Lagrangian Formulation of $\tilde{U}(12)$ Symmetry and the Bargmann-Wigner Equations*

G. S. GURALNIK† AND T. W. B. KIBBLE
Department of Physics, Imperial College, London, England
(Received 22 March 1965)

The problem of finding Lagrangian functions which yield the Bargmann-Wigner equations is discussed, and is solved explicitly for the case of third-rank spinors. The formalism provides a field-theoretic realization of the $\tilde{U}(12)$ symmetry theory proposed by Salam, Delbourgo, and Strathdee. A general expression for the residue at the physical particle pole corresponding to an arbitrary multiplet is given, in a simple form which exhibits the symmetries of the theory. The propagators for the **143** and **364** representations are analyzed in detail.

1. INTRODUCTION

THE successes of the $SU(6)$ symmetry theory[1] for strongly interacting particles have led a number of authors to look for a relativistic generalization of the theory.[2] In particular, Salam, Delbourgo, and Strathdee (SDS)[3,4] have recently proposed such a theory, based on the group $\tilde{U}(12)$. In this paper, we present an interpretation of this theory from a slightly different, and more strictly field-theoretic viewpoint, and discuss the forms of the Lagrangian function and the propagator for the various multiplets.

In Sec. 2, we discuss the group structure of $\tilde{U}(12)$ and its relation to $SU(6)$, and show how $\tilde{U}(12)$ symmetry arises naturally out of $SU(6)$. The results of this section are not new, but are given in a rather different form which is well suited to our later discussions. The predictions of $\tilde{U}(12)$ symmetry are in no way dependent on the existence of quarks,[5] but the group structure is most easily described in terms of them, and we shall therefore begin by regarding the known physical particles as composite objects formed from the quarks and their antiparticles.

It is a characteristic of this theory that the interaction terms in the Lagrangian are invariant under $\tilde{U}(12)$, while the kinetic terms are not. The interaction terms therefore have a relatively simple structure, described by SDS. The essential problem associated with finding a Lagrangian formulation of the theory is therefore that of finding a free Lagrangian for each multiplet. We make a preliminary study of this problem in Sec. 3, and discuss certain general features of the Lagrangian function and the propagator for an arbitrary multiplet. In particular, we obtain a simple and general expression for the residue at the pole $p^2=m^2$. This is the quantity which is needed in making peripheral model calculations. We also discuss in detail the case of the lowest meson representation **143**, for which the Bargmann-Wigner equations[6] reduce to the well-known Kemmer-Duffin equation.[7]

The baryon representation **364** is described by a field with three indices, corresponding to bound states of three quarks. This case is treated in Secs. 4 and 5. It is shown that to obtain a Lagrangian which yields the Bargmann-Wigner equations, it is necessary to introduce auxiliary field variables, described by fields with lower symmetry. Such Lagrangians are constructed for the spin-$\frac{3}{2}$ part of the field in Sec. 4, and for the spin-$\frac{1}{2}$ part in Sec. 5. The corresponding propagators are evaluated, and shown to have a very simple structure. The relationship with the field variables introduced by SDS is discussed in the appendix. It is shown that our propagator has the same residue at the pole $p^2=m^2$ as theirs, but differs from it in the contact terms.

The possible extension of the formalism to higher representations is discussed in Sec. 6, which also contains some concluding remarks.

2. $\tilde{U}(12)$ AND THE BARGMANN-WIGNER EQUATIONS

In a relativistic extension of the $SU(6)$ symmetry theory, the quarks must be described by a 12-component field $\psi_A=\psi_{a\alpha}$, where $a=1, 2, 3$ is the $SU(3)$ index, and $\alpha=1, 2, 3, 4$ is the Dirac spinor index. The group $\tilde{U}(12)$ acting on the quark field ψ is generated by the 144 matrices $\lambda_i\Gamma_r$, where λ_i are the 9 generators of $U(3)$, and Γ_r are the 16 Dirac matrices[8] which generate $\tilde{U}(4)$.

* The research reported in this document has been sponsored in part by the Air Force Office of Scientific Research OAR through the European Office of Aerospace Research, U. S. Air Force.
† National Science Foundation Postdoctoral Fellow.

[1] F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964); A. Pais, *ibid.* **13**, 175 (1964); F. Gürsey, A. Pais, and L. A. Radicati, *ibid.* **13**, 299 (1964); B. Sakita, *ibid.* **13**, 643 (1964); Phys. Rev. **136**, B1759 (1964).

[2] R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters **13**, 678 (1964); K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, *ibid.* **13**, 698 (1964); **14**, 48 (1965); R. E. Marshak and S. Okubo, *ibid.* **13**, 818 (1964); T. Fulton and J. Wess, Phys. Letters **14**, 57 (1965); J. J. Agasshi and P. Roman, *ibid.* **14**, 68 (1965); M. A. B. Bég and A. Pais; Phys. Rev. **137**, B1514(1965) K. T. Mahanthappa and E. C. G. Sudarshan, Phys. Rev. Letters **14**, 163 (1965).

[3] A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965).

[4] A. Salam, R. Delbourgo, M. A. Rashid, and J. Strathdee Proc. Roy. Soc. (London), in press.

[5] M. Gell-Mann, Phys. Letters **8**, 216 (1964); G. Zweig, CERN Report No. 8182TH 401, 1964 (unpublished).

[6] V. Bargmann and E. P. Wigner, Proc. Nat. Acad. Sci. **34**, 211 (1948).

[7] N. Kemmer, Proc. Roy. Soc. (London) **A173**, 91 (1939); R. J. Duffin, Phys. Rev. **54**, 1114 (1938).

[8] We use a metric with signature $(+---)$, and Dirac matrices defined by $\{\gamma_\mu,\gamma_\nu\}=2g_{\mu\nu}$. The sixteen matrices 1, γ_μ, $\sigma_{\mu\nu}=\frac{1}{2}i[\gamma_\mu,\gamma_\nu]$, $\sigma_{\mu5}=i\gamma_\mu\gamma_5$, $\gamma_5=\gamma_0\gamma_1\gamma_2\gamma_3$ are denoted by Γ_r. All 16 matrices $\beta\Gamma_r$ are Hermitian; the matrices $\beta\gamma_\mu$, $\beta\sigma_{\mu\nu}$ are symmetric, while β, $\beta\sigma_{\mu5}$, $\beta\gamma_5$ are antisymmetric.

Thus, $\tilde{U}(12)$ consists of all linear transformations of ψ which leave invariant the bilinear quantity $\bar{\psi}\psi=\psi^{\dagger}\beta\psi$.

This group $\tilde{U}(12)$ is of course not an invariance group of the theory because the $\tilde{U}(4)$ symmetry is broken by the kinetic term in the quark Lagrangian

$$L_0=\bar{\psi}(\gamma p-m)\psi. \tag{1}$$

(Here we have introduced the notation $\Gamma_A{}^B=\Gamma_{a\alpha}{}^{b\beta}=\delta_a{}^b\Gamma_\alpha{}^\beta$ for any Dirac matrix Γ; also $\gamma p\equiv\gamma_\mu p^\mu$.) If we require $SU(6)$ invariance in the static limit, then the covariant four-quark interaction term is limited to a combination of the two possible terms $(\bar{\psi}\psi)(\bar{\psi}\psi)$ and $(\bar{\psi}\gamma_5\psi)(\bar{\psi}\gamma_5\psi)$. However, it is the first of these which is the more natural extension of the static $SU(6)$ invariant interaction.[9] Moreover, this choice is one way of avoiding the difficulties of parity doubling and indefinite metric for the bound states which would arise in the more general case. Thus, we shall choose the interaction Lagrangian to be

$$L_1=\lambda(\bar{\psi}\psi)(\bar{\psi}\psi). \tag{2}$$

With this choice, the mass term and the interaction term are both invariant under the group $\tilde{U}(12)$. The only noninvariance arises from the kinetic term of (1). It is easy to see that, in the quark model, a similar statement must apply to the bound states. If quarks exist, they must be extremely heavy,[10] and the binding energy of quark bound states must be large. Thus the kinetic energies of the quarks in such a state will be small compared to their masses and potential energies. Under these conditions, the bound states will exhibit $SU(6)$ symmetry.

Let us consider a bound state of n quarks. In the center-of-mass frame, the quarks are almost at rest, and therefore their Dirac wave functions must contain only the "large" pair of components, and must satisfy the equation $\gamma_0\psi=\psi$. It follows that the bound-state wave function $\Psi_{AB\cdots D}$ must satisfy this equation with respect to each of its n indices. Then in any other frame, in which the center of mass is moving with momentum p, the function Ψ must satisfy the equations

$$(\gamma p)_k\Psi=m\Psi, \tag{3}$$

where m is the mass of the bound state, and

$$(\gamma p)_k=1\times\cdots\times1\times(\gamma p)\times1\times\cdots\times1, \tag{4}$$

with the factor (γp) in the kth position of the direct product. The equations (3) are of course the Bargmann-Wigner equations. These are therefore the equations which arise naturally in seeking a relativistic generalization of the $SU(6)$ symmetry theory.

We shall consider in the following sections the problem of finding an effective Lagrangian for the field Ψ, but some remarks may be useful at this stage. Since the only $\tilde{U}(12)$ noninvariant term in the quark Lagrangian—the kinetic term—does not contribute appreciably to the bound-state masses, the effective-mass term in the Lagrangian for Ψ must be $\tilde{U}(12)$ invariant. If we decompose Ψ into its irreducible components under $\tilde{U}(12)$, then for each component the mass term has the unique form $-m\bar{\Psi}\Psi$. In the static limit, where each four-component spinor reduces to two components, we have a set of particles corresponding to the representation of $U(6)$ with the same symmetry type.[11]

Similarly, since the quark interaction is $\tilde{U}(12)$ invariant, we may expect the effective interaction terms between the bound states to exhibit this same invariance. Their structure is then of the type described by SDS. As in the case of the quark Lagangian itself, the only term in the Lagrangian which lacks $\tilde{U}(12)$ invariance is the kinetic term. The determination of the form of this term is the main concern of the following sections.

The two essential elements in the theory are the $\tilde{U}(12)$ invariance of the interaction terms, and the Bargmann-Wigner equations which describe the way in which this $\tilde{U}(12)$ symmetry is broken. As we have seen, both these elements arise very naturally out of a quark model. However, they are independent of it, and may be taken as a postulational basis for the theory. In fact, there are some obvious difficulties in the way of a genuine quark model. For, the baryons are assigned to the totally symmetric representation **364**, whereas an S-wave bound state of three quarks must be totally antisymmetric. Thus a quark model would lead more naturally to the representation **220**.

The relationship between the group $\tilde{U}(12)$ and the symmetries of the particle states may be elucidated by introducing, for each value of p, the orthogonal projection operators

$$\Lambda_\pm(p)=\tfrac{1}{2}(1\pm\gamma p/p), \tag{5}$$

where $p=(p^2)^{1/2}$. These operators are of course undefined when $p^2=0$, but this is of no consequence at this stage because there are no massless strongly interacting particles. We also introduce projection operators

$$\begin{aligned} P_\pm(p)&=\Lambda_\pm\times\Lambda_\pm\times\cdots\times\Lambda_\pm,\\ P_0(p)&=1-P_+(p)-P_-(p), \end{aligned} \tag{6}$$

and note that the Bargmann-Wigner equations can be written in the form

$$\begin{aligned} (p-m)P_+(p)\Psi&=0,\\ -mP_0(p)\Psi&=0,\\ (-p-m)P_-(p)\Psi&=0, \end{aligned} \tag{7}$$

except possibly when $p^2=0$, where these equations are undefined.

---

[9] J. M. Charap and P. T. Matthews, Proc. Roy. Soc. (London), in press.

[10] L. B. Peipuner, W. T. Chu, R. C. Larsen, and Robert K. Adair, Phys. Rev. Letters **12**, 423 (1964).

[11] See also, Abdus Salam, J. Strathdee, J. M. Charap, and P. T. Matthews (unpublished).

Now for any momentum p, the set of operators $\Lambda_+(p)\lambda_i\Gamma_r\Lambda_+(p)$ generate a subgroup of $\tilde{U}(12)$ isomorphic to $U(6)$, which we may call $U_p(6)$. Consider a field transforming according to a given irreducible representation of $\tilde{U}(12)$. Then the Bargmann-Wigner equations (7) require that for any given p only the components $P_+(p)\Psi$ should be nonvanishing. These components clearly transform according to the corresponding irreducible representation of $U_p(6)$. Thus, for each value of the momentum the particle states possess $U(6)$ symmetry. In the static limit, the entire system must of course exhibit this symmetry. However, the intrinsically broken $\tilde{U}(12)$ theory also makes definite predictions about the interaction of particles with different momenta, as shown by SDS, because it specifies the manner in which the $\tilde{U}(12)$ symmetry is broken—namely, by the kinetic terms alone.

3. PRELIMINARY DISCUSSION OF THE LAGRANGIAN FUNCTION

There are no known particles corresponding to representations of $SU(3)$ with nonzero triality,[12] but it will be useful to consider first the mathematically simple case of a two-index field Ψ_{AB}, which may be resolved into the irreducible components **78**⊕**66**. For both representations together the Lagrangian function may be taken to be[13]

$$L=\bar{\Psi}\{\tfrac{1}{2}[(\gamma p)_1+(\gamma p)_2]-m\}\Psi\,, \tag{8}$$

where $(\gamma p)_k$ is defined by Eq. (4). It yields the equation of motion

$$\tfrac{1}{2}[(\gamma p)_1+(\gamma p)_2]\Psi=m\Psi\,, \tag{9}$$

which is the sum of the two Bargmann-Wigner equations. However, multiplying by either $(\gamma p)_1$ or $(\gamma p)_2$ we find that

$$(\gamma p)_1\Psi=(\gamma p)_2\Psi\,, \tag{10}$$

whence both Bargmann-Wigner equations follows. (We assume here and in what follows that $m\neq 0$.) This conclusion is unaffected by restricting Ψ to either of its two irreducible components.

The propagator corresponding to this Lagrangian is

$$\langle\Psi\bar{\Psi}\rangle_+=(\gamma p+m)_1(\gamma p+m)_2/[2m(p^2-m^2)]-(1/2m)\,. \tag{11}$$

Note that the residue at the pole is simply the projection operator onto positive-energy components, as one might expect. Of course, if Ψ has definite symmetry, then (11) must be appropriately symmetrized or antisymmetrized, to yield

$$\langle\Psi_{AB}\bar{\Psi}^{CD}\rangle_+ =(\gamma p+m)_A{}^C(\gamma p+m)_B{}^D\pm(\gamma p+m)_A{}^D(\gamma p+m)_B{}^C/[4m(p^2-m^2)]-(1/4m)(\delta_A{}^C\delta_B{}^D\pm\delta_A{}^D\delta_B{}^C)\,. \tag{12}$$

Before going on to consider higher multispinors, it may be well to point out the difference between this two-quark bound state and a quark-antiquark bound state, described by a field $\Phi_A{}^B$ belonging to the adjoint representation **143**. (We shall ignore the traceless condition, which is an inessential complication.) The Lagrangian may be taken to be

$$L=\tfrac{1}{2}\,\mathrm{tr}[\Phi(\gamma p-m)\Phi]\,. \tag{13}$$

In finding the equations of motion, one must remember that p_μ really stands for $i\partial_\mu$, and there is therefore a change of sign when it acts to the left. The equations of motion may therefore be written

$$\tfrac{1}{2}[\gamma p,\Phi]=m\Phi\,,$$

from which we may deduce, by pre- and post-multiplication by γp, that

$$\tfrac{1}{2}\{\gamma p,\Phi\}=0\,.$$

Thus we obtain the two Bargmann-Wigner equations in the form

$$\gamma p\Phi=m\Phi=-\Phi\gamma p\,. \tag{14}$$

The difference in sign is to be expected, since we must project onto the negative-energy components of the antiquark field $\bar{\psi}$, and the positive-energy components of ψ.

The corresponding propagator is

$$\langle\Phi_A{}^B\Phi_C{}^D\rangle_+=(\gamma p+m)_A{}^D(-\gamma p+m)_C{}^B/[2m(p^2-m^2)]-(1/2m)\delta_A{}^D\delta_C{}^B\,. \tag{15}$$

If Φ is restricted from the outset to be traceless, then the only difference is the appearance of an extra contact term

$$+\frac{1}{48m}\delta_A{}^B\delta_C{}^D\,.$$

Now, when we go to higher multispinors, this same simple procedure fails to work. A Lagrangian with kinetic term involving

$$\frac{1}{n}[(\gamma p)_1+(\gamma p)_2+\cdots+(\gamma p)_n] \tag{16}$$

would yield as before the sum of all n Bargmann-Wigner equations, but it is no longer true that these imply the n equations separately. For $n=3$, for example, the expression (16) has as eigenvalues not only $\pm p$ but also $\pm p/3$, so that the equations have solutions with $p^2=9m^2$ as well as $p^2=m^2$.

Clearly, what is needed is a kinetic term which is nonzero only when the eigenvalues of the terms of

[12] G. E. Baird and L. C. Biedenharn in *Proceedings of the 1964 Coral Gables Conference on Symmetry Principles at High Energy* (W. H. Freeman and Company, San Francisco, 1964), p. 58.

[13] With the identification $\beta_\mu=\frac{1}{2}[(\gamma_\mu)_1+(\gamma_\mu)_2]$, this is the Lagrangian for the Kemmer equation. See Ref. 7.

(16) are all positive, or all negative. Thus, in terms of the projection operators (6) we might take, as a preliminary guess,

$$L=\bar{\Psi}[K(p)-m]\Psi\,, \tag{17}$$

where

$$K(p)=p[P_+(p)-P_-(p)]. \tag{18}$$

Let us write

$$(\gamma p)^{(r)}=\sum(\gamma p)_{k_1}(\gamma p)_{k_2}\cdots(\gamma p)_{k_r}\,,$$

where the sum is over all $\binom{n}{r}$ selections of different indices $(k_1\cdots k_r)$ from $(1\cdots n)$. Then an alternative expression for $K(p)$ is

$$K(p)=\frac{1}{2^{n-1}}\sum_r\frac{(\gamma p)^{(2r+1)}}{(p^2)^r}.$$

We note also that

$$P(p)=P_+(p)+P_-(p)=\frac{1}{2^{n-1}}\sum_r\frac{(\gamma p)^{(2r)}}{(p^2)^r}.$$

The "Lagrangian" (17) is still not altogether satisfactory, because it is nonlocal, owing to the appearance of the factors of p^2 in the denominator. (To replace these factors by m^2 would destroy the projection character of $P_\pm$, and lead to another multi-mass equation.) Nevertheless, it has many of the features of the correct Lagrangian, and it will prove useful to carry the discussion a stage further. Evidently, the equations of motion

$$[K(p)-m]\Psi=0$$

are identical with (7), so to this extent (17) is correct. The corresponding "propagator" is readily found to be

$$\langle\Psi\bar{\Psi}\rangle_+=\frac{P_+(p)}{p-m}+\frac{P_-(p)}{-p-m}+\frac{P_0(p)}{-m}\,,$$
$$=\frac{K(p)+mP(p)}{p^2-m^2}-\frac{P_0(p)}{m}. \tag{19}$$

It evidently has a spurious pole at $p^2=0$ which arises from the p^2 denominators in L. This is clearly an unphysical and undesirable feature which must be removed in a correct propagator. However, the important feature which is illustrated by the form (19) is the residue of the pole at $p^2=m^2$, namely,

$$(p^2-m^2)\langle\Psi\bar{\Psi}\rangle_+|_{p^2=m^2}=2mP_+'=2m(\Lambda_+')_1\cdots(\Lambda_+')_n\,, \tag{20}$$

with $\Lambda_\pm'=(m\pm\gamma p)/2m$.

For example, for $n=3$, the explicit form of this "propagator" is

$$\langle\Psi\bar{\Psi}\rangle_+=\frac{(\gamma p+m)_1(\gamma p+m)_2(\gamma p+m)_3}{4m^2(p^2-m^2)}-\frac{(\gamma p)_1(\gamma p)_2(\gamma p)_3}{4m^2p^2}-\frac{3}{4m}. \tag{21}$$

We shall see in the following sections that the true propagator has a remarkably similar structure.

The expression (20) for the residue at the physical particle pole provides a simple means of reducing peripheral model calculations with exchanged particles corresponding to an arbitrary representation to traces of Dirac matrices. One must of course apply symmetrization or antisymmetrization, and also reverse the sign of p in any factors referring to antiquark indices. For example, the residue at the pole for the meson representation **4212** is

$$(p^2-m^2)\langle\Phi_{AB}{}^{CD}\Phi_{EF}{}^{GH}\rangle_+|_{p^2=m^2}$$
$$=2m\tfrac{1}{4}[(\Lambda_+')_A{}^G(\Lambda_+')_B{}^H-(\Lambda_+')_A{}^H(\Lambda_+')_B{}^G]$$
$$\times[(\Lambda_-')_E{}^C(\Lambda_-')_F{}^D-(\Lambda_-')_E{}^D(\Lambda_-')_F{}^C]. \tag{22}$$

Note that the trace on any pair of indices like A and C is automatically zero, as it should be.

4. THE SPIN-$\frac{3}{2}$ FIELD

It is well known that to find a Lagrangian for many higher spin equations it is necessary to introduce extra fields which are set equal to zero by the equations of motion.[14] We must expect this to be true also of the Bargmann-Wigner equations. Thus we shall look for Lagrangians involving the fields we want, and also some auxiliary fields. Without loss of generality we can require that the Lagrangian function shall be linear in the derivatives, since this may always be achieved by adding more fields.[15]

In a free Lagrangian, each $SU(3)$ multiplet within any $\tilde{U}(12)$ representation must enter separately if the Lagrangian is to retain $SU(3)$ invariance. Thus the $SU(3)$ indices are an irrelevant complication, and we shall drop them and concentrate on an irreducible representation of $\tilde{U}(4)$.

Let us consider first a spin-$\frac{3}{2}$ particle described by a totally symmetric spinor $\Psi_{\alpha\beta\gamma}$. As we have seen in the preceding section no simple Lagrangian can be constructed out of Ψ alone which will yield the Bargmann-Wigner equations. Thus, we are forced to include extra fields in the Lagrangian. The most natural choice for these are other three-index fields with different symmetries. Thus, we shall consider in addition to Ψ a field Φ of symmetry type [2,1] and a field Ω of symmetry type [1^3]. It is not difficult to write down the most general form of bilinear Lagrangian which can be constructed from Ψ, Φ, Ω and the matrix γp. The requirement that this Lagrangian should yield the Bargmann-Wigner equations for Ψ, and set Φ and Ω equal to zero, then fixes all the coefficients uniquely, apart from arbitrary normalization constants on Φ and Ω.

We begin by introducing some notation. We shall use parentheses and square brackets to denote complete

[14] M. Fierz and W. Pauli, Proc. Roy. Soc. (London) **A173**, 211 (1939).
[15] J. Schwinger, Phys. Rev. **91**, 713 (1953).

symmetrization or antisymmetrization, respectively, of all the indices enclosed within them. Thus, the symmetries of Ψ and Ω are represented by $\Psi_{(\alpha\beta\gamma)}$ and $\Omega_{[\alpha\beta\gamma]}$. The field Φ of mixed symmetry may be represented in either of the two equivalent forms $\Phi_{(\alpha\beta)\gamma}$ and $\Phi'_{[\alpha\beta]\gamma}$, which satisfy the cyclic identities

$$\begin{aligned}\Phi_{(\alpha\beta)\gamma}+\Phi_{(\beta\gamma)\alpha}+\Phi_{(\gamma\alpha)\beta}&=0\,,\\ \Phi'_{[\alpha\beta]\gamma}+\Phi'_{[\beta\gamma]\alpha}+\Phi'_{[\gamma\alpha]\beta}&=0\,.\end{aligned}\tag{23}$$

They are related by

$$\begin{aligned}\Phi_{(\beta\gamma)\alpha}-\Phi_{(\gamma\alpha)\beta}&=\sqrt{3}\Phi'_{[\alpha\beta]\gamma}\,,\\ \Phi'_{[\beta\gamma]\alpha}-\Phi'_{[\gamma\alpha]\beta}&=-\sqrt{3}\Phi_{(\alpha\beta)\gamma}\,.\end{aligned}\tag{24}$$

Clearly, by using these symmetry properties we can always reduce any linear combination of components of Φ to a linear combination of $\Phi_{(\alpha\beta)\gamma}$ and $\Phi'_{[\alpha\beta]\gamma}$ with the indices written in standard order. The normalization factors in (24) are so chosen that

$$\bar{\Phi}'\Phi'=\bar{\Phi}\Phi\,.$$

From these symmetry relations, it is easy to establish the following identities:

$$\begin{aligned}\bar{\Phi}'(\gamma p)_1\Phi'&=\bar{\Phi}'(\gamma p)_2\Phi'=\tfrac{1}{3}\bar{\Phi}(\gamma p)_1\Phi+\tfrac{2}{3}\bar{\Phi}(\gamma p)_3\Phi\,,\\ \bar{\Phi}'(\gamma p)_3\Phi'&=\tfrac{4}{3}\bar{\Phi}(\gamma p)_1\Phi-\tfrac{1}{3}\bar{\Phi}(\gamma p)_3\Phi\,,\end{aligned}\tag{25}$$

together with an identical set in which Φ and Φ' are interchanged. Similarly, we have

$$\begin{aligned}&\bar{\Psi}(\gamma p)_1\Phi=\bar{\Psi}(\gamma p)_2\Phi=-\tfrac{1}{2}\bar{\Psi}(\gamma p)_3\Phi\,,\\ &\bar{\Psi}(\gamma p)_1\Phi'=-\bar{\Psi}(\gamma p)_2\Phi'=\tfrac{1}{2}\sqrt{3}\bar{\Psi}(\gamma p)_3\Phi\,,\\ &\bar{\Psi}(\gamma p)_3\Phi'=0\,,\end{aligned}\tag{26}$$

and also

$$\begin{aligned}&\bar{\Omega}(\gamma p)_1\Phi'=\bar{\Omega}(\gamma p)_2\Phi'=-\tfrac{1}{2}\bar{\Omega}(\gamma p)_3\Phi'\,,\\ &\bar{\Omega}(\gamma p)_1\Phi=-\bar{\Omega}(\gamma p)_2\Phi=-\tfrac{1}{2}\sqrt{3}\bar{\Omega}(\gamma p)_3\Phi'\,,\\ &\bar{\Omega}(\gamma p)_3\Phi=0\,.\end{aligned}\tag{27}$$

The appropriate Lagrangian for a spin-$\frac{3}{2}$ field may now be written in the form

$$\begin{aligned}L=&-m\bar{\Psi}\Psi+2m\bar{\Phi}\Phi-m\bar{\Omega}\Omega\\ &+\bar{\Psi}(\gamma p)_3\Psi-\tfrac{1}{2}\bar{\Phi}(\gamma p)_3\Phi+\tfrac{1}{2}\bar{\Phi}'(\gamma p)_3\Phi'-\bar{\Omega}(\gamma p)_3\Omega\\ &+\tfrac{1}{2}[\bar{\Psi}(\gamma p)_3\Phi+\bar{\Phi}(\gamma p)_3\Psi]\\ &\qquad+\tfrac{1}{2}[\bar{\Omega}(\gamma p)_3\Phi'+\bar{\Phi}'(\gamma p)_3\Omega]\,.\end{aligned}\tag{28}$$

The most general possible structure has precisely this form but with arbitrary coefficients. To show that (28) is unique, one may follow through the arguments below keeping these coefficients arbitrary, and show that for all other choices there are additional unwanted solutions to the field equations.

The equations of motion may be obtained by varying Ψ,Φ,Ω subject to the appropriate symmetry constraints. Thus, for Ψ and Ω, we have

$$m\Psi_{\alpha\beta\gamma}=(\gamma p)_{(\gamma}{}^{\delta}\Psi_{\alpha\beta)\delta}+\tfrac{1}{2}(\gamma p)_{(\gamma}{}^{\delta}\Phi_{\alpha\beta)\delta}\,,\tag{29}$$

$$m\Omega_{\alpha\beta\gamma}=-(\gamma p)_{[\gamma}{}^{\delta}\Omega_{\alpha\beta]\delta}+\tfrac{1}{2}(\gamma p)_{[\gamma}{}^{\delta}\Phi'_{\alpha\beta]\delta}\,.\tag{30}$$

The equation for Φ is more complicated, and may be written

$$\begin{aligned}-2m\Phi_{\alpha\beta\gamma}=&-\tfrac{1}{3}[(\gamma p)_\alpha{}^\delta\Phi_{\gamma\delta\beta}+(\gamma p)_\beta{}^\delta\Phi_{\delta\gamma\alpha}+(\gamma p)_\gamma{}^\delta\Phi_{\alpha\beta\delta}]\\ &+\tfrac{1}{6}[2(\gamma p)_\gamma{}^\delta\Psi_{\alpha\beta\delta}-(\gamma p)_\alpha{}^\delta\Psi_{\delta\beta\gamma}-(\gamma p)_\beta{}^\delta\Psi_{\alpha\delta\gamma}]\\ &-(2\sqrt{3})^{-1}[(\gamma p)_\alpha{}^\delta\Omega_{\delta\beta\gamma}-(\gamma p)_\beta{}^\delta\Omega_{\alpha\delta\gamma}]\,,\end{aligned}\tag{31}$$

or, equivalently,

$$\begin{aligned}-2m\Phi'_{\alpha\beta\gamma}=&\tfrac{1}{3}[(\gamma p)_\alpha{}^\delta\Phi'_{\gamma\delta\beta}+(\gamma p)_\beta{}^\delta\Phi'_{\delta\gamma\alpha}+(\gamma p)_\gamma{}^\delta\Phi'_{\alpha\beta\delta}]\\ &+\tfrac{1}{6}[2(\gamma p)_\gamma{}^\delta\Omega_{\alpha\beta\delta}-(\gamma p)_\alpha{}^\delta\Omega_{\delta\beta\gamma}-(\gamma p)_\beta{}^\delta\Omega_{\alpha\delta\gamma}]\\ &+(2\sqrt{3})^{-1}[(\gamma p)_\alpha{}^\delta\Psi_{\delta\beta\gamma}-(\gamma p)_\beta{}^\delta\Psi_{\alpha\delta\gamma}]\,.\end{aligned}\tag{32}$$

Now we may use the symmetry properties to rearrange all the indices in standard order, and then rewrite these equations in a more symbolic notation in the form

$$\begin{aligned}m\Psi=&\tfrac{1}{3}[(\gamma p)_1+(\gamma p)_2+(\gamma p)_3]\Psi\\ &+\tfrac{1}{12}[2(\gamma p)_3-(\gamma p)_1-(\gamma p)_2]\Phi\\ &+(4\sqrt{3})^{-1}[(\gamma p)_1-(\gamma p_2)]\Phi'\,,\end{aligned}\tag{33}$$

$$\begin{aligned}m\Omega=&-\tfrac{1}{3}[(\gamma p_1)+(\gamma p)_2+(\gamma p)_3]\Omega\\ &-(4\sqrt{3})^{-1}[(\gamma p)_1-(\gamma p_2)]\Phi\\ &+\tfrac{1}{12}[2(\gamma p)_3-(\gamma p)_1-(\gamma p)_2]\Phi'\,,\end{aligned}\tag{34}$$

and

$$\begin{aligned}-2m\Phi=&\tfrac{1}{6}[2(\gamma p)_3-(\gamma p)_1-(\gamma p)_2](\Psi-\Phi)\\ &-(2\sqrt{3})^{-1}[(\gamma p)_1-(\gamma p)_2](\Omega+\Phi')\,,\end{aligned}\tag{35}$$

or

$$\begin{aligned}-2m\Phi'=&(2\sqrt{3})^{-1}[(\gamma p)_1-(\gamma p)_2](\Psi-\Phi)\\ &+\tfrac{1}{6}[2(\gamma p)_3-(\gamma p)_1-(\gamma p)_2](\Omega+\Phi')\,.\end{aligned}\tag{36}$$

Multiplying these equations by $(\gamma p)_1+(\gamma p)_2$, we obtain two pairs of uncoupled equations for Ψ,Φ and Ω,Φ'. A straightforward elimination of Φ and Φ' then yields the equations

$$\begin{aligned}&m[(\gamma p)_1+(\gamma p)_2]\Psi\\ &=\tfrac{1}{2}[p^2+(\gamma p)_1(\gamma p)_2+(\gamma p)_2(\gamma p)_3+(\gamma p)_3(\gamma p)_1]\Psi\,,\end{aligned}\tag{37}$$

$$\begin{aligned}&m[(\gamma p)_1+(\gamma p)_2]\Omega\\ &=-\tfrac{1}{2}[p^2+(\gamma p)_1(\gamma p)_2+(\gamma p)_2(\gamma p)_3+(\gamma p)_3(\gamma p)_1]\Omega\,,\end{aligned}\tag{38}$$

whence it follows from the symmetries that

$$\begin{aligned}(\gamma p)_1\Psi&=(\gamma p)_2\Psi=(\gamma p)_3\Psi\,,\\ (\gamma p)_1\Omega&=(\gamma p)_2\Omega=(\gamma p)_3\Omega\,.\end{aligned}\tag{39}$$

Thus, the Eqs. (37) and (38) have nonzero solutions only in two cases. Either (a) $p^2=m^2$ and

$$(\gamma p-m)\Psi=(\gamma p-m)\Omega=0\,,\tag{40}$$

or (b) $p^2=0$ and

$$\gamma p\Psi=\gamma p\Omega=0\,.$$

In either case, it follows immediately that $\Omega=0$, because a totally antisymmetric spinor cannot be a simultaneous eigenstate of all three of the operators $(\gamma p)_k$

with the same eigenvalue.[16] Then, comparing Eqs. (34) and (36), and using (39), we find that Φ' (and therefore Φ) also vanishes. Finally, returning to Eq. (33) we see that in case (b) $\Psi=0$. Hence we are left with a unique solution

$$(\gamma p)_1\Psi=(\gamma p)_2\Psi=(\gamma p)_3\Psi=m\Psi\,, \tag{41}$$

$$\Phi=\Phi'=\Omega=0\,. \tag{42}$$

Thus, we have shown that the Lagrangian (28) yields the correct Bargmann-Wigner equations, and sets the auxiliary fields equal to zero.

It is straightforward, if tedious, to compute the propagators. We find for the physical field Ψ,

$$\langle\Psi\bar{\Psi}\rangle_+=\frac{(\gamma p+m)_1(\gamma p+m)_2(\gamma p+m)_3}{4m^2(p^2-m^2)}-\frac{(\gamma p)_1+(\gamma p)_2+(\gamma p)_3}{12m^2}-\frac{3}{4m}, \tag{43}$$

in which total symmetrization is understood. This expression may be compared with our earlier form (21). It will be noted that it differs from it only in the momentum-dependent contact term, and that even the coefficients of the contact terms agree. The remaining propagators all consist of pure contact terms, and have no physical significance unless the auxiliary fields Φ and Ω are coupled in some way to other fields. For completeness, we list them below:

$$\langle\Psi\bar{\Phi}\rangle_+=\langle\Phi\bar{\Psi}\rangle_+=\langle\Omega\bar{\Phi}'\rangle_+=\langle\Phi'\bar{\Omega}\rangle_+=\frac{(\gamma p)_1+(\gamma p)_2-2(\gamma p)_3}{12m^2}, \tag{44}$$

$$\langle\Omega\bar{\Omega}\rangle_+=\frac{(\gamma p)_1+(\gamma p)_2+(\gamma p)_3}{3m^2}-\frac{1}{m}, \tag{45}$$

$$\langle\Phi\bar{\Phi}\rangle_+=\frac{2(\gamma p)_3-(\gamma p)_1-(\gamma p)_2}{12m^2}+\frac{1}{2m}. \tag{46}$$

These equations are to be understood in a symbolic sense. In each of them, appropriate symmetrization or antisymmetrization is implied, and it is thus not really true that $\langle\Psi\bar{\Phi}\rangle_+=\langle\Phi\bar{\Psi}\rangle_+$. Written out in full, each is a sum of 18 terms. For example,

$$\langle\Psi_{\alpha\beta\gamma}\bar{\Phi}^{\delta\epsilon\xi}\rangle_+=-\frac{1}{12m^2}[2\delta_{(\alpha}{}^{\delta}\delta_{\beta}{}^{\epsilon}(\gamma p)_{\gamma)}{}^{\xi}-\delta_{(\alpha}{}^{\epsilon}\delta_{\beta}{}^{\xi}(\gamma p)_{\gamma)}{}^{\delta}-\delta_{(\alpha}{}^{\xi}\delta_{\beta}{}^{\delta}(\gamma p)_{\gamma)}{}^{\epsilon}].$$

[16] Because of the use of this condition, it would be incorrect to generalize the Lagrangian (28) directly to the description of the $\tilde{U}(12)$ multiplet **364**.

The relationship of this propagator to that given by SDS is discussed in the Appendix.

5. THE 20-COMPONENT SPIN-$\frac{1}{2}$ FIELD

Next we turn to the case of a spin-$\frac{1}{2}$ particle described by a field with symmetry type [2,1]. The Lagrangian may be expressed in terms of the same variables we used in the preceding section, but without the totally symmetric field Ψ. It is

$$L=-m\bar{\Phi}'\Phi'+\tfrac{1}{2}m\bar{\Omega}\Omega+\bar{\Phi}'(\gamma p)_1\Phi'+\bar{\Omega}(\gamma p)_3\Omega-\tfrac{1}{2}[\bar{\Phi}'(\gamma p)_3\Omega+\bar{\Omega}(\gamma p)_3\Phi']. \tag{47}$$

The equations of motion read

$$m\Phi'_{\alpha\beta\gamma}=\tfrac{1}{2}[(\gamma p)_\alpha{}^\delta\Phi'_{\delta\beta\gamma}+(\gamma p)_\beta{}^\delta\Phi'_{\alpha\delta\gamma}]+\tfrac{1}{6}[(\gamma p)_\alpha{}^\delta\Phi'_{\beta\gamma\delta}+(\gamma p)_\beta{}^\delta\Phi'_{\gamma\alpha\delta}+(\gamma p)_\gamma{}^\delta\Phi'_{\alpha\beta\delta}]+\tfrac{1}{6}[(\gamma p)_\alpha{}^\delta\Omega_{\delta\beta\gamma}+(\gamma p)_\beta{}^\delta\Omega_{\alpha\delta\gamma}-2(\gamma p)_\gamma{}^\delta\Omega_{\alpha\beta\delta}],$$

and

$$-\tfrac{1}{2}m\Omega_{\alpha\beta\gamma}=-\tfrac{1}{2}(\gamma p)_{[\alpha}{}^\delta\Phi'_{\beta\gamma]\delta}+(\gamma p)_{[\alpha}{}^\delta\Omega_{\beta\gamma]\delta}.$$

Written in the same symbolic form as before, they become

$$m\Phi=\tfrac{1}{4}[(\gamma p)_1+(\gamma p)_2+2(\gamma p)_3]\Phi-(4\sqrt{3})^{-1}[(\gamma p)_1-(\gamma p)_2]\Phi'+(2\sqrt{3})^{-1}[(\gamma p)_1-(\gamma p)_2]\Omega, \tag{48}$$

or

$$m\Phi'=-(4\sqrt{3})^{-1}[(\gamma p)_1-(\gamma p)_2]\Phi+\tfrac{1}{12}[5(\gamma p)_1+5(\gamma p)_2+2(\gamma p)_3]\Phi'+\tfrac{1}{6}[(\gamma p)_1+(\gamma p)_2-2(\gamma p)_3]\Omega, \tag{49}$$

and

$$-\tfrac{1}{2}m\Omega=(4\sqrt{3})^{-1}[(\gamma p)_1-(\gamma p)_2]\Phi+\tfrac{1}{12}[(\gamma p)_1+(\gamma p)_2-2(\gamma p)_3]\Phi'+\tfrac{1}{3}[(\gamma p)_1+(\gamma p)_2+(\gamma p)_3]\Omega. \tag{50}$$

Note that from (49) and (50) it follows that

$$m(\Phi'-\tfrac{1}{2}\Omega)=\tfrac{1}{2}[(\gamma p)_1+(\gamma p)_2](\Phi'+\Omega)\,. \tag{51}$$

Operating on (50) with $[(\gamma p)_1+(\gamma p)_2]$ and eliminating Φ' between this equation and (51) we obtain the equation for Ω,

$$-\tfrac{1}{2}m^2[(\gamma p)_1+(\gamma p)_2]\Omega=\tfrac{1}{4}m[p^2+(\gamma p)_1(\gamma p)_2+(\gamma p)_2(\gamma p)_3+(\gamma p)_3(\gamma p)_1]\Omega-\tfrac{1}{2}[p^2\{(\gamma p)_1+(\gamma p)_2+(\gamma p)_3\}+(\gamma p)_1(\gamma p)_2(\gamma p)_3]\Omega\,,$$

from which it follows immediately that

$$(\gamma p)_1\Omega=(\gamma p)_2\Omega=(\gamma p)_3\Omega\,.$$

Then as in the previous section we can conclude that

$\Omega=0$. It then follows from (51) that

$$(\gamma p)_1\Phi'=(\gamma p)_2\Phi',$$

and from (50) that both expressions are equal to $(\gamma p)_3\Phi'$. Finally, using (49) we see that Φ' satisfies the Bargmann-Wigner equations

$$(\gamma p)_1\Phi'=(\gamma p)_2\Phi'=(\gamma p)_3\Phi'=m\Phi'. \qquad (52)$$

We can now evaluate the propagator as before. We find

$$\langle\Phi'\bar{\Phi}'\rangle_+=\frac{(\gamma p+m)_1(\gamma p+m)_2(\gamma p+m)_3}{4m^2(p^2-m^2)} -\frac{(\gamma p)_1+(\gamma p)_2-(\gamma p)_3}{4m^2}-\frac{3}{4m}. \qquad (53)$$

Note that this expression is very similar to the spin-$\frac{3}{2}$ propagator, Eq. (43). The propagators involving Ω are

$$\langle\Phi'\bar{\Omega}\rangle_+=\langle\Omega\bar{\Phi}'\rangle_+=\frac{2(\gamma p)_3-(\gamma p)_1-(\gamma p)_2}{3m^2}, \qquad (54)$$

$$\langle\Omega\bar{\Omega}\rangle_+=-4\frac{(\gamma p)_1+(\gamma p)_2+(\gamma p)_3}{3m^2}+\frac{2}{m}. \qquad (55)$$

6. DISCUSSION

There is no difficulty of principle in extending the discussion of the preceding sections to higher representations, though in practice the number of auxiliary fields required increases so rapidly that the algebra would soon become prohibitive. In any case, there is probably little point in going further. Even if one believes that some of the multiplets should be represented by fundamental fields, this is hardly likely to be true except for the simplest representations. If the higher multiplets are regarded as bound states, there is no need to look for a local Lagrangian function to describe them. In an S-matrix theory, we require only the free equations of motion (the Bargmann-Wigner equations) to describe the asymptotic particle states, and the residue of the pole in the propagator, which is given by the formulas of Sec. 3. From these and the $\tilde{U}(12)$—invariant form of the vertex, we can calculate the appropriate discontinuity functions to feed into the S-matrix calculations. The only additional information one would obtain from a Lagrangian formulation concerns the contact terms in the propagator, which in an S-matrix theory would appear as additional four-particle vertices.

It is worth noting that with the simple form (20) for the residue, it is very easy to calculate pole diagrams in the approximation where the mass splittings of the $SU(6)$ multiplets are neglected. In general, however, one would wish to take account of the mass splittings, and it is then necessary to project out from this expression the various $SU(3)$, and even $SU(2)$, multiplets, so that the poles can be placed in their correct position.

The necessity of introducing auxiliary fields to obtain a Lagrangian formulation of the Bargmann-Wigner equations should not be taken to imply that these fields have any direct physical significance. When interactions are introduced, they will in general no longer be zero. However, provided that the interaction terms involve only the physical fields (Ψ rather than Φ or Ω for the spin-$\frac{3}{2}$ case), then the auxiliary fields will still be constraint variables whose values are fixed in terms of the dynamical fields by the equations of motion. The $\tilde{U}(12)$ invariant interactions of SDS are of course of this type. So long as this condition is satisfied, the precise structure of the propagator for these auxiliary fields is physically irrelevant, because they never appear in the evaluation of any physical matrix element. Any interaction which actually excites these fields as additional dynamical variables is liable to encounter difficulties with the positivity of the metric. In this connection, the question of the consistency of the electromagnetic interactions deserves further investigation.

ACKNOWLEDGMENTS

We are indebted to Professor Abdus Salam for a helpful discussion on this problem. One of us (G.S.G.) would like to thank Professor P. T. Matthews for hospitality.

APPENDIX

Here we wish to relate the propagators obtained above to the field variables of the type introduced by SDS.

For the representation **143** we may write

$$\Phi_A{}^B=(2\sqrt{2})^{-1}(\Gamma_r\lambda_i)_A{}^B\phi^{ri}$$

and find for the propagators

$$\langle\phi^{ri}\phi^{sj}\rangle_+ =\delta^{ij}\frac{\frac{1}{4}\,\mathrm{tr}[(\gamma p+m)\Gamma^s(-\gamma p+m)\Gamma^r]}{2m(p^2-m^2)}-\delta^{ij}\frac{\frac{1}{4}\,\mathrm{tr}(\Gamma^s\Gamma^r)}{2m}.$$

This yields the familiar spin-0 and spin-1 propagators, identical with those found by SDS.

Next, let us turn to the three-index fields. Any third-rank spinor can be written (in a Majorana representation with $C=\beta$) in the form

$$\Psi_{\alpha\beta\gamma}=(2\sqrt{2})^{-1}(\Gamma_r\beta^{-1})_{\alpha\beta}\psi^r{}_\gamma,$$
$$\bar{\Psi}^{\alpha\beta\gamma}=-(2\sqrt{2})^{-1}(\beta\Gamma^r)^{\beta\alpha}\bar{\psi}_r{}^\gamma.$$

For the totally symmetric spin-$\frac{3}{2}$ field, we have

$$\Psi_{\alpha\beta\gamma}=(2\sqrt{2})^{-1}(\gamma_\mu\beta^{-1})_{\alpha\beta}\psi^\mu{}_\gamma+(4\sqrt{2})^{-1}(\sigma_{\mu\nu}\beta^{-1})_{\alpha\beta}\psi^{\mu\nu}{}_\gamma.$$

The corresponding propagators are easily found to be

$$\langle\psi^r\bar{\psi}_s\rangle=[24m^2(p^2-m^2)]^{-1}\{(\gamma p+m)\ \mathrm{tr}[(\gamma p+m)\Gamma_s(\gamma p-m)\Gamma^r]+2(\gamma p+m)\Gamma_s(\gamma p-m)\Gamma^r(\gamma p+m)\}$$
$$+[72m^2]^{-1}\{\gamma p\ \mathrm{tr}[\Gamma_s\Gamma^r]+2\ \mathrm{tr}[\gamma p\Gamma_s\Gamma^r]+2\gamma p\Gamma_s\Gamma^r-2\Gamma_s\gamma p\Gamma^r+2\Gamma_s\Gamma^r\gamma p\}+[8m]^{-1}\{\mathrm{tr}[\Gamma_s\Gamma^r]+2\Gamma_s\Gamma^r\}.$$

Similarly, for the spin-($\frac{1}{2}$) field we may write

$$\Phi'_{[\alpha\beta]\gamma}=(2\sqrt{2})^{-1}(\beta^{-1})_{\alpha\beta}\psi_\gamma+(2\sqrt{2})^{-1}(\sigma_{\mu 5}\beta^{-1})_{\alpha\beta}\psi^{\mu 5}\gamma+(2\sqrt{2})^{-1}(\gamma_5\beta^{-1})_{\alpha\beta}\psi^5\gamma\,,$$

and find for the propagators

$$\langle\psi^r\bar{\psi}_s\rangle_+=[12m^2(p^2-m^2)]^{-1}\{(\gamma p+m)\ \mathrm{tr}[(\gamma p+m)\Gamma_s(\gamma p-m)\Gamma^r]-(\gamma p+m)\Gamma_s(\gamma p-m)\Gamma^r(\gamma p+m)\}$$
$$-[36m^2]^{-1}\{\gamma p\ \mathrm{tr}[\Gamma_s\Gamma^r]-4\ \mathrm{tr}[\gamma p\Gamma_s\Gamma^r]+\gamma p\Gamma_s\Gamma^r+5\Gamma_s\gamma p\Gamma^r+\Gamma_s\Gamma^r\gamma p\}+[4m]^{-1}\{\mathrm{tr}[\Gamma_s\Gamma^r]-\Gamma_s\Gamma^r\}.$$

The explicit evaluation of these propagators is straightforward but the result is not particularly illuminating, and we omit it. We note that the residues at the pole $p^2=m^2$ are identical with those of SDS, but that the contact terms are different. The asymptotic behavior for large p is no worse than linear.

PHYSICAL REVIEW VOLUME 139, NUMBER 3B 9 AUGUST 1965

Low-Energy K^--Meson Interactions in Hydrogen*

M. SAKITT,† T. B. DAY, R. G. GLASSER, AND N. SEEMAN
University of Maryland, College Park, Maryland and Naval Research Laboratory, Washington, D. C.

AND

J. FRIEDMAN, W. E. HUMPHREY, AND R. R. ROSS
Lawrence Radiation Laboratory, University of California, Berkeley, California

(Received 25 March 1965)

Low-energy K^--meson interactions in hydrogen are studied in the following channels:

$$K^-+p\to K^-+p,$$
$$K^-+p\to\Sigma^-+\pi^+,$$
$$K^-+p\to\Sigma^++\pi^-,$$

and cross sections, as a function of momentum, are presented in the region of 60–300 MeV/c K^- laboratory momentum. These cross sections, combined with existing data, are used to fit the zero-effective-range theory of Dalitz and Tuan. Two possible solutions are obtained; the preferred one agrees with previous higher energy data. The favored solution also suggests an S-wave bound state at 1410 MeV, which could be associated with the Y_0^* at 1405 MeV whose spin is still undetermined. Various properties of the two solutions are presented for K^-p interactions and $K_2{}^0p$ interactions.

I. INTRODUCTION

DURING the past several years there have been several theoretical investigations of low-energy $\bar{K}N$ interactions. Jackson and Wyld,[1] and Dalitz and Tuan[2] have both developed an S-wave zero-effective-range formalism, taking into account the Coulomb interactions and mass-difference effects. Humphrey and Ross[3] measured the cross sections, in the various channels, at K^- laboratory momentum below 275 MeV/c, and using the Dalitz and Tuan formalism determined two possible sets of scattering lengths, $HR1$ and $HR2$. The favored solution, $HR1$, predicts a positive phase angle between the $T=1$ and $T=0$ amplitudes for $\Sigma\pi$ production from K^-P, while $HR2$ predicts a negative phase angle. Higher energy data[4] seem to imply a negative phase angle.[5] Attempts to explain K^-D interactions in terms of $\bar{K}N$ scattering lengths have been made[6–8] and seem to yield better agreement with $HR2$ than with $HR1$. Faced with these problems, it seemed interesting to redo the low-energy experiment with higher statistics.

---

* Work supported in part by the U. S. Atomic Energy Commission.

† Based in part on a Ph.D. dissertation submitted to the University of Maryland, 1965. Present address: Brookhaven National Laboratory, Upton, New York.

[1] J. D. Jackson, D. G. Ravenhall and H. W. Wyld, Nuovo Cimento **10**, 834 (1958).

[2] R. H. Dalitz and S. F. Tuan, Ann. Phys. **10**, 307 (1960).

[3] W. E. Humphrey and R. R. Ross, Phys. Rev. **127**, 1305 (1962).

[4] M. B. Watson, M. Ferri-Luzzi, and R. D. Tripp, Phys. Rev. **131**, 2248 (1963).

[5] T. Akiba and R. H. Capps, Phys. Rev. Letters **8**, 457 (1962).

[6] T. B. Day, G. A. Snow, and J. Sucher, Nuovo Cimento **14**, 637 (1959).

[7] R. Chand and R. H. Dalitz, Ann. Phys. (N. Y.) **20**, 1 (1962).

[8] A. K. Bhatia and J. Sucher, Phys. Rev. **132**, 855 (1963).