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## Some $(d,\alpha)$ Differential Cross Sections for $\text{Na}^{23}$ , $\text{Al}^{27}$ , and $\text{P}^{31}$ at About 9.3 MeV\*†

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The differential cross sections for the  $(d,\alpha)$  reactions which lead to various nuclear states in  $\text{Ne}^{21}$  (ground, 1, and 2),  $\text{Mg}^{25}$  (ground, 1, 2, 3, and 4), and  $\text{Si}^{29}$  (ground, 1, 2, 3, 4, 5, and 6) were measured at nominal deuteron energies of 9.2, 9.2, and 9.5 MeV, respectively, using a silicon surface-barrier detector. Targets of various thicknesses ranging from 70 to 642  $\mu\text{g}/\text{cm}^2$  were used. In each case the differential cross section was determined at more than 34 angles from 10 to 172.5°. Although the shape of each angular distribution is different from any of the others in detail, all exhibit the following characteristics: pronounced forward and backward peaking; an over-all oscillatory structure; an asymmetry about 90°; and minima whose magnitudes differ significantly from zero. Comparisons are made with  $(d,\alpha)$  results which have been previously obtained by other investigators using deuteron energies in the 7–15-MeV range. No correlation could be established between any specific detail in shape or general feature of these angular distributions and any physical quantity associated with the nuclear states involved. Analyses of these differential cross sections are described using an expression consisting of the sum of an isotropic component and a plane-wave Born approximation component which arises from the coherent action of a two-nucleon pickup and a heavy-particle-stripping mechanism.

### I. INTRODUCTION

THE dearth of experimental information concerning the  $(d,\alpha)$  reaction precludes the formulation of anything but the most broad generalizations concerning the nature of the reaction mechanism itself. The experimental results obtained at low incident deuteron energies (1–2 MeV) suggest that the compound-nucleus mechanism may be dominant<sup>1–6</sup> while those at higher

incident energies (7–15 MeV) imply the simultaneous operation of direct-interaction (DI) and compound-nucleus (CN) mechanisms with the former possibly dominant.<sup>7–12</sup> Although the number of reported cases in which the experimental differential cross sections are of good statistical quality over the complete angular range is limited, the low- and high-energy results display at least one major qualitative difference. The low-energy angular distributions appear to be relatively flat with some smooth structure appearing in the 2-MeV data.<sup>1–5</sup> The higher energy angular distributions may be characterized by the following features: pronounced forward and backward peaking; an asymmetry with respect to 90°; an over-all oscillatory structure; and minima which differ significantly from zero. It should be emphasized that the general occurrence of backward peaking is not nearly as well established as the other characteristics. This study, as in a previous one con-

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<sup>1</sup> J. A. Biggerstaff, R. F. Hood, H. Scott, and M. T. McEllistrem, *Nucl. Phys.* **36**, 631 (1962).

<sup>2</sup> K. H. Purser and B. H. Wildenthal, *Nucl. Phys.* **44**, 22 (1963).

<sup>3</sup> M. A. Abuzeid, Y. P. Antoufiev, A. T. Baranik, M. I. El-Zaiki, T. M. Nower, and P. V. Sorokin, *Nucl. Phys.* **54**, 315 (1964).

<sup>4</sup> A. Z. El-Behay, M. A. Farouk, M. H. Nassef, and I. I. Zaloubovsky, *Nucl. Phys.* **61**, 282 (1965).

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<sup>6</sup> M. A. Abuzeid, A. T. Baranik, M. I. El-Zaiki, V. J. Gontchar, S. M. Morsy, and I. I. Zaloubovsky, *Nucl. Phys.* **60**, 264 (1964).

<sup>7</sup> C. Hu, *J. Phys. Soc. Japan* **15**, 1741 (1960).

<sup>8</sup> N. Cindro, M. Cerineo, and A. Strzalkowski, *Nucl. Phys.* **24**, 107 (1961).

<sup>9</sup> F. Pellegrini, *Nucl. Phys.* **24**, 372 (1961).

<sup>10</sup> K. Takamatsu, *J. Phys. Soc. Japan* **17**, 896 (1962).

<sup>11</sup> T. Yanabu, S. Yamashita, T. Nakamura, K. Takamatsu, A. Masaike, S. Kakigi, Dai Ca Nguyen, and K. Takimoto, *J. Phys. Soc. Japan* **17**, 914 (1962).

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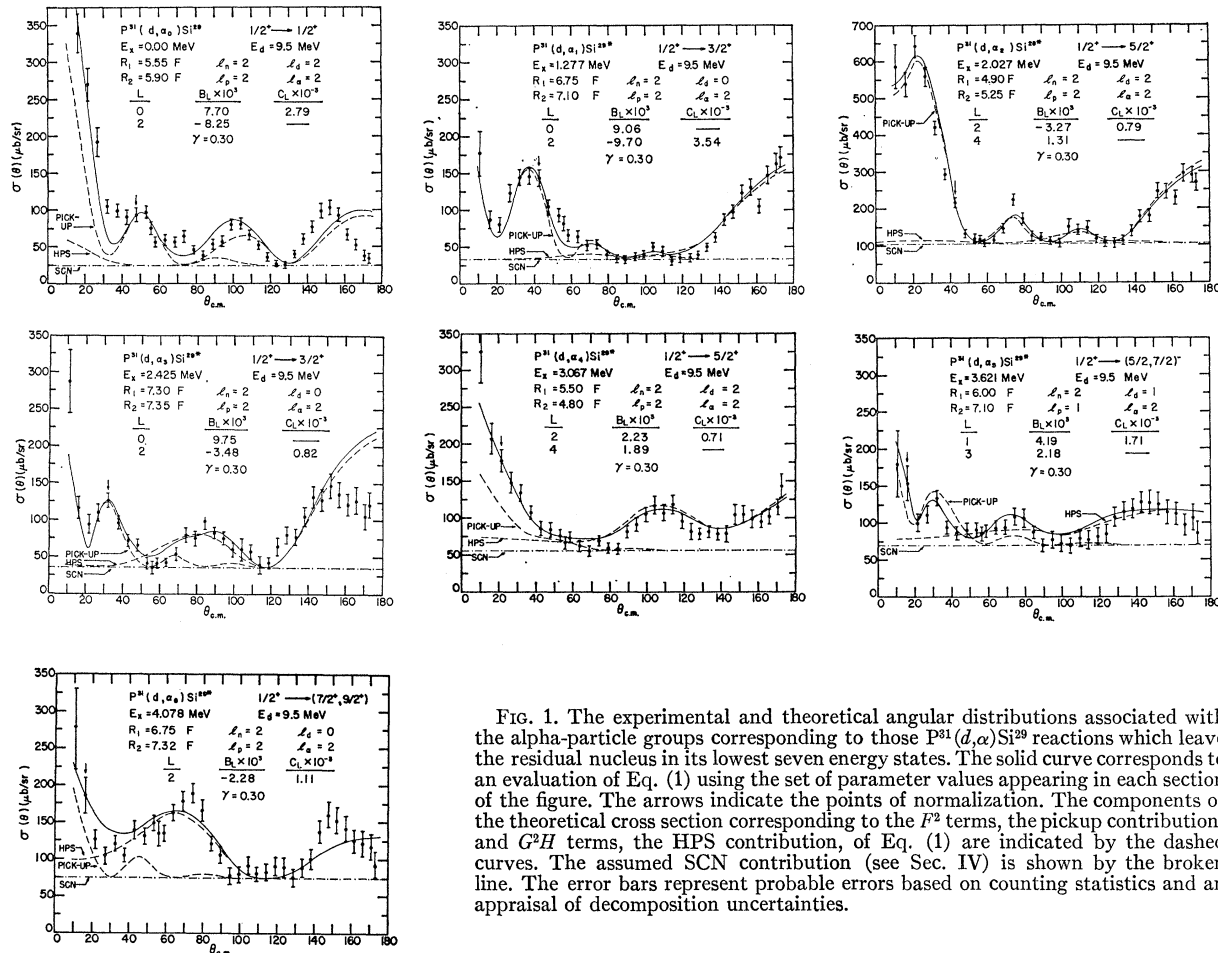


FIG. 1. The experimental and theoretical angular distributions associated with the alpha-particle groups corresponding to those  $P^{31}(d, \alpha)Si^{29}$  reactions which leave the residual nucleus in its lowest seven energy states. The solid curve corresponds to an evaluation of Eq. (1) using the set of parameter values appearing in each section of the figure. The arrows indicate the points of normalization. The components of the theoretical cross section corresponding to the  $F^2$  terms, the pickup contribution, and  $G^2H$  terms, the HPS contribution, of Eq. (1) are indicated by the dashed curves. The assumed SCN contribution (see Sec. IV) is shown by the broken line. The error bars represent probable errors based on counting statistics and an appraisal of decomposition uncertainties.

cerning the  $F^{19}(d, \alpha)O^{17}$  reaction,<sup>13</sup> was undertaken in part to investigate the occurrence and nature of this particular feature of the differential cross sections.

The presence of this pronounced backward peaking is of interest because there appears to be no generally accepted theoretical model which can quantitatively account for it and provide a physical basis for its interpretation. It has been suggested<sup>14</sup> that distortion effects might give rise to strong backward peaking in the  $(d, \alpha)$  reaction, as was found in the cases of some single-nucleon stripping reactions. To date this surmise has not been demonstrated in the open literature. Another alternative is to assume the presence of a heavy-particle or exchange-stripping mechanism. However, Austern<sup>15</sup> has pointed out that aside from the question as to whether or not heavy-particle stripping

(HPS) actually occurs, the existing models for this process are so crude as to be of questionable value; and that what is required is a detailed treatment of recoil and antisymmetrization within the framework of a finite-range distorted-wave formalism. Consequently, as far as the backward-angle behavior of the differential cross section is concerned there is little choice in the selection of a theory for comparison purposes.

Although the inadequacies of and/or objections to plane-wave theories have been extensively discussed in the open literature, the moderate success of these theories in describing single-particle transfer reactions has prompted many investigators to apply these ideas to multiparticle transfer processes. In a previous investigation at this laboratory the differential cross sections for the  $F^{19}(d, \alpha)O^{17}$  reactions which leave  $O^{17}$  in its ground and lowest four excited states were measured, and prior to attempting a distorted-wave Born approximation (DWBA) analysis, these data were analyzed in terms of a plane-wave Born approximation (PWBA) theory.<sup>13</sup> Because of the presence of pronounced backward peaking in the differential cross sections, the analyses were made within the framework

<sup>13</sup> S. W. Cosper, B. T. Lucas, and O. E. Johnson, Phys. Rev. **138**, B51 (1965).

<sup>14</sup> D. A. Bromley, *Nuclear Spectroscopy with Direct Interactions, II Proceedings, March, 1964*, edited by F. E. Throw (Argonne National Laboratory, Argonne, Illinois, 1964), p. 376.

<sup>15</sup> N. Austern, *Nuclear Spectroscopy with Direct Interactions, II Proceedings, March, 1964*, edited by F. E. Throw (Argonne National Laboratory, Argonne, Illinois, 1964), pp. 12, 14.

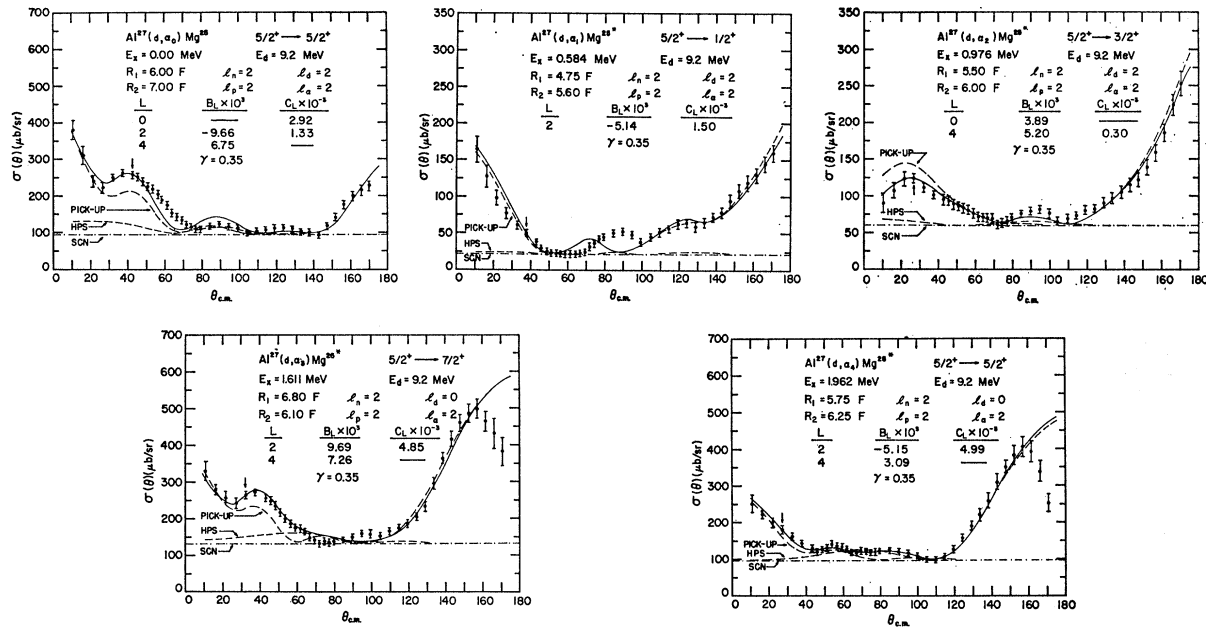


FIG. 2. The experimental and theoretical angular distributions associated with the alpha-particle groups corresponding to those  $\text{Al}^{27}(d, \alpha)\text{Mg}^{25}$  reactions which leave the residual nucleus in its lowest five energy states. The solid curve corresponds to an evaluation of Eq. (1) using the set of parameter values appearing in each section of the figure. The arrows indicate the points of normalization. The components of the theoretical cross section corresponding to the  $P^2$  terms, the pickup contribution and  $G^2H$  terms, the HPS contribution, of Eq. (1) are indicated by the dashed curves. The assumed SCN contribution (see Sec. IV) is shown by the broken line. The error bars represent probable errors based on counting statistics and an appraisal of decomposition uncertainties.

of the two-nucleon pickup theory due to Newns<sup>16</sup> as modified by Manning and Aitken<sup>17</sup> to include a HPS mechanism.<sup>18</sup> The fitted theoretical cross sections reproduced somewhat more than the gross characteristics of the experimental angular distributions. The present study constitutes an extension of this previous investigation. In attempting to reproduce the experimental differential cross sections over the *complete* angular range, full advantage is taken of the parametrization of this PWBA theory.

## II. EXPERIMENTAL

A detailed description of the experimental area, beam handling system, and 30-in. scattering chamber associated with the Purdue University cyclotron facility has been presented elsewhere.<sup>19</sup> The charged-particle spectra were measured using a conventional electronic spectrometer system which incorporated a silicon surface-barrier detector as an energy-sensitive element and a multichannel pulse-height analyzer. Information concerning the experimental procedures used in the execution of these measurements, the characteristics

and number of targets used, and the experimental geometry has been previously reported.<sup>13,20</sup>

An appraisal of possible uncertainties associated with target characteristics, charge integration, and experimental geometry has resulted in the assignment of a 15% probable systematic error to the absolute cross sections.

## III. RESULTS

The  $(d, \alpha)$  differential cross sections which were derived from the measurements of the present investigation are shown graphically in Figs. 1, 2, and 3. In each section of the figures the subscripted  $\alpha$  in the labeling reaction denotes the state of the residual nucleus. With each angular distribution are presented the excitation energy of the residual nucleus,  $E_x$ ; the spins and parities of the initial and final nuclear states; and the incident deuteron energy,  $E_d$ .<sup>21</sup> These cross

<sup>20</sup> S. W. Cosper and O. E. Johnson, Phys. Rev. **138**, B610 (1965).

<sup>21</sup> Unless otherwise specified, the level structure and individual level properties proposed in the following compilations will be assumed for the nuclei of interest; P. M. Endt and C. Van der Leun, Nucl. Phys. **34**, 1 (1962); T. Lauritsen and F. Ajzenberg-Selove, *Nuclear Data Sheets—Energy Levels of Light Nuclei, May 1962*, edited by K. Way *et al.* (National Academy of Sciences, National Research Council, Washington, D. C., 1962), NRC 61-5,6-3. However, in instances where more current information is available, or specific quantitative values and/or interpretations are relevant to the discussion, detailed bibliographical references will be given.

<sup>16</sup> H. C. Newns, Proc. Phys. Soc. (London) **76**, 489 (1960).

<sup>17</sup> I. Manning and A. H. Aitken, Nucl. Phys. **32**, 524 (1962).

<sup>18</sup> A detailed discussion of the circumstances and considerations associated with the selection of the pickup rather than the knock-out mechanism for the analyses of these data has been presented in Ref. 13.

<sup>19</sup> B. T. Lucas, S. W. Cosper, and O. E. Johnson, Phys. Rev. **133**, B963 (1964).

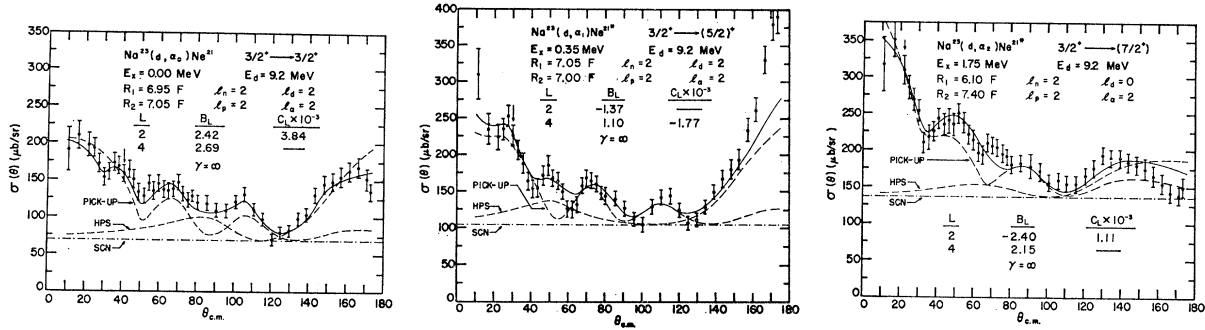


FIG. 3. The experimental and theoretical angular distributions associated with the alpha-particle groups corresponding to those  $\text{Na}^{23}(d, \alpha)\text{Ne}^{21}$  reactions which leave the residual nucleus in its lowest three energy states. The solid curve corresponds to an evaluation of Eq. (1) using the set of parameter values appearing in each section of the figure. The arrows indicate the points of normalization. The components of the theoretical cross section corresponding to the  $F^2$  terms, the pickup contribution, and  $G^2H$  terms, the HPS contribution, of Eq. (1) are indicated by the dashed curves. The assumed SCN contribution (see Sec. IV) is shown by the broken line. The error bars represent probable errors based on counting statistics and an appraisal of decomposition uncertainties.

sections have been corrected to first order for finite experimental geometry. The vertical bars on the data points represent the probable errors determined from counting statistics and the uncertainties associated with spectral decomposition.

Detailed discussions of the individual angular distributions will be presented in another section of this report, however, some remarks of a more or less general nature can be made at this point. For completeness the differential cross sections determined in the previous  $\text{F}^{19}(d, \alpha)\text{O}^{17}$  investigation<sup>13</sup> will be considered and referred to along with those of the present investigation. It is clear that the angular distributions associated with the  $(d, \alpha)$  reactions on  $\text{F}^{19}$ ,  $\text{Na}^{23}$ ,  $\text{Al}^{27}$ , and  $\text{P}^{31}$  at  $E_d \approx 9.3$  MeV may be characterized by the same general features (see Sec. I) as those obtained by other investigators at incident deuteron energies in the 7–15 MeV range. It should also be noted that of the 20 angular distributions associated with the above four nuclei, those corresponding to the  $\text{Al}^{27}(d, \alpha)\text{Mg}^{25}$  reactions appear to be the most washed out.

#### IV. ANALYSIS

In the course of the PWBA analyses of  $(d, \alpha)$  data at this laboratory it was generally found that better fits to the experimental data could be achieved if an isotropic component were subtracted from each of the experimental angular distributions. The magnitude of this component was taken in each case to be equal to the value of the differential cross section at its lowest observed minimum. The association of this isotropic component of the cross section with an incoherent contribution due to the statistical compound-nucleus (SCN) mechanism has been discussed in great detail elsewhere.<sup>20</sup>

The present data have been analyzed in terms of the two-nucleon pickup theory of Newns<sup>16</sup> as extended by Manning and Aitken<sup>17</sup> to include HPS. The isotropic components discussed above were subtracted from the experimental angular distributions prior to determining

the parameter values which give the best fits. The theoretical differential cross section may be written

$$\frac{d\sigma}{d\Omega} = \frac{64\pi^2}{3\hbar^4} \sum_L (F^2 + 2DFG + G^2H), \quad (1)$$

where

$$F = B_L \left[ \frac{M_I^* M_F^* k_\alpha}{k_d} \right]^{1/2} (\gamma/4R_1^2) \exp(-K^2/16\gamma^2) j_L(kR_1),$$

$$D = (-1)^L \left[ \frac{(2l_\alpha + 1)}{4\pi(2L + 1)} \right]^{1/2} (l_\alpha, 0, l_d, 0 | L, 0) Y_{L,0}^0(\hat{K}_0 \cdot \hat{q}_0),$$

$$G = C_L \left[ \frac{M_I^* M_F^* k_\alpha}{k_d} \right]^{1/2} j_{l_d}(K_0 R_2) j_{l_\alpha}(q_0 R_2),$$

and

$$H = \sum_{L'} \left[ \frac{(2l_\alpha + 1)(2l_d + 1)}{(4\pi)^{3/2}(2L' + 1)^{1/2}} \right] (l_d, 0, l_d, 0 | L', 0) (l_\alpha, 0, l_\alpha, 0 | L', 0) \\ \times W(l_\alpha, l_d, l_\alpha, l_d; L, L') Y_{L',0}^0(\hat{K}_0 \cdot \hat{q}_0).$$

$L$  is the angular-momentum transfer in the reaction;  $B_L$  and  $C_L$  are treated as adjustable parameters but are actually combinations of various constants such as fractional-parentage coefficients, configuration-mixing parameters, the potential parameters, and radial integrals<sup>22</sup>;  $R_1$  and  $R_2$  are the pickup and HPS interaction radii, respectively;  $\gamma$  is the width parameter associated with the Gaussian wave function used for the deuteron;  $l_p$  and  $l_n$  are the orbital angular momenta of the picked-up proton and neutron; and  $l_\alpha$  and  $l_d$  are the orbital angular momenta of the alpha particle before and the deuteron after HPS. The vectors  $\mathbf{K}$ ,  $\mathbf{k}$ ,  $\mathbf{K}_0$ , and  $\mathbf{q}_0$  are defined as follows:

$$\mathbf{K} \equiv \frac{1}{2}\mathbf{K}_\alpha - \mathbf{K}_d; \quad \mathbf{K}_0 \equiv \mathbf{K}_d + (M_d/M_F)\mathbf{K}_\alpha; \\ \mathbf{k} \equiv \mathbf{K}_\alpha - (M_F/M_I)\mathbf{K}_d; \quad \text{and} \quad \mathbf{q}_0 \equiv (M_\alpha/M_I)\mathbf{K}_d + \mathbf{K}_\alpha.$$

<sup>22</sup> A redefinition of these constants prohibits the direct comparison of the quoted  $B_L$  and  $C_L$  values of this report with those given in Ref. 13.

The subscripts  $I$  and  $F$  refer to the initial and final nucleus, and  $\mathbf{K}_d$  and  $\mathbf{K}_\alpha$  are the wave vectors associated with the incident deuteron and the outgoing alpha particle. The initial- and final-state reduced masses are denoted by  $M_I^*$  and  $M_F^*$ , respectively. The selection rules for the use of Eq. (1) are  $\mathbf{J}_F = \mathbf{J}_I + \mathbf{L} + \mathbf{S}$ ,  $\mathbf{L} = \mathbf{I}_n + \mathbf{I}_p$  for pickup, and  $\mathbf{L} = \mathbf{I}_d + \mathbf{I}_\alpha$  for HPS,  $|\mathbf{S}| = 1$ . There is interference between the pickup terms  $F^2$  and the HPS terms  $G^2H$  in Eq. (1) only if the same  $L$  value is allowed for both processes. It is a basic assumption of this model that all nucleons of the initial nucleus except the picked-up pair form an inert core which takes no part in the reaction.

At this point it is essential that a few comments be made concerning the general philosophy adopted in making the comparison between the theoretical and experimental differential cross sections. First, in attempting to reproduce the experimental angular distributions, those sets of parameter values were sought which yielded the best agreement over the *entire* angular range. Secondly, in endeavoring to accomplish this end, when necessary and within the limits of calculational practicability, the full freedom allowed by the formalism was utilized. In exploiting the parametrization of the theory, judgment as to the physical reasonableness of the acceptable parameter values was somewhat relaxed. It is obvious that judgment as to the "best" fits could be highly subjective since no formal or quantitative criteria had been established. However, experience gained in performing this type of analysis indicates that no simple set of criteria could be set down which would yield sensibly good overall fits.

The same format is used in Figs. 1, 2, and 3. The "best-fitting" theoretical differential cross section is represented by a solid curve which was normalized to the experimental data at the point indicated by an arrow. The parameter values used in the evaluation of Eq. (1) are also given. The components of the theoretical cross section corresponding to the  $F^2$  terms, the pickup contribution, and the  $G^2H$  terms, the HPS contribution, of Eq. (1) are indicated by appropriately labeled, dashed curves. The assumed SCN contribution is indicated by a broken line.

## V. DISCUSSION

### A. $P^{31}(d, \alpha)Si^{29}$ : $E_d = 9.5$ MeV

The experimental angular distributions of the alpha-particle groups which leave  $Si^{29}$  in its lowest seven states are presented in Fig. 1. Aside from the general characteristics detailed in Sec. I, these angular distributions exhibit no other similarities. The integrated cross sections ( $10^\circ$ – $170^\circ$ ) for these seven reactions differ by less than a factor of three, while the ratios of the integrated backward-angle ( $90^\circ$ – $170^\circ$ ) to the integrated forward-angle ( $10^\circ$ – $90^\circ$ ) cross section vary from 0.65

for  $\alpha_2$  to 1.05 for  $\alpha_4$ .<sup>23</sup> It is interesting to note that the integrated cross section ( $10^\circ$ – $170^\circ$ ) for the 2.027-MeV ( $\alpha_2$ ) state is approximately a factor of two larger than any of the others. No satisfactory explanation for the relative strength of this transition has been found.

Broude, Green, and Willmott<sup>24</sup> have considered  $P^{31}$  in terms of the unified model of Nilsson<sup>25</sup> and identified the ground state as the base of a  $K = \frac{1}{2}$  (Nilsson orbit 9) rotational band.<sup>26</sup> Bromley, Gove, and Litherland<sup>27</sup> have applied this same model to  $Si^{29}$  and assigned the 2.027- and 2.425-MeV levels (second and third excited states) to the  $K = \frac{1}{2}$  (Nilsson orbit 9) rotational band built on the ground state and the 3.067-MeV level (fourth excited state) to a  $K = \frac{3}{2}$  (Nilsson orbit 8) rotational band built on the 1.277-MeV level (1st excited state). Hu<sup>7</sup> argues that ( $d, \alpha$ ) transitions to states in the  $K = \frac{1}{2}$  rotational band of  $Si^{29}$  should be more probable than those to states in the  $K = \frac{3}{2}$  rotational band. The present data suggest little if any inhibition of the ( $d, \alpha$ ) transitions to the 1.277- and 3.067-MeV levels of  $Si^{29}$ , contrary to Hu's expectation. However, discrepancies between the experimental results and predictions based on an assumed rotational character for the initial and final states might be expected since Harris and Seagondollar,<sup>28</sup> on the basis of a comparison of the empirical level scheme of  $P^{31}$  with those predicted by a weak-coupling vibrational model<sup>29</sup> and a Nilsson rotational model,<sup>30</sup> have suggested that the rotational effects which are more or less well established near  $A = 25$  may have given way to vibrational effects in the vicinity of  $A = 31$  rendering a rotational-model description of  $P^{31}$  inappropriate.

Only two studies of this reaction in which angular distributions were measured are relevant to the present discussion. The investigations of Hu<sup>7</sup> at 11.1 and 11.4 MeV and of Takamatsu<sup>10</sup> at 14.7 MeV yielded only the ground-state differential cross sections. The extreme forward-angle behavior or the  $\alpha_0$  angular distributions of the present investigation and those of Hu and Takamatsu are qualitatively similar. The local maximum at about  $50^\circ$  in the 9.5-MeV data appears to be much more pronounced in the higher energy data. The backward-angle behavior of these four angular distri-

<sup>23</sup> For complete information concerning integrated cross sections, see Table II of Ref. 20.

<sup>24</sup> C. Broude, L. L. Green, and J. C. Willmott, Proc. Phys. Soc. (London) **72**, 1122 (1958).

<sup>25</sup> S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 16 (1955).

<sup>26</sup> The error in the original calculations does not change the considerations concerning the nature of the ground state of  $P^{31}$ . See L. L. Green, J. C. Willmott, and G. Kayne, Nucl. Phys. **25**, 278 (1961).

<sup>27</sup> D. A. Bromley, H. E. Gove, and A. E. Litherland, Can. J. Phys. **35**, 1057 (1957).

<sup>28</sup> G. I. Harris and L. W. Seagondollar, Phys. Rev. **131**, 787 (1963).

<sup>29</sup> V. K. Thankappan and S. P. Pandya, Nucl. Phys. **19**, 303 (1960), and V. K. Thankappan, Phys. Letters **2**, 122 (1962).

<sup>30</sup> The rotational level scheme was based on a corrected version of the calculations originally presented in Ref. 24.

butions ( $\alpha_0$  at 9.5, 11.1, 11.4, and 14.7 MeV) is quite similar and differs only by a scale factor, with the structure of the 14.7-MeV data somewhat washed out. It should be noted that the unique and pronounced backward-angle oscillations in the  $\alpha_0$  angular distribution are almost identical to those previously observed and reported<sup>13</sup> for the  $F^{19}(d,\alpha_1)O^{17}$ , ( $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$ ) reaction. In terms of the shell model the  $F^{19}$  and  $P^{31}$  ground states are described as three nucleons outside a closed core,  $O^{16}$  and  $Si^{28}$ , respectively, while the  $O^{17}$  first excited state and the  $Si^{29}$  ground state are pure single-particle states consisting of a ( $2s$ ) neutron outside a closed core,  $O^{16}$  and  $Si^{28}$ , respectively. These structural similarities might provide a tenuous basis for expecting over-all similarities in the corresponding ( $d,\alpha$ ) angular distributions. However, the near identity in shape occurs only at backward angles, a fact that is not readily interpretable on this basis.

### B. $Al^{27}(d,\alpha)Mg^{25}: E_d=9.2$ MeV

The experimental angular distributions of alpha-particle groups which leave  $Mg^{25}$  in its lowest five states are presented in Fig. 2. In addition to the previously mentioned common characteristics of these distributions (see Sec. I), there are other similarities among these data. The most striking of these is the similar strong backward-angle peaking in the  $\alpha_3$  and  $\alpha_4$  data. Closer inspection reveals the nearly identical forward-angle behavior of the  $\alpha_0$  and  $\alpha_3$  distributions, and the presence of a small maximum at approximately  $90^\circ$  in all five angular distributions. The integrated cross sections ( $10-170^\circ$ ) vary from 0.68 mb for  $\alpha_1$  to 2.75 mb for  $\alpha_3$ , and the ratios of the integrated backward-angle ( $90-170^\circ$ ) to the integrated forward-angle ( $10-90^\circ$ ) cross section range from 0.64 for  $\alpha_0$  to 1.43 for  $\alpha_1$ .<sup>23</sup>

Analyses of  $Mg^{25}$  in terms of the unified model of Nilsson<sup>25</sup> have been performed by Litherland *et al.*<sup>31</sup> and by Bhatt.<sup>32</sup> They identify the 1.611-MeV level (third excited state) of  $Mg^{25}$  as the second member of a  $K=\frac{5}{2}$  (Nilsson orbit 5) rotational band based on the ground state, and the 0.976- and 1.962-MeV levels (second and fourth excited states) as members of a  $K=\frac{1}{2}$  (Nilsson orbit 9) rotational band built on the 0.584-MeV level (first excited state). Litherland *et al.*<sup>31</sup> report that there may be a considerable admixture of the  $K=\frac{1}{2}$  (Nilsson orbit 11) rotational band with the  $K=\frac{1}{2}$  (Nilsson orbit 9) rotational band, while Bhatt<sup>32</sup> asserts that the 0.976-MeV level contains a  $K=\frac{3}{2}$  (Nilsson orbit 8) component. It is tempting to speculate whether or not the forward-angle similarity in the  $\alpha_0$  and  $\alpha_3$  distributions and the forward-angle dissimilarity among the  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_4$  distributions are related to the rotational character and band purity of the various final states.

<sup>31</sup> A. E. Litherland, H. McManus, E. B. Paul, D. A. Bromley, and H. E. Gove, *Can. J. Phys.* **36**, 378 (1958).

<sup>32</sup> K. H. Bhatt, *Nucl. Phys.* **39**, 375 (1962).

Although there have been numerous studies involving the  $Al^{27}(d,\alpha)Mg^{25}$  reaction at incident deuteron energies in the 1.4–27.5-MeV range, many dealt with matters which did not require an exposition of the angular distributions. Consequently, only the experiments of Cassagnou *et al.*<sup>33</sup> at 10 MeV and Yanabu *et al.*<sup>11</sup> at 14.7 MeV are both close enough in deuteron energy and are reported in sufficient detail to allow and warrant comparison with the present results. While there is a marked similarity between the 10- and 9.2-MeV angular distributions there are two major points of difference: a maximum at about  $60^\circ$  in the 10-MeV  $\alpha_1$  distribution does not appear in the 9.2-MeV data; and the backward-angle peaking in the 10-MeV  $\alpha_3$  and  $\alpha_4$  distributions is less pronounced than that of the present investigation. The forward-angle shapes of the 9.2- and 14.7-MeV angular distributions are roughly the same, but the higher energy backward-angle cross sections are smaller and exhibit very little structure. It is interesting to note that the near identity of shape observed in the forward-angle 9.2-MeV  $\alpha_0$  and  $\alpha_3$  distributions persists at 10 and 14.7 MeV.

### C. $Na^{23}(d,\alpha)Ne^{21}: E_d=9.2$ MeV

The experimental angular distributions of alpha-particle groups which leave  $Ne^{21}$  in its ground and lowest two excited states are shown in Fig. 3. These angular distributions differ from those presented in Secs. V-A and -B in two essential respects: Each distribution exhibits three more or less regularly spaced, washed-out peaks forward of  $90^\circ$  whose magnitudes decrease monotonically; and the rate of decrease with angle of the forward-angle cross sections is much smaller. Because of this latter characteristic it was necessary that  $\gamma$ , the width parameter, be taken very large in order to fit the data. In the analyses  $\gamma$  was taken to be infinite. This corresponds to using a point deuteron, and in effect removes the two terms involving  $\gamma$  from the expression for  $F$  in Eq. (1). The integrated cross sections ( $10-170^\circ$ ) vary from 1.59 mb for  $\alpha_0$  to 2.40 mb for  $\alpha_2$ , while the ratios of the integrated backward-angle ( $90-170^\circ$ ) to integrated forward-angle ( $10-90^\circ$ ) cross section remain fairly constant (0.77, 0.88, and 0.76 for  $\alpha_0$ ,  $\alpha_1$ , and  $\alpha_2$ , respectively).<sup>23</sup>

Rakavy,<sup>34</sup> Paul and Montague,<sup>35</sup> and Braben *et al.*<sup>36</sup> have analyzed  $Na^{23}$  in terms of the collective model and characterize the ground state as the relatively pure base of a  $K=\frac{3}{2}$  (Nilsson orbit 7) rotational band. Collective-model analyses of  $Ne^{21}$  have been performed

<sup>33</sup> Y. Cassagnou, I. Iori, C. Levi, T. Mayer-Kuckuk, M. Mermaz, and L. Papineau, *Phys. Letters* **6**, 209 (1963).

<sup>34</sup> G. Rakavy, *Nucl. Phys.* **4**, 375 (1957).

<sup>35</sup> E. B. Paul and J. H. Montague, *Nucl. Phys.* **8**, 61 (1958).

<sup>36</sup> D. W. Braben, L. L. Green, and J. C. Willmott, *Nucl. Phys.* **32**, 584 (1962).

by Freeman,<sup>37</sup> Bhatt,<sup>32</sup> and Chi and Davidson.<sup>38</sup> These investigators agree that the first three excited states of  $\text{Ne}^{21}$  arise from three interacting rotational bands: the  $K = \frac{3}{2}$  (Nilsson orbit 7); the  $K = \frac{5}{2}$  (Nilsson orbit 5); and the  $K = \frac{1}{2}$  (Nilsson orbit 9). Although the ground state is believed to possess a fairly pure  $K = \frac{3}{2}$  rotational character, because both excited states of interest are predicted to be of mixed character, the intercomparison of these angular distributions provides no further information concerning a possible relationship between the shape of the angular distribution and rotational properties of the final state.

There have been no previously reported angular distributions of alpha particles from the  $\text{Na}^{23}(d, \alpha)\text{Ne}^{21}$  reaction.

## VI. CONCLUSIONS

The ( $d, \alpha$ ) angular distributions resulting from the present investigation (including those of Ref. 13) display all the same general features which have been found to characterize those of the previous 7–15 MeV ( $d, \alpha$ ) studies (see Sec. I). It does however appear that as the incident deuteron energy is increased the backward-angle cross section tends to decrease in relative magnitude and show less structure. Aside from the general characteristics referred to above, these angular distributions possess no other generally occurring discernible similarities, that is, their *over-all* shapes appear to be unique. Furthermore, it has not been possible to correlate any general trend with physical quantities which describe the nuclear states involved or the reaction itself. However, in two of the three instances in which rather striking similarities between corresponding portions of two angular distributions were observed [the forward-angle behavior of the  $\text{Al}^{27}(d, \alpha_0)\text{Mg}^{25}$  and  $\text{Al}^{27}(d, \alpha_3)\text{Mg}^{25}$  data, and the backward-angle behavior of the  $\text{F}^{19}(d, \alpha_1)\text{O}^{17}$  and  $\text{P}^{31}(d, \alpha_0)\text{Si}^{29}$  data] there are intriguing structural relationships. A future investigation might be directed toward studying the prevalence of this behavior as a function of energy for these and similarly related nuclei.

<sup>37</sup> J. M. Freeman, *Proceedings of the International Conference on Nuclear Structure at Kingston, 1960*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, 1960), p. 477.

<sup>38</sup> B. E. Chi and J. P. Davidson, *Phys. Rev.* **131**, 366 (1963).

Although somewhat more than the gross structure of the experimental angular distributions is usually well reproduced by the fitted theoretical curves, important features could not be reproduced with reasonable parameter values such as the sharp drop in some of the cross sections at extreme backward angles; the rather pronounced local maxima in certain angular distributions; and the general concavity or convexity in certain angular regions. The importance of the HPS-pickup interference in accounting for certain features of the angular distributions and its surprisingly large contribution at forward angles in some cases should be noted. While some physical judgment was exercised in determining the reasonable range for various parameter values, the lack of firm physical bases of PWBA theories precludes any comment on their final values in so far as physical interpretations are concerned.

While it is hoped that the distorted-wave analyses which are in their preliminary stages at this laboratory will provide some basis for the codification of the shapes of these angular distributions, there are some factors which may prove to be severe limitations. First, there is little basis for optimism in achieving a description of the backward-angle behavior of these differential cross sections with distorted-wave theories in their present form. Secondly, the physical circumstances associated with the entrance channel in these studies are not favorable since it has not been convincingly demonstrated that the optical model provides an adequate description of elastic deuteron scattering from light nuclei ( $A \leq 31$ ) at about 9 MeV.

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