ing in terms of a spin-independent, purely imaginary amplitude. The incorrect energy dependence of vector-meson-exchange amplitudes is the most important shortcoming, and prevents a meaningful determination of vector-meson coupling constants.[61] Another difficulty is the violation (or near violation) of the unitarity bound in some reactions (e.g., $Kp \rightarrow K^*N^*$).

[61] A further illustration of this remark is provided by the unsuccessful calculations of π-N and $\bar{K}$-N charge exchange scattering [V. Barger and M. Ebel, Phys. Rev. **138**, B1148 (1965)]. While one may attribute this failure to the vector-meson-exchange mechanism, it is probably also due to the inapplicability of the model to processes that are, in essence, elastic. From the viewpoint of unitary symmetry (SU_3), calculations of associated ropduction and reactions such as $\bar{p}p \rightarrow \bar{\Lambda}\Lambda$ are open to the same doubts. Of course, within the framework of SU_6 all the processes considered in this paper are elastic scatterings!

(11) The physical assumptions of the model (and also the mathematical approximations) are most reliable at small production angles and at energies such that many partial waves participate in the reaction. Precise data in this domain would allow definitive tests of theory.

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Mass Relations of SU_3 in a Soluble ω-ϕ Mixing Model*

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A soluble model of SU_3-invariant meson-vecton interactions is investigated. ω-ϕ mixing is introduced as a soluble, symmetry-breaking interaction. Small form-factor effects in this interaction are also treated. Mass renormalization is carried out to study higher order effects in the Gell-Mann–Okubo formula for mesons. Two new mass formulas for vectons and an equation relating vecton masses to meson masses is derived. Finally, relations between bare coupling constants are studied. The agreement with experiment is generally good.

1. INTRODUCTION

IN view of the apparent success of the octet version of unitary symmetry (SU_3)[1–3] it is of increasing importance to obtain some understanding for the surprising validity of the Gell-Mann–Okubo mass formula (GMO formula).[2,4] It has been derived in first-order perturbation theory with respect to the symmetry-breaking interaction, and nothing is known so far about the behavior of higher order contributions. We therefore set up a model which can be solved exactly and derive mass relations from it. In order to do this, we have to decide what kind of symmetry-breaking interaction we will choose. There are, of course, several ways to introduce symmetry-breaking effects. For reasons, specified below, we will choose ω-ϕ mixing[5–7] for our model: (1) Since we do not know the exact type of the basic symmetry-breaking interaction, ω-ϕ mixing will serve as a good phenomenological description. Even if the fundamental interaction is of a completely different nature, ω-ϕ mixing has to emerge as an "effective interaction." (2) A more technical reason for choosing ω-ϕ mixing is that it is bilinear in the field operators and can thus be solved exactly.

The model we propose is an SU_3-invariant version of the Zachariasen-Thirring[8–11] model with ω-ϕ mixing. In its Lagrangian version, the Zachariasen-Thirring model is nothing but a restriction on the type of Feynman graphs which have to be summed up. Our results can therefore be viewed either as the exact solution of a Zachariasen-Thirring model or as the chain approximation to a full-fledged theory. In any case they are valid to every order in the symmetry-breaking interaction.

We will study the interactions of (pseudoscalar) mesons and vector mesons ("vectons"). Their propaga-

* Supported in part by the U. S. Atomic Energy Commission.
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tors will be derived and their physical masses will be obtained as the poles in these propagators. Unless otherwise stated, notation and definitions follow those of Ref. 12.)

2. THE MODEL

The model under consideration is defined through the interaction-Hamiltonian density together with the prescription that only chains of bubble diagrams have to be considered.[11] The SU_3-invariant interaction between mesons and vectons is given by[6] ($M=$ bare vecton mass and $A_{;\mu}\equiv\partial_\mu A$)

$$H_I=\epsilon^{\mu\nu\sigma\lambda}\left(\frac{f_0}{M}(\partial_\mu\omega_\nu^{(0)})\operatorname{Tr}(\mathcal{V}_{\lambda;\sigma}\mathcal{P})+\frac{f_1}{M}\operatorname{Tr}(\mathcal{V}_{\nu;\mu}\mathcal{V}_{\lambda;\sigma}\mathcal{P})\right)+ig\operatorname{Tr}(\mathcal{V}_\mu[\mathcal{P}_{;\mu},\mathcal{P}]),\quad(1)$$

where $\mathcal{V}$ and $\mathcal{P}$ are the familiar 3 by 3 matrices of vectons and mesons with their appropriate Clebsch-Gordan coefficients.[5] Using 8-dimensional vectors P^i and V^i, (1) can be cast into the form

$$H_I=\epsilon^{\mu\nu\sigma\lambda}\left(\frac{f_0}{M}\omega_{\nu;\mu}^{(0)}V_{\lambda;\sigma}{}^iP^jM_{ij}+\frac{f_1}{M}V_{\nu;\mu}{}^jV_{\lambda;\sigma}{}^kP^iD_{jk}{}^i\right)+igV_\mu{}^iP_{;\mu}{}^jP^kF_{jk}{}^i,\quad(2)$$

where the coefficient matrices M, D^i, and F^i display the following symmetry properties

$$M_{ij}=M_{ji},\quad D_{jk}{}^i=D_{kj}{}^i,\quad -F_{jk}{}^i=F_{kj}{}^i,\quad(3)$$

and

$$\begin{aligned}\textstyle\sum_i(D^iD^i)_{lk}&=\operatorname{Tr}(D^lD^k)=\tfrac{5}{6}\delta_{kl},\\ \textstyle\sum_i(F^iF^i)_{lk}&=\operatorname{Tr}(F^lF^k)=-6\delta_{kl}.\end{aligned}\quad(4)$$

As discussed in the Introduction, the symmetry-breaking interaction we assume is ω-ϕ mixing[5]:

$$H_A=\tfrac{1}{2}m_{\phi\omega}{}^2(\omega_\mu{}^{(0)}\phi^{(0)\mu}+\phi_\mu{}^{(0)}\omega^{(0)\mu});\quad(5)$$

the total Hamiltonian density will thus be

$$H=H_0+H_I+H_A,\quad(6)$$

where H_0 is the free Hamiltonian density of the eight mesons and vectons. The bare mass μ of the singlet $\omega^{(0)}$ need not be the same as the bare vecton-mass M and thus will be taken as an extra parameter in the model.

In Eq. (5), $m_{\phi\omega}{}^2$ has been taken as a constant. However, if ω-ϕ mixing is looked upon as the effective result of some other basic symmetry-breaking interaction, it can depend on the coordinates in the following way

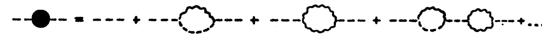

FIG. 1. The meson propagator. Dashed lines denote mesons and wiggly lines vectons (including ω).

(H_A' being the integrated Hamiltonian):

$$H_A'=\frac{1}{2}\int d^4xd^4y\,m_{\phi\omega}{}^2(x-y)\times[\omega_\mu{}^{(0)}(x)\phi^{(0)\mu}(y)+\phi_\mu{}^{(0)}(y)\omega^{(0)\mu}(x)],\quad(7)$$

thus leading to a momentum dependence in momentum space. We will return to this point later.

Obviously, Eq. (6) leads to highly divergent integrals in the renormalized propagators. In order to get a consistent model, we will therefore complement it by a set of rules for obtaining meaningful results:

(1) A cutoff Λ will be introduced but will eventually be taken to be infinite; thus,

(2) only those results shall be considered which are independent of Λ.

(3) At least one of the bare masses has to tend to infinity together with Λ; it will be chosen to be the bare pseudoscalar mass m. Thus also m must be eliminated in all the results.

3. DIAGONALIZATION OF THE MASS MATRIX

As stated in the introduction, the interaction (5) [or (7)] can be dealt with exactly by diagonalizing the mass matrix. This has been carried out in detail by Harte and Sachs[13] and we therefore only restate some of their important results for further reference.

The vecton propagator can be written as

$$\Delta_{ll'}{}^{\mu\nu}(k)=\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{z_\alpha}\right)\frac{(-1)}{k^2-z_\alpha}\langle l|\psi_\alpha\otimes\psi_\alpha{}^T|l'\rangle,\quad(8)$$

where, in the notation of Harte and Sachs,[13]

$$\psi_\alpha=|\alpha\rangle,\quad z_\alpha=M^2\quad\text{if }\alpha\neq\omega,\phi\quad(9)$$

and

$$\begin{aligned}\psi_\phi&=\sin\lambda|\omega\rangle+\cos\lambda|\phi\rangle,\\ \psi_\omega&=\cos\lambda|\phi\rangle-\sin\lambda|\omega\rangle.\end{aligned}\quad(10)$$

z_ϕ and z_ω are defined through

$$z_\phi=\tfrac{1}{2}[M^2+\mu^2+r(z_\phi)],\quad(11)$$

$$z_\omega=\tfrac{1}{2}[M^2+\mu^2-r(z_\omega)],\quad(12)$$

where

$$r(z)=[\Delta^2+4m_{\phi\omega}{}^4(z)]^{1/2}\quad(13)$$

and

$$\Delta=M^2-\mu^2.\quad(14)$$

λ is the ω-ϕ mixing angle. Note that we have here taken into account a possible momentum dependence of $m_{\phi\omega}$ according to Eq. (7). We will however take it to be very small; thus

$$m_{\phi\omega}{}^2(z)=\eta^2+\epsilon z\quad\epsilon\ll1.\quad(15)$$

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[12] W. Thirring, *Principles of Quantum Electrodynamics* (Academic Press Inc., New York, 1958).

[13] J. Harte and R. G. Sachs, Phys. Rev. **135**, B459 (1964).

The unrenormalized propagators for ω and ϕ vectons will thus be

$$\Delta_\phi{}^{\mu\nu}=-\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{z_\phi}\right)\frac{\cos^2\lambda}{k^2-z_\phi}-\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{z_\omega}\right)\frac{\sin^2\lambda}{k^2-z_\omega}, \quad (16)$$

$$\Delta_\omega{}^{\mu\nu}=-\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{z_\phi}\right)\frac{\sin^2\lambda}{k^2-z_\phi}-\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{z_\omega}\right)\frac{\cos^2\lambda}{k^2-z_\omega}. \quad (17)$$

In addition, there will be a "mixed" propagator, connecting a ϕ vertex to an ω vertex.

$$\Delta_{\omega\phi}{}^{\mu\nu}=-\tfrac{1}{2}\sin2\lambda\left\{\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{z_\phi}\right)\frac{1}{k^2-z_\phi}-\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{z_\omega}\right)\frac{1}{k^2-z_\omega}\right\}. \quad (18)$$

4. THE MESON PROPAGATOR

Contributions to the renormalized meson propagator are shown in Fig. 1. Neglecting, for the moment, the interaction H_A, the renormalized meson propagator $P(x-y)$ is the solution of the following integral equation:

$$P(x-y)=\Delta^F(x-y)-i\int d^4x_1d^4x_2\Delta^F(x-x_1)\times\Sigma(x_1-x_2)P(x_2-y), \quad (19)$$

where $\Sigma(x)$ is the sum of the vecton-meson bubble, the vecton-vecton bubble, and the vecton-ω bubble, i.e.,

$$\Sigma(x)=\Sigma^{VV}(x)+\Sigma^{VP}(x)+\Sigma^{V\omega}(x). \quad (20)$$

Standard Feynman rules together with Eq. (2) yield

$$\Sigma^{VV}(x)=-\frac{5}{6}\frac{f_1{}^2}{M^2}4\{\Delta^F{}_{;\mu\mu}(x,M^2)\Delta^F{}_{;\nu\nu}(x,M^2)-\Delta^F{}_{;\mu\nu}(x,M^2)\Delta^F{}_{;\mu\nu}(x,M^2)\}, \quad (21)$$

$$\Sigma^{V\omega}(x)=\frac{f_0{}^2}{M^2}4\{\Delta^F{}_{;\mu\mu}(x,M^2)\Delta^F{}_{;\nu\nu}(x,\mu^2)-\Delta^F{}_{;\mu\nu}(x,M^2)\Delta^F{}_{;\mu\nu}(x,\mu^2)\}, \quad (21')$$

$$\Sigma^{VP}(x)=-6g^2\{4\Delta^F(x,M^2)\Delta^F{}_{;\mu\mu}(x,m^2)+\Delta^F{}_{;\mu\mu}(x,M^2)\Delta^F(x,m^2)+2\Delta^F{}_{;\mu}(x,m^2)\Delta^F{}_{;\mu}(x,M^2)$$
$$+\frac{1}{M^2}[4\Delta^F{}_{;\mu\nu}(x,M^2)\Delta^F{}_{;\mu\nu}(x,m^2)+\Delta^F{}_{;\mu\mu\nu\nu}(x,M^2)\Delta^F(x,m^2)+2\Delta^F{}_{;\mu\mu\nu}(x,M^2)\Delta^F{}_{;\nu}(x,m^2)]\}. \quad (22)$$

The product of two Δ^F's at the same space-time point has to be handled with great care. We can, however, derive spectral representations for two Δ^+'s (or Δ^-'s) and then define the Δ^F product with the same spectral function. In this way we obtain (see Appendix II of Ref. 12)

$$\Delta^F(x,M^2)\Delta^F{}_{;\mu\mu}(x,m^2)=\frac{-im^2}{16\pi^2}\int_{(M+m)^2}^\infty\frac{ds}{s}w(M,m,s)\Delta^F(x,s); \quad (23a)$$

$$\Delta^F{}_{;\mu}(x,M^2)\Delta^F{}_{;\mu}(x,m^2)=\frac{i}{32\pi^2}\int_{(M+m)^2}^\infty\frac{ds}{s}(M^2+m^2-s)w(M,m,s)\Delta^F(x,s), \quad (23b)$$

$$\Delta^F{}_{;\mu\nu}(x,M^2)\Delta^F{}_{;\mu\nu}(x,m^2)=\frac{i}{64\pi^2}\int_{(M+m)^2}^\infty\frac{ds}{s}(M^2+m^2-s)^2w(M,m,s)\Delta^F(x,s), \quad (23c)$$

$$\Delta^F{}_{;\mu\mu\nu\nu}(x,M^2)\Delta^F(x,m^2)=\frac{iM^4}{16\pi^2}\int_{(M+m)^2}^\infty\frac{ds}{s}w(M,m,s)\Delta^F(x,s), \quad (23d)$$

$$\Delta^F{}_{;\mu\nu\nu}(x,M^2)\Delta^F{}_{;\mu}(x,m^2)=\frac{iM^2}{32\pi^2}\int_{(M+m)^2}^\infty\frac{ds}{s}(s-M^2-m^2)w(M,m,s)\Delta^F(x,s), \quad (23e)$$

$$\Delta^F{}_{;\mu\mu}(x,M^2)\Delta^F{}_{;\nu\nu}(x,m^2)=\frac{im^2M^2}{16\pi^2}\int_{(M+m)^2}^\infty\frac{ds}{s}w(M,m,s)\Delta^F(x,s), \quad (23f)$$

where

$$w(M,m,s)=[M^4+m^4+s^2-2(M^2m^2+M^2s+m^2s)]^{1/2}. \quad (24)$$

Inserting (23) in (21) and (22) gives

$$\Sigma^{VV}(x)=-\frac{5}{6}\frac{f_1^2}{M^2}\frac{i}{16\pi^2}\int_{4M^2}^{\infty}\frac{ds}{s}[w(M,s)]^3\Delta^F(x,s)\,, \tag{25}$$

$$\Sigma^{V\omega}(x)=-\frac{f_0^2}{M^2}\frac{i}{16\pi^2}\int_{(M+\mu)^2}^{\infty}\frac{ds}{s}[w(M,\mu,s)]^3\Delta^F(x,s)\,, \tag{25'}$$

with

$$w(M,s)\equiv w(M,M,s) \tag{26}$$

and

$$\Sigma^{VP}(x)=6\frac{g^2}{M^2}\frac{i}{16\pi^2}\int_{(M+m)^2}^{\infty}\frac{ds}{s}[w(M,m,s)]^3\Delta^F(x,s)\,. \tag{27}$$

In momentum space, the solution of (19) is simply the sum of a geometrical series given by

$$P(k)=-\left[k^2-m^2-6\frac{g^2}{M^2}I(k^2;M^2,m^2)-\frac{5}{6}\frac{f_1^2}{M^2}I(k^2;M^2)-\frac{f_0^2}{M^2}I(k^2;M^2,\mu^2)\right]^{-1}, \tag{28}$$

where

$$I(k^2;M^2,m^2)=\frac{1}{16\pi^2}\int_{(M+m)^2}^{\infty}\frac{ds}{s}[w(M,m,s)]^3\frac{1}{s-k^2-i\epsilon} \tag{29}$$

and

$$I(k^2;M^2)\equiv I(k^2;M^2,M^2)\,. \tag{30}$$

Clearly, all mesons have the same renormalized mass in the absence of H_A.

Taking into account the symmetry-breaking interaction H_A now, we have to change all ϕ and ω propagators to their forms (16) and (17) and also properly include contributions from the mixed propagator (18). Since the coupling of ϕ will differ for the three types of mesons, K, π, and η, by a Clebsch-Gordan coefficient of SU_3, we now have different propagators and thus different mass renormalization. The physical masses are defined through the zeros of the denominator of the renormalized propagators; thus,

$$\begin{aligned}m_K^2-m^2&-\frac{9}{2}\frac{g^2}{M^2}I(m_\alpha^2;M^2,m^2)-\frac{3}{4}\frac{f_1^2}{M^2}I(m_K^2;M^2)\\&-\frac{1}{M^2}I(m_K^2;z_\phi,m^2)[\tfrac{3}{2}g^2\cos^2\lambda+\tfrac{1}{12}f_1^2\cos^2\lambda+f_0^2\sin^2\lambda-(1/\sqrt{6})f_0f_1\sin2\lambda]\\&-\frac{1}{M^2}I(m_K^2;z_\omega,m^2)[\tfrac{3}{2}g^2\sin^2\lambda+\tfrac{1}{12}f_1^2\sin^2\lambda+f_0^2\cos^2\lambda+(1/\sqrt{6})f_0f_1\sin2\lambda]=0;\end{aligned} \tag{31a}$$

$$\begin{aligned}m_\pi^2-m^2&-6\frac{g^2}{M^2}I(m_\pi^2;M^2,m^2)-\frac{1}{2}\frac{f_1^2}{M^2}I(m_\pi^2;M^2)\\&-\frac{1}{M^2}I(m_\pi^2;z_\phi,m^2)[\tfrac{1}{3}f_1^2\cos^2\lambda+f_0^2\sin^2\lambda+(2/\sqrt{6})f_0f_1\sin2\lambda]\\&-\frac{1}{M^2}I(m_\pi^2;z_\omega,m^2)[\tfrac{1}{3}f_1^2\sin^2\lambda+f_0^2\cos^2\lambda-(2/\sqrt{6})f_0f_1\sin2\lambda]=0;\end{aligned} \tag{31b}$$

$$\begin{aligned}m_\eta^2-m^2&-6\frac{g^2}{M^2}I(m_\eta^2;M^2,m^2)-\frac{2}{3}\frac{f_1^2}{M^2}I(m_\eta^2;M^2)\\&-\frac{1}{M^2}I(m_\eta^2;z_\phi)\cos^2\lambda[3f_0^2\sin^2\lambda+\tfrac{1}{6}f_1^2\cos^2\lambda-(2/\sqrt{6})f_0f_1\sin2\lambda]\\&-\frac{1}{M^2}I(m_\eta^2;z_\omega)\sin^2\lambda[\tfrac{1}{6}f_1^2\sin^2\lambda+3f_0^2\cos^2\lambda+(2/\sqrt{6})f_0f_1\sin2\lambda]\\&-\frac{1}{M^2}I(m_\eta^2;z_\phi,z_\omega)[f_0^2(1-\tfrac{3}{2}\sin^22\lambda)+\tfrac{1}{12}f_1^2\sin^22\lambda+(1/\sqrt{6})f_0f_1\sin4\lambda]=0.\end{aligned} \tag{31c}$$

From (29) it is seen that $I(k^2,M^2,m^2)$ is divergent with the 4th power of energy $s^{1/2}$. In fact,

$$I(k^2;M^2,m^2)=\frac{1}{16\pi^2}\lim_{\Lambda\to\infty}\left\{\frac{\Lambda^4}{2}+\Lambda^2[k^2-3(M^2+m^2)]\right\}+O\left(\ln\frac{\Lambda}{M}\right). \tag{32}$$

According to the rules set up at the end of Sec. 2, we have to insert (32) in (31) and eliminate Λ^2 and m^2. It turns out that one more parameter can be eliminated; we choose it to be $f_0f_1\sin 2\lambda$ which is the term containing all lowest order contributions from H_A. The result is

$$m_K{}^2=\frac{3m_\eta{}^2+m_\pi{}^2}{4}-\frac{3}{4}\frac{f_1{}^2+3f_0{}^2-6g^2}{\frac{5}{6}f_1{}^2+f_0{}^2+6g^2}\xi, \tag{33}$$

where

$$\xi=z_\phi\cos^2\lambda+z_\omega\sin^2\lambda-M^2. \tag{34}$$

Note that $\xi=0$ if $m_{\phi\omega}{}^2$ is a constant. We thus get the very important result that within our model, the GMO relation for mesons is true to every order in the symmetry breaking interaction. If $m_{\phi\omega}{}^2$ is given by (15), ξ will be pro-

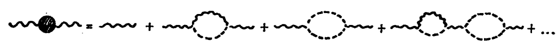

FIG. 2. The vecton propagator.

portional to ϵ and thus be a small parameter in our theory. This is, in fact, the only place where we will not neglect the momentum dependence of $m_{\phi\omega}{}^2$. With g taken to be zero, the result (33) has already been discussed in a communication.[14]

5. THE VECTON PROPAGATOR

Starting again with the SU_3-invariant interaction H_I only, we obtain the following integral equation for the vecton propagator (Fig. 2)

$$P_{\mu\nu}(x-y)=\Delta_{\mu\nu}{}^V(x-y)+i\int d^4x_1d^4x_2\Delta_{\mu\sigma}{}^V(x-x_1)\Sigma^{\sigma\lambda}(x_1-x_2)P_{\lambda\nu}(x_2-y), \tag{35}$$

where

$$\Delta_{\mu\nu}{}^V(x-y)=(g_{\mu\nu}+\partial_\mu\partial_\nu/M^2)\Delta^F(x-y). \tag{36}$$

$\Sigma_{\mu\nu}(x)$ is again a sum of three contributions:

$$\begin{aligned}\Sigma_{\mu\nu}{}^{VP}(x)=-\tfrac{5}{6}(f_1{}^2/M^2)2\{[\Delta^F{}_{;\sigma\sigma}(x,m^2)\Delta^F{}_{;\lambda\lambda}(x,M^2)-\Delta^F{}_{;\sigma\lambda}(x,m^2)\Delta^F{}_{;\sigma\lambda}(x,M^2)]g_{\mu\nu}\\+\Delta^F{}_{;\nu\lambda}(x,m^2)\Delta^F{}_{;\mu\lambda}(x,M^2)+\Delta^F{}_{;\mu\lambda}(x,m^2)\Delta^F{}_{;\nu\lambda}(x,M^2)\\-\Delta^F{}_{;\mu\nu}(x,m^2)\Delta^F{}_{;\sigma\sigma}(x,M^2)-\Delta^F{}_{;\sigma\sigma}(x,m^2)\Delta^F{}_{;\mu\nu}(x,M^2)\};\end{aligned} \tag{37a}$$

a similar term, $\Sigma_{\mu\nu}{}^{\omega P}(x)$ with the following replacements

$$\tfrac{5}{6}(f_1{}^2/M^2)2\to(f_0{}^2/M^2)\quad\Delta^F(x,M^2)\to\Delta^F(x,\mu^2) \tag{37b}$$

and

$$\Sigma_{\mu\nu}{}^{PP}(x)=6g^2[\Delta^F{}_{;\mu\nu}(x,m^2)\Delta^F(x,m^2)-\Delta^F{}_{;\mu}(x,m^2)\Delta^F{}_{;\nu}(x,m^2)]. \tag{37c}$$

In addition to (23) we need the following spectral representations:

$$\Delta^F{}_{;\mu}(x,m^2)\Delta^F{}_{;\nu}(x,m^2)=\frac{i}{48\pi^2}\int_{4m^2}^\infty ds\left\{\left(m^2-\frac{s}{4}\right)g_{\mu\nu}+\left(m^2+\frac{s}{4}\right)\frac{\partial_\mu\partial_\nu}{s}\right\}[(s-4m^2)/s]^{-1/2}\Delta^F(x,s); \tag{38a}$$

$$\begin{aligned}\Delta^F{}_{;\mu\nu}(x,m^2)\Delta^F(x,M^2)=\frac{i}{48\pi^2}\int_{(M+m)^2}^\infty\frac{ds}{s^2}w(M,m,s)\\\times\{[\tfrac{1}{4}(s+m^2-M^2)^2-sm^2]g_{\mu\nu}+[s^{-1}(s+m^2-M^2)^2-m^2]\partial_\mu\partial_\nu\}\Delta^F(x,s);\end{aligned} \tag{38b}$$

$$\begin{aligned}\Delta^F{}_{;\nu\sigma}(x,m^2)\Delta^F{}_{;\mu\sigma}(x,M^2)=\frac{i}{48\pi^2}\int_{(M+m)^2}^\infty\frac{ds}{s^2}w(M,m,s)[s-(M^2+m^2)]\\\times\{\tfrac{1}{8}[w(M,m,s)]^2g_{\mu\nu}-(1/4s)[s^2+s(M^2+m^2)-2(M^2-m^2)^2]\partial_\mu\partial_\nu\}\Delta^F(x,s),\end{aligned} \tag{38c}$$

[14] H. Pietschmann, Phys. Letters 11, 352 (1964).

where $w(M,m,s)$ is given by (24). Inserting Eqs. (23) and (38) in Eq. (37), one can derive the following solution of Eq. (35) in momentum space:

$$P_{\mu\nu}(k)=G(k)P_{\mu\nu}{}^{t}(k)+G(0)P_{\mu\nu}{}^{l}(k)\,, \tag{39}$$

where $P_{\mu\nu}{}^{t}$ and $P_{\mu\nu}{}^{l}$ are projection operators for transverse and longitudinal polarization, given by

$$\begin{aligned} P_{\mu\nu}{}^{t}(k)&=g_{\mu\nu}-(k_\mu k_\nu/k^2)\,,\\ P_{\mu\nu}{}^{l}(k)&=k_\mu k_\nu/k^2\,. \end{aligned} \tag{40}$$

$G(k)$ has the following form:

$$G(k)=-\left[k^2-M^2-g^2J(k^2;m^2)-\frac{5}{6}\frac{f_1{}^2}{M^2}\frac{1}{3}I(k^2;M^2,m^2)-\frac{f_0{}^2}{M^2}\frac{1}{6}I(k^2;\mu^2,m^2)\right]^{-1}, \tag{41}$$

where $I(k^2,M^2,m^2)$ was defined in (29) and

$$J(k^2;m^2)=\frac{1}{16\pi^2}\int_{4m^2}^{\infty}ds\frac{1}{6s^{1/2}}[s-4m^2]^{3/2}\frac{1}{s-k^2-i\epsilon}\,. \tag{42}$$

The renormalized $\omega^{(0)}$ propagator can be derived in an entirely similar way with the only change in (41) being

$$G_\omega(k)=-[k^2-\mu^2-(f_0{}^2/M^2)\tfrac{4}{3}I(k^2;M^2,m^2)]^{-1}\,. \tag{43}$$

It is clear from Eqs. (41) and (43) that even without symmetry-breaking mixing effects, the renormalized mass of ω and ϕ will be different. The assumption of the same bare mass for ω and ϕ (and hence for all vectons) thus appears to be an artificial constraint. Switching on the interaction H_A, we obtain the physical masses of $M(\equiv K^*)$ and ρ as the zeros in the denominator of their propagators.

$$\begin{aligned} &m_M{}^2-M^2-g^2J(m_M{}^2,m^2)-(19/24)(f_1{}^2/M^2)\tfrac{1}{3}I(m_M{}^2;M^2,m^2)\\ &\quad-(1/3M^2)I(m_M{}^2;z_\phi,m^2)[(1/24)f_1{}^2\cos^2\lambda+\tfrac{1}{2}f_0{}^2\sin^2\lambda-(1/4\sqrt{6})f_0f_1\sin2\lambda]\\ &\quad-(1/3M^2)I[(m_M{}^2;z_\omega,m^2)[(1/24)f_1{}^2\sin^2\lambda+\tfrac{1}{2}f_0{}^2\cos^2\lambda+(1/4\sqrt{6})f_0f_1\sin2\lambda]=0; \end{aligned} \tag{44a}$$

$$\begin{aligned} &m_\rho{}^2-M^2-g^2J(m_\rho{}^2;m^2)-\tfrac{2}{3}(f_1{}^2/M^2)\tfrac{1}{3}I(m_\rho{}^2;M^2,m^2)\\ &\quad-(1/3M^2)I(m_\rho{}^2;z_\phi,m^2)[\tfrac{1}{6}f_1{}^2\cos^2\lambda+\tfrac{1}{2}f_0{}^2\sin^2\lambda+(1/2\sqrt{6})f_0f_1\sin2\lambda]\\ &\quad-(1/3M^2)I(m_\rho{}^2;z_\omega,m^2)[\tfrac{1}{6}f_1{}^2\sin^2\lambda+\tfrac{1}{2}f_0{}^2\cos^2\lambda-(1/2\sqrt{6})f_0f_1\sin2\lambda]=0\,. \end{aligned} \tag{44b}$$

$\omega-\phi$ mixing will introduce off-diagonal elements also in the renormalized propagator matrix. To obtain $m_\omega{}^2$ and $m_\phi{}^2$ we have yet to diagonalize this matrix. But first, we use (32) together with

$$J(k^2;m^2)=(1/16\pi^2)\lim_{\Lambda\to\infty}\Lambda^2/6+O(\ln\Lambda/M) \tag{45}$$

to eliminate Λ and m in Eqs. (31a), (31b), and (44a), (44b). In this way we obtain [recall Eq. (34)]

$$m_M{}^2-m_\rho{}^2=-\frac{1}{2}\frac{\frac{5}{6}f_1{}^2+f_0{}^2+6g^2}{(5/3)f_1{}^2+f_0{}^2}(m_K{}^2-m_\pi{}^2)-\frac{3}{8}\frac{f_1{}^2+6g^2}{(5/3)f_1{}^2+f_0{}^2}\xi\,. \tag{46}$$

This is a quantitative extension of the qualitative result $(m_M{}^2-m_\rho{}^2)(m_K{}^2-m_\pi{}^2)>0$ obtained by Sakurai.[5] In order to check Eq. (46) we have to extract relations between the coupling constants from our model. Since they appear in fractions only, the dependence on them is not very critical and approximate information on the coupling constants will be sufficient, see Sec. 6.

Turning to ϕ and ω, we first note that $D_\omega\equiv[-G_\omega]^{-1}$ and $D_\phi\equiv[-G_\phi]^{-1}$ are given by

$$D_\omega(k^2)=k^2-\mu^2-(f_0{}^2/M^2)[(7/6)I(k^2;M^2,m^2)+\tfrac{1}{6}I(k^2;z_\phi,m^2)\cos^2\lambda+\tfrac{1}{6}I(k^2;z_\omega,m^2)\sin^2\lambda]; \tag{47a}$$

$$\begin{aligned} D_\phi(k^2)=&k^2-M^2-g^2J(k^2;m^2)-\tfrac{2}{3}(f_1{}^2/M^2)\tfrac{1}{3}I(k^2;M^2,m^2)\\ &-(1/3M^2)I(k^2;z_\phi,m^2)[\tfrac{1}{6}f_1{}^2\cos^2\lambda+\tfrac{1}{2}f_0{}^2\sin^2\lambda-(1/2\sqrt{6})f_0f_1\sin2\lambda]\\ &-(1/3M^2)I(k^2;z_\omega,m^2)[\tfrac{1}{6}f_1{}^2\sin^2\lambda+\tfrac{1}{2}f_0{}^2\cos^2\lambda+(1/2\sqrt{6})f_0f\sin2\lambda]. \end{aligned} \tag{47b}$$

A typical graph contributing to the off-diagonal element (to lowest order in $m_{\phi\omega}{}^2$) is shown in Fig. 3. The full contribution is given by

$$E(k^2) = -m_{\phi\omega}^2 - (f_0^2/M^2)[I(k^2; z_\phi, m^2) - I(k^2; z_\omega, m^2)]\tfrac{1}{12}\sin 2\lambda. \tag{48}$$

m_ϕ^2 and m_ω^2 are the two solutions of the quadratic equation

$$D_\phi(k^2)D_\omega(k^2) - [E(k^2)]^2 = 0. \tag{49}$$

Computing these solutions and eliminating Λ and m in them and Eq. (44) yields the following two equations

$$m_\phi^2 + m_\omega^2 - 2m_M^2 = \frac{3f_0^2}{(5/3)f_1^2 + f_0^2}\Delta + \frac{6g^2}{(5/3)f_1^2 + f_0^2}M^2 + \frac{9}{8}\frac{3f_0^2 + f_1^2}{(5/3)f_1^2 + f_0^2}\xi; \tag{50}$$

$$m_\phi^2 m_\omega^2 = \{\tfrac{1}{9}(2m_M^2 + m_\rho^2) + \tfrac{1}{3}(m_\phi^2 + m_\omega^2 - 2m_M^2) - \tfrac{1}{4}(3f_1^2 + 10f_0^2/(5/3)f_1^2 + f_0^2)\xi\}$$
$$\times\left(4m_M^2 - m_\rho^2 + \frac{3f_1^2}{(5/3)f_1^2 + f_0^2}\xi\right) - \frac{(5/3)f_1^2 + f_0^2}{12f_1^2}\left[(m_M^2 - m_\rho^2) + \frac{3}{4}\frac{f_0^2}{(5/3)f_1^2 + f_0^2}\xi\right]^2, \tag{51}$$

where Δ and ξ are given by Eqs. (34) and (14).

Equation (50) is not really of the type of a mass relation. Rather it gives some information on the bare masses. Equation (51) is the true mass relation for vectons. It should be noted that the GMO relation for vectons has no meaning in a pure $\omega-\phi$ mixing model, because without mixing, all vecton masses degenerate. It is thus inconsistent to try to compute the "unmixed" ϕ mass from the physical ρ and M masses.

Note, that for constant $m_{\phi\omega}^2$, i.e., $\xi=0$, Eq. (51) goes into

$$m_\phi^2 m_\omega^2 = (4m_M^2 - m_\rho^2)\{\tfrac{1}{9}(2m_M^2 + m_\rho^2) + \tfrac{1}{3}(m_\phi^2 + m_\omega^2 - 2m_M^2)\} - \{[(5/3)f_1^2 + f_0^2]/12f_1^2\}(m_M^2 - m_\rho^2)^2, \tag{52}$$

which depends on the coupling constants only through the last term; this, however, is very small because of the small M-ρ mass difference.

6. ON THE COUPLING CONSTANTS

Up to now, we have derived our results completely within the framework of the model. We note, however, that all propagators are real in this model. In order to obtain additional information on decay widths and thus coupling constants, we simply add the imaginary part of the propagators we could obtain if the intermediate particles carried their physical masses. We will show presently that this is exactly equivalent to correcting for phase space. By definition

$$\mathrm{Im}[-G_M(k^2)]^{-1} = 3g^2\,\mathrm{Im}J(k^2; m_K^2, m_\pi^2) = \tfrac{1}{2}(g^2/k^4)[w(m_K, m_\pi, \sqrt{(k^2)})]^{3/2}, \tag{53a}$$

$$\mathrm{Im}[-G_\rho(k^2)]^{-1} = 4g^2\,\mathrm{Im}J(k^2; m_\pi^2) = \tfrac{2}{3}(g^2/\sqrt{(k^2)})[k^2 - 4m_\pi^2]^{3/2}. \tag{53b}$$

Since we shall face an infinite coupling constant renormalization, only ratios of decay width shall have any meaning according to the rules set up at the end of Sec. 2.

Following our standard procedure, we obtain

$$\frac{\Gamma_{\rho^+}}{\Gamma_{M^+}} = \frac{4}{3}\frac{m_\rho}{m_M}\frac{\mathrm{Im}J(m_\rho^2; m_\pi^2)}{\mathrm{Im}J(m_M^2; m_K^2, m_\pi^2)} = \frac{4}{3}\frac{p_\rho^3/m_\rho^2}{p_M^3/m_M^2}, \tag{54}$$

where p_ρ and p_M are the c.m. momenta of their decay

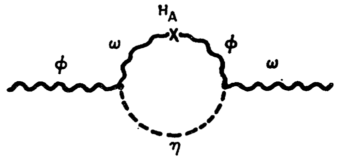

FIG. 3. Off-diagonal contribution to lowest order in H_A.

products. Equation (54) is exactly the result one derives from a phase space correction.[15]

With the masses taken from Ref. 16, we obtain $\Gamma_\rho/\Gamma_M = 3.5$ which is to be compared with the experimental value[16] 2.1 ± 0.9. This gives an estimate of the accuracy of coupling-constant relations.

Because of (49), decay widths of the ϕ will diverge more badly than those of ρ and M and we can hence only compare $\phi\to K+\bar{K}$ with $\phi\to\rho+\pi$. Carrying out the algebra yields

$$\frac{\Gamma_{\phi\to\rho\pi}}{\Gamma_{\phi\to K\bar{K}}} = \frac{f_1^2 - 3f_0^2}{6g^2}\frac{\mathrm{Im}I(m_\phi^2; m_\rho^2, m_\pi^2)}{6\,\mathrm{Im}J(m_\phi^2; m_K^2)}. \tag{55}$$

The fact that a difference of coupling constants appears

[15] J. J. Sakurai, in *Theoretical Physics* (International Atomic Energy Agency, Vienna, 1963), p. 227.

[16] A. M. Rosenfeld, A. Barbaro-Galtieri, W. M. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **36**, 977 (1964).

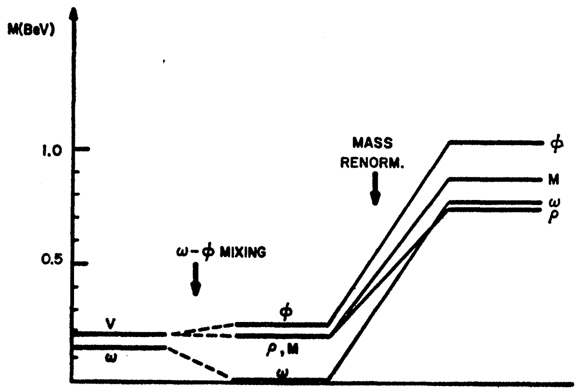

FIG. 4. The energy-level diagram for vectons.

in the numerator of (55) allows us to understand the exceptionally small experimental branching ratio.[16] Though the value seems to increase,[17] the errors are still large and within our approximations it will be good enough to take

$$f_1^2 = 3f_0^2. \tag{56}$$

7. DISCUSSION OF RESULTS

Our results are compiled in Eqs. (33), (46), (50), (51), (52), and (56). Let us first discuss them under the assumption of constant $m_{\phi\omega}^2$, i.e., $\xi=0$. We then obtain the GMO formula for mesons, the validity of which is no longer a mystery. Inserting (56) and the experimental masses[16] of the (uncharged) vectons in Eq. (52) gives (in BeV^4)

$$\text{left-hand side: } 0.637\,, \quad \text{right-hand side: } 0.673\,, \tag{57}$$

which shows about the same deviation as the GMO formula for mesons Eq. (33). This encouraging result is good support for our model. Equation (46) with $\xi=0$ can be used to compute g^2 with the result

$$g^2 = 1.3 f_0^2\,, \tag{58}$$

which is very reasonable. We cannot directly derive the bare mass difference from Eq. (50) since M^2 enters as extra parameter. But with the assumption that Δ should be smaller than M^2, Eq. (50) can be satisfied with the following numbers:

$$M = 200 \text{ MeV}\,, \quad \mu = 150 \text{ MeV}. \tag{59}$$

ω-ϕ mixing will decrease the bare mass of the ω and it turns out that the angle λ, for which the bare ω mass vanishes, is just 37°. If this angle has anything to do with the one derived in first-order perturbation theory,[5] we can explain the latter within our model through "maximal ω-ϕ mixing." Note, however, that within our model, λ is undetermined and only its maximum is set at 37°. The energy level diagram for vectons (assuming maximal mixing) is shown in Fig. 4. Notice that the fact that the physical vecton masses are larger than the bare masses does not contradict general inequalities derived from the Lehmann representation.[18]

Taking $\xi \neq 0$, which means including a small momentum dependence of $m_{\phi\omega}^2$, we can compute its value from Eq. (51) and, selecting the smaller root, we find

$$\xi = 0.021 \text{ BeV}^2. \tag{60}$$

Inserting Eqs. (58) and (60) in (33) allows us to compute the correction to the GMO meson formula. It has the right sign but is about six times too small to account for the full difference between GMO prediction and experiment. We thus conclude, that graphs neglected in our model make a contribution of the order of a few percent to mass renormalization.

[17] S. Lichtman, M. Goldberg, T. Kikuchi, J. Leitner, M. Primer, E. L. Hart, V. W. Lai, G. W. London, N. P. Samios, and S. S. Yamamoto, Bull. Am. Phys. Soc. **10**, 66 (1965).

[18] K. Johnson, Nucl. Phys. **25**, 435 (1961).