New Tests for the Invariance of the Vacuum State Under the Lorentz Group

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We assume that Lorentz invariance is broken in the simplest possible way by the existence of a constant vector field λ^μ which is coupled as if it were an ordinary quantized field. The presence of λ^μ could explain the decay of K_2^0 into $\pi^+\pi^-$ and avoid the difficulties which accompany a similar theory put forward by Bell and Perring. If the coupling is through axial- as well as polar-vector terms a new set of low-energy experiments will serve to elucidate the form of λ^μ in much greater detail than is possible by using K_2^0. Present experimental limits are discussed and we conclude that their improvement by several orders of magnitude is feasible.

1. INTRODUCTION

ONE of the main foundations of modern physical theory is the requirement of Lorentz invariance. From the time of the Michelson-Morley experiment until the present day, no effect has been discovered which definitely violates this principle, and only one, to be discussed below, which may eventually turn out to do so. Nevertheless, Lorentz invariance, like any other scientific idea, can never be completely verified, and is always liable to be overturned by more refined measurements. Since it is so fundamental to our thinking we should continually look for new ways of checking it, with the intention both of increasing the accuracy of previous work and of discovering unsuspected effects through which a violation might be detected. In this paper we point out one of the simplest ways in which the invariance might fail, and show that a clear-cut set of experiments can be used to test this supposition. They are low-energy experiments which are analogous to the classical investigations in gravitation and electrostatics; it is possible that they will give a positive result even if the Michelson-Morley experiment were null to any order of accuracy.

2. EFFECTS OF A PREFERRED FRAME OF REFERENCE

Lorentz invariance implies that all inertial frames of reference are equivalent. The simplest violation consists in picking out one set of frames as preferred; in the days when one spoke of the "ether," these were the frames in which the "ether" appeared to be at rest. This suggests that in our theory we should introduce a four-vector λ^μ, which is simply the velocity vector of our frame of reference in the preferred frame, and hence has the same value throughout space.[1] Except for one conjecture in Sec. 7, we will not consider the nature of theories which provide such a vector as a property of the vacuum state. We restrict ourselves instead to a discussion of the probable form that λ^μ will have in a laboratory on the earth and of possible experiments to detect it.

We will introduce terms involving λ^μ into our Lagrangian by treating the vector as if it were an ordinary quantized field; if our conjecture of Sec. 7 turns out to be correct this will inevitably be the right way to proceed. A Lagrangian containing terms of the form $\lambda^\mu j^\mu$, where j^μ is a neutral current, appears at first sight to be Lorentz invariant, and there is indeed a sense in which this is so. We have invariance if both λ^μ and j^μ are transformed; this corresponds to moving the observer while keeping the system and the "ether" fixed. On the other hand, we can consider transformations in which j^μ is changed but λ^μ remains unaltered; here the system is moved but the observer and the "ether" are not. These two types of transformations are usually distinguished as "passive" and "active," respectively, and in a strictly Lorentz-invariant theory there is no observable difference between them. In field theory, where one considers Lorentz transformations as mappings of the vectors and operators of Hilbert space, it is the "active" transformations which most naturally arise. This is because λ^μ is not quantized, and so will commute with all the operators in the Hilbert space. This method of violating Lorentz invariance is a "minimal" one, in that it introduces the simplest entity (a constant four-vector) and preserves invariance under "passive" transformations.

3. THE FORM OF THE VECTOR λ^μ

The preferred frames of reference are those in which λ^μ takes the simple form (1,0,0,0). The following argument leads to the probable form of λ^μ in a laboratory on the earth. We know that, apart from the general recession, the relative velocity of galaxies is small. If we assume that the recession is an effect of general relativity and can eventually be assimilated into the theory, we can say that for our purposes the galaxies are at rest with respect to each other. It is then natural to identify this rest frame with the preferred frame, so

[1] See also D. I. Blokhintsev, Phys. Letters 12, 272 (1964). A vector similar to our λ^μ is introduced in this letter, but the emphasis is on high-energy experiments, and the conclusion is drawn that it will be impossible to detect such a field until we can perform colliding-beam experiments and compare their results with those obtained under normal conditions. This work is completely independent of ours, and we do not believe that so pessimistic a conclusion is justified.

that an observer at the center of our galaxy would find that λ^μ had no nonzero space components. The earth, however, is by no means at the galactic center, and will acquire a velocity in the preferred frame because of the galactic rotation. This velocity is about 10^{-3} of the velocity of light, and will be nearly tangential to our galactic arm, which lies in the direction of Cygnus. The spatial component of λ^μ in this direction will then be about 10^{-3}. It will be modified to some extent during the course of a year by the earth's motion round the sun. This would be a 10% effect if the plane of the orbit contained the line of the arm. The orbit is, in fact, considerably tilted, and the expected modulation is reduced to 3–4%. Any proper motion of the galaxy as a whole will modify λ^μ. It has been estimated[2] that the velocity of proper motion for most galaxies lies between $3\times10^{-4}c$ and $10^{-3}c$, so that it is comparable with, but generally smaller than, the velocity due to the galactic rotation. We will use a velocity of $10^{-3}c$ in subsequent work, bearing in mind that its direction may be considerably different from the one we expect, and that fortuitous cancellations may reduce its value.

4. THE SIMPLEST FORMS OF INTERACTION

The simplest interaction terms which can appear in our Lagrangian are

$$G_1\lambda^\mu(\phi^\dagger\partial_\mu\phi-(\partial_\mu\phi^\dagger)\phi)\,, \tag{a}$$

$$G_2\lambda^\mu\bar{\psi}\gamma_\mu\psi\,, \tag{b}$$

$$G_3\lambda^\mu\bar{\psi}\gamma_\mu\gamma_5\psi\,. \tag{c}$$

Here ϕ is a scalar field and ψ a spinor field. G_1, G_2, and G_3 are coupling constants with the dimension of energy. Many other interaction terms are possible, but these are the natural ones; they are all linear in λ^μ.

The simplest effect of the vector couplings (a) and (b) is a splitting of the masses of particle and antiparticle. It is interesting that this is one of the interpretations which have been given[3] to the recent high-energy experiment[4] demonstrating the decay mode:

$$K_2{}^0\rightarrow\pi^+ + \pi^-.$$

The mass splitting needed here is of order 10^{-17} of the kaon mass. No measurement of the masses of any other particle-antiparticle pair can yet approach this sensitivity.

Another result of the vector coupling is a modification of the usual relation $E^2-P^2=M^2$. With $G_2\approx10^{-17}M$, the change becomes important only at energies far beyond those of present accelerators.

The situation improves remarkably if we include axial-vector coupling. The axial-vector current is spacelike, and for a particle at rest reduces to a three-vector pointing in the direction of the spin. To realize the possibilities open to us we need only assume that G_3 has about the same magnitude as G_1, and hence may be as large as 10^{-17} of the kaon mass. In a laboratory in which the spatial components of λ^μ are about 10^{-3} this splits the two spin states of a spin-$\frac{1}{2}$ particle by 10^{-11} eV. The effect is the same as that of a magnetic field which, for protons, has a strength of 0.1 G. However, λ^μ interacts solely with the spins and does not exert a force on a wire carrying a current.

We are of course only using the $K_2{}^0$ decay as an example; there are several other explanations which could be true. But it demonstrates neatly that a violation of Lorentz invariance might be detectable by looking at axial-vector terms, but by no other experiments yet known, except possibly the $K_2{}^0$ decay.

It is interesting that the photon field provides no natural gauge-invariant vector to couple with λ^μ, so that even if a fixed frame exists we may be unable to detect it by means of experiments using light.

5. PRESENT LIMITS ON $\boldsymbol{\lambda}^\mu$ FROM MAGNETIC MEASUREMENTS

We will examine in turn the limits which we can set on the value of $|G_3\boldsymbol{\lambda}|$ for protons, neutrons and electrons. Here $\boldsymbol{\lambda}$ represents the spatial part of the four-vector λ^μ. The limits will be expressed in gauss.

A. Protons

By far the most accurate measurements seem to be those recently carried out at the Fredericksburg Magnetic Observatory[5] to compare a proton-resonance magnetometer with the standard sine galvanometer. Agreement between the two was obtained down to 0.5×10^{-5} G, which represented the limit of the equipment.

B. Neutrons

These are accessible only when bound in nuclei such as D or He[3]. The low-field work with D seems never to have been carried to the degree of precision noted above for protons. The most accurate determination of the deuteron magnetic moment[6] has a precision of 3 parts in 10^7, and was done in a field approaching 10^4 G, so that an anomalous field on neutrons of 1 mG would not have been detected. A much more delicate experiment using He^3 is being assembled by Fairbank[7] and will have an ultimate sensitivity of 10^{-14} G. A simpler experiment, based on the proton-resonance magnetometer, seems capable of extending the present limit down to about

[2] R. H. Dicke (private communication).

[3] J. S. Bell and J. K. Perring, Phys. Rev. Letters **13**, 348 (1964).

[4] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).

[5] J. L. Bottum, R. E. Gebhardt, and J. B. Townshend, J. Geophys. Res. **66**, 4319 (1961).

[6] B. Smaller, E. Yasaitis, and H. L. Anderson, Phys. Rev. **80**, 137 (1950).

[7] W. M. Fairbank (private communication). This experiment is actually designed to set a limit on the electric dipole moment of He^3 by taking advantage of its very long relaxation time. However, it turns out to be admirably adapted to detecting a field such as λ^μ.

10^{-5} G. We hope to carry out this experiment in the next few months.

C. Electrons

Sensitive magnetometers using electron spins are well known and are widely used for monitoring the earth's magnetic field. However, to pick out a very small anomalous component it is essential to shield or compensate the magnetic field very carefully. The most precise experiment of this kind is one that has just been completed by us at the Kettering Magnetics Laboratory.[8] We can say that the anomalous field on electrons is unlikely to be greater than 10^{-6} G. Details of this experiment will be published elsewhere. A second version of the equipment is nearly complete, and with it we hope to extend the precision by up to three orders of magnitude.

6. LIMITS ON λ^μ FROM PREVIOUS EXPERIMENTS TO LOOK FOR SPATIAL ANISOTROPY

A paper by Feinberg and Goldhaber[9] makes a general survey of the limits of our confidence in symmetry principles. The last section is concerned with the conservation of angular momentum. One new experiment is quoted, a search by Sunyar for γ rays emitted in a 0^+-0^+ transition. The argument is that if angular momentum were not conserved then we should expect an admixture of states with $J\neq 0$ into the predominantly 0^+ states, and that this mixing would cause the emission of γ rays, which are otherwise strictly forbidden. The presence of a vector field such as λ^μ will cause such mixing. If the 0^+ state is separated from the nearest spin-1 level by E, and the matrix element connecting the two is V, then the resultant state contains a $J=1$ amplitude of order V/E. Assuming normal matrix elements, the limits set in the previous section on the coupling strength of λ^μ allow us to set $V\approx 10^{-14}$ eV. This implies that even if E were as small as 1eV, the mixing amplitude would be only 10^{-14}. Sunyar's limit, obtained under much less favorable conditions, is 3×10^{-4}. Similar arguments can be used to show that mixing is also unimportant in the experiments described in the following paragraphs.

Other experiments on spatial anisotropy[10–12] were stimulated by the suggestion of Cocconi and Salpeter[13] that mass may have a tensorial character dependent on the distribution of matter in the galaxy. The experiments were designed to check a specific hypothesis, which is different from ours, so their relevance must be checked individually.

The experiment of Sherwin *et al.*[10] using the Mössbauer effect is sensitive to energy shifts down to about 10^{-10} eV. The limits quoted in the previous section for neutrons and electrons are about four orders of magnitude beyond this, and the limit for protons is two orders farther still.

Experiments carred out by Hughes *et al.*[11] and by Drever[12] have greater precision, that of Drever setting the stronger limit. In these experiments a search was made for a splitting of the Zeeman line from a nucleus with $J=\frac{3}{2}$. Such a splitting can be caused by any effect which eliminates J_z as a constant of the motion. In a field λ^μ, J_z is such a constant; however, if a magnetic field $\mathbf{H}$ is simultaneously present, we no longer have a single direction defined in space, but two, and J_z is not necessarily conserved. This actually has no observable effect in the present case. Since λ^μ, acting alone, splits the levels equally, it has precisely the same effect as an equivalent magnetic field $\mathbf{H}_\lambda$. The combined effect of $\mathbf{H}_\lambda$ and an applied magnetic field $\mathbf{H}$ is simply to define a resultant field $(\mathbf{H}+\mathbf{H}_\lambda)$; quantization will occur along this direction and the levels will again be equally split. We conclude that the Drever experiment has no bearing on the existence of λ^μ.

7. EXPECTED BEHAVIOR OF λ^μ

The spatial part of λ^μ is expected to lie along the line joining the earth and the constellation Cygnus. Viewed from a laboratory on the earth, the field will appear to turn through one complete revolution every sidereal day. By taking readings for a year or so we will be able to distinguish this variation from any background effects which vary with a period of one solar day.

By a simple extension of the experiment we may hope to determine not only the line along which the solar system is traveling in the preferred frame, but also the sense and magnitude of its velocity along that line. It is easy to see that this is possible if we can detect the small modulation of λ^μ by the motion of the earth round the sun (see Sec. 3). This development would be interesting because it would show directly the noninvariance of the vacuum under Lorentz transformations; the experiments listed earlier are concerned with violation of rotational invariance or CP invariance.

In concluding this account it is intriguing to consider the possible role of the vector λ^μ in a future theory. One of the most tempting alternatives is to associate it with a vector field η^μ, so that the vacuum expectation value of η^μ is nonzero, being in fact equal to λ^μ. Fields with a nonzero vacuum expectation value are well known in connection with "superconductor" theories, though up till now the only fields with this property have been scalar.[14] That a theory of elementary particles could be

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[8] This laboratory has recently been moved to Oakland University, Rochester, Michigan, and is described in an article by G. G. Scott, Research Laboratories, General Motors Corporation, Detroit, Michigan, Report No. GMR-291 (unpublished).

[9] G. Feinberg and M. Goldhaber, Proc. Natl. Acad. Sci. U. S. **45**, 1301 (1959).

[10] C. W. Sherwin, H. Frauenfelder, E. L. Garwin, E. Lüscher, S. Margulies, and R. N. Peacock, Phys. Rev. Letters **4**, 399 (1960).

[11] V. W. Hughes, H. G. Robinson, and V. Beltran-Lopez, Phys. Rev. Letters **4**, 342 (1960).

[12] R. W. P. Drever, Phil. Mag. **6**, 683 (1961).

[13] G. Cocconi and E. Salpeter, Nuovo Cimento **10**, 646 (1958).

[14] An exception is the photon field, which, as a result of gauge invariance, can be given a nonzero expectation value without causing any new physical effects.

constructed in analogy to the BCS theory of superconductivity[15] was first pointed out by Nambu.[16] Such theories have always suffered from a peculiar difficulty, the apparently inevitable appearance of massless bosons as a direct consequence of the broken symmetry. This prediction, known as the Goldstone theorem,[17] has recently been closely examined by several people.[18] The conclusion seems to be that the theorem is true in any theory which is Lorentz-invariant; however, it is not true in the BCS theory, in which the bosons which one might have expected to be massless are made massive by the long-range interactions. It is an attractive possibility that the fundamental symmetry which is broken is in fact Lorentz invariance, so that the Goldstone theorem is avoided. Because of the extreme weakness of the coupling, it is not clear that the masses of particles could be generated in the symmetry-breaking process, as Nambu originally hoped. However, in the light of recent work in electrodynamics[19] it seems possible that vector fields can give large effects even when they are weakly coupled, because of the singular kernel in the Dyson equations.

A theory such as this is too complicated and speculative to be taken seriously at the present time. Its main virtue is that it led us to a new experiment just at the time when the *CP*-violating decays were announced. Until the basic idea is confirmed by experiment, further theoretical work is out of place.

One final remark to compare our theory with that of Bell and Perring[3]: These authors predict that the decay rate for $K_2^0 \to 2\pi$ should vary as the square of the total energy of the K meson. The theory presented here leads to a similar prediction.[20] Moreover, it avoids the objection which can be made[21] to any theory involving vector fields of small but nonzero rest mass, because the field η^μ may well have a large mass. In fact, if η^μ is coupled in an approximately universal way, a large rest mass and a small coupling constant are essential if we are to avoid unwanted effects in experiments on weak interactions. If axial-vector coupling is included, the violation of current conservation is severe, and the objections raised in Ref. 21 become even stronger. One can imagine experiments using the axial-vector coupling which could distinguish between our theory and that of Ref. 3. But because of the weakness of the coupling these experiments would involve equipment of a heroic scale, and it is to be hoped that indirect evidence would settle the matter.

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[15] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **106**, 162 (1957).

[16] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).

[17] J. Goldstone, Nuovo Cimento **19**, 155 (1961).

[18] J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. **127**, 965 (1962); A. Klein and B. W. Lee, Phys. Rev. Letters **12**, 266 (1964); W. Gilbert, *ibid.* **12**, 713 (1964).

[19] K. Johnson, M. Baker, and R. S. Willey, Phys. Rev. Letters **11**, 518 (1963).

[20] Recent experiments at Harwell, Brookhaven, and CERN Laboratories rule out a γ^2 dependence of the decay rate: W. Galbraith, G. Manning, A. E. Taylor, B. D. Jones, J. Malos, A. Astbury, N. H. Lipman, and T. G. Walker, Phys. Rev. Letters **14**, 383 (1965); V. L. Fitch (private communication). However, a variation of this type is not the only one possible in a superconducting theory. At least equally likely is a rate proportional to $(1+b\gamma)^2$, where b is a constant. In other words, the masses of K^0 and $\bar{K}^0$ may be split by a scalar term as well as the one considered in this paper. There is no need to invoke a new cause for this; the same mechanism may serve for both. To see this qualitatively, we recall that the Nambu theory involves a dense background of massless quanta, which is usually unobservable. In the modification suggested here, this background defines a rest frame and provides us with λ^μ. But it does more: Since it is not Lorentz-invariant, it provides a fixed (though very large) energy E_0 in the preferred frame, just as the electron sea in a metal defines a Fermi energy. Self-energy integrals will then produce terms in the mass proportional to $(E_0)_\mu\lambda^\mu$; such terms are scalars, since they do not refer at all to the momentum of the particle whose mass we are trying to calculate. On the other hand, being linear in λ^μ, they will have opposite signs for particle and antiparticle. Present statistics in the K_2^0 experiments are not sufficient to detect the presence of b if it has a magnitude smaller than about 1/100. On the other hand, the experiments proposed in Sec. 5 are unaffected by the scalar term, and look directly at a term analogous to b. Consequently, the arguments of this footnote should not be understood as a plea for more statistics in the K_2^0 experiment, though these will be of great interest. The low-energy experiments seem to offer a more economical way to the answer.

[21] Steven Weinberg, Phys. Rev. Letters **13**, 495 (1964).