# Neutron Form Factors from Inelastic Electron-Deuteron Scattering*

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The electromagnetic form factors of the neutron have been measured in the $q^2$ range from 1.0 to 30.0 F$^{-2}$. It is found that for values of $q^2$ greater than about 6.0 F$^{-2}$ the square of the neutron's charge form factor $(G_{En})^2$ is consistent with zero to within an error of the order of 5% in the existing theoretical cross section. For values of $q^2$ less than 6.0 F$^{-2}$ it is concluded that meaningful values of the neutron form factors cannot be given until the necessary corrections to the theory are better understood. The present results on the neutron form factors are combined with recent measurements of the proton form factors to provide experimental determinations of the isotopic nucleon form factors. It is found that the $\omega$ and $\varphi$ mesons provide a good approximation to the observed isoscalar form factors but that it is difficult to understand the behavior of the isovector form factors in terms of the $\rho$ meson alone.

## I. INTRODUCTION

THE form factors of the proton have been established by means of experiments on elastic electron-proton scattering. The recent work of Janssens *et al.*[1] appears to be the most comprehensive and accurate in the range of four-momentum transfer squared $q^2$ from 4.0 to 30.0 F$^{-2}$. The form factors of the neutron have been measured by means of experiments on elastic electron-deuteron scattering for values of $q^2$ up to about 8.0 F$^{-2}$,[2] and by the method of quasielastic electron-deuteron scattering at higher values of $q^2$.[3] In the latter category of experiments the work of de Vries *et al.* has provided the most extensive series of measurements in the range of $q^2$ from 3.0 to 22.0 F$^{-2}$.

The variation with $q^2$ of both the charge and magnetic form factors of the proton and the magnetic form factor of the neutron are now well established for the values of $q^2$ up to about 30.0 F$^{-2}$, but there is still uncertainty as to both the magnitude and sign of the electric form factor of the neutron. At higher values of $q^2$ the work of Chen *et al.*[4] has provided information on the proton form factors for values of $q^2$ up to 125 F$^{-2}$, but there is at present no information on the neutron in this $q^2$ range except for the work of Dunning *et al.*[5] which is not extensive. The electric form factor of the neutron has proven a difficult quantity to measure by the method of quasielastic electron-deuteron scattering, particularly for low values of $q^2$, because it is a small quantity, probably smaller in abolute magnitude than 0.2 for values of $q^2$ up to 30.0 F$^{-2}$. The measured value of this form factor is therefore very sensitive to errors in the experiment and to errors in the theoretical treatment of the scattering process.

The principle of the method of quasielastic electron scattering is very familiar; measurements are made of the inelastic electron-deuteron cross section at the quasi-elastic peak, from which the electron-neutron cross section is derived with the help of a theoretical treatment to allow for the scattering from the proton and the internal motion of the nucleons in the deuteron. Since the work of de Vries a number of improvements have been made to the theory of the quasielastic electron-deuteron scattering process, notably to the theory of the radiative corrections and to the theory of the final-state interaction. In view of these improvements it was thought worthwhile to repeat and extend the original measurements using an improved experimental arrangement and aiming at a significantly higher experimental precision. In the present experiment we have made measurements of the ratio of the elastic electron-proton cross section to quasi-elastic electron-deuteron cross section over a range of $q^2$ from 0.5 to 35.0 F$^{-2}$. At each value of $q^2$ this ratio has been measured to an accuracy of about 3% for at least two electron scattering angles in the range 45° to 135°. Exceptions to this statement are at $q^2=0.5$ F$^{-2}$ where the measurements are limited to a scattering angle of 45°, at $q^2=17.5$ F$^{-2}$ where the measurements are limited to 60° and at $q^2=35.0$ F$^{-2}$ where the measurements are limited to 135°.

In Sec. II of this paper we describe the experimental technique used in the present experiment. In Sec. III we discuss the method by which the data were analyzed and give our results for the form factors of the neutron. In Sec. IV we combine these results on the neutron with the measurements of Janssens on the proton and give experimental determinations of the nucleon isotopic form factors. In Sec. IV we discuss the degree to which the nucleon form factors can be represented by a pole approximation to the dispersion theory of nucleon form factors. Finally, the conclusions of the present work are summarized in Sec. V.

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[2] J. I. Friedman, H. W. Kendall, and P. A. M. Gram, Phys. Rev. **120**, 992 (1960); E. Erickson (to be published).

[3] C. de Vries, R. Hofstadter, A. Johansson, and R. Herman, Phys. Rev. **134**, B848 (1964); E. B. Hughes, T. A. Griffy, M. R. Yearian, and R. Hofstadter, Proceedings of the International Conference on High Energy Physics, Dubna, 1964; (to be published).

[4] K. W. Chen, A. A. Cone, J. R. Dunning, Jr., S. G. F. Frank, N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. Letters **11**, 561 (1963).

[5] J. R. Dunning, K. W. Chen, A. A. Cone, G. Hartwig, N. F. Ramsey, J. K. Walker, and Richard Wilson, Phys. Rev. Letters **13**, 631 (1964).

## II. EXPERIMENTAL METHOD

The electron beam from the Stanford Mark III linear accelerator, with an energy resolution of 0.5%, was scattered from a thin liquid-hydrogen, liquid-deuterium or empty target. The targets were 0.375 in. thick in the beam direction and the target walls were made of 0.001 in. stainless steel foils. The design of these targets is described by Chambers *et al.*[6] and their use in the present experiment was preferred to the liquid targets used by de Vries (which were 7.5 in. in length) because they defined the geometry of the experiment more clearly.

After passing through the target the intensity of the electron beam was monitored by means of a Faraday cup. The scattered electrons were focussed and analyzed in momentum by a 72-in. double-focussing magnetic spectrometer and detected by an array of ten plastic scintillation counters located in the focal plane of the spectrometer and operated in coincidence with a single large Cherenkov counter.[7] Each scintillator accepted a momentum width of 0.37% and the total counter a width of about 4.0%.

The experimental procedure consisted of measuring an elastic electron-proton cross section, then the peak of the inelastic electron-deuteron cross section at the same value of $q^2$ and finally a second measurement of the elastic electron-proton cross section to check the reproducibility of the data. The reproducibility check was not invariably performed but was applied at least once during every run and, in all, for about 50% of the data taken throughout the experiment. The observed widths of the elastic electron-proton peaks showed the over-all experimental resolution to be about 0.7%. It was therefore possible to measure the entire profile of an elastic cross section with a single momentum setting of the spectrometer. Usually, however, measurements were made for three such settings, the subsequent settings differing from the initial one by a factor of $\pm 1.33$ times the difference in the central momentum accepted by adjacent counters in the ladder.

A detailed knowledge of the effective solid angle accepted by the spectrometer and the absolute detection efficiencies of the counters was not required in this experiment since only a ratio was measured and absolute electron-deuteron cross sections were obtained by normalization to the absolute electron-proton cross sections given by Janssens.[1] An accurate knowledge was required of the relative detection efficiencies of the various channels of the ladder and this was obtained by comparing the counting rates of the different channels when the broad peak of the inelastic electron-deuteron cross section was observed. A series of ten measurements was made at this peak such that a particular scattered momentum was accepted successively by each of the ten channels of the ladder. A knowledge of the slight curvature of the inelastic peak was therefore not required. This measurement was performed periodically throughout each run.

For values of $q^2$ greater than 10 $F^{-2}$, particularly at the smaller scattering angles, the measurement of the quasielastic electron-deuteron cross section was complicated by the production of negative pions of the same momentum and at the same angle as the scattered electrons. One method of dealing with the problem was to reverse the polarity of the spectrometer in order to measure the number of positive pions produced in the target and then to deduce the number of negative pions using the ratios by Neugebauer *et al.*[8] for the relative rates of photo-production of positive and negative pions in deuterium. This method was unsatisfactory because at the larger values of $q^2$ the correction for pion production rapidly became very large and uncertain. We chose instead to prevent the detection of pions by our counter system by inserting a thickness of two radiation lengths of lead immediately in front of the plastic scintillators. The electrons initiated cascade showers in the lead and gave rise to larger pulses in the scintillators, whereas the pulse height produced by the pions was unaffected and could be suppressed by suitable discrimination. With this technique we reduced the electron-detection efficiency to about 75% of its original value but were able to eliminate the detection of pions entirely. We applied this method whenever preliminary investigation showed the negative pion contamination to be greater than 10%, and we verified, for a small number of data points when the pion contamination was negligible, that the experimental ratios measured with and without the lead were identical.

The ratios which we desired to measure were directly dependent on both the density and the thickness of the liquid targets. The temperature, and hence the density, of the liquid hydrogen and the liquid deuterium were measured in the target cells using carbon-resistance thermometers. These measurements were not made when the electron beam was passing through the target but extensive measurements by Janssens[1] have shown that the presence of the beam has substantially no effect on the target density. The ratio of thicknesses of the two targets was accurately measured by filling both targets with liquid hydrogen and measuring the ratio of the counts in an elastic electron-proton peak as the beam was scattered in turn from the two targets. For this purpose a single counter of large momentum acceptance was used.

In quoting $q^2$ values we have adopted the usual convention of quoting the $q^2$ value appropriate to the proton peak. Due to the 2.2-MeV binding energy of the deuteron, the maximum of the deuteron spectrum is at a slightly lower energy than the energy at which the proton

[6] B. Chambers, R. Hofstadter, A. Marcum and M. R. Yearian, Rev. Sci. Instr. **30**, 1019 (1963).

[7] D. Aitken, R. Hofstadter, E. B. Hughes, T. Janssens, and M. R. Yearian, *Proceedings of the 1962 Annual International Conference on High-Energy Nuclear Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 185.

[8] G. Neugebauer, W. Wales and R. W. Walker, Phys. Rev. **119** 1726 (1960).

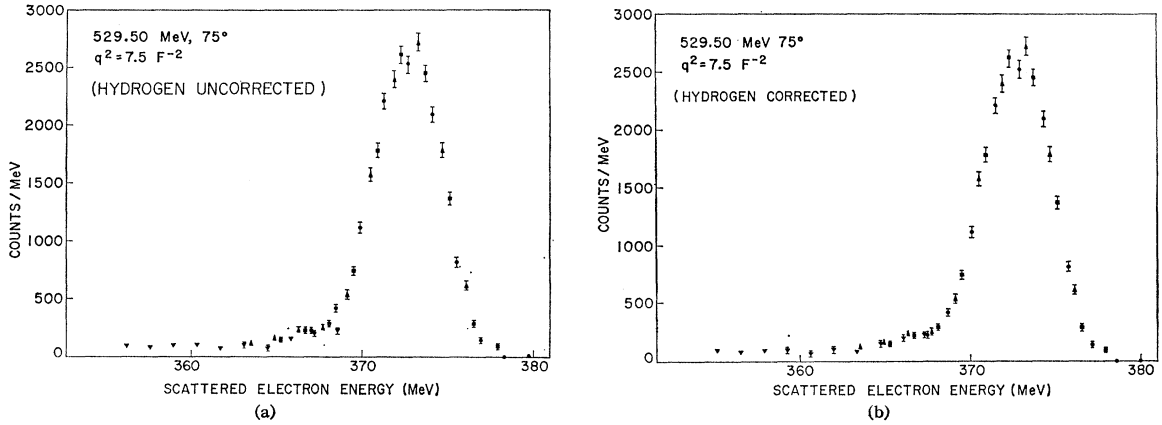


FIG. 1. (a) A typical elastic electron-proton peak measured by the 72-in. magnetic spectrometer. The data points corresponding to four momentum settings of the spectrometer are distinguished by different plotting symbols. (b) The data of Fig. 1(a) after correction for small shifts in setting the spectrometer.

peak appears. This introduces a very small error into the quoted value of $q^2$ in the case of inelastic scattering from the deuteron. This error is typically only 0.25%. In the theoretical calculation of deuteron peaks the correct values of $q^2$ were always used.

## III. ANALYSIS AND RESULTS

The first step in the analysis was the correction of the data for counting rate losses and for relative channel efficiencies. Typical data from the hydrogen target are shown in Fig. 1(a), where the corrected counts for each of four ladder settings, corresponding to four different momentum settings of the spectrometer (distinguished in the plot by different plotting symbols) are displayed as a function of scattered electron momentum. It can be seen from this particular example that the various settings do not in general combine to give a sharply defined elastic peak. However, by shifting the central momentum of the various settings by small independent amounts the definition of the peak can be significantly improved. The result of applying this procedure to the plot shown in Fig. 1(a) is shown in Fig. 1(b). We think it reasonable that such shifts between settings can occur, for example, as a result of hysteresis effects in adjusting the spectrometer, and we have applied this correction to the hydrogen data when necessary. The data shown in Figs. 1(a) and 1(b) illustrate this effect. In this particular situation the correction was such as to decrease the observed cross section by 1.0% and usually the correction was of this order of magnitude.

Figure 2 gives an example of the experimental data, an elastic hydrogen peak and the corresponding inelastic deuterium peak, for a value of $q^2$ equal to 2.5 $F^{-2}$ and a scattering angle of 75°. Figure 3 shows similar data for $q^2$ equal to 17.5 $F^{-2}$ and 60°. In this latter situation preliminary investigations showed the negative-pion contamination to be about 50% of the inelastic deuterium peak but this was reduced to less than 3% with the help of the small lead radiators. The areas under the hydrogen peak were integrated numerically using an IBM 7090 computer and the deuterium peak heights were obtained by fitting a second-order curve to the observed data points. Also a small correction was applied

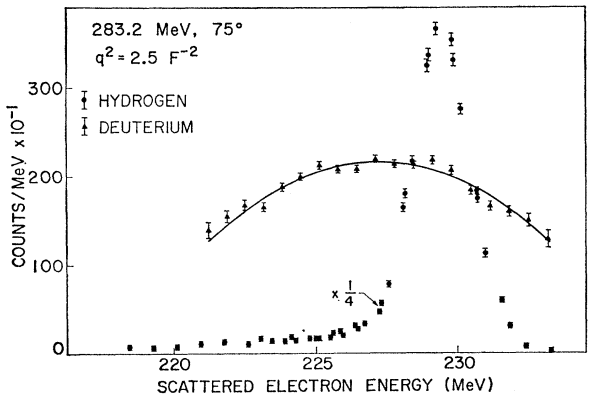


FIG. 2. An example of an elastic electron-proton peak and the corresponding quasielastic electron-deuteron peak for $q^2=2.5$ $F^{-2}$. The experimental points are not corrected for the effects of target density and target thickness.

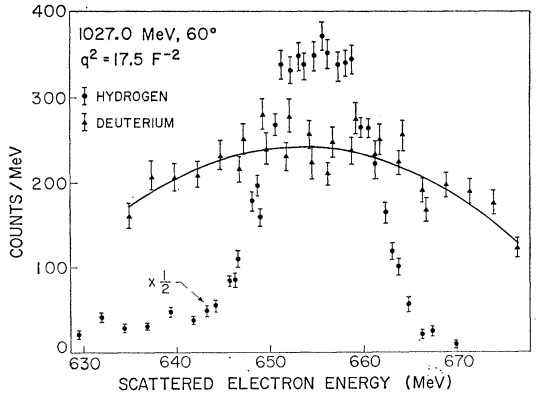


FIG. 3. Typical experiment data for $q^2=17.5$ $F^{-2}$ displayed as in Fig. 2.

TABLE I. The experimental ratios of the proton area to the deuterium peak height as a function of $q^2$ and scattering angle. Also shown are the ratios of the elastic electron-neutron and electron-proton cross sections which follow from the experimental ratios by means of Eq. (3). In determining the errors on the ratios of the free-nucleon cross sections no allowance has been made for possible systematic errors in the deuteron theory.

| $q^2$ (F$^{-2}$) | $E$ (MeV) | Angle (degrees) | Ratio | $\sigma_n/\sigma_p$ |
|---|---|---|---|---|
| 0.5 | 187.2 | 45.0 | 26.80±0.80 | −0.131±0.027 |
| 1.0 | 268.0 | 45.0 | 27.61±0.70 | −0.019±0.025 |
| | 149.9 | 90.0 | 22.54±0.99 | 0.109±0.049 |
| 1.5 | 331.4 | 45.0 | 28.54±0.71 | 0.057±0.026 |
| | 331.4 | 45.0 | 29.67±0.74 | 0.017±0.025 |
| | 331.4 | 45.0 | 28.95±0.65 | 0.042±0.023 |
| | 240.6 | 60.0 | 27.13±0.64 | 0.085±0.026 |
| | 186.8 | 90.0 | 24.85±0.59 | 0.106±0.027 |
| | 155.6 | 120.0 | 21.41±0.66 | 0.216±0.039 |
| | 146.9 | 135.0 | 18.45±0.54 | 0.379±0.046 |
| 2.5 | 434.2 | 45.0 | 34.09±0.78 | 0.050±0.024 |
| | 434.2 | 45.0 | 33.23±0.87 | 0.077±0.029 |
| | 338.7 | 60.0 | 29.96±0.68 | 0.147±0.027 |
| | 338.7 | 60.0 | 30.34±0.76 | 0.132±0.029 |
| | 283.2 | 75.0 | 28.33±0.65 | 0.167±0.928 |
| | 283.2 | 75.0 | 28.70±0.63 | 0.152±0.027 |
| | 283.2 | 75.0 | 27.20±0.65 | 0.215±0.031 |
| | 247.7 | 90.0 | 24.88±0.61 | 0.282±0.035 |
| | 247.7 | 90.0 | 25.66±0.63 | 0.243±0.033 |
| | 207.6 | 120.0 | 22.53±0.54 | 0.333±0.037 |
| | 207.6 | 120.0 | 21.37±0.63 | 0.405±0.047 |
| | 196.4 | 135.0 | 18.43±0.44 | 0.591±0.051 |
| | 196.4 | 135.0 | 19.45±0.46 | 0.507±0.046 |
| 4.6 | 602.6 | 45.0 | 40.61±1.02 | 0.081±0.028 |
| | 602.6 | 45.0 | 38.02±0.91 | 0.154±0.029 |
| | 473.4 | 60.0 | 36.52±0.82 | 0.141±0.027 |
| | 473.4 | 60.0 | 35.65±0.77 | 0.169±0.027 |
| | 473.4 | 60.0 | 35.98±0.82 | 0.159±0.028 |
| | 401.0 | 75.0 | 33.15±0.71 | 0.197±0.028 |
| | 401.0 | 75.0 | 32.96±0.71 | 0.204±0.028 |
| | 352.0 | 90.0 | 30.20±0.67 | 0.258±0.032 |
| | 352.0 | 90.0 | 30.04±0.74 | 0.265±0.034 |
| | 296.4 | 120.0 | 24.99±0.66 | 0.422±0.045 |
| | 281.4 | 135.0 | 23.19±0.63 | 0.499±0.050 |
| 7.5 | 788.2 | 45.0 | 44.24±1.08 | 0.224±0.032 |
| | 623.7 | 60.0 | 40.92±0.98 | 0.240±0.033 |
| | 529.5 | 75.0 | 38.52±0.85 | 0.237±0.031 |
| | 529.5 | 75.0 | 38.25±0.83 | 0.245±0.030 |
| | 468.7 | 90.0 | 34.99±0.76 | 0.284±0.033 |
| 7.5 | 399.2 | 120.0 | 30.41±0.78 | 0.344±0.040 |
| | 399.2 | 120.0 | 28.65±0.82 | 0.429±0.048 |
| | 380.2 | 135.0 | 27.16±0.69 | 0.457±0.045 |
| | 380.2 | 135.0 | 27.72±0.87 | 0.429±0.051 |
| | 380.2 | 135.0 | 26.50±0.62 | 0.495±0.045 |
| 10.0 | 925.8 | 45.0 | 51.75±1.30 | 0.179±0.031 |
| | 738.0 | 60.0 | 44.94±1.01 | 0.260±0.032 |
| | 738.0 | 60.0 | 43.91±0.97 | 0.289±0.033 |
| | 627.8 | 75.0 | 40.16±0.88 | 0.312±0.034 |
| | 558.4 | 90.0 | 35.93±0.86 | 0.345±0.038 |
| | 558.4 | 90.0 | 36.17±0.85 | 0.363±0.038 |
| | 479.9 | 120.0 | 31.75±0.95 | 0.395±0.048 |
| | 479.9 | 120.0 | 31.07±0.80 | 0.425±0.044 |
| | 456.9 | 135.0 | 28.05±0.79 | 0.523±0.052 |
| | 456.9 | 135.0 | 28.70±0.79 | 0.489±0.049 |
| 12.5 | 839.4 | 60.0 | 48.00±1.18 | 0.286±0.035 |
| | 717.2 | 75.0 | 41.04±0.92 | 0.388±0.038 |
| | 639.8 | 90.0 | 37.51±0.92 | 0.412±0.042 |
| | 552.8 | 120.0 | 32.89±1.00 | 0.432±0.050 |
| | 528.8 | 135.0 | 28.90±0.75 | 0.564±0.052 |
| 15.0 | 935.8 | 60.0 | 48.49±1.33 | 0.365±0.043 |
| | 802.5 | 75.0 | 43.37±1.09 | 0.399±0.042 |
| | 718.1 | 90.0 | 37.67±1.00 | 0.488±0.048 |
| | 623.5 | 120.0 | 32.97±0.95 | 0.497±0.051 |
| | 597.5 | 135.0 | 30.47±1.33 | 0.550±0.074 |
| 17.5 | 1027.0 | 60.0 | 54.30±1.54 | 0.293±0.040 |
| 20.0 | 1114.4 | 60.0 | 52.33±1.59 | 0.412±0.049 |
| | 961.7 | 75.0 | 49.88±1.98 | 0.340±0.057 |
| | 865.3 | 90.0 | 44.46±1.36 | 0.373±0.047 |
| | 757.8 | 120.0 | 33.42±1.27 | 0.582±0.069 |
| 25.0 | 1034.7 | 85.0 | 44.08±1.85 | 0.522±0.070 |
| | 853.3 | 135.0 | 31.93±2.29 | 0.642±0.123 |
| 30.0 | 1108.6 | 95.0 | 45.93±2.33 | 0.436±0.077 |
| | 974.2 | 135.0 | 36.75±3.09 | 0.469±0.126 |
| 35.0 | 1092.4 | 135.0 | 29.05±2.85 | ... |

to the deuterium data to allow for the finite experimental resolution. This correction was deduced from the observed width of the elastic hydrogen data and was usually of the order of 1%.

The observed elastic electron-proton cross sections and the inelastic electron-deuteron cross sections were corrected for the effects of electron radiation during the interaction according to the corrections given, respectively, by Meister and Yennie[9] and Tsai[10] and by Meister and Griffy.[11] The result for the inelastic electron-deuteron process may be written in the form

$$(d^2\sigma/d\Omega dE_f)_{\text{exp}}=(d^2\sigma/d\Omega dE_f)_0(1+\delta_D), \quad (1)$$

where $\delta_D$ is the radiative correction and $E_f$ is the scattered electron energy. A similar equation holds for the elastic electron-proton cross section and involves a radiative correction $\delta_p$. Combining these two results, the correction to the experimental ratio may be written in the form

$$R_{\text{exp}}=(R_0(1+\delta_p)/1+\delta_D), \quad (2)$$

from which the corrected ratio $R_0$ can be obtained.

An additional correction for the electron bremsstrahlung during passage through the thin targets was folded into the above results but this was always a small effect. Typically the value of $\delta_p$ was about 0.2 and $\delta_D$ about 0.1, but the value of the corresponding bremsstrahlung correction term was never more than 0.03. The correction to the ratio due to bremsstrahlung was usually less than 1%.

The ratios measured during the course of the present experiment are given in Table I and shown as a function of $q^2$ in Figs. 4(a), 4(b), and 4(c). The measurements

[9] N. Meister and D. Yennie, Phys. Rev. **130**, 1210 (1963).
[10] Y. S. Tsai, Phys. Rev. **122**, 1898 (1961).
[11] N. Meister and T. A. Griffy, Phys. Rev. **133**, B1032 (1964).

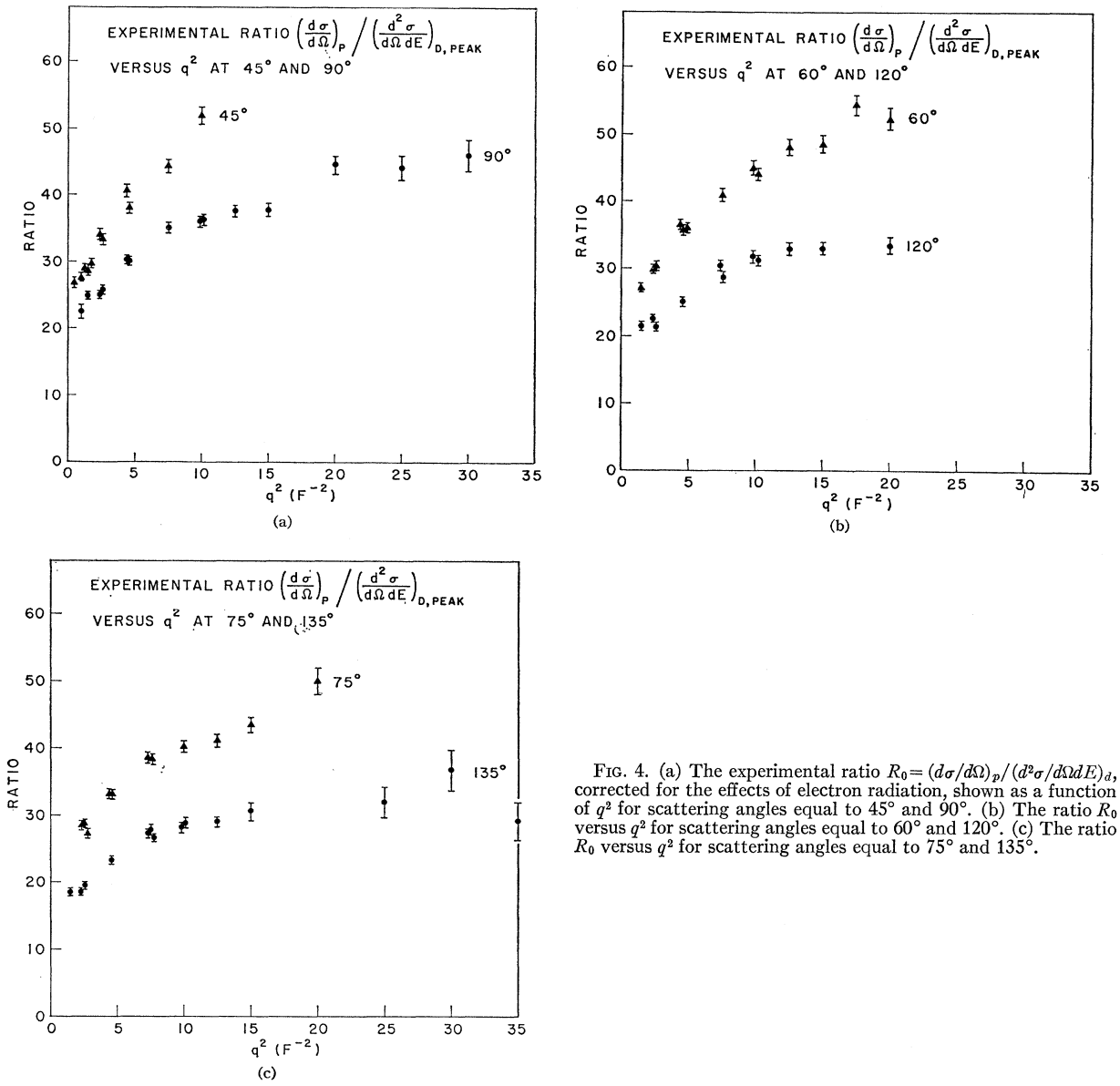


FIG. 4. (a) The experimental ratio $R_0=(d\sigma/d\Omega)_p/(d^2\sigma/d\Omega dE)_d$, corrected for the effects of electron radiation, shown as a function of $q^2$ for scattering angles equal to 45° and 90°. (b) The ratio $R_0$ versus $q^2$ for scattering angles equal to 60° and 120°. (c) The ratio $R_0$ versus $q^2$ for scattering angles equal to 75° and 135°.

were made as a function of scattering angle for a series of selected values of $q^2$ so that no interpolation was necessary in the subsequent analysis leading to the neutron form factors. The errors due to counting statistics range from about 2% at the lower values of $q^2$ to about 5% at the higher values. Counting statistics are believed to be the major source of experimental error since many errors of a systematic nature cancel in forming the ratio. A study of those ratios whose measurement was repeated during the course of the experiment indicated that an additional error of 2% was necessary to allow for errors in the determination of the relative efficiencies and for variations in other experimental parameters, such as target thickness and target density. This error has been included in Table I. The reproducibility of the data was frequently demonstrated for the lower values of $q^2$ where the same ratio was measured on two and sometimes three different runs.

Figure 5 shows the results of the present experiment at 60° and 135° and also the results of de Vries for comparison. The variation of the experimental ratios with $q^2$ shows the same behavior in the two experiments but the results of the present experiment appear to differ systematically from the earlier measurements of de Vries especially at the higher values of $q^2$. We have made a considerable effort to account for this difference. For instance, the ratios at $q^2=4.6$ $F^{-2}$ and scattering angles of 60° and 75° have been remeasured using the 7.5-in.-long liquid targets employed by de Vries. The results

were consistent with those of the present experiment. We have, however, been able to explain the difference between the two experiments in terms of other refinements in the experimental technique, notably a better determination of the target density, and to improvements in the theory of radiative corrections which have taken place since the work of de Vries.

The inelastic electron-deuteron cross section can be determined from the ratio $R_0$ and the known absolute electron-proton cross section. A formula for the deuteron cross section at the quasi-elastic peak in terms of the proton and neutron form factors has been given by Durand,[12,13]

$$\frac{d^2\sigma}{d\Omega dE_f}=\sigma_{\text{Mott}}(4.81\times10^{-3})(1\pm0.03)\frac{M^2}{p(p^2+M^2)^{1/2}}\times C(q^2,\theta)(G_p+G_n), \quad (3)$$

where

$$G_i=\frac{G_{Ei}^2+(q^2/4M^2)G_{Mi}^2}{1+q^2/4M^2}+\frac{2q^2}{4M^2}G_{Mi}^2\tan^2(\theta/2),$$

with $i$=proton or neutron, $\sigma_{\text{Mott}}$=the Mott cross section=$(e^2/2E_0)^2(\cos^2(\theta/2)/\sin^4(\theta/2))$, $E_0$=the initial electron energy, $M$=the nucleon mass, $p=q/2$=the relative momentum of either of the nucleons in their final center-of-mass system.

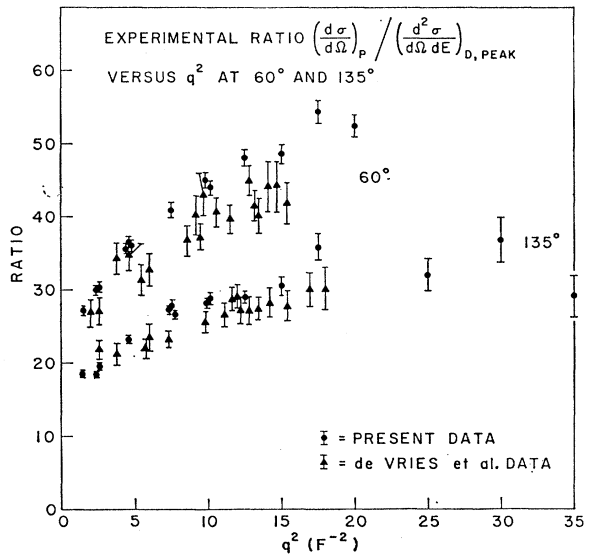


FIG. 5. A comparison between the experimental ratios $R_0$ measured in the present experiment at 60° and 135° and the ratios measured by de Vries *et al.* (Ref. 3).

[12] L. Durand, III, Phys. Rev. **123**, 1393 (1961).

[13] The first numerical factor in Eq. (3) was given by Durand (Ref. 12) as $4.57\times10^{-3}$. This was a normalization of the cross section to correspond to a 95% $^3S_1$ deuteron theory. We have renormalized this constant to correspond to a 100% $^3S_1$ deuteron, putting the effects of the $D$-state contribution into the factor $C(q^2,\theta)$.

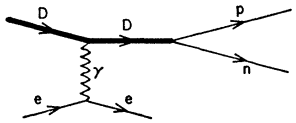


FIG. 6. The diagram corresponding to the deuteron pole term which makes a significant contribution to the quasielastic electron-deuteron cross section for values of $q^2$ less than 5 $F^{-2}$.

This formula was obtained assuming a Hulthén model for the $^3S_1$ component of the deuteron wave function and is expected to be uncertain to about 3% owing to imperfect knowledge of the deuteron wave function. The effects of the $D$-state component of the deuteron wave function and the final-state interaction of the neutron and proton are included in the factor $C(q^2,\theta)$. A recent calculation of this factor, using a Gammel-Thaler potential for the neutron-proton interaction in the final state, has been performed by Nuttall and Whippman[14] and we have used their results in the analysis of our data. A $D$-state component of the deuteron wave function of 7% has been assumed.[15]

We have obtained the following representation of the factor $C(q^2,\theta)$ by a process of curve fitting to the numerical results given by Nuttall and Whippman.[14]

$$C(q^2,\theta)=1-0.01(7.0+(a_1/q^2)+a_2+a_3q^2e^{-a_4q^2}) \quad (4)$$

where

$$a_1=1.49,\quad a_2=1.22,$$
$$a_3=0.94,\quad a_4=0.15;$$

this correction appears to be substantially independent of the scattering angle.

At the lower values of $q^2$ considered in this experiment, the wavelength of the virtual photon becomes comparable to the size of the deuteron and hence the impulse approximation used in obtaining Eq. (3) is no longer valid. We have partially corrected the impulse approximation theory by adding the "deuteron pole term" contribution shown in Fig. 6. Writing Eq. (3) in the form

$$(d^2\sigma/d\Omega dE_f)=\sigma_{\text{Mott}}[A(q^2)+B(q^2)\tan^2(\tfrac{1}{2}\theta)] \quad (5)$$

the effect of the deuteron pole term can be written as

$$(d^2\sigma/d\Omega dE_f)=\sigma_{\text{Mott}}[A(1+\Delta A)+B(1+\Delta B)\tan^2(\tfrac{1}{2}\theta)]. \quad (6)$$

The approximate expression used for $\Delta A$ is

$$\Delta A=\frac{-\alpha^2}{\mathbf{q}^2}\ln\frac{|\mathbf{q}|^2}{\alpha^2}\frac{G_{Ed}}{G_{Ep}}+\frac{\alpha^2}{|\mathbf{q}|^2}\left(\frac{G_{Ed}}{G_{Ep}}\right)^2, \quad (7)$$

where $\alpha^2=M\epsilon$, and $\epsilon$ is the binding energy of the deuteron. $G_{Ep}$ and $G_{Ed}$ are the charge form factors of

[14] J. Nuttall and M. L. Whippman, Phys. Rev. **130**, 2495 (1963).

[15] R. Wilson, *The Nucleon-Nucleon Interaction* (Interscience Publishers, Inc., New York, 1963); F. J. Hadjioannou, Phys. Rev. **125**, 1414 (1962).

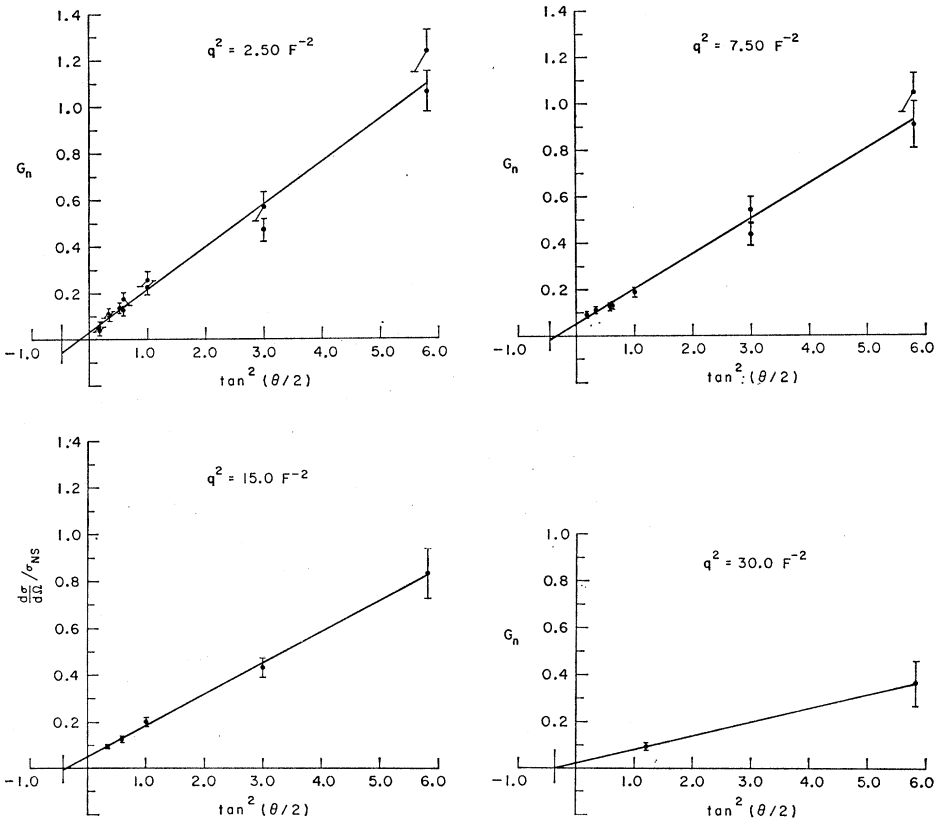


FIG. 7. Examples of Rosenbluth plots for the neutron, showing $G_n$ versus $\tan^2(\theta/2)$ for values of $q^2$ equal to 2.5, 7.5, 15.0, and 30.0 F$^{-2}$.

the proton and deuteron, respectively. Typical numerical values of $\Delta A$ are 0.08 at $q^2=1.0$ F$^{-2}$ and 0.01 at $q^2=5.0$ F$^{-2}$. The $\Delta B$ correction term is much less important and was neglected.

The corrections applied to Eq. (3) for the effects of the $D$-state component of the deuteron wave function, the final-state interaction and the deuteron pole term are given in Table II as a function of $q^2$.

Figure 7 shows examples of our plots of $G_n$ versus $\tan^2(\theta/2)$ for four values of $q^2$. At each value of $q^2$, including those not shown in Fig. 7, the experimental points can be fitted by a straight line, which, as pointed out by Gourdin,[16] is consistent with the assumption of one-photon exchange between electron and deuteron. The goodness of fit, as measured by the $\chi^2$ test, is given in Table III. The experimental points uncorrected for the final-state interaction can also be fitted by straight lines at all values of $q^2$, which is also consistent with the one-photon assumption, since the linearity of these plots should not be affected by corrections to the theory of deuteron electrodisintegration.[15] The magnetic form factor of the neutron $G_{Mn}$ is given by the slope of those lines, and the square of the electric form factor $(G_{En})^2$ by the intercept at the small negative value of $\tan^2(\frac{1}{2}\theta)$ indicated in Fig. 7. $G_{Mn}$ is always a real number but at lowest value of $q^2$ the fitted line corresponds to a small negative value of $(G_{En})^2$, which would imply a purely imaginary value of $G_{En}$.

TABLE II. The corrections applied to the theoretical deuteron cross section for the final-state interaction, the $D$ state and the deuteron pole term.

| $q^2$ (F$^{-2}$) | Correction for final state $+D$ state (%) | Correction for deuteron pole term (%) | Total correction (%) |
|---|---|---|---|
| 1.0 | −18.8 | −6.5 | −25.3 |
| 1.5 | −14.7 | −3.9 | −18.6 |
| 2.5 | −11.2 | −2.1 | −13.3 |
| 4.6 | −8.5 | −0.6 | −9.1 |
| 7.5 | −7.5 | 0.0 | −7.5 |
| 10.0 | −7.3 | 0.0 | −7.3 |
| 20.0 | −7.9 | 0.0 | −7.9 |
| 30.0 | −8.2 | 0.0 | −8.2 |

The values we find for the charge and magnetic form factors of the neutron are given in Table III and shown as a function of $q^2$ in Figs. 8 and 9. Figure 8(a) shows the values of the square of the neutron's charge form factor $(G_{En})^2$ that are obtained by setting the factor $C(q^2,\theta)$ in the Durand formula equal to unity and ignoring the contribution from the deuteron pole term. The indicated errors include only the statistical errors in the ratios and a 4% uncertainty in the absolute electron-proton cross sections. Figure 8(b) shows how the form

[16] M. Gourdin, Nuovo Cimento **21**, 1094 (1961).

TABLE III. The neutron form factors as a function of $q^2$.

| $q^2$ ($F^{-2}$) | $(G_{En})^2$ | Statistical error | Total error (including 5% theoretical uncertainty) | $-(G_{Mn}/\mu_n)$ | Statistical error | Total error (including 5% theoretical uncertainty) | Goodness of fit $\chi^2/N$ |
|---|---|---|---|---|---|---|---|
| 1.0 | −0.117 | ±0.023 | ±0.058 | 1.360 | ±0.290 | ±0.355 | ··· |
| 1.5 | −0.038 | ±0.010 | ±0.044 | 0.931 | ±0.049 | ±0.112 | 0.9 |
| 2.5 | −0.058 | ±0.009 | ±0.035 | 0.948 | ±0.028 | ±0.084 | 1.5 |
| 4.6 | −0.057 | ±0.006 | ±0.023 | 0.701 | ±0.024 | ±0.070 | 0.5 |
| 7.5 | −0.018 | ±0.007 | ±0.018 | 0.496 | ±0.013 | ±0.051 | 1.0 |
| 10.0 | −0.026 | ±0.006 | ±0.013 | 0.432 | ±0.011 | ±0.042 | 0.7 |
| 12.5 | −0.023 | ±0.009 | ±0.014 | 0.382 | ±0.016 | ±0.042 | 1.2 |
| 15.0 | −0.010 | ±0.009 | ±0.013 | 0.331 | ±0.015 | ±0.038 | 0.4 |
| 20.0 | −0.013 | ±0.014 | ±0.016 | 0.249 | ±0.031 | ±0.049 | 3.1 |
| 25.0 | −0.019 | ±0.024 | ±0.023 | 0.224 | ±0.026 | ±0.040 | ··· |
| 30.0 | −0.003 | ±0.033 | ±0.033 | 0.156 | ±0.028 | ±0.041 | ··· |

factor is modified by the inclusion of the factor $C(q^2,\theta)$ and the deuteron pole term. At this step a significant scatter appears among the data points at the lower values of $q^2$ which could be due to errors in the theoretical corrections applied to the Durand formula. The sensitivity of $(G_{En})^2$ to errors in the theory is illustrated in Fig. 8(c) where the indicated errors include, in addition to the errors mentioned above, a possible systematic error of 5% in the theoretical cross section associated with uncertainties in the deuteron wave function, with the $D$-state scattering and with the final-state interaction. The corresponding values of the neutron's magnetic form factor are shown in Fig. 9. In this figure the full error bars indicate the statistical error and the dashed extensions show the additional error due to a systematic error of 5% in the theory.

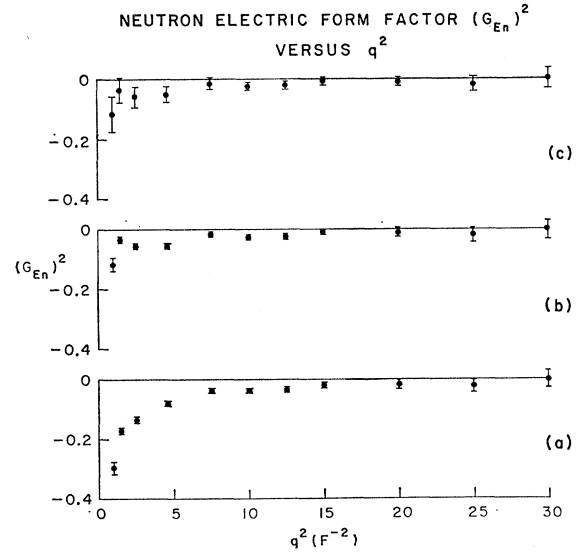


FIG. 8. (a)–(c) The square of the neutron's charge form factor $(G_{En})^2$ as a function of $q^2$. The lower plot shows the result obtained when the corrections to the theoretical cross section for the final-state interaction, the $D$-state component of the wave function and deuteron pole term are ignored. The middle plot shows how the results are changed by the inclusion of these corrections, and the upper plot shows how the error bars are increased when allowance is made for a possible systematic error of 5% in the theoretical cross section.

These results suggest that for values of $q^2$ greater than about 6 $F^{-2}$ the square of the neutron's charge form factor $(G_{En})^2$ is equal to zero to within an error of the order of 5% in the existing theoretical cross section. On the other hand for values of $q^2$ less than 6 $F^{-2}$, in order to avoid the unattractive result of a negative value of $(G_{En})^2$, a correction to the theory is required which is larger than 5% at certain angles. It should be pointed out that Nuttall and Whippman's correction to the Durand theory does not presently extend to values of $q^2$ less than 4 $F^{-2}$ and that the extrapolation of their results into the low $q^2$ region, where the correction rapidly increases, is not regarded as reliable. It is probable therefore that conclusions regarding the size of the neutron form factors in the low $q^2$ region cannot be made from experiments on quasielastic electron-deuteron scattering until the necessary corrections to the theory are better understood. Accordingly the

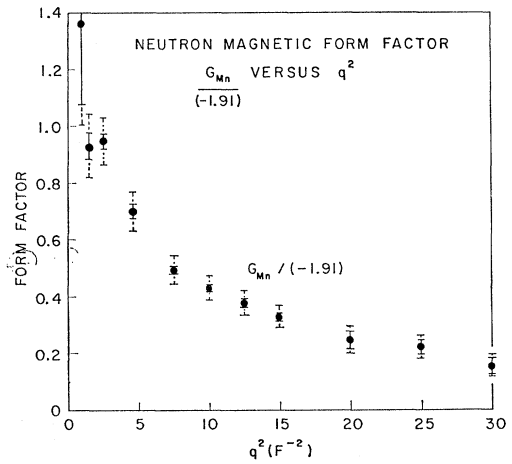


FIG. 9. The neutron's magnetic form factor $G_{Mn}$ as a function of $q^2$. The full error bars show the statistical errors involved in the experiment and the dashed extensions the additional error due to a possible systematic error of 5% in the deuteron theory.

analysis presented in later sections of this paper will be based solely on the results of the present experiment for values of $q^2$ greater than 6 $F^{-2}$ and will disregard completely the measurements at lower values of $q^2$.

On the basis of these more accurate data we should like to point out a rather simple relationship which exists between the form factors of the proton and neutron when these quantities are expressed in terms of the charge and magnetic form factors, as in this paper, as opposed to a description in terms of the related Dirac and Pauli form factors. Figure 10 shows a plot of the proton form factors given by Janssens and the neutron form factors found in the present experiment, with the magnetic form factors normalized to unity at $q^2=0$. The errors attached to the neutron form factors include a possible systematic error of 5% in the deuteron theory. It is clear that to within the indicated errors in the $q^2$ range from 7.5 to 30.0 $F^{-2}$.

$$G_{Ep}=G_{Mp}/\mu_p=G_{Mn}/\mu_n \quad \text{and} \quad G_{En}=0, \qquad (8)$$

where $\mu_p$ and $\mu_n$ are the static magnetic moments of the proton and neutron, respectively.[17] The validity of the former of these two equations in the above $q^2$ range is illustrated in Fig. 11 which shows the ratios of the form factors as a function of $q^2$. We conclude that

$$\begin{aligned} G_{Ep}/(G_{Mp}/\mu_p)&=0.996\pm0.013, \\ G_{Ep}/(G_{Mn}/\mu_n)&=0.92\pm0.08, \\ (G_{Mp}/\mu_p)/(G_{Mn}/\mu_n)&=0.90\pm0.10, \end{aligned} \qquad (9)$$

where the errors shown on the latter two ratios include the systematic error due to the deuteron theory. Figure 11 also shows measurements of the form factor ratios taken from the work of Lehmann *et al.*,[18] Dudelzak *et al.*,[19] Drickey *et al.*,[20] and Benaksas *et al.*[21] for values of $q^2$ less than 4 $F^{-2}$. These results support the validity of Eqs. (9) in the low $q^2$ range. The Orsay group has shown that the first of Eqs. (9) is well obeyed up to $q^2$ values of 10 $F^{-2}$.

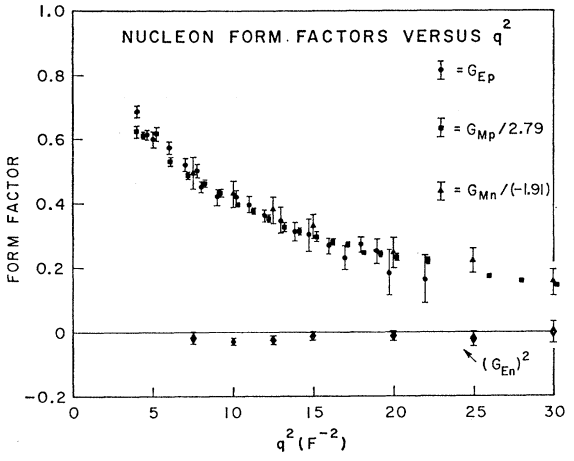


FIG. 10. A comparison between the proton form factors measured by Janssens *et al.* (Ref. 1) and the neutron form factors measured in the present experiment. The magnetic form factors are normalized to unity at $q^2=0$.

[17] The quantities $G_{Ep}$, $G_{Mp}/\mu_p$, $G_{Mn}/\mu_n$, and $G_{En}$ are frequently also called $F_{ch}^p$, $F_{mag}^p$, $F_{mag}^n$, and $F_{ch}^n$, respectively.

## IV. THE ISOTOPIC NUCLEON FORM FACTORS

Theoretical attempts to calculate the nucleon form factors usually consider the proton and neutron as an isotopic doublet and it is therefore convenient to resolve the observed nucleon form factors into their isoscalar and isovector components. These are defined in Eq.

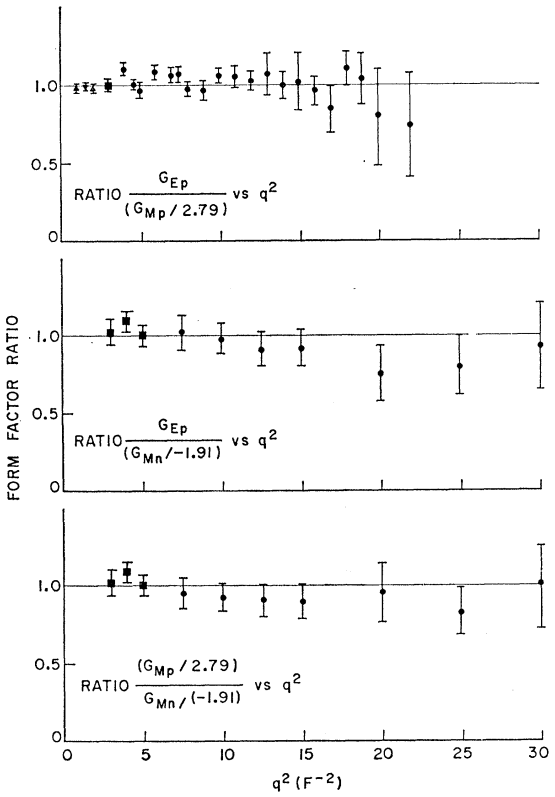


FIG. 11. This figure illustrates the common dependence on $q^2$ of the form factors $G_{Ei}$, $G_{Mp}$, and $G_{Mn}$. For values of $q^2$ less than 4.0 $F^{-2}$ the figure also includes form-factor ratios taken from the work of Lehmann *et al.* (Ref. 18), Dudelzak *et al.* (Ref. 19), and Drickey *et al.* (Ref. 20) on the proton and from Benaksas *et al.* (Ref. 21) on the neutron. The key to the symbols used to identify the work of the various authors in the top graph is given in Fig. 14. For the second and third graphs the key is given in Fig. 15.

[18] P. Lehmann, R. Taylor and R. Wilson, Phys. Rev. **126**, 1183 (1962).

[19] B. Dudelzak, G. Sauvage, and P. Lehmann, Nuovo Cimento **28**, 18 (1963).

[20] D. J. Drickey and L. N. Hand, Phys. Rev. Letters **9**, 521 (1962).

[21] D. Benaksas, D. Drickey, and D. Frerejacque, Phys. Rev. Letters **13**, 353 (1964).

(10),

$$\begin{aligned} G_{ES}&=\tfrac{1}{2}(G_{Ep}+G_{En}),\\ G_{EV}&=\tfrac{1}{2}(G_{Ep}-G_{En}),\\ G_{MS}&=\tfrac{1}{2}(G_{Mp}+G_{Mn}),\\ G_{MV}&=\tfrac{1}{2}(G_{Mp}-G_{Mn}). \end{aligned} \tag{10}$$

By combining the present results on neutron form factors with the proton form factors measured by Janssens it is possible to give experimental determinations of the isotopic form factors in the $q^2$ range 7.5 to 30.0 $F^{-2}$. In accordance with these results we take $G_{En}$ to be equal to zero in this $q^2$ range, to within an error determined by the present experiment, and make no attempt to estimate the values of the magnetic isotopic form factors for smaller values of $q^2$. The experimental values of the isotopic form factors are given in Tables IV and V and shown as a function of $q^2$ in Figs. 12(a) to 12(d). $G_{ES}$ is equal to $G_{EV}$, since $G_{En}$ is taken equal to zero.

TABLE IV. The electric isoscalar and isovector form factors as a function of $q^2$. The errors include a possible 5% systematic error in the deuteron theory.

| $q^2$ ($F^{-2}$) | $G_{ES}=G_{EV}$ |
|---|---|
| 4.0 | 0.345±0.037 |
| 4.6 | 0.308±0.036 |
| 5.0 | 0.300±0.039 |
| 6.0 | 0.289±0.037 |
| 7.0 | 0.261±0.037 |
| 7.5 | 0.252±0.037 |
| 8.0 | 0.227±0.037 |
| 9.0 | 0.211±0.038 |
| 10.0 | 0.212±0.036 |
| 11.0 | 0.200±0.037 |
| 12.0 | 0.182±0.037 |
| 13.0 | 0.170±0.041 |
| 14.0 | 0.158±0.038 |
| 15.0 | 0.152±0.044 |
| 16.0 | 0.136±0.037 |
| 17.0 | 0.117±0.041 |
| 18.0 | 0.137±0.038 |
| 19.0 | 0.127±0.041 |
| 20.0 | 0.094±0.050 |
| 22.0 | 0.083±0.051 |

If Eqs. (8) are true it is easily shown that all four isotopic form factors when normalized to unity at $q^2=0$ have a common dependence on $q^2$. Figure 13 shows a plot of the isotopic form factors normalized in this way and confirms that to within the experimental error they do have a common variation with $q^2$.

According to the ideas of dispersion theory[22] the isotopic form factors in the space-like region of $q^2$ reached in electron-scattering experiments can be expressed in terms of dispersion integrals over the time-like region of $q^2$ reached in annihilation experiments. Thus

$$\begin{aligned} G_E^{S,V}(q^2)&=\frac{1}{\pi}\int_{9m_\pi^2,4m_\pi^2}^{\infty}\frac{g_E^{S,V}(q'^2)}{q'^2-q^2}dq'^2,\\ \text{and}\quad G_M^{S,V}(q^2)&=\frac{1}{\pi}\int_{9m_\pi^2,4m_\pi^2}^{\infty}\frac{g_M^{S,V}(q'^2)dq'^2}{q'^2-q^2}, \end{aligned} \tag{11}$$

where the spectral weight functions $g^{S,V}(q'^2)$ are related to the matrix elements for processes in which a photon couples to intermediate states of total mass $q'$ which in turn annihilate to form a nucleon-antinucleon pair.

At the present time it is expected that intermediate states consisting of the $\omega$ and $\varphi$ mesons will give large contributions to the isoscalar form factors and that the $\rho$ meson will give a large contribution to the isovector form factors. If it is assumed that the behavior of the isoscalar and the isovector form factors in the $q^2$ range less than about 30.0 $F^{-2}$ is dominated by these particular intermediate states, and that they contribute to the dispersion integrals only at their average mass, then the expressions for the isotopic form factors can be written in the following way:

$$\begin{aligned} G_{ES}&=0.5\left\{\frac{s_{e1}}{1+q^2/15.7}+\frac{s_{e2}}{1+q/26.7}+(1-s_{e1}-s_{e2})\right\},\\ G_{MS}&=0.44\left\{\frac{s_{m1}}{1+q^2/15.7}+\frac{s_{m2}}{1+q^2/26.7}+(1-s_{m1}-s_{m2})\right\},\\ G_{EV}&=0.5\left\{\frac{v_{e1}}{1+q^2/M_\rho^2}+(1-v_{e1})\right\},\\ G_{MV}&=2.353\left\{\frac{v_{m1}}{1+q^2/M_\rho^2}+(1-v_{m1})\right\}, \end{aligned} \tag{12}$$

where the constant terms represent the contributions from nonresonant intermediate states or states of higher mass and the parameters $s_{e1}$, $s_{e2}$, $s_{m1}$, $s_{m2}$, $v_{e1}$, and $v_{m1}$ determine the strength which the various terms contribute. The constant terms can be expressed in terms of the parameters $s_{e1}\cdots v_{m1}$ by the requirement that the isotopic form factors reduce to their known static values.

TABLE V. The magnetic isoscalar and isovector form factors as a function of $q^2$. The errors include a possible 5% systematic error in the deuteron theory.

| $q^2$ ($F^{-2}$) | $G_{MS}$ | $G_{MV}$ |
|---|---|---|
| 7.5 | 0.185±0.056 | 1.133±0.056 |
| 10.0 | 0.146±0.045 | 0.971±0.045 |
| 12.5 | 0.114±0.047 | 0.844±0.047 |
| 15.0 | 0.098±0.047 | 0.732±0.047 |
| 20.0 | 0.093±0.051 | 0.569±0.051 |
| 25.0 | 0.046±0.040 | 0.474±0.040 |
| 30.0 | 0.065±0.040 | 0.365±0.040 |

[22] S. Fubini, *Proceedings of the Aix en Provence International Conference on Elementary Particles, 1961* (Centre d'Etudes Nucléaires de Saclay, Seine et Oise, 1961).

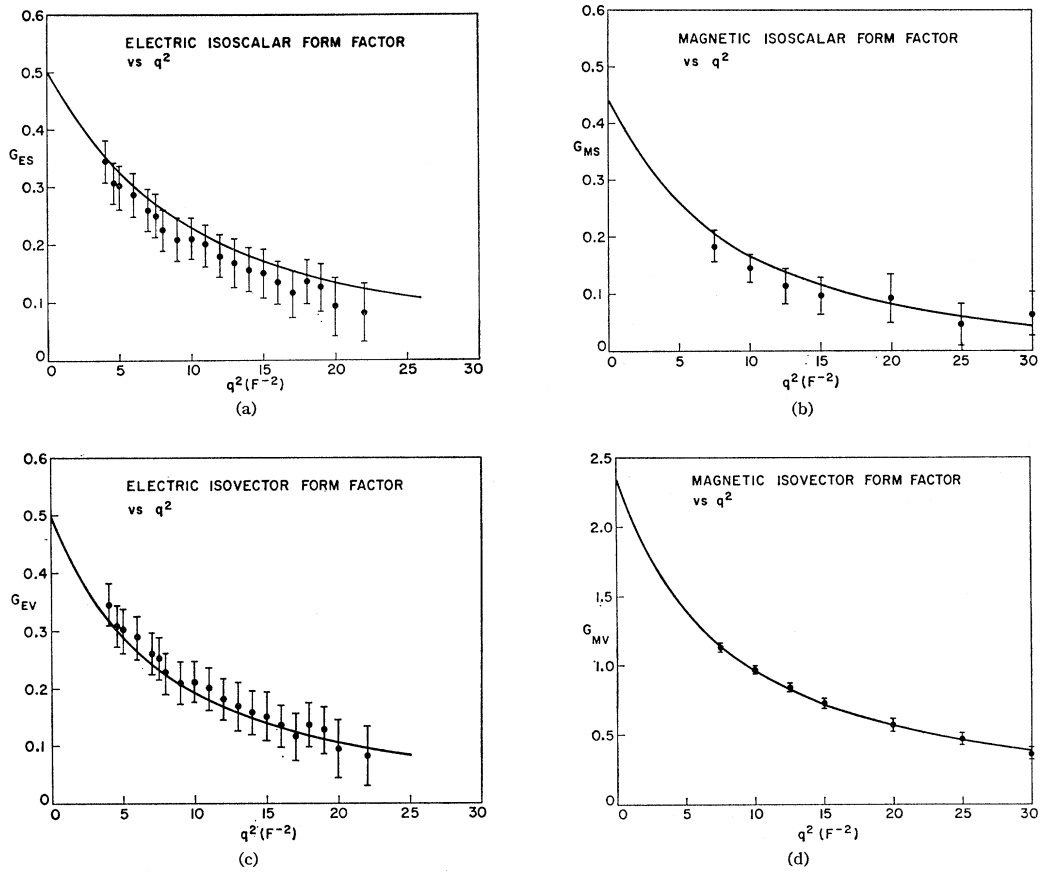


FIG. 12. (a)–(d) Experimental determinations of the isotopic nucleon form factors versus $q^2$. The isotopic form factors are derived from the proton form factors measured by Janssens *et al.* (Ref. 1) and from the neutron form factors measured in the present experiment. The full lines show the isotopic form factors predicted by a three-pole fit to the data of Janssens *et al.* (Ref. 1) and the present experiment.

We have investigated the degree to which a three-pole approximation to the nucleon form factors of the type given by Eq. (12) can be made to fit the experimental data. The $\omega$ and $\varphi$ mesons are assigned their well-defined observed masses but the mass of the $\rho$ meson is treated as an adjustable parameter in view of the large experimental width of this resonance. The total number of free parameters is reduced to six by imposing the condition

$$(dG_{En}/dq^2)_{q^2=0}=0.021\ \mathrm{F}^{-2} \qquad (13)$$

as required by the neutron-electron interaction.[23]

The fitting procedure compares the experimental electron-proton cross sections measured by Janssens and the elastic electron-proton to quasi-elastic electron-deuterium ratios measured in the present experiment with those computed from an initial set of parameters through Eqs. (10) and (12). The statistical function $\chi^2$ is computed and then minimized as a function of the six free parameters using an IBM 7090 computer. This is basically the same procedure used by de Vries *et al.*[3] The following best fit is obtained which corresponds to a value of $\chi^2$ of 175 for 151 degrees of freedom.

$$G_{ES}=0.5\left\{\frac{2.18\pm0.06}{1+q^2/15.7}-\frac{1.11\pm0.14}{1+q^2/26.7}-0.07\pm0.15\right\},$$

$$G_{MS}=0.44\left\{\frac{2.42\pm0.05}{1+q^2/15.7}-\frac{1.35\pm0.09}{1+q^2/26.7}-0.07\pm0.10\right\},$$

$$G_{EV}=0.5\left\{\frac{1.05\pm0.07}{1+q^2/(7.51\pm0.32)}-0.05\pm0.07\right\},$$

$$G_{MV}=2.353\left\{\frac{1.05\pm0.01}{1+q^2/(7.51\pm0.32)}-0.05\pm0.01\right\}. \qquad (14)$$

A satisfactory representation of the form factor behavior for values of $q^2$ up to about 30.0 $\mathrm{F}^{-2}$ can therefore be given by such a three-pole model. Two features of this fit are of interest. Firstly, with the exception of $G_{MV}$, the constant terms appearing in the expressions for the isotopic form factors are zero to within the error of the fit. Secondly, the optimum fit is obtained for an effective mass of the $\rho$ meson equal to 548±24 MeV,

[23] D. J. Hughes, L. A. Harvey, M. D. Goldberg, and M. J. Stafne, Phys. Rev. **90**, 497 (1953).

which is considerably less than the average observed mass of 760 MeV. This low effective mass of the single vector meson confirms the earlier result of de Vries, but it is significantly less than the effective mass of 625 MeV predicted by Ball and Wong.[24] We also find that removal of the condition imposed by the neutron-electron interaction has substantially no effect on the best fit parameters.

Bergia and Brown[25] have recently suggested two additional constraints which can be applied to parametric representations of the nucleon form factors. They argue that the charge and magnetic form factors of each nucleon should be equal at the threshold for nucleon-antinucleon production in the crossed channel, which in turn implies that $G_{ES}=G_{MS}$ and $G_{EV}=G_{MV}$ at $q^2=-4M^2$. We have chosen not to use these conditions in the fitting procedure because the value of $q^2$ at which they apply is far removed from the region of $q^2$ investigated in the present experiment. If Eqs. (14) are extrapolated to $q^2=-4M^2$ the form factors obtained are as follows:

$$G_{ES}=0.03\pm0.15, \quad G_{MS}=0.19\pm0.08, \quad G_{EV}=-0.10\pm0.05, \quad G_{MV}=-0.38\pm0.15. \tag{15}$$

We can claim therefore that Eqs. (14) are consistent with the constraint at $q^2=-4M^2$ to within the errors with which the adjustable parameters are determined.

The solid curves in Figs. 12(a) to 12(d) show the comparison between the measured isotopic form factors and those predicted by the three-pole model defined in Eqs. (14). Figures 14 and 15 show a similar comparison between the measured proton and neutron form factors and the predictions of the three-pole fit. The proton and neutron form factors are satisfactorily reproduced, as might be expected since the fitting procedure required

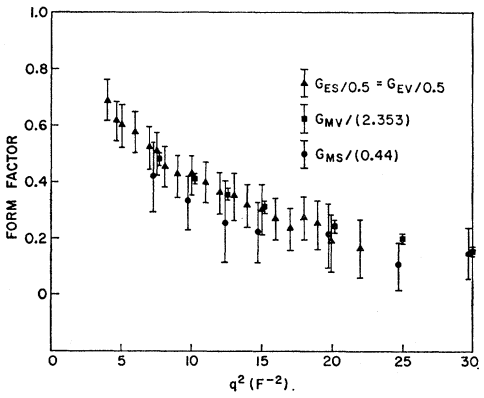


FIG. 13. A comparison between the isotopic nucleon form factors which illustrates their common dependence on $q^2$. The isotopic form factors are normalized to unity at $q^2=0$.

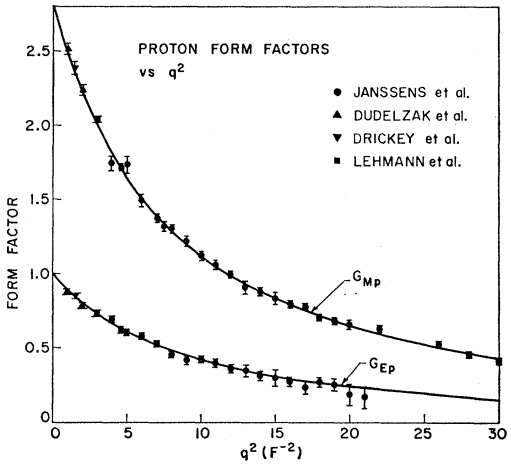


FIG. 14. A comparison between the proton form factors $G_{Ep}$ and $G_{Mp}$ measured by Janssens *et al.* (Ref. 1), Lehmann *et al.* (Ref. 18), Dudelzak *et al.* (Ref. 19), Drickey *et al.* (Ref. 20) and the predictions of the three-pole fit defined by Eqs. (14).

an optimum fit to the experimental data on the proton and neutron, but the fit to the electric isotopic form factors $G_{ES}$ and $G_{EV}$, although quite acceptable to within the errors on these form factors, could clearly be improved. This small systematic difference between the measured isotopic form factors and those predicted by the three-pole fit can be attributed to the assumption that $G_{En}$ is equal to zero which was made in determining the isotopic form factors but which *was not included* in making the three-pole fit. If, however, the expressions for the electric isotopic form factors given in Eq. (12) are separately fitted to the measured values of these form factors then it is possible to remove this small systematic difference with only small changes in the best fit parameters.

Figures 14 and 15 also show the comparison between previous measurements of the proton and neutron form factors[18–21] and the predictions of the three-pole fit to the present data for values of $q^2$ less than 5 $F^{-2}$. Even

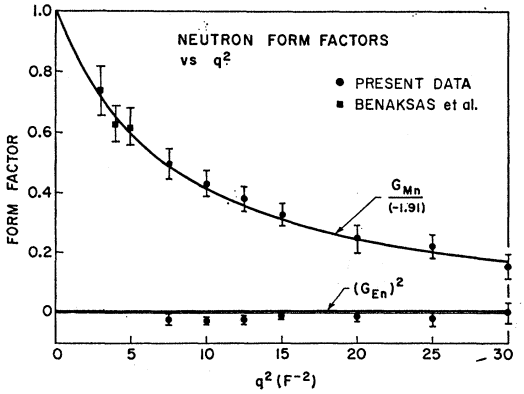


FIG. 15. A comparison between the neutron form factors $(G_{En})^2$ and $G_{Mn}$, including the data of Benaksas *et al.* (Ref. 21), with the predictions of the three-pole fit defined by Eqs. (14).

[24] J. S. Ball and D. Y. Wong, Phys. Rev. **130**, 2112 (1963).

[25] S. Bergia and L. Brown, *Proceedings of the International Conference on Nucleon Structure*, edited by R. Hofstadter and L. I. Schiff (Stanford University Press, Stanford, California, 1963), p. 320.

though these data were not included in making the fit they are well reproduced by an extrapolation of the fit into the low $q^2$ region. The results of the present experiment are also in good agreement with earlier but less accurate measurements of the neutron form factors made by Akerlof *et al.*[26] We have also investigated the agreement between Eqs. (14) and the data of Chen *et al.*[4] who have measured the elastic electron-proton cross section for values of $q^2$ up to 125 $F^{-2}$. When these data are included in making the three-pole fit, only small changes in the best fit parameters are required in order to give a minimum value of $\chi^2$ of 191 for 157 degrees of freedom. Undoubtedly a severer test of the parameters given in Eqs. (14), and of the three-pole approximation itself, will be provided when the accuracy of the experimental data at such high values of $q^2$ increases.[27]

We conclude from the above analysis that the $\omega$ and $\varphi$ mesons provide a good approximation to the behavior of the isoscalar form factors. On the other hand we regard the unexpectedly low effective mass of the $\rho$ meson as evidence that the $\rho$ meson alone cannot account for the behavior of the isovector form factors. We should point out, however, that the effective mass of the $\rho$ meson is determined primarily by the magnetic rather than the electric isotopic form factor since the magnetic isotopic form factor is much more precisely determined by the present experiments. Also, it is reasonable to expect that the magnetic isovector form factor would be more sensitive than the electric isovector form factor to contributions from intermediate states more massive than the $\rho$ meson because the anomalous moments of the proton and neutron reinforce each other in the isovector form factor. We therefore take the low effective mass of the $\rho$ meson to indicate the presence of such heavier intermediate states.

There is a possibility that the recently reported $B$ meson,[28] a $\pi\omega$ resonance of mass 1220 MeV and width about 100 MeV, might have the quantum numbers required for it to contribute to the isovector nucleon form factors. The present experimental information on this resonance is consistent with it being a vector meson with the correct spin and parity but these assignments are not unique.[29] In view of this possibility we have investigated the degree to which a four-pole approximation to the nucleon form factors can be made to fit the experimental data. We assign the $\rho$ meson its average observed mass and compute the minimum value of $\chi^2$ for a series of values of the mass of a second vector meson. Figure 16 shows the minimum value of $\chi^2$ which is obtained as a function of the mass of the second vector meson. The best fit is obtained for a mass of about 1200 MeV which corresponds to a value of $\chi^2$ of 274 for 149 deg of freedom. For masses less than 1200 MeV the fit becomes progressively worse but for higher masses the minimum value of $\chi^2$ increases only relatively slowly. This latter feature is expected since intermediate states of such high mass do not contribute strongly to the $q^2$ dependence of the form factors in the region of the present experiments.

It is possible therefore to give an approximate representation of the form factor behavior for values of $q^2$ up to 30.0 $F^{-2}$ by means of a four-pole approximation using the $\omega$, $\varphi$, and $\rho$ mesons and a second vector meson of mass of the order of or greater than that of the $B$ meson. Equations (16) give the parameters of such a fit when the mass of the second vector meson is taken to be that of the $B$ meson. The constant terms in the expressions for $G_{ES}$, $G_{MS}$, and $G_{EV}$ are small and consistent with zero to within the accuracy of the fit. However, the constant term in the expression for $G_{MV}$ is definitely nonzero and suggests that the behavior of the isovector form factors cannot be accounted for solely in terms of pole contributions due to the $\rho$ and $B$ mesons.[30]

$$G_{ES}=0.5\left\{\frac{1.93\pm0.06}{1+q^2/15.7}-\frac{0.72\pm0.10}{1+q^2/26.7}-0.21\pm0.12\right\},$$

$$G_{MS}=0.44\left\{\frac{2.10\pm0.08}{1+q^2/15.7}-\frac{0.91\pm0.11}{1+q^2/26.7}-0.19\pm0.14\right\},$$

$$G_{EV}=0.5\left\{\frac{3.28\pm0.07}{1+q^2/14.1}-\frac{3.51\pm0.70}{1+q^2/37.2}+1.23\pm0.71\right\},$$

$$G_{MV}=2.353\left\{\frac{2.23\pm0.02}{1+q^2/14.1}-\frac{1.54\pm0.04}{1+q^2/37.2}+0.31\pm0.05\right\}. \tag{16}$$

In the preceding analysis we have implicitly assumed that it is the charge and magnetic form factors $G_E$ and $G_M$ and not the Dirac and Pauli form factors $F_1$ and $F_2$ which can be formulated in terms of dispersion theory according to Eqs. (11). However it is not clear theoretically to which set of form factors the dispersion theory more closely applies, although in general it should not be possible to express both sets of form factors simultaneously in this form. In order to investigate this question we have attempted to fit the data of Janssens *et al.* and of the present experiment to a three-pole approximation to the Dirac and Pauli form factors.

[26] C. W. Akerlof, K. Berkelman, G. Rouse and M. Tigner, Phys. Rev. **135**, B810 (1964).

[27] The values of the neutron and proton form factors predicted by Eq. (14) as $q^2\to\infty$ are as follows: $G_{Ep}=-0.06\pm0.08$, $G_{En}=-0.01\pm0.08$, $G_{Mp}/\mu_p=-0.05\pm0.02$ and $G_{Mn}/\mu_n=0.05\pm0.03$. These can be compared with the experimental limits given by Dunning *et al.* (Ref. 5) which are $G_{Ep}\leq0.07$, $G_{En}\leq0.08$, $G_{Mp}/\mu_p\leq0.013$ and $G_{Mp}/\mu_n\leq0.013$.

[28] M. Abolins, R. L. Lander, W. Mahlhop, N. Xuong and P. Yager, Phys. Rev. Letters **11**, 381 (1963).

[29] D. D. Carmony, R. L. Lander, C. Rindfleisch, N. Xuong, and P. Yager, Phys. Rev. Letters **12**, 254 (1964).

[30] A similar conclusion was reached by Akerlof *et al.* (Ref. 26) and by Dunning *et al.* (Ref. 5) from a study of earlier experimental data. The decidedly less acceptable values of $\chi^2$ found by these groups can be attributed to their assumption that the constant terms in Eqs. (16) be identically equal to zero. This was not required in the present investigation.

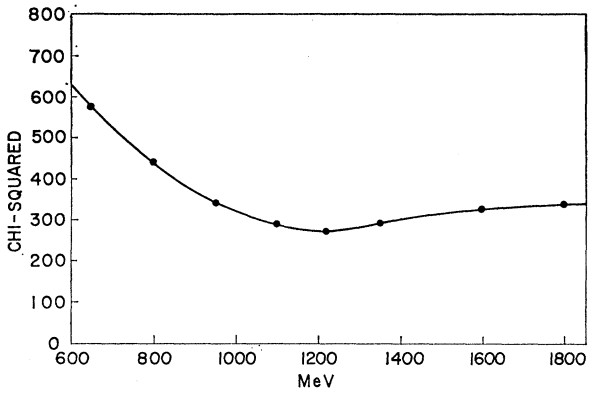


FIG. 16. The minimum value of $\chi^2$ obtained by a four-pole fit to the data of Janssens *et al.* (Ref. 1) and the present experiment as a function of the mass of the second vector meson.

When the isotopic form factors are defined as in Eqs. (17),

$$\begin{aligned} F_{1S}&=F_{1p}+F_{1n}, \\ F_{1V}&=F_{1p}-F_{1n}, \\ F_{2S}&=(1.79F_{2p}-1.91F_{2n})/(-0.12), \\ F_{2V}&=(1.79F_{2p}+1.91F_{2n})/(3.70), \end{aligned} \tag{17}$$

the following best fit is obtained which corresponds to a value of $\chi^2$ of 168 for 151 degrees of freedom.

$$F_{1S}=\left\{\frac{2.19\pm0.03}{1+q^2/15.7}-\frac{1.04\pm0.05}{1+q^2/26.7}-0.15\pm0.06\right\},$$

$$F_{2S}=\left\{\frac{-8.03\pm0.60}{1+q^2/15.7}+\frac{11.43\pm0.84}{1+q^2/26.7}-2.40\pm1.02\right\},$$

$$F_{1V}=\left\{\frac{0.81\pm0.07}{1+q^2/(8.09\pm0.35)}+0.19\pm0.07\right\},$$

$$F_{2V}=\left\{\frac{1.13\pm0.02}{1+q^2/(8.09\pm0.35)}-0.13\pm0.02\right\}. \tag{18}$$

It is possible therefore to give a satisfactory representation of the experimental results in terms of a three-pole approximation to the Dirac and Pauli form factors. Moreover the features of this fit are very similar to those of the three-pole fit to the charge and magnetic form factors. The effective mass of the $\rho$ meson is again required to be about 200 MeV less than the average observed mass. On the basis of the present experimental results the dispersion approximation appears to work equally well for both sets of form factors.

## V. CONCLUSIONS

In the present experiment the form factors of the neutron have been measured in the $q^2$ range from 1.0 to 30.0 $F^{-2}$.

It is found that for values of $q^2$ greater than about 6.0 $F^{-2}$ the square of the neutron's charge form factor $(G_{En})^2$ is consistent with zero to within an error or the order of 5% at certain angles and it is concluded that meaningful values of the neutron form factors in the low $q^2$ range cannot be obtained from experiments on quasielastic electron-deuteron scattering until the necessary corrections to the theory are better understood.

The present results on the neutron form factors are combined with the results of Janssens on the proton form factors to provide experimental determinations of the isotopic nucleon form factors. It is found that the $\omega$ and $\varphi$ mesons provide a good approximation to the observed isoscalar form factors but that it is difficult to understand the behavior of the isovector form factors in terms of the $\rho$ meson alone. It is shown that an alternative but less accurate description of the isovector form factors can be given in which the contribution from the $\rho$ meson is supplemented by a contribution from a second vector meson of mass approximately equal to or greater than that of the recently discovered $B$ meson. Although the latter representation does not provide as satisfactory a goodness of fit as the former it does have the advantage that the $\rho$ meson is not required to have an effective mass of the order of 200 MeV less than its average observed mass but can be assigned an effective mass equal to its observed mass.

It is finally pointed out that equally acceptable representations of the available experimental data for values of $q^2$ up to 30.0 $F^{-2}$ can be provided by three-pole approximations to either the charge and magnetic form factors or to the Dirac and Pauli form factors of the nucleons.

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