing data[2] do not contain information on the absolute value of our "effective" coupling constant a_r. For example, one could attempt to relate our a_r to the strength of the "vertex" Γ_{33} of Eq. (10) of Gürsey, Pais, and Radicati[30] derived on the basis of $SU(6)$ symmetry considerations. As soon as data on absolute values are available, tests of the symmetry group and other predictions of the basic t-matrix elements involved in our calculations become possible and important.

To the resonant component of Eq. (8) we could add a nonresonant term $\langle\mathbf{q}_1|t|\mathbf{q}_0\rangle_{\text{dir}}=A_d$, taken for simplicity to be a constant, and we first consider the sum of the two to be our total t matrix. If one considers a t-matrix element of our elementary process corresponding to a higher resonance such as Y_1^{**} and to the resonance $N_{1/2\ 5/2}$ in the crossed channel ($\bar{K}+\bar{\Lambda}\rightarrow\bar{n}+\pi^-$ with $T=\frac{1}{2}$ and $J=\frac{5}{2}$), it turns out[31] that the resulting $\langle\mathbf{q}_1|t|\mathbf{q}_0\rangle$ contribution to our final R_3 is a flat, slowly varying function of the momenta involved—almost like a constant A_d.

In his first paper[32] Block assumed the presence of the latter ("direct") component only, and in his most recent article,[33] in which only the branching ratios of the total reaction rates for the bound-hyperfragment final state and the hyperon in the continuum have been estimated, he considers a Breit-Wigner $\langle|t|\rangle_{\text{res}}$ only (as in I, however, his $\langle|t|\rangle_{\text{res}}$ is nonrelativistic).

In the present article both the "direct" and the resonant terms are included, and the ratio A_d/a_r is treated as an adjustable parameter (it is naturally expected that the a_r component is the most important one). Their relative importance varies with energy. The $P_{3/2}$-resonant term alone gives the well-known $(1+3\cos^2\theta)$ angular distribution where $\theta=\measuredangle(\mathbf{q}_0,\mathbf{q}_1)$, i.e., the angle between $\mathbf{p}_3$ and the Λ-π relative momentum. On the other hand, such a simple form is insufficient to explain the corresponding observed angular distributions in a wide $\mathbf{p}_3$- and $\mathbf{p}_\pi$-momenta range. In particular, being symmetrical about 90°, it cannot account for the observed (rather small) forward-backward asymmetry.[2]

Let our $\bar{M}$ be the sum of the "direct" and the resonant terms ($\bar{M}=\bar{M}_{\text{dir}}+\bar{M}_{\text{res}}$). The reaction rate is proportional to the trace with respect to σ_Λ of $\bar{M}^\dagger\bar{M}$. With an arbitrary normalization we can express it as

$$R_{3\pi}=d\mathbf{p}_3d\mathbf{p}_\pi\int d\mathbf{p}_\Lambda\delta(\mathbf{p}_\Lambda+\mathbf{p}_3+\mathbf{p}_\pi)\delta(T_\Lambda+T_3+E_\pi{}^0-Q)\times\tfrac{1}{2}\,\mathrm{Tr}_{(\sigma_\Lambda)}(\bar{M}^\dagger\bar{M}). \quad (9)$$

In the simplest case of the nS capture we find

$$\tfrac{1}{2}\,\mathrm{Tr}_{(\sigma_\Lambda)}(\bar{M}^\dagger\bar{M})_{nS}=N_{nS}{}^2|\mathfrak{F}_{(3,n,\Lambda)}|^2\left\{|A_d|^2+\tfrac{1}{4}|a_r|^2\times\frac{q_0^2q_1^2}{(s^{1/2}-M_1^*)^2+\frac{1}{4}\Gamma_1^2}(1+3\cos^2\theta)+2\,\mathrm{Re}\left(\frac{A_da_r^*}{s^{1/2}-M_1^*-\frac{1}{2}i\Gamma_1}\right)q_0q_1\cos\theta\right\}. \quad (10)$$

---

[30] F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters **13**, 299 (1964).

[31] We are indebted to Dr. R. Vinh Mau and Dr. J. Letessier for this information.

[32] M. M. Block, Nuovo Cimento **20**, 715 (1961).

[33] M. M. Block, Proceedings of the CERN International Conference on Hyperfragments, St. Cergue, 1963 (unpublished).

A similar formula is obtained in the mP case.

The $|A_d|^2$ term gives an isotropic contribution while the interference (cross) term gives a $\cos\theta$ distribution. An angular distribution of the form of Eq. (10) can easily account for the observed asymmetry about 90°.

We have now to specify our Ψ_i and Ψ_f functions in order to calculate the form factor $\mathfrak{F}$. It is generally reasonable to separate the n-^{3}He and the Λ-^{3}He relative motions from the ξ intrinsic coordinates of the ^{3}He by assuming the factorized form:

$$\Psi_i=\psi_{(n-{}^3\mathrm{He})}(\mathbf{u})\phi_3(\xi)$$

and

$$\Psi_f=\psi_{(\Lambda-{}^3\mathrm{He})}{}^{(-)}(\mathbf{r})\phi_3(\xi).$$

The $\phi_3(\xi)$ function represents the ^{3}He component unchanged in the reaction (ground state). Such a simple form corresponds to well-known reasonable variational trial functions. A Gaussian $\Psi_{(n-{}^3\mathrm{He})}(\mathbf{u})$ S-state function is known to always give too sharp falloffs of $R_3(p_3)$ at large p_3 (cf., I). We have examined several other $\Psi_{(n-{}^3\mathrm{He})}$ and have finally chosen the Hulthén form $\Psi_{(n-{}^3\mathrm{He})}(u)=N_H(e^{-\mu u}-e^{-\nu u})u^{-1}$, where $\mu\cong0.8$ F^{-1} and $\nu\cong1.25$ F^{-1} so that it fits the ^{4}He$(p,2p)^3$H momentum distribution as given in Ref. 28. In addition, it turns out that this parameterization also assures approximately the correct nuclear mean-square radius of ^{4}He.

The Λ-^{3}He wave function is taken in the distorted-wave form (cf., Chand[5] or Kotani and Ross[19]):

$$\Psi_{(\Lambda-{}^3\mathrm{He})}{}^{(-)}(\mathbf{r})=e^{i\mathbf{q}_{\Lambda3}\mathbf{r}} +f^{(0)\dagger}(q_{\Lambda3})(e^{-iq_{\Lambda3}r}-e^{-\lambda r})/q_{\Lambda3}r, \quad (11)$$

where $\mathbf{q}_{\Lambda3}=(m_\Lambda\mathbf{p}_3-M_3\mathbf{p}_\Lambda)(M_3+m_\Lambda)^{-1}$; only the S wave is assumed to be distorted. The distortion amplitude can be expressed as

$$f^{(0)}(q_{\Lambda3})=(1/2i)(e^{i2\delta^{(0)}}-1),$$
$$\tan\delta^{(0)}=-q_{\Lambda3}a_0{}^{(\Lambda)}(1-\tfrac{1}{2}r_0a_0{}^{(\Lambda)}q_{\Lambda3}{}^2)^{-1}. \quad (12)$$

In the zero-energy ($q_{\Lambda3}\to0$) limit, $f^{(0)}\to-q_{\Lambda3}a_0{}^{(\Lambda)}\times(1+iq_{\Lambda3}a_0{}^{(\Lambda)})^{-1}$; here $a_0{}^{(\Lambda)}$ is the S-state scattering length and r_0 is the effective range; in the above formula $\delta^{(0)}$ is assumed real (elastic Λ-^{3}He scattering). The singlet S-state scattering appears to be the most important, as it corresponds to the ^{4}He bound state (spin $J=0$, cf., Ref. 34).

We have also examined a variant of this theory in which one makes the hypothesis of a purely surface character of the absorption process, i.e., a Butler-type stripping cutoff for the wave functions involved [$\Psi_{(n-{}^3\mathrm{He})}$ or $\Psi_{(\Lambda-{}^3\mathrm{He})}$ in our case]. This totally or partially eliminates the inner region from the Fourier integration (in $\mathfrak{F}_{(3,n,\Lambda)}$); we believe that this region contributes essentially only to other processes. Such a procedure has been applied by Dowker[35] for ^{4}He and by Adair[23] to the case of ^{12}C. However, we have found that the corresponding numerical results are not satisfactory at all, while on the other hand the theoretical justification of the method is not well enough established. Therefore we do not present any corresponding results for this variant of our theory.

The p_3 momentum distribution $R_3(p_3)$ is obtained by integrating $R_{3\pi}$ over $\mathbf{p}_\pi$, i.e., over $\mathbf{q}_1$; there is no restriction on the limits of the angular integration, which gives, therefore, a trivial constant. The integration over $q_1=|\mathbf{q}_1|$ is equivalent to the integration over $\sqrt{s}$ according to the modified energy conservation [cf., Eq. (8) of I]:

$$T_3+p_3{}^2/2(m_\Lambda+m_\pi)+[s(q_1)]^{1/2}-(Q+m_\Lambda)$$
$$=0=p_3{}^2/2m+[s(q_1)]^{1/2}-(Q+m_\Lambda), \quad (13)$$

where $m^{-1}=M_3{}^{-1}+(m_\Lambda+m_\pi)^{-1}$; here only the π-Λ relative motion is taken to be relativistic.

Actually, the integrand is a function of

$$\mathbf{P}_f(=-\mathbf{p}_\pi=-\mathbf{q}_1+m_\pi(m_\Lambda+m_\pi)^{-1}\mathbf{p}_3)$$

and of

$$\mathbf{q}_{\Lambda3}(=[-1+(\mu_{\Lambda3}\mu_{\pi\Lambda}/m_\Lambda{}^2)]\mathbf{p}_3-(\mu_{\Lambda3}/m_\Lambda)\mathbf{q}_1);$$

here

$$\mu_{\Lambda3}=M_3m_\Lambda(M_3+m_\Lambda)^{-1},$$
$$\mu_{\pi\Lambda}=m_\pi m_\Lambda(m_\pi+m_\Lambda)^{-1},$$

i.e., of p_3, q_1 and $\cos\theta(=\mathbf{p}_3\cdot\mathbf{q}_1/p_3q_1)$. Therefore, on expressing all the variables in terms of the last three and using Eq. (10) in the same way as we used Eq. (9) in I, we finally find

$$R_3(p_3)=q_1(\sqrt{s_0})\frac{dq_1{}^2}{d\sqrt{s}}\bigg|_{\sqrt{s}=\sqrt{s_0}} \cdot\int_{-1}^{1}d\cos\theta\tfrac{1}{2}\,\mathrm{Tr}_{(\sigma_\Lambda)}(\bar{M}^\dagger\bar{M})\bigg|_{\sqrt{s_0}}p_3{}^2dp_3, \quad (14)$$

where as in I,

$$\sqrt{s_0}=m_\Lambda+Q-p_3{}^2/2m,$$
$$q_1(\sqrt{s_0})=\tfrac{1}{2}[((\sqrt{s_0})^2-m_\Lambda{}^2-m_\pi{}^2)^2-4m_\Lambda{}^2m_\pi{}^2]^{1/2}s_0{}^{-1/2},$$
$$(dq_1{}^2/d\sqrt{s})=\tfrac{1}{2}[(\sqrt{s})^4-(m_\Lambda{}^2-m_\pi{}^2)^2]s^{-3/2}.$$

The analytic form of $\frac{1}{2}\mathrm{Tr}_{(\sigma_\Lambda)}\bar{M}^\dagger\bar{M}$ is given in Eq. (10) for the case of nS capture. Another possible way of calculating $R_3(p_3)$ would be to express all the momenta in terms of $\mathbf{q}_{\Lambda3}$ and $\mathbf{p}_\pi$; one integrates over $\cos\beta$ [$\beta\equiv\measuredangle(\mathbf{q}_{\Lambda3},\mathbf{p}_\pi)$] trivially and $q_{\Lambda3}$ is fixed by energy conservation:

$$q_{\Lambda3}{}^2/2\mu_{\Lambda3}+p_\pi{}^2/2(M_3+m_\Lambda)+E_\pi{}^0-Q=0. \quad (15)$$

In order to calculate the p_π distribution, $R_\pi(p_\pi)$, it is convenient to perform the $\mathbf{q}_{\Lambda3}$ integration ($d\mathbf{p}_3=-d\mathbf{q}_{\Lambda3}$; $\mathbf{p}_3=-\mathbf{q}_{\Lambda3}-\mu_{\Lambda3}m_\Lambda{}^{-1}\mathbf{p}_\pi$). The details of the direct $\mathbf{p}_3$ integration are given in I [Eqs. (10)–(14) of I].

[34] R. H. Dalitz and C. Rajasekharan, Phys. Letters **1**, 58 (1962); R. Prem and P. Steinberg, Phys. Rev. **136**, B1803 (1964).

[35] J. S. Dowker, Nuovo Cimento **22**, 218 (1961).

B. Contribution of the Σ-Λ Conversion

As pointed out already in Ref. 3 in the case of the analogous reaction with deuterium, the two-stage process with a Σ formation in the $\bar{K}$ absorption and its subsequent conversion into the final-state Λ in a collision with another nucleon of the nucleus may be of extreme importance for the far-off tail (large-momentum region) of the recoiling final-nucleus momentum distribution. We should expect a similar situation in the ^{4}He case. In their theoretical treatments Karplus and Rodberg[18] and Kotani and Ross[19] make the extreme hypothesis of even neglecting the direct Λ production amplitude altogether.

The process, being of the multiple-scattering kind, and as such corresponding to some lowest order cluster terms in the usual t-matrix expansion, can be visualized as in Fig. 1.

In the language of the multiple-scattering formalism one can write this second-order "cluster" amplitude as

$$\Delta M_{\rm conv}=\sum_{\Sigma',{}^3I'}\langle\Lambda^0\,{}^3{\rm He}|t_2|\Sigma'{}^3I'\rangle(E_f-E_{\Sigma'+{}^3I'})^{-1}\times\langle\pi^-\Sigma'{}^3I'|t_1|\bar{K}{}^4{\rm He}\rangle \qquad (16)$$

which corresponds to the two-stage process:

$$\bar{K}+{}^4{\rm He}\rightarrow\pi^-+\Sigma'+{}^3I'\rightarrow\pi^-+\Lambda^0+{}^3{\rm He};$$

$^3I'$ is the intermediate three-nucleon system (two neutrons and one proton or two protons and one neutron); the intermediate Σ' is a $\Sigma^{+\prime}$ or a $\Sigma^{0\prime}$, respectively. Any other processes involving an initial $\Sigma^{-\prime}+\pi^{0\prime}$ or $\Sigma^{-\prime}+\pi^{+\prime}$ production with the subsequent Σ' and pion reabsorption and conversion are of higher order and are not considered here. A calculation of $\Delta M_{\rm conv}$ based on Eq. (16) is presented in Appendix I. No directly corresponding numerical results are presented in the present publication. Instead, we perform below a calculation along the lines of the distorted-wave two-channel formalism of Karplus and Rodberg[18] and Kotani and Ross.[19] This calculation corresponds to replacing the intermediate state in Eq. (16) by one $\pi^-+\Sigma+{}^3I$ on-energy-shell state and the corresponding $\langle|t_2|\rangle(E_f-E_{\Sigma+{}^3I})^{-1}$ factor by an "equivalent" conversion probability amplitude $f_{\Sigma\Lambda}$.

In this formalism the $\Sigma+{}^3I$ wave function is represented by a one-column two-element matrix whose first element corresponds to the directly produced and elastically scattered Σ, and whose second element corresponds to the Σ-Λ conversion. The first one is that of Ref. 18 and does not interest us here; the $g_{\Sigma\Lambda}^{(-)}$ conversion function depends on $\mathbf{q}_{\Sigma3}=(M_3\mathbf{p}_\Sigma-m_\Sigma\mathbf{p}_3)\times(M_3+m_\Sigma)^{-1}$, the relative Σ-$^3I$ momentum. This latter function in the $\langle|t_1|\rangle$ reaction amplitude is then equivalent to the $\Sigma\langle|t_2|\rangle(E_f-E_{\Sigma+{}^3I})^{-1}$ factor in Eq. (16). Thus our new $\Delta M_{\rm conv}$ is now essentially of the form of the $\bar{M}$ of Eq. (4). Only inside the $\bar{M}$ of Eq. (4) does the Λ kinematics have to be replaced by the Σ kinematics related to $\mathbf{q}_{\Sigma3}$. The Σ-Λ conversion

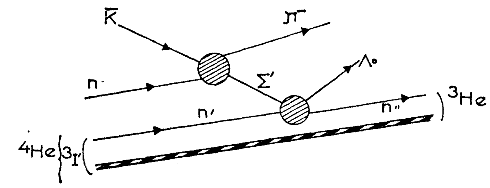

FIG. 1. A diagrammatic representation of the two-stage Σ-Λ conversion process: $\bar{K}+{}^4{\rm He}\rightarrow\pi^-+\Sigma'+{}^3I'\rightarrow\pi^-+\Lambda^0+{}^3{\rm He}$.

wave function is taken in analogy with Eq. (11) in the form:

$$\Psi_{\Sigma\Lambda}^{(-)}(\mathbf{r})=f_{\Sigma\Lambda}^{(0)\dagger}(q_{\Sigma3})(v_{\Lambda3}/v_{\Sigma3})^{1/2}\times\left((e^{-iq_{\Sigma3}r}-e^{-\lambda' r})/q_{\Lambda3}r\right), \qquad (17)$$

where the Σ-Λ conversion is assumed to take place in the S wave only; λ' is quite analogous to λ of Eq. (11), and the actual conversion amplitude $f_{\Sigma\Lambda}^{(0)}$ is determined in terms of the imaginary and the real parts of the complex S-wave Σ-N phase shift $\delta_\Sigma^{(0)}$ as in Eq. (35) of Ref. 18. In the lowest order approximation

$$\tan\delta_\Sigma^{(0)}\cong-q_{\Sigma3}(a_{\Sigma3}^{(0)}-i\eta_{\Sigma3}^{(0)})\equiv-q_{\Sigma3}(a_0^{(\Sigma)}-i\eta_0).$$

In this approximation we have for the physical Σ region (of real $q_{\Sigma3}$):

$$|f_{\Sigma\Lambda}^{(0)}|^2=\frac{q_{\Sigma3}\eta_0}{(1+q_{\Sigma3}\eta_0)^2+(q_{\Sigma3}a_0^{(\Sigma)})^2}. \qquad (18)$$

In the region in which only physical Λ's may be produced we take the absolute value squared of the analytic continuation of the function $f_{\Sigma\Lambda}$ from the region of real $q_{\Sigma3}$ to that in which $q_{\Sigma3}=-i\kappa_{\Sigma3}$, $\kappa_{\Sigma3}$ real. In this way we find[36]

$$|f_{\Sigma\Lambda}^{(0)}|^2|_{q=i\kappa}=\kappa_{\Sigma3}\eta_0/[(1-\kappa_{\Sigma3}a_0^{(\Sigma)})^2+(\kappa_{\Sigma3}\eta_0)^2]. \qquad (18a)$$

This represents the virtual (unphysical) intermediate Σ.

If we denote the $\bar{K}+{}^4{\rm He}\rightarrow\Sigma+{}^3I$ matrix element with the $f_{\Sigma\Lambda}$ conversion amplitude by $\Delta\bar{M}_{\rm conv}(\mathbf{q}_{\Sigma3},\mathbf{p}_\pi)$, we can write the total correction matrix element with the $\pi^-+\Lambda^0+{}^3{\rm He}$ final-state momentum and energy-conservation delta functions in the form

$$\Delta M_{\rm conv}\cong\delta\left((p_3^2/2m)+s^{1/2}-m_\Lambda-Q\right)\times\delta(\mathbf{p}_\Lambda+\mathbf{p}_\pi+\mathbf{p}_3)\int d\mathbf{p}_{3'}\delta(\mathbf{p}_{3'}+\mathbf{p}_{\Sigma'}+\mathbf{p}_\pi)\times\int d\left(\frac{q_{\Sigma'3'}^2}{2\mu_{\Sigma3}}\right)\delta\left(\frac{q_{\Sigma'3'}^2}{2\mu_{\Sigma3}}+\frac{p_\pi^2}{2(M_3+m_\Sigma)}+E_\pi^0-Q'\right)\times\Delta\bar{M}_{\rm conv}(\mathbf{q}_{\Sigma'3'},\mathbf{p}_\pi), \qquad (19)$$

[36] This is in the following sense: we can put $f_{\Sigma\Lambda}^{(0)}(q_{\Sigma3})$ in the form $f_{\Sigma\Lambda}^{(0)}(q)=(q\eta_0)^{1/2}(1+q\eta_0+iqa_0^{(\Sigma)})^{-1}$; we write now $f_{\Sigma\Lambda}^{(0)}(i\kappa)=(i\kappa\eta_0)^{1/2}(1-\kappa a_0^{(\Sigma)}+i\kappa\eta_0)^{-1}$; Eq. (18a) gives just the square of the absolute value of this function. It should be pointed out that this formula (Ref. 18) corresponds to the analytic continuation of the amplitude squared as in the unitarity relation, i.e., to $f_{\Sigma\Lambda}^{(0)}(q)f_{\Sigma\Lambda}^{(0)*}(q^*)$; the asterisk denotes complex conjugation.

where $Q' \equiv M_4 - M_3 + m_K - m_\Sigma \equiv Q - \Delta Q$, $\Delta Q = m_\Sigma - m_\Lambda$. In Eq. (19) we simply set ^{3}He in place of $^3I'$. In this sense the ^{3}He (ground state) and the final π^- are already produced in the first stage of the reaction; by charge conservation our Σ' means then a Σ^0. However, we consider the parameter η_0 as an "equivalent" parameter from the point of view of the combined effect of both these channels $\Sigma' = \Sigma^{0'}$ and $\Sigma' = \Sigma^{+'}$ in a way which is manifestly isospin-independent; consequently, we also take $m_\Sigma = \bar{m}_\Sigma$, the average mass of Σ^0 and Σ^+. The inserted integrated δ functions fix the total $\Sigma + {}^3$He ($\Sigma' + {}^3I'$) momentum as equal to $-\mathbf{p}_\pi$ and the kinetic energy (or $|q_{\Sigma'3'}|$) of the $\Sigma - {}^3$He relative motion in the way required by total energy conservation [in an interval of p_π (real), $q_{\Sigma'3'}$ even becomes imaginary]. In this way $q_{\Sigma'3'}$ is expressed as a function of p_π, and so also $\Delta\bar{M}_{\text{conv}}$. In calculating the interesting correction term to the function $R_3(p_3)$, $\Delta R_3^{(\text{conv})}(p_3)$, we again employ the method of I or of Eq. (14). As $\mathbf{p}_\pi = \mathbf{q}_1 - m_\pi(m_\Lambda + m_\pi)^{-1}\mathbf{p}_3$ and $|\mathbf{q}_1| = q_1$ is determined from $\sqrt{s_0}$ of the final and initial total energy conservation, the only integration left is over $\cos\theta$, where $\theta = \sphericalangle(\mathbf{p}_3, \mathbf{q}_1)$. In this way we finally obtain

$$\Delta R_3^{\text{conv}}(p_3) = q_1(s_0^{1/2}(p_3)) \frac{dq_1^2}{d\sqrt{s}}\bigg|_{\sqrt{s}=s_0^{1/2}(p_3)} \times \int_{-1}^{1} d\cos\theta |\Delta\bar{M}_{\text{conv}}|^2 \bigg|_{\sqrt{s}=s_0^{1/2}(p_3)}, \quad (20)$$

where $|\Delta\bar{M}|^2|_{\sqrt{s}=s_0^{1/2}(p_3)}$ is a function of the following variables:

$$\{q_{\Sigma'3'}(p_\pi), p_\pi\}$$

where

$$p_\pi = p_\pi[q_1(\sqrt{s_0}(p_3)), p_3, \cos\theta].$$

The corresponding interference correction term with the "direct" plus the Y_1^* amplitude is given by the same formula of Eq. (20) with $|\Delta\bar{M}_{\text{conv}}|^2$ replaced by $2\,\text{Re}\{\bar{M}^*\Delta\bar{M}_{\text{conv}}\}$ (always in the sense of $\frac{1}{2}\,\text{Tr}_{(\sigma_\Lambda)}$).

We replace the relevant t_1 matrix element involved in $\Delta\bar{M}_{\text{conv}}$ by an "equivalent" constant A_c in our numerical computations. However, we also discuss below the possible Y_0^* (1405 MeV, $S_{1/2^-}$, $\frac{1}{2}\Gamma_0 \cong 25$ MeV) and also the Y_0^{**} resonance formation for the (Σ,π) system in a Breit-Wigner form of the corresponding t_1. From the corresponding energy conservation with $(s_0')^{1/2}(p_3)$ and $M_0^* = 1405$ MeV, we find the value of p_3 corresponding to this Y_0^* as $p_{3\text{res}}(Y_0^*) \cong 112$ MeV/c. However, owing to the large value of Γ_0 and to the kinematics of our conversion amplitude, the effect of this Y_0^* is important, not around this $p_{3\text{res}}(Y_0^*)$, but only for high p_3 (>250 MeV/c). The Y_0^{**} (1519 MeV, $\frac{3}{2}^-$, $\frac{1}{2}\Gamma_0 = 8.2$ MeV) resonance corresponds to an almost "flat" t_1 element which is quite compatible with A_c.

On replacing $\langle|t_1|\rangle$ by A_c, employing the approximation of Eq. (17) for the hyperon wave function, and taking our $\Psi_{(n-{}^3\text{He})}(u)$ as the Hulthén function of Sec. 2(A), we can put $\Delta\bar{M}_{\text{conv}}$ in the form

$$\begin{aligned}\Delta\bar{M}_{\text{conv}} = {} & A_c\{\mu_{\Sigma 3}q_{\Sigma'3'}/\mu_{\Lambda 3}q_{\Lambda 3}\}^{1/2} f_{\Sigma\Lambda}^{(0)}(q_{\Sigma'3'}) \\ & \times (4\pi i/2q_{\Sigma'3'}\sigma)\{\ln[(q_{\Sigma'3'}+\sigma+i\mu)(q_{\Sigma'3'}-\sigma+i\nu)/ \\ & (q_{\Sigma'3'}-\sigma+i\mu)(q_{\Sigma'3'}+\sigma+i\nu)] - \ln[(\sigma+i(\mu+\lambda')) \\ & \times(-\sigma+i(\nu+\lambda'))/(-\sigma+i(\mu+\lambda')) \\ & \times(\sigma+i(\nu+\lambda'))]\}, \quad (21)\end{aligned}$$

where $q_{\Lambda 3}$ is determined as a function of p_π as $q_{\Sigma'3}$ is, and $\sigma \equiv M_3(M_3 + m_\Sigma)^{-1}p_\pi$. The difference between the Σ and the Λ kinematics, caused mainly by the Σ-Λ mass difference ΔQ and the structure of $\Delta\bar{M}_{\text{conv}}$ of Eq. (21), shifts the peak of $\Delta R_3^{(\text{conv})}(p_3)$ to a value of p_3 much higher than that of the maximum of $R_3(p_3)$. As a consequence, also, the corresponding interference (cross) term is rather small. It should be remarked that higher resonances such as $Y_0^{**} = Y_{0,3/2}^*$(1519 MeV) and $Y_{0,5/2}^*$(1815 MeV) cannot produce any peak of $\Delta R_3^{(\text{conv})}(p_3)$, since they give only "flat" contributions to $\langle|t_1|\rangle$, just compatible with our A_c (a complex constant).

So far we have not discussed the distribution in the pion momentum $R_\pi(p_\pi)$. In calculating $R_\pi(p_\pi)$ we express all the vectors involved in terms of $\mathbf{p}_\pi$ and $\mathbf{q}_{\Lambda 3}$ (cf. I), i.e., in terms of p_π, $q_{\Lambda 3}$, and the angle $\beta = \sphericalangle(\mathbf{p}_\pi, \mathbf{q}_{\Lambda 3})$; the absolute value of $\mathbf{q}_{\Lambda 3}$, $q_{\Lambda 3}$, is fixed as a function of p_π from the energy conservation of Eq. (15); the variable $q_{\Sigma'3'}$ occurring in $\Delta\bar{M}_{\text{conv}}$ is expressed in terms of p_π as in Eq. (19) in the same way; thus the only integration left is that over $\cos\beta$:

$$R_\pi(p_\pi) = 4\pi\mu_{\Lambda 3}q_{\Lambda 3}(p_\pi)p_\pi^2 dp_\pi \times \int_{-1}^{1} d\cos\beta |\bar{M} + \Delta\bar{M}_{\text{conv}}|^2 \bigg|_{\substack{q_{\Lambda 3}=q_{\Lambda 3}(p_3),\\ q_{\Sigma'3}=q_{\Sigma'3}(p_3)}} (\cos\beta). \quad (22)$$

We know from I that it is rather easy to explain the large-p_π ($\geqslant 256$ MeV/c) region of the $R_\pi(p_\pi)$ spectrum. It is only the contribution of $\Delta\bar{M}_{\text{conv}}$ which can account for the low-p_π part of the experimental histogram, as follows from the corresponding kinematics.

3. NUMERICAL RESULTS AND DISCUSSION

A. Λ Production without the Σ-Λ Conversion

All the numerical computations of $R_3(p_3)$ and $R_\pi(p_\pi)$ presented below were performed on the UNIVAC computer of the Faculté d'Orsay. For these calculations the nuclear wave function $\psi_{(n-{}^3\text{He})}(u)$ in its Hulthén form has been specified in Sec. 2(A). For the hypernuclear wave function $\Psi_{(\Lambda-{}^3\text{He})}^{(-)}(\mathbf{r})$ of Eq. (11) we have considered several cases of the distortion scattering amplitude $f^{(0)}$ and of the parameter λ. In our first study we considered $\bar{A}_0^{-1} = \bar{a}_0^{-1} - \frac{1}{2}r_0 q_{\Lambda 3}^2$ as a sort of adjustable parameter with several positive and negative values of $\bar{a}_0 \equiv a_0^{(\Lambda)}$ within a "reasonable" interval and with "reasonable" r_0 of the order of the ^{3}He nuclear radius. We consider here $\eta_0^{(\Lambda)} = 0$ [i.e., we take only the real

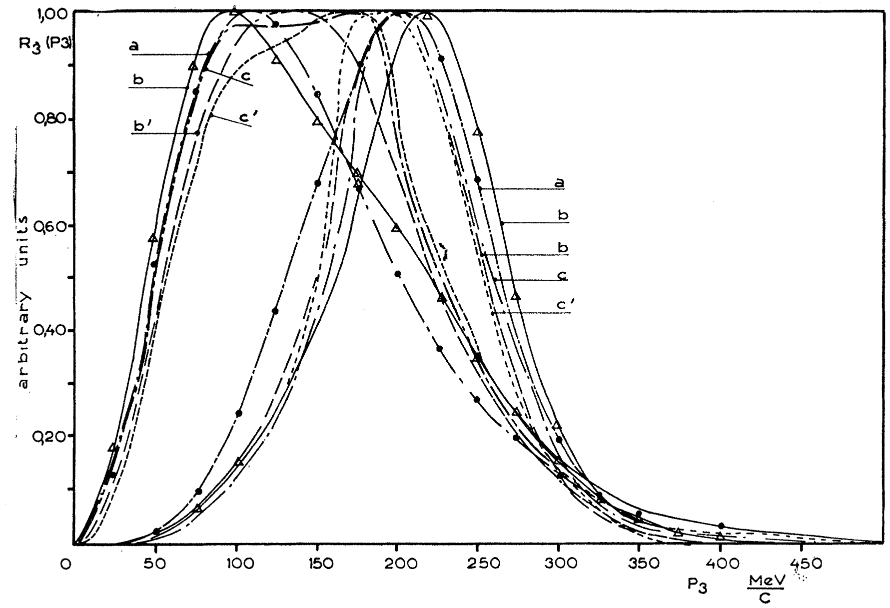

FIG. 2. The recoiling ^{3}He momentum distribution $R_3(p_3)$ for the $\bar{K}+{}^4\text{He}\to\pi^-+\Lambda^0+{}^3\text{He}$ reaction (nS $\bar{K}$ orbit) with an arbitrary normalization and with the following Λ-^{3}He distortion parameters:

(a)	$a_0^{(\Lambda)}=0$,	$r_0=0$	$\lambda=0$ (no distortion);
(b)	$a_0^{(\Lambda)}=4.54$ F,	$r_0=1.76$ F	$\lambda=1.35$ F^{-1};
(b′)	$a_0^{(\Lambda)}=-4.54$ F,	$r_0=1.76$ F	$\lambda=1.85$ F^{-1};
(c)	$a_0^{(\Lambda)}=10.0$ F,	$r_0=1.76$ F	$\lambda=1.55$ F^{-1};
(c′)	$a_0^{(\Lambda)}=-10.0$ F,	$r_0=1.76$ F	$\lambda=1.55$ F^{-1}.

The first five curves from the left (small p_3) refer to the "direct" ($\propto|A_d|^2)R_3(p_3)$ the other five ones to the Y_1*-"resonant" ($\propto|a_r|^2)R_3(p_3)$.

part of $\tan\delta^{(0)}=-q_{\Lambda 3}\bar{A}_0$]. In our crude theory we do not consider definite final Λ spin states. As a consequence, either $\bar{a}_0=a_0^{(\Lambda)}$ can be considered an "average equivalent" scattering length or we limit ourselves to one (singlet or triplet) particular final-spin scattering state. Out of several possibilities we have decided to fix the parameter λ from the effective-range theory in terms of the corresponding $a_0^{(\Lambda)}$ and r_0 as was done by Chand[5] in the case of deuterium:

$$\lambda=(3/2r_0)[1+(1-16r_0/9a_0^{(\Lambda)})^{1/2}].$$

The corresponding $R_3(p_3)$ results are compared in Fig. 2 with those with no distortion ($a_0^{(\Lambda)}=0=r_0$). The case of $a_0^{(\Lambda)}=+4.54$ F corresponds to the (seemingly most important) singlet Λ-^{3}He ($S=0$) scattering; in fact the Λ-^{3}He bound state, $_\Lambda{}^4$He, is now well known[34] to be a singlet state, $J=0$. This value $a_0^{(\Lambda)}=4.54$ F has been decided on from the following analysis:

(a) We take $r_0=1.76$ F, the mean-square radius in ^{4}He from Ref. 37, and from the $_\Lambda{}^4$He Λ binding energy $=2.25$ MeV[38] we determine $a_0^{(\Lambda)}$.

(b) We take a Gaussian Λ-nucleon potential from Eq. (23) of Ref. 29. We then calculate the Λ-^{3}He range $b'\cong2.24$ F and the well depth from the parameter $K=60$ MeV of Ref. 29; the value of $b'=2.24$ F corresponds to $b=1.48$ F of Ref. 29 and the ^{3}He radius $=1.60$ F; taking the Λ-^{3}He $J=0$, we find, with the help of the tables of Levee and Pexton,[39] $a_0^{(\Lambda)}\cong4.10$ F, $r_0\cong1.61$ F.

(c) The same procedure, with the ^{3}He radius $=1.94$ F,[37] yields $a_0^{(\Lambda)}=5.70$ F and $r_0=1.95$ F.

(d) With $b'\cong2.53$ F ($R_{^3\text{He}}=1.94$ F) and the Λ binding energy $=2.25$ MeV, we find $a_0^{(\Lambda)}=4.53$ F, $r_0=1.80$ F.

Finally, we decide to take the result (a). Although such a detailed analysis is probably not warranted with our crude approximations, it is interesting to see that our best $a_0^{(\Lambda)}$ can even be "derived" from known Λ-nucleon phenomenological potentials. Finally, we determined λ from our $a_0^{(\Lambda)}=4.54$ F and $r_0=1.76$ F from the above formula as $\lambda=1.35$ F^{-1}.

We see from Fig. 2 that $R_3(p_3)$ is rather insensitive to $a_0^{(\Lambda)}$, i.e., to the details of the Λ-^{3}He elastic-scattering distortion in our S-wave low-energy approximation; in fact, no fundamental improvement has been obtained in this way relative to the results of I, contrary to previous speculations (cf., I).

All the curves presented in Fig. 2 are normalized to the same common maximum. The Y_1*-resonance curve [corresponding to the term $\propto|a_r|^2$ in Eq. (10)] with no distortion ($a_0^{(\Lambda)}=r_0=0$) is not quite equal to the corresponding curves of Fig. 1 of I, not only because

[37] F. Beck and U. Gutsch, Phys. Letters **14**, 133 (1965).
[38] K. Dietrich, H. J. Mang and R. Folk, Nucl. Phys. **50**, 177 (1964).
[39] R. D. Levee and R. L. Pexton, University of California Radiation Laboratory Report UCRL-7155, 1963 (unpublished).

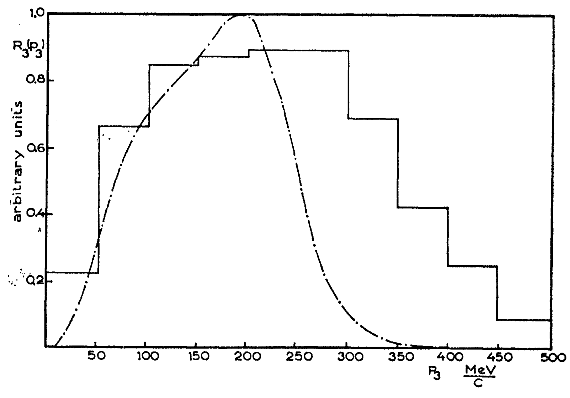

FIG. 3. The recoiling ^{3}He momentum distribution $R_3(p_3)$ for the $\bar{K}+{}^4\text{He}\rightarrow\pi^-+\Lambda^0+{}^3\text{He}$ reaction (nS $\bar{K}$ orbit) with an arbitrary normalization and with the following parameter values: $A_d/a_r=(-0.209+i0.273)\times10^3$ (with all the momenta in units MeV/c, $\sqrt{s}$ in MeV), $a_0^{(\Lambda)}=4.54$ F, $r_0=1.76$ F, $\lambda=1.35$ F, and with no Σ-Λ conversion ($\eta_0=0$); the experimental histogram is given for the weighted sum of all the data of Ref. 2.

of the difference in the respective nuclear wave functions $\Psi_{(n-{}^3\text{He})}(u)$, but also because of the factor $q_0^2q_1^2$ which was absent in I. However, the difference is quite small; in fact, $q_0^2q_1^2\cong\text{const}\times p_3^2$ $(p_{3\,\max}^2-p_3^2)$ and its variation with p_3 is almost negligible compared with the variation of other factors (its value at the resonance Y_1^* is 0.64 of its maximum value).

In Fig. 2, we have not plotted the direct-resonant interference term [$\propto A_da_r^*$ in Eq. (10)]. In fact, it is our aim to first determine the ratio a_r/A_d, i.e., its absolute value $|a_r/A_d|$ and its phase angle ϕ; the complex number a_r/A_d is practically our only adjustable parameter. We determine it from two data: (1) the angular distribution as given by Eq. (10) and (2) the best fit possible to the experimental histogram $R_3(p_3)$. As for (1), we actually limit ourselves to fixing the phase angle ϕ from the "fore-aft" asymmetry (the difference $F-A\equiv\int_0^1 d\cos\theta|\bar{M}|_{\sqrt{s_0}}{}^2-\int_{-1}^0 d\cos\theta|\bar{M}|_{\sqrt{s_0}}{}^2$). From the θ-angular-distribution histogram of Ref. 2 for $p_3\leqslant200$ MeV/c, the ratio $(F-A)(F+A)^{-1}$ appears to be 0.176 (the angle θ^* of Ref. 2 is actually $\theta^*=180°-\theta$ with our definition of θ). Fixing our p_3 at 150 MeV/c we determined our ϕ from this with a "reasonable" value of $|a_r/A_d|$. The resulting interference term is quite small, and the values of $R_3^{(\text{dir})}(p_3)$ and $R_3^{(\text{res})}(p_3)$ are about equal at $p_3=150$ MeV/c. The "best fit" so obtained is presented in Fig. 3. We see that our "best" elastic distortion is still quite incapable of explaining the large-p_3 behavior of $R_3(p_3)$. Unfortunately, the interference term with the phase ϕ so determined is destructive for $p_3\gtrsim p_{3\text{res}}$, and so the stringent requirement of fitting the data[2] for the θ angular distribution imposed on A_d worsens the agreement with the observed $R_3(p_3)$ distribution. On the other hand, the θ-asymmetry (θ-distribution) data mentioned appear to be very poor (e.g., they refer to wide p_3 intervals such as $0\leqslant p_3\leqslant200$ MeV/c, and the statistics are very poor), and in the following, in our final discussion, we consider also the case where the above ϕ phase condition on A_d has been relaxed in order to improve the agreement with $R_3(p_3)$ of Ref. 2.

Let us see now how the distributions change when we consider K nuclear capture from an mP Bohr orbit instead of an nS one. In order to get an idea of the situation it appears sufficient to consider the cases with no distortion in the final states. In fact, as is seen from Fig. 2, such distortions modify the $R_3(p_3)$ distribution only a little, and on the other hand, the formula of Eq. (7) simplifies greatly. If one considers the direct ($\propto|A_d|^2$) and the Y_1^*-resonant ($\propto|a_r|^2$) terms of $R_3(p_3)$ separately, one finds for the mP capture in this approximation the following respective final expressions:

$$R_3^{(\text{dir})mP}(p_3)=\tfrac{3}{16}N_{mP}^2|A_d|^2q_1(s_0^{1/2})(dq_1^2/d(s)^{1/2})|_{\sqrt{s}=\sqrt{s_0}}\times p_3^2[\partial I(p_3)/\partial p_3]^2. \quad (23)$$

$$R_3^{(\text{res})mP}(p_3)=N_{mP}^2\frac{|a_r|^2q_1^3(\sqrt{s_0})p_3^2}{(\sqrt{s_0}-M_1^*)^2+\frac{1}{4}\Gamma_1^2}\left(\frac{dq_1^2}{d(s)^{1/2}}\right)_{\sqrt{s}=\sqrt{s_0}}\times\{\tfrac{3}{16}q_0^2[\partial I(p_3)/\partial p_3]^2+I^2(p_3)-\tfrac{1}{2}\cdot q_0I(p_3)[\partial I(p_3)/\partial p_3]\}, \quad (24)$$

where $I(p_3)\equiv(2\pi)^{-3}\int e^{i\mathbf{p}_3\cdot\mathbf{r}}\psi_{(n-{}^3\text{He})}(r)d\mathbf{r}$, and where all the other symbols have been defined above [cf., Eq. (14), etc.].

The curves (1a) and (1b) in Fig. 4 refer to the "direct" and the "resonant" nS capture; the ones labeled (2a) and (2b) to the same for mP capture, respectively. The "direct" $R_3(p_3)$ for mP is displaced only a little towards higher p_3 relative to the corresponding nS curve, especially in the interval $p_3\lesssim100$ MeV/c, and is slightly narrower than that "direct"

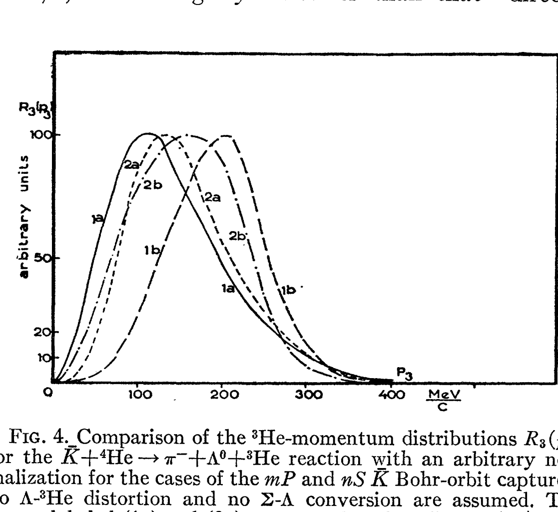

FIG. 4. Comparison of the ^{3}He-momentum distributions $R_3(p_3)$ for the $\bar{K}+{}^4\text{He}\rightarrow\pi^-+\Lambda^0+{}^3\text{He}$ reaction with an arbitrary normalization for the cases of the mP and nS $\bar{K}$ Bohr-orbit captures; no Λ-^{3}He distortion and no Σ-Λ conversion are assumed. The curves labeled (1a) and (2a) correspond to the "direct" ($\propto|A_d|^2$) $R_3^{(\text{dir})}(p_3)$ for the nS and mP cases, respectively; the curves (1b) and (2b) correspond to the Y_1^* "resonant" ($\propto|a_r|^2$) $R_3^{(\text{res})}(p_3)$ for the same cases, respectively.

nS. The situation is just reversed for the corresponding Y_1*-"resonant" $R_3(p_3)$: the *mP* curve is shifted towards lower p_3 relative to its *nS* counterpart and forms a broader peak; the difference between the two is particularly marked in the region up to $p_3 \cong 200$ MeV/c. The general over-all effect of the combined "direct" and "resonant" $R_3(p_3)$ for the *mP* capture would result in a narrower p_3 distribution than that for the *nS* case, thus giving a worse agreement with the observed $R_3(p_3)$ of Ref. 2.

As for the absolute values of $R_3(p_3)$ in the case of the *mP* capture, we find (for large n and m) the following ratios of the maxima of $R_3(p_3)$ for the *mP* and the *nS* cases:

(a) for the "direct" capture:

$$R_3^{(\mathrm{dir})mP}(p_3)_{\max}/R_3^{(\mathrm{dir})nS}(p_3)_{\max} \cong 4.8\times 10^{-5}(n/m)^3$$

(b) for the "resonant" capture:

$$R_3^{(\mathrm{res})mP}(p_3)_{\max}/R_3^{(\mathrm{res})nS}(p_3)_{\max} \cong 0.5\times 10^{-2}(n/m)^3.$$

Thus we see that unless $m \ll n$, if the populations of the respective *mP* and *nS* orbits are equal, the *mP* contribution is quite negligible as compared with the *nS* one in the "direct" case and very small in the "resonant" case. Consequently, we confine ourselves to the *nS* case in the following (only a great depletion of an *nS* population relative to an *mP* one could make the latter one non-negligible).

The θ angular distribution in the *mP* "resonant" case is

$$F_{mP}^{(\mathrm{res})}(\theta) = \tfrac{1}{4}(1+3\cos^2\theta)\left[\tfrac{3}{16}q_0^2(\partial I(p_3)/\partial p_3)^2 - \tfrac{1}{2}q_0 I(p_3)(\partial I(p_3)/\partial p_3)\right] + \tfrac{1}{2}I^2(p_3). \quad (25)$$

In the case of the $R_\pi(p_\pi)$ distribution the *mP*-*nS* difference is less marked, as the pion kinematics is the essential determining effect there.

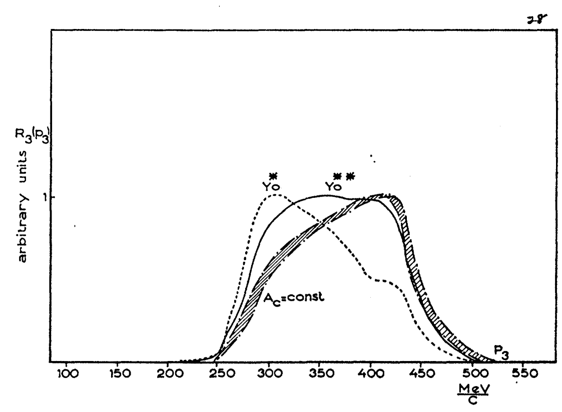

FIG. 5. The contribution from the Σ-Λ conversion to the ^{3}He momentum distribution, $R_3^{(\mathrm{conv})}(p_3)$, with an arbitrary normalization and with the following parameters for the wave function of Eq. (17): the curves within the hatched area correspond to: $-4.0\ \mathrm{F} \leqslant a_0^{(\Sigma)} \leqslant -2.0\ \mathrm{F}$, $0.1\ \mathrm{F} \leqslant \eta_0 \leqslant 1.0\ \mathrm{F}$, $\lambda' = 1.35\ \mathrm{F}^{-1}$, A_c=const, the curves labeled Y_0* and Y_0** correspond to $a_0^{(\Sigma)} = -2.0$ F, $\eta = 1.0$ F, $\lambda' = 1.35\ \mathrm{F}^{-1}$ and A_c replaced by the appropriate Y_0*- and Y_0**-resonance t matrix elements, respectively.

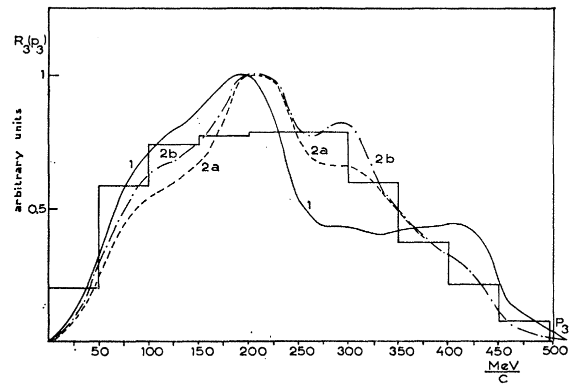

FIG. 6. The ^{3}He momentum distribution $R_3(p_3)$ for the $\bar{K}+{}^4\mathrm{He} \rightarrow \pi^- + \Lambda^0 + {}^3\mathrm{He}$-reaction (*nS* $\bar{K}$ orbit) with an arbitrary normalization and the following parameters: (1) $a_0^{(\Lambda)} = 4.54$ F, $r_0 = 1.76$ F, $\lambda = 1.35\ \mathrm{F}^{-1}$, $A_d/a_r = (-0.209+i0.273)\times 10^3$ (all the momenta in MeV/c, $\sqrt{s}$ in MeV) and for the Σ-Λ conversion: $a_0^{(\Sigma)} = -2.0$ F, $\eta_0 = 1.0$ F, $\lambda' = \lambda$ and $A_c/|A_d| = 2.1$. In (2a) the interference terms corresponding to all the amplitudes involved are neglected, and all the parameters are as in curve (1) except that A_d/a_r is adjusted to assure $R_3^{(\mathrm{dir})}(p_3) = R_3^{(\mathrm{res})}(p_3)$ at $p_3 = 150$ MeV/c and A_c is replaced by the appropriate Y_0* t-matrix element, and the relative "Σ/Λ normalization" is defined by $R_3^{(\mathrm{conv})}(p_3)_{\max} = \frac{1}{2}R_3^{(\mathrm{nonconv})}(p_3)_{\max}$; finally, (2b) is identical with (2a) except that the Y_1*-resonance width $\Gamma_1 = 40$ MeV is replaced by $\Gamma_1 = 53$ MeV as in Ref. 52. The experimental histogram is given for the weighted sum of all the data of Ref. 2 and is normalized so that its area is equal to that of our curve (1).

B. Contribution of the Σ-Λ Conversion

With our formalism of Sec. 2(B) the parameters to be determined are $a_{\Sigma 3}^{(0)} \equiv a_0^{(\Sigma)}$, $\eta_{\Sigma 3}^{(0)} \equiv \eta_0$, λ', and the ratio A_c/A_d. No experimental data seem to exist as yet from which one could determine $a_0^{(\Sigma)}$ and η_0. One could either "derive" them from the quite uncertain theoretical Σ-nucleon potentials available or via the basic "plausible" Σ-nucleon scattering lengths. Unfortunately, even the latter are not directly available to our knowledge, and could at best be roughly estimated from the known low-energy total Σ-nucleon cross sections. In the extreme zero-energy limit an average $a_{\Sigma N}^{(0)}$ could be estimated from the elastic Σ-nucleon scattering cross section, and the corresponding $\eta_{\Sigma N}^{(0)}$ could be estimated approximately from the inelastic ("absorption") Σ-N cross sections.

Another point of view is to consider $a_0^{(\Sigma)}$ and η_0 rather as adjustable parameters of a sort within a reasonable interval (sign and order of magnitude). The absence of a Σ-^{3}He bound state tends to indicate a negative sign for $a_0^{(\Sigma)}$, while η_0 is obviously positive. The order of magnitude of $a_0^{(\Sigma)}$ should be the same as that of $a_0^{(\Lambda)}$, and that of η_0 can be seen from the above-mentioned average available low-energy $\Sigma + N \rightarrow \Lambda + N'$ experimental (and theoretical) conversion cross sections and from other (e.g., symmetry) considerations. Thus we can establish the "reasonable" ranges of variation for our adjustable $a_0^{(\Sigma)}$ and η_0. This is the approach which we adopted in our computations leading to the results presented in Figs. 5 and 6.

As for η_0, which is most important, let us consider first the corresponding hyperon-nucleon parameter. For $\eta_{\Sigma N}^{(0)} \equiv \eta_{\Lambda N}^{(0)}$ (by assumption as in the formalism of Refs. 18 and 19) we can consider first, e.g., the data due to Crawford *et al.*[40] $[\sigma_{\text{tot}}(\Lambda^0 p \to \Sigma^+ n)]$ and Alexander *et al.*[41] $[\sigma_{\text{tot}}(\Lambda^0 p \to \Sigma^0 p)]$. If we take an average of these two cross sections, $\bar{\sigma}_r$, and apply the zero-energy limit formula $\bar{\eta}_{\Lambda N}^{(0)} = (4\pi)^{-1} \bar{\sigma}_r q_{\Lambda N}^2 q_{\Sigma N}^{-1}$, we obtain: $\bar{\eta}_{\Lambda N}^{(0)} = \bar{\eta}_{\Sigma N}^{(0)} \cong 0.3$–$0.75$ F; from the theoretical $\bar{\sigma}_r$ $(\Sigma N \to \Lambda N)$ of de Swart and Dullemond[42] we obtain $\bar{\eta}_{\Sigma N}^{(0)} \cong 0.5$–$0.9$ F; a speculation based on the ratios of the pertinent fundamental coupling constants following from the $SU(3)$ group symmetry (cf., Refs. 43–45) as well as those following from the ratio of the conversion and the Σ-N elastic scattering total cross sections together with the observed value of the latter (cf., Stannard[46]) gives the same order of magnitude $\bar{\eta}_{\Sigma N}^{(0)}$. By analogy, with this we feel it reasonable to consider in our Σ-^{3}He case $\eta_0 = 0.1$, 0.5, and 1.0 F, which all fall within the same "reasonable" interval. Actually, as we see from Fig. 5, the general p_3 distribution of our $R_3^{(\text{conv})}(p_3)$ is quite insensitive to η_0; in fact, only the absolute value of A_c/A_d has to be varied considerably with a variation of η_0.

As for $a_{\Sigma N}^{(0)}$, the total $\sigma_{\text{el}}(\Sigma^+ N)$ as measured by Stannard[46] seems to indicate[47]

$$|\bar{a}_{\Sigma N}^{(0)}| = \{\tfrac{1}{4}[a_{\Sigma N}^{(0)}(s=0)]^2 + \tfrac{3}{4}[a_{\Sigma N}^{(0)}(s=1)]^2\}^{1/2} \cong 0.6 \text{ F}.$$

For $\bar{a}_{\Sigma 3}^{(0)} \equiv a_0^{(\Sigma)}$ we have considered $a_0^{(\Sigma)} = -2$ and -4 F which appear to be reasonable, particularly in view of the fact that here again only the absolute value of $R_3^{\text{conv}}(p_3)$ is relatively sensitive to $a_0^{(\Sigma)}$.

In Fig. 5 the net (quadratic) contribution of the conversion amplitude to $R_3(p_3)$, $R_3^{(\text{conv})}(p_3)$, is presented for several cases with an arbitrary common normalization and with $a_0^{(\Sigma)}$ between -2 and -4 F; the cluster of curves corresponding to values of η_0 falling between 0.1 and 1 F all lie so close to each other that we have marked them as a shaded area. The curves labeled Y_0^* and Y_0^{**}, corresponding to A_c being replaced by the Y_0^*- and the Y_0^{**}-resonance t-matrix elements, respectively, are not very different from the ones with A_c=const. The Y_0^{**} resonance (and the pertinent higher resonances) give contributions rather compatible with A_c=const., as expected. The Y_0^* distribution is shifted towards smaller p_3 relative to the others [its peak falls at $p_3 \cong 300$ MeV/c instead of at $\cong 400$ MeV/c as for $R_3(p_3)$ with A_c=const], but the bulk of the $R_3^{(\text{conv})}(p_3)$ distribution remains quite similar. The conversion contribution appears to be negligible below $p_3 \lesssim 250$ MeV/c and is peaked only at high p_3, dropping to zero at $p_{3\max}$. The kinematical conditions are decisive for the form of $R_3^{(\text{conv})}(p_3)$.

As for the ratio A_c/A_d, we choose it so as to give the best fit of the over-all final $R_3(p_3)$ to the observed one, remaining at the same time within a "reasonable" range (to an order of magnitude), i.e., it has to be compatible with the observed or expected Σ-to-Λ $\bar{K}$-production-rate ratios. From the 1960 data[2] the ratio of all the $\bar{K}$-He absorption events with the Σ^+ in the final state to those with the Λ appears to be 136:165. This, however, (with the suggested[2] "conversion ratio" $4\epsilon = 0.57$) could at best give only an idea of the order of magnitude of the ratio $|A_c|^2/|A_d|^2$, where the "equivalent" A_c comprises the effect of all the Σ-π resonances, while A_d does not represent the contribution of Y_1^*. In fact, also, the corresponding Σ- and Λ-channel respective theoretical momentum distributions and thus their pertinent integrals are different, and the number of "speculative" points in such a comparison with the branching ratio of Ref. 2 becomes too great. As for the $\bar{K}$-N absorption Σ-to-Λ branching ratio, Fry *et al.*[48,49] suggest $\bar{\sigma}_{\text{tot}}(\bar{K}N \to \Sigma\pi)/\bar{\sigma}_{\text{tot}}(\bar{K}N \to \Lambda\pi) \cong 0.5$. In the absence of the Y_1^* contribution it would give an idea of the order of magnitude of our $|A_c|^2/|A_d|^2$. According to Table 4 of Burhop *et al.*,[50] the $\bar{K}$-N Σ-to-Λ branching ratios at zero momentum are $\sigma(\Sigma^+\pi^-):\sigma(\Sigma^0\pi^-):\sigma(\Lambda^0\pi^-) = 0.20:0.08:0.16$. The same authors[50] quote $\bar{K}+D$ data from which it appears (Table 14 of Sec. 3.1.1. and Fig. 40 of Ref. 50) that the ratio of the total numbers of the "conversion" $(\Lambda^0\pi^-)$ events to the "direct" $(\Lambda^0\pi^-)$ events is about 1.15:1 in the particular case of deuterium (cf. also Ref. 3). Finally, we can add that a speculation based on the

[40] F. S. Crawford, M. Cresti, M. L. Good, F. F. Schmitz, M. L. Stevenson, and H. K. Ticho, Phys. Rev. Letters **2**, 174 (1959).

[41] G. Alexander, J. Anderson, F. S. Crawford, W. Laskar, and L. Lloyd, Phys. Rev. Letters **7**, 348 (1961).

[42] J. J. de Swart and C. Dullemond, Ann. Phys. (N.Y.) **16**, 263 (1961); **19**, 458 (1962); cf. also J. J. de Swart and C. K. Iddings, Phys. Rev. **128**, 2810 (1962); **130**, 319 (1963); C. Dullemond and J. J. de Swart, Nuovo Cimento **25**, 1072 (1962). For the Λ-nucleon scattering lengths cf., also B. Sechi-Zorn, R. A. Burnstein, T. B. Day, B. Kehee, and G. A. Snow, Phys. Rev. Letters **13**, 282 (1964); for other experimental data, cf. T. H. Groves, Phys. Rev. **120**, 1372 (1963); L. Piekenbrock and F. Oppenheimer, Phys. Rev. Letters **12**, 625 (1964).

[43] J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963).

[44] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963); cf., also R. L. Anderson and S. N. Gupta, Nucl. Phys. **60**, 521 (1964).

[45] P. McNamee and F. Chilton, Rev. Mod. Phys. **36**, 1005 (1964).

[46] F. R. Stannard, Phys. Rev. **121**, 1513 (1961); cf. also R. A. Bumstein *et al.*, in *Proceedings of the International Conference on High Energy Physics, Dubna, 1964* (Moscow, 1965).

[47] A calculation based on the Bryan-Gartenhaus potentials gives for the theoretical singlet and triplet Σ-N complex scattering lengths: $a_{\Sigma N}^{(0)}(s=0) = (-1.1+i0.4)$F and $a_{\Sigma N}^{(0)}(s=1) = (-0.1+i2.8)$F (the isospin $T=\frac{1}{2}$); cf. R. H. Dalitz, University of Chicago, 1961 (unpublished).

[48] W. F. Fry, J. Schneps, G. A. Snow, and M. S. Swami, Phys. Rev. **100**, 950 (1955); however, D. M. Haskin, T. Bowen, and M. Schein, Phys. Rev. **103**, 1512 (1956) give the Σ/Λ production ratio as of the order of 2; still higher possible values are discussed by N. Dallaporta and F. Ferrari, Nuovo Cimento **5**, 742 (1957).

[49] D. E. Neville, Phys. Rev. **130**, 327 (1963).

[50] E. H. S. Burhop, D. H. Davis, and J. Zakrzewski, Progr. Nucl. Phys. **9**, (1964), p. 163.

ratios of combinations of the pertinent fundamental coupling constants as "derived" from the $SU(3)$ group symmetry[43,44,51] would seem to indicate $\bar{\sigma}_{\text{tot}}(\bar{K}N\to\Sigma\pi)/\bar{\sigma}_{\text{tot}}(\bar{K}N\to\Lambda\pi)\sim\frac{1}{3}$.

In Fig. 6 we compare the combined effect of the non-Σ-Λ conversion $R_3(p_3)$, as in Fig. 3, with the contributions of the corresponding Σ-Λ conversion amplitude as discussed above. The curve labeled "1" corresponds to the same values of the parameters $a_0^{(\Lambda)}$, r_0, λ, A_d, a_r as those of Fig. 3; $a_0^{(\Sigma)}$ has been chosen to be $=-2$ F and $\eta_0=1$ F, $\lambda'=\lambda$; $A_c=0.72$ gives $A_c/A_d=2.1$, which appears to fall within the "reasonable" range for this parameter, and at the same time it gives a reasonably good $R_3(p_3)$ distribution; all the interference terms involved are kept here. The curve labeled "2a" refers to the same parameters as those of Fig. 3 for the non-Σ-Λ conversion amplitudes, and the Σ-Λ conversion amplitude is that of the pure Y_0* resonance t-matrix element; $a_0^{(\Sigma)}=-2$ F, $\eta_0=1$ F, $\lambda'=\lambda$; all the (uncertain and rather small anyway) interference terms are left out; the normalization of $R_3^{(\text{conv})}(p_3)$ is fixed as

$$R_{3\max}{}^{(\text{conv})}/R_{3\max}{}^{(\text{nonconv})}=\tfrac{1}{2}(R_3{}^{(\text{nonconv})}\equiv R_3{}^{(\text{dir})}+R_3{}^{(\text{res})}).$$

The curve labeled "2b" differs from "2a" only in the choice of $\Gamma_1=53$ MeV for the Y_1* resonance instead of $\Gamma_1=40$ MeV; the former value has been most recently suggested by Rosenfeld *et al.*[52]; in our case it gives a slightly wider $R_3(p_3)$ distribution more consistent with the data. The curves 1, 2a, and 2b reproduce most of the features of the experimental histogram indicated in Fig. 6. The worst agreement with the presently existing (still rather poor) data is presented by our curve 1, for which the phase of A_d chosen produces a destructive interference and too sharp a drop of $R_3(p_3)$ for 200 MeV/$c\lesssim p_3\lesssim 250$ MeV/c. A similar but less pronounced drop is present even in our curves 2a and 2b. This may be due to our exaggeration of the Y_1* effect, which produces the common (maximum) peak of our curves 1, 2a, and 2b and/or perhaps partly to our "zero-range" approximation and to the neglect of the initial state $\bar{K}$-nuclear multiple scattering correction. In a most recently published article (see after the completion of most of this paper), Hetherington and Schick[53] show that such multiple scattering is important for the elastic $\bar{K}+D$ scattering (cf. however, our first *Note added in proof*). The region of large p_3 values appears to be explained by the Σ-Λ conversion. The second peak of our curves 2a and 2b at $p_3\cong 300$ MeV/c corresponds to the Y_0* resonance. Owing to the disappearance of $R_3^{(\text{conv})}(p_3)\cong 0$ at $p_3\lesssim 250$ MeV/c there are practically no conversion-nonconversion interference terms present at least in our approximation.

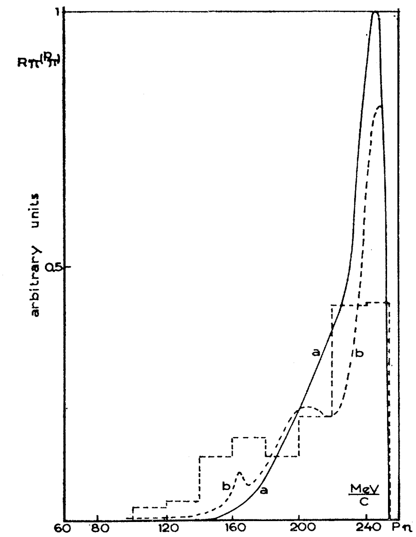

FIG. 7. The π^--momentum distribution $R_\pi(p_\pi)$ for the $\bar{K}+{}^4\text{He}\to\pi^-+\Lambda^0+{}^3\text{He}$-reaction ($nS$ $\bar{K}$ orbit) with an arbitrary normalization and with the parameters: $a_0^{(\Lambda)}=4.54$ F, $r_0=1.76$ F, $\lambda=1.35$ F^{-1}, $A_d/a_r=(-0.209+i0.273)\times10^3$ (all the momenta in MeV/c, $\sqrt{s}$ in MeV) and: (a) no Σ-Λ conversion, (b) $a_0^{(\Sigma)}=-2.0$ F, $\eta_0=1.0$ F, $A_c/|A_d|=2.1$. The experimental histogram is given for the weighted sum of all the data of Ref. 2 and its area is equal to that of the curve (b).

The pion momentum distribution $R_\pi(p_\pi)$ is displayed in Fig. 7 as calculated with the same parameters as described above. The curve a refers to the case with no Σ-Λ conversion, and b corresponds to $a_0^{(\Sigma)}=-2.0$ F, $\eta_0=1.0$ F, $\lambda'=\lambda$, and $A_c/|A_d|=2.1$ as in Fig. 6. The striking difference between the two is the presence in the latter of a small peak at $p_\pi\cong 160$ MeV/c which corresponds to the Λ-Σ threshold ($q_{\Sigma3}=0$). For higher values of p_π, the quantity $q_{\Sigma3}$ is imaginary, which means that no real physical Σ can be produced. In this interval (up to the maximum p_π, $p_{\pi\max}\cong 254$ MeV/c) where no physical Σ but only physical Λ can be produced, we employ the model approximation of Eq. (18a) for the "virtual" Σ, as a result of which $\Delta\bar{M}_{\text{conv}}\neq 0$ at $p_\pi=p_{\pi\max}$ while it should actually be $=0$. Consequently, the effect of $\Delta\bar{M}_{\text{conv}}$ is exaggerated in the interval 200 MeV/$c\lesssim p_\pi\leqslant p_{\pi\max}$ as is also the case in a similar analysis of Refs. 18 and 19. The effect of the Σ-Λ conversion gives an improvement in the agreement with the experimental histogram over the no-Σ-Λ-conversion approximation [the difference between our curves (b) and (a) in Fig. 7], especially in the region of the above-mentioned small peak and near $p_\pi\lesssim p_{\pi\max}$. Our small peak at about 165 MeV/c corresponds to a small bump in the experimental histogram in the same p_π region. However, the height of our small peak is insufficient to fit the histogram well, which is due to our choice of the value of the parameter A_c appropriate to Fig. 6 (curve

[51] P. D. de Souza, G. A. Snow, and S. Meshkov, Phys. Rev. **135**, B565 (1964).

[52] A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, and M. Ross, Rev. Mod. Phys. **36**, 977 (1964).

[53] J. H. Hetherington, and L. H. Schick, Phys. Rev. **137**, B395 (1965). It is interesting to observe that while the multiple-scattering corrections to the elastic-scattering cross sections obtained by these authors are extremely important, the *reaction* cross sections of their "MS" calculation differ relatively only little from their "IA" (impulse approximation) counterparts.

1). It appears that a satisfactory shape of $R_\pi(p_\pi)$ would be obtained with our constant A_c replaced by an appropriate Y_0^*-resonance t matrix element. With our choice of phases of the parameters involved (as in Fig. 6) the Σ-Λ-conversion–non-Σ-Λ-conversion interference terms are destructive at $p_\pi \gtrsim 200$ MeV/c. As a result we have a small dip at $p_\pi = 220$ MeV/c and a reduction of the height of our $R_\pi(p_\pi)_{\max}$. In Fig. 7 only the curve (b) is arbitrarily normalized [the area under (b) is equal to that under the experimental histogram] so that the magnitude of the difference (b)−(a) can be directly appreciated. It is interesting to observe the difference between the quite small peak at $p_\pi \cong 160$ MeV/c of the experimental $R_\pi(p_\pi)$ histogram in our case of ^{4}He and the quite large corresponding peak due to Σ-Λ conversion in the case of deuterium (cf., Fig. 40 of Ref. 50). This difference is due to different kinematical conditions and a different nuclear form factor.

In conclusion we can state that our crude oversimplified model of the very complex $\bar{K}+{}^4\mathrm{He}\rightarrow\pi^- +\Lambda^0+{}^3\mathrm{He}$ reaction exhibits the following features: it appears that within reasonable ranges of all the parameters involved; it is quite possible to obtain a semiquantitative agreement with the ^{3}He and π^- momenta and the angular distributions. In particular we note that the contribution of the formation of the Y_1^* resonance and the Σ-Λ conversion are both most important for the reaction mechanism. The final-state interactions, especially the elastic Λ-^{3}He distortion, appear to be unimportant.

The Σ-Λ conversion contribution is quite essential for the explanation of the ^{3}He momentum distribution $R_3(p_3)$ for large p_3 values and partly also of the π^- momentum distribution $R_\pi(p_\pi)$ for medium p_π values. The $\bar{K}$ nuclear capture from mP Bohr orbits appears to be unimportant and the nS orbits appear to be the most likely ones. Incidentally, we should stress the fact that the presently available data are very poor, and, in view of the importance of the reaction, new data would be most desirable. In particular, the absolute values of the cross sections would make possible a determination of the coupling constants involved.

As for the shortcomings of our theoretical model, we should stress the "zero-range" approximation employed. A future detailed study of our reaction should be free of this suspect approximation. Such a study should also contain calculations of other channels of the absorption reaction such as that of the ^{4}He bound hyperfragment formation, and those with a Σ hyperon in the final state, with all the corresponding branching ratios. The effects of nuclear structure could be best exhibited by a simultaneous study of the same reactions with several other neighboring light nuclei (D^3, He^3, H, etc.).

ACKNOWLEDGMENTS

We are particularly grateful to Mlle. M. T. Commault for her invaluable help with the numerical computations. It is a great pleasure to acknowledge a discussion with Professor R. H. Dalitz and one with Professor M. M. Block. One of us (J. S.) wishes to thank Professor M. Jean for his kind hospitality at the Laboratoire de Physique Théorique at Orsay.

APPENDIX: AN ALTERNATIVE CALCULATION OF THE Σ-Λ CONVERSION AMPLITUDE

The most satisfactory calculation should be a direct one of the Σ-Λ conversion amplitude based on the "multiple-scattering" formula of Eq. (16).

In an approximate analytic evaluation of ΔM_{conv} of Eq. (16) we can neglect any elastic hyperon-nucleus distortions and apply the usual method of Fourier analysis; here again $\mathbf{P}_i=0$.

On taking the simplest nS capture in the usual approximation and neglecting the recoil effect, we can rewrite Eq. (16) as

$$\Delta M_{\mathrm{conv}}=N_{nS}^2\sum_{\mathrm{spins}}\int\frac{d\mathbf{p}_{\Sigma'}}{(2\pi)^3}\sum_{^3I'}\int\frac{d\mathbf{p}_N d\mathbf{p}_{N'}d\mathbf{p}_{N''}}{(2\pi)^9}\int d(1')d(2')d(3')d(4')$$

$$\times e^{-i\mathbf{p}_N\cdot\mathbf{r}_1'}\Psi_{^3I'}^*(2',3',4')\Psi_4(1',2',3',4')\int d(3)d(4)\int d(2'')e^{-i\mathbf{p}_{N'}\cdot\mathbf{r}_2''}\Psi_{^3I'}(2'',3,4)$$

$$\times\int d(2''')e^{-i\mathbf{p}_{N''}\cdot\mathbf{r}_2'''}\Psi_{^3\mathrm{He}}^*(2''',3,4)\langle\mathbf{p}_\Lambda\mathbf{p}_{N''}|t_2|\mathbf{p}_{\Sigma'}\mathbf{p}_{N'}\rangle(E_f-E_{\Sigma'+{}^3I'})^{-1}$$

$$\times\langle\mathbf{p}_\pi\mathbf{p}_{\Sigma'}|t_1|\mathbf{p}_K=0\cdot\mathbf{p}_N\rangle(2\pi)^4\delta(T_\Lambda+T_3+E_\pi{}^0-Q).\quad\text{(A1)}$$

We introduce the intrinsic coordinates of the three-system:

$$\mathbf{v}'\equiv\tfrac{1}{6}(\mathbf{r}_3'+\mathbf{r}_4')-\tfrac{1}{3}\mathbf{r}_2',\quad\mathbf{w}'\equiv\mathbf{r}_3'-\mathbf{r}_4'$$

and

$$\mathbf{v}'''\equiv\tfrac{1}{6}(\mathbf{r}_3+\mathbf{r}_4)-\tfrac{1}{3}\mathbf{r}_2''',\quad\mathbf{w}\equiv\mathbf{r}_3-\mathbf{r}_4,\quad\mathbf{v}=\tfrac{1}{6}(\mathbf{r}_3+\mathbf{r}_4)-\tfrac{1}{3}\mathbf{r}_2''.$$

Consequently, we obtain

$$\Delta M_{\text{conv}} \cong (2\pi)^4 \delta(\mathbf{p}_\pi + \mathbf{P}_f)\delta(T_\Lambda + T_3 + E_{\pi}{}^0 - Q) N_{nS}{}^2 \times 9 \sum_{\text{spins}} \int \frac{d\mathbf{p}_N d\mathbf{p}_{N'} d\mathbf{p}_{N''}}{(2\pi)^9}$$

$$\times I(p_N) \int d\mathbf{w} \int d\mathbf{v} e^{i(\mathbf{p}_{N'}-\mathbf{p}_N)\cdot\mathbf{v}} \phi_{^3I'}(\mathbf{v},\mathbf{w}) \int d\mathbf{v}''' e^{-i(\mathbf{p}_{N''}+\mathbf{p}_3)\cdot\mathbf{v}'''} \phi_3{}^*(\mathbf{v}''',\mathbf{w}) \int d\mathbf{v}' d\mathbf{w}' \phi_{^3I'}{}^*(\mathbf{v}',\mathbf{w}') \phi_3(\mathbf{v}',\mathbf{w}')$$

$$\times \langle \mathbf{q}_f | t_2 | \mathbf{q}' \rangle \cdot \left[\frac{p_\Lambda{}^2}{2m_\Lambda} - \frac{(\mathbf{p}_N - \mathbf{p}_\pi)^2}{2m_\Sigma} + \frac{p_3{}^2}{2M_3} - \frac{p_N{}^2}{2M_{^3I'}} + \bar{Q}' \right]^{-1} \langle \mathbf{q}'' | t_1 | \mathbf{q}_i \rangle, \quad \text{(A2)}$$

where

$$\mathbf{q}_i \equiv -m_K(M+m_K)^{-1}\mathbf{p}_N,$$

$$\mathbf{q}'' \equiv [m_\Sigma \mathbf{p}_\pi - m_\pi(\mathbf{p}_N - \mathbf{p}_\pi)](m_\Sigma + m_\pi)^{-1} = \mathbf{p}_\pi - m_\pi(m_\Sigma + m_\pi)^{-1}\mathbf{p}_N,$$

$$\mathbf{q}' \equiv (M\mathbf{p}_\Sigma - m_\Sigma \mathbf{p}_{N'})(M+m_\Sigma)^{-1} = [M(\mathbf{p}_N - \mathbf{p}_\pi) - m_\Sigma \mathbf{p}_{N'}](M+m_\Sigma)^{-1},$$

$$\mathbf{q}_f \equiv (M\mathbf{p}_\Lambda - m_\Lambda \mathbf{p}_{N''})(M+m_\Lambda)^{-1},$$

$$\bar{Q}' \equiv M_3 - M_{^3I'} + m_\Lambda - m_{\Sigma'}.$$

In the above we have assumed that the factorization of the ^{4}He wave function as $\Psi_4(\mathbf{u},\xi) \equiv \Psi_{(n-{}^3\text{He})}(u)\phi_3(\mathbf{v},\mathbf{w})$; $\phi_{^3I'}(\mathbf{v},\mathbf{w})$ represents the intermediate $^3I'$ nuclear intrinsic wave function. If one suppresses the $^3I'$ dependence of the energy denominator and employs the closure relation $\Sigma_{^3I'}\phi_{^3I'}{}^*(\mathbf{v}',\mathbf{w}')\phi_{^3I'}(\mathbf{v},\mathbf{w}) = \delta(\mathbf{v}-\mathbf{v}')\delta(\mathbf{w}-\mathbf{w}')$, one double integration disappears, yielding the form factor $\int d\mathbf{v}d\mathbf{w}\, e^{i(\mathbf{p}_{N'}-\mathbf{p}_N)\mathbf{v}'}\phi_3(\mathbf{v},\mathbf{w}) \int d\mathbf{v}''' e^{-i(\mathbf{p}_{N''}-\mathbf{p}_3)\cdot\mathbf{v}'''}\phi_3{}^*(\mathbf{v}''',\mathbf{w})$. One further simplification is to replace the t_1 matrix element by a constant $\bar{A}_c$ (an "average equivalent" t_1 matrix). The t_2 element of the type $\langle \Lambda N'' | t_2 | \Sigma' N' \rangle$ can be deduced from the fundamental group symmetry considerations for baryons in terms of the elementary nucleon-nucleon interactions. Such relations have been discussed for the global symmetry by e.g., de Swart and Dullemond[42] and for the $SU(3)$ symmetry by de Swart,[43] Martin and Wali[44] (ratios of the effective pertinent baryon-meson coupling constants), by McNamee and Chilton,[45] and by de Souza *et al.*[51] (directly in terms of the pertinent t-matrix elements).

Another method, like that employed by Neville,[49] would be to employ directly the t_2 matrix elements calculated in the effective-range theory in Ref. 42.

In the extreme approximation of suppressing even the $\mathbf{p}_{N'}$ and $\mathbf{p}_{N''}$ dependence of t_2, one is finally left with only one (triple) $\mathbf{p}_N$ integration (the $\mathbf{p}_{N'}$ and $\mathbf{p}_{N''}$ integrations give two δ functions). When one goes over to the calculation of $R_3(p_3)$, one is finally faced with a triple numerical integration even in the "caricature" model with extremely crude approximations. The results of this analysis and those for other multiple scattering corrections may be the subject of another publication.

Finally, we should mention the Σ-nuclear Coulomb (elastic) distortion effect, which we have neglected throughout. This effect, being a result of a hypernuclear interaction, would be present only in the intermediate states in our case, i.e., while the Σ particle practically stays within the nucleus before the Σ-Λ conversion. It is then reasonable to believe that the Σ nuclear interaction predominates in this situation, and the Coulomb effect should be of relatively little importance. This would be in contrast to the situation in the reaction K+nucleus $\to$ $\pi^\pm + \Sigma^\mp$+recoil, where the Coulomb distortion has been found rather important by Friedmann *et al.*[54]

---

[54] M. Friedmann, D. Kessler, A. Levy, and A. Perlmutter, Nuovo Cimento **35**, 355 (1965).