Bootstrap Model of Meson and Baryon Supermultiplets

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The $SU(6)$-symmetric bootstrap model of the baryon supermultiplet is discussed and extended. The relation between the static limit of the model and relativistic interactions is discussed. A hypothesis is made concerning the meson-meson-meson interactions. The V_8 (vector octet) exchange forces in the V_8+V_8 states may lead to the bootstrapping of the X_0 meson in this model. The bootstrapping of the other mesons is discussed. It is postulated that the lightest odd-parity baryon isobars are produced by meson exchange forces in meson-baryon S states. The lightest odd-parity resonances predicted from this postulate are a spin-$\frac{1}{2}$ singlet and a spin-$\frac{3}{2}$ octet. These may be identified with the 1405-MeV Y_0^* and the "$D_{3/2}$" baryon-meson resonance octet. Recent data concerning the various decay amplitudes of the $\frac{3}{2}^-$ resonances are compared with the predictions of $SU(3)$ symmetry.

I. INTRODUCTION

IN a recent letter, it was shown that a baryon bootstrap model based on approximate $SU(6)$ invariance is self-consistent, and is simpler than previous baryon bootstrap models in the sense that only one baryon multiplet is involved.[1] The purpose of the present paper is to clarify and extend this model. In Sec. II, a simple method for calculating the ratios of the interaction constants of the model is outlined, and the relation between the static limit of the model and relativistic meson-baryon-baryon interactions is discussed. A simple hypothesis concerning the nature of the meson-meson-meson interaction is made in Sec. III. The manner in which the various mesons may be bootstrapped is discussed. The proposed meson-meson-meson interaction is not $SU(6)$-invariant, but is suggested by the way that the $SU(6)$ symmetry of the meson-baryon-baryon interactions is broken. In Sec. IVA the meson exchange forces in S-wave meson baryon states is investigated. It is predicted that the lightest odd-parity resonances are a spin-$\frac{1}{2}$ singlet and a spin-$\frac{3}{2}$ octet. Section IVB contains a discussion of the consistency of the measured decay amplitudes of the $\frac{3}{2}^-$ particles and the $SU(3)$ octet assignments.

II. THE MODEL

In the $SU(6)$-symmetric bootstrap model, each spin state of the ground-state baryon octet and the $J^P=\frac{3}{2}^+$ baryon decuplet is associated with bound state and resonance poles in scattering amplitudes connecting the meson-baryon states of the appropriate quantum numbers. The mesons involved are the pseudoscalar and vector (P and V) octets, and an $SU(3)$ singlet, which may be either a pseudoscalar or a vector particle. For the reasons given in Ref. 1, we choose the singlet to be pseudoscalar, and indentify it with the 960-MeV X_0 particle. The mesons are emitted and absorbed in P states. The $SU(6)$ invariance may be made exact only in the static limit, so we discuss this limit and assume that the full baryon (B) and meson (μ) multiplets are separately degenerate. The interactions of the P-wave P mesons transform under space rotations in the same manner as those of S-wave axial-vector mesons. It is assumed that the P-wave V mesons are emitted and absorbed in states of total angular momentum zero; i.e., the V interactions transform under space rotations as those of scalar mesons in S states. The effective axial-vector and scalar mesons are assumed to interact as the 35-fold (regular) representation of $SU(6)$.

The dependence of the meson-baryon wave functions on the various meson-baryon channels may be written,

$$\psi(B_i)=\sum_{jk} C(ijk)B_j\mu_k\,, \tag{1}$$

where $C(ijk)$ are Clebsch-Gordan coefficients of $SU(6)$. We describe below a simple method of determining these coefficients. An advantage of this method is that it illustrates the unusual manner in which the $SU(6)$-invariance is broken by the baryon mass splitting, and suggests an hypothesis concerning the $\mu\mu\mu$ interactions.

The baryons correspond to the 56-fold, completely symmetric representation formed from the cube of the fundamental representation. It is convenient to think of each baryon as formed from three fundamental "quarks," although it is assumed here that no physical quarks exist. We use the numbers 1, 3, and 5 to refer to the spin-up states of quarks of hypercharge and I_z equal to $(\frac{1}{3},\frac{1}{2})$, $(\frac{1}{3},-\frac{1}{2})$ and $(-\frac{2}{3},0)$, respectively. The even integers 2, 4, and 6 refer to the corresponding states of $J_z=-\frac{1}{2}$. The quark wave functions χ for the baryon states of $J_z=\pm\frac{1}{2}$, $Y=0$, and $I_z=0$ are then[2]

$$\begin{aligned}
\chi(Y^*_{0,1/2})&=(\tfrac{1}{3})^{1/2}[(136)+(145)+(235)]\,,\\
\chi(Y^*_{0,-1/2})&=(\tfrac{1}{3})^{1/2}[(245)+(236)+(146)]\,,\\
\chi(\Sigma_{0,1/2})&=(\tfrac{1}{6})^{1/2}[2(136)-(145)-(235)]\,,\\
\chi(\Sigma_{0,-1/2})&=(\tfrac{1}{6})^{1/2}[2(245)-(236)-(146)]\,,\\
\chi(\Lambda_{0,1/2})&=(\tfrac{1}{2})^{1/2}[(145)-(235)]\,,\\
\chi(\Lambda_{0,-1/2})&=(\tfrac{1}{2})^{1/2}[(236)-(146)]\,,
\end{aligned} \tag{2}$$

[2] The relative phases assumed here between different members of the same spin and $SU(3)$ multiplets differ from the conventional phases. However, if one uses Eqs. (2) and (3) to compute only "diagonal" coefficients $C(iij)$, and then uses $SU(2)$ and $SU(3)$ coefficients in order to determine the other C's, the phase convention of Eqs. (2) and (3) is irrelevant.

* Supported in part by the National Science Foundation.
[1] R. H. Capps, Phys. Rev. Letters 14, 31 (1965); see also J. G. Belinfante and R. E. Cutkosky, *ibid.* 14, 33 (1965).

TABLE I. Eigenvalues of the states of the fundamental representation for the five diagonal generators of $SU(6)$.

Quark	X_{00}	ρ_0	φ_0	π_{00}	η_{00}
1	α	β	γ	β	γ
2	$-\alpha$	β	γ	$-\beta$	$-\gamma$
3	α	$-\beta$	γ	$-\beta$	γ
4	$-\alpha$	$-\beta$	γ	β	$-\gamma$
5	α	0	-2γ	0	-2γ
6	$-\alpha$	0	-2γ	0	2γ

$\alpha=(1/135)^{1/2}$, $\beta=(1/90)^{1/2}$, $\gamma=(1/270)^{1/2}$

where the two subscripts following the baryon symbols denote the values of I_z and J_z, respectively, and (abc) denotes the completely symmetric state formed from the quarks a, b, and c. The corresponding wave functions for all spin states of the particles $N^*_{3/2}$, $N^*_{-3/2}$, and Ω, and the $J_z=\pm\frac{3}{2}$ states of the other decuplet members may be determined immediately, since only one three-quark state is of the appropriate Y, I_z, and J_z for these particles. The wave functions χ for the other baryons (states of $J_z=\pm\frac{1}{2}$ and $I_z=\pm1$ or $\pm\frac{1}{2}$) may be written symbolically in the form,[2]

$$\begin{aligned}\chi(B_{\frac{3}{2}i})&=(\tfrac{1}{3})^{1/2}(aab)_i+(\tfrac{2}{3})^{1/2}(abc)_i\,,\\ \chi(B_{\frac{1}{2}i})&=(\tfrac{2}{3})^{1/2}(aab)_i-(\tfrac{1}{3})^{1/2}(abc)_i\,,\end{aligned}\qquad(3)$$

where $B_{1/2}$ and $B_{3/2}$ denote spin $\frac{1}{2}$ and $\frac{3}{2}$ baryons, i denotes the the values of Y, I_z, and J_z, the symbol $(abc)_i$ denotes the state of quantum numbers i formed from three distinct quarks, and $(aab)_i$ denotes the corresponding state that contains one of the quarks twice.

We now resume with the description of a method for computing the $C(ijk)$ of Eq. (1). The μ_k in this equation may be thought of as 35 operators in the space of the baryons, with matrix elements $C(ijk)$. Since the 35-fold representation is contained only once in the direct product $56\otimes56^*$, these matrix elements must be equal to the corresponding matrix elements of the 35 generators of $SU(6)$. It is convenient to work only with the diagonal generators. These may be identified with the five states of $Y=I_z=J_z=0$ of the meson multiplet, i.e., with φ_0, ρ_0, X_{00}, π_{00}, and η_{00}. The second subscript refers to J_z (the angular momentum being orbital for the P mesons). We use φ_0 to refer to the isoscalar member of the V octet. The eigenvalues of these five generators for the six basic quarks are shown in Table I. The values of α, β, and γ have been chosen so that the eigenvalues for the three-quark states of the representation 56 are numerically equal to the appropriate $BB\mu$ Clebsch-Gordan coefficients.

The quantum numbers associated with the generators φ_0, ρ_0, and X_{00} are Y, I_z, and J_z, respectively. Since these quantum numbers are additive, one can write immediately the Clebsch-Gordan coefficients of Eq. (1) associated with these particles, i.e.,

$$\begin{aligned}C(ij\varphi_0)&=(1/30)^{1/2}\delta_{ij}Y(i)\,,\\ C(ij\rho_0)&=(2/45)^{1/2}\delta_{ij}I_z(i)\,,\\ C(ijX_{00})&=(4/135)^{1/2}\delta_{ij}J_z(i)\,.\end{aligned}\qquad(4)$$

The unusual manner in which $SU(6)$ is broken is illustrated by the fact that the generators η_{00} and π_{00} do not commute with J^2 and F^2, where F is the $SU(3)$ spin. Since the decuplet-octet mass splitting is large, η_{00} and π_{00} are not associated with approximately conserved quantum numbers. However, the simple three-quark states are eigenfunctions of these generators; the eigenvalues are additive and may be determined from Table I. The various Clebsch-Gordan coefficients involving the η_{00} and π_{00} may then be determined from the quark structure of the baryon wave functions. Finally, $SU(2)$ and $SU(3)$ Clebsch-Gordan coefficients may be used to determine those $C(ijk)$ not associated with the diagonal generators.[3]

A similar technique may be used to derive many of the electromagnetic interaction ratios predicted by $SU(6)$. For example, one may define the diagonal generator η', whose eigenvalues for the quarks are proportional to J_zQ, the charge Q being given by $Q=I_z+\frac{1}{2}Y$. The predicted ratios of magnetic moments of the various baryons may be determined from the matrix elements of η'.

The ratio of the residues of the N and N^* poles in the P-wave pion-nucleon scattering amplitudes, $(\gamma_{NN\pi}^2/\gamma_{N^*N\pi}^2)$, is equal to the ratio of the appropriate sums of $C(ijk)^2$ over the spin and isospin states of the π-N system. The $SU(6)$ prediction is $\gamma_{NN\pi}^2/\gamma_{N^*N\pi}^2=25/8$. This leads to a predicted N^* width of about 60 MeV, roughly half the experimental value.[4,5]

One may relate the static-model interaction constants to coefficients of Lorentz-invariant interactions by taking the static limit of the relativistic interactions. We illustrate this procedure for the ρ_0-proton-proton interaction.[6] We assume that the ρ_0pp vertex factor is $F_\rho\gamma\cdot e$, where e is the polarization four-vector of the ρ. The term of highest order in the baryon-meson mass ratio (which dominates in the static limit) is the γ_0 term. Since the V mesons are real, the component e_0 (defined in the center-of-mass system) is related to the polarization three-vector in the meson rest system by the relation $e_0=\mathbf{k}\cdot\mathbf{e}/m$, where m is the meson mass. Since $\mathbf{k}\cdot\mathbf{e}$ corresponds to V mesons of zero total angular momentum, it is seen that the $\gamma\cdot e$ interaction leads to the V interactions assumed in the $SU(6)$ model in the static limit.

[3] For tables of $SU(3)$ Clebsch-Gordan coefficients, see P. McNamee, S. J. Chilton, and Frank Chilton, Rev. Mod. Phys. **36**, 1005 (1964).

[4] F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters **13**, 299 (1964).

[5] M. A. B. Bég and A. Pais, Phys. Rev. **137**, B1514 (1965).

[6] A similar treatment may be applied to the interactions involving the $B_{3/2}$. A convenient formalism for such a treatment is that of W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).

We now compare the predicted values of the strengths of the relativistic $\pi_0 pp$ and $\rho_0 pp$ interactions, represented by the vertices $G\gamma_5$ and $F_\rho \gamma \cdot e$. The ratio R of the $p \to p+\rho_0$ and $p \to p+\pi_0$ probabilities is given in the static limit by

$$R = F_\rho^2 (2M)^2 / G^2 m^2 \left(\int (\mathbf{e}\cdot\mathbf{k})^2 \Big/ \int (\boldsymbol{\sigma}\cdot\mathbf{k})^2 \right), \quad (5)$$

where the integral signs denote integration over solid angle and the sum over spin states. In the $SU(6)$ model, this ratio is given by

$$R = \frac{C^2(p_i p_i \rho_0)}{\sum_j C^2(p_i p_j \pi_{0,i-j})} = \frac{3}{25}, \quad (6)$$

where i and j are spin indices. Since the integral terms of Eq. (5) are equal, the predicted F^2/G^2 ratio is

$$\frac{F_\rho^2}{4\pi} = \frac{3}{25}\left(\frac{G^2}{4\pi}\right)\left(\frac{m}{2M}\right)^2. \quad (7)$$

It is interesting to note that the leading terms in M/m that arise in the relativistic generalization of $SU(6)$ of Bég and Pais and of Sakita and Wali, lead to a value of F_ρ^2 three times larger than that predicted by Eq. (7).[5,7] However, if the first- and second-order correction terms in m/M are included, the F_ρ^2 predicted in the relativistic model is fairly close to the value of Eq. (7).[7]

The constant F_ρ^2 is not known experimentally. However, if the ρ_0 is coupled universally to the I_z current, F_ρ^2 may be determined from the $\rho \to 2\pi$ decay width. If $m_\rho = 763$ MeV and $\Gamma = 106$ MeV, the result is[8]

$$(F_\rho^2/4\pi) = 3m_\rho^2 \Gamma / (m_\rho^2 - 4m_\pi^2)^{3/2} \approx 0.52. \quad (8)$$

If one sets the average V and baryon masses in Eq. (7) equal to 860 and 1150 MeV, and takes $G^2/(4\pi) = 14$, the result is $F_\rho^2/(4\pi) \approx 0.24$. Thus the predicted F_ρ^2/G^2 ratio is roughly a factor of 2 too small, but, as seen from the discussion of the N^* width, this implies that the ratio $F_\rho^2/\Gamma(N^*)$ is about right. Deviations of factors of 2 between predicted and experimental interaction constants cannot be considered serious until the effects of the large mass splittings within the supermultiplets are understood better.

The coupling of the massive φ and ρ_0 mesons to the Y and I_z currents in our model, and the known coupling of massless photons to the charge current, are both of the $\gamma \cdot e$ type, but are somewhat different in effect. Some physicists have worried about the propriety of coupling massive vector mesons to conserved or nearly conserved currents. In view of this, it is interesting that the coupling of the φ and ρ_0 to the Y and I_z currents in the $SU(6)$ model occurs naturally only because these particles are not massless. It may be seen from our discussion of the diagonal generators of $SU(6)$ that V mesons coupled to the currents corresponding to internal quantum numbers must be associated with mesons in states of $J=0$. However, the V mesons could not occur in states of $J=0$ if they were massless.

[7] K. C. Wali (private communication), and B. Sakita and K. C. Wali, Phys. Rev. (to be published).

[8] Our F_ρ^2 is identical to the γ_ρ^2 of M. Gell-Mann, D. Sharp and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962).

III. REMARKS CONCERNING THE MESON BOOTSTRAPS

In a complete bootstrap model the mesons, as well as the baryons, must be considered particle compounds. It is reasonable to suppose that the most important constituent states of the meson wave functions are of the $\mu\mu$ and $B\bar{B}$ types, and that the dominant forces are transmitted by meson exchange. We consider here the question of why the lightest meson states include a pseudoscalar octet and singlet and a vector octet and singlet. $SU(6)$ symmetry of the interactions leading to the meson states is not assumed, for the two reasons listed below. (1) One cannot write the $\mu\mu\mu$ interactions in a form such that the static limit involves $SU(6)$ symmetry in a simple manner analogous to that assumed in Sec. II for the μBB interactions. (2) If the PBB and VBB interactions are related in the manner discussed in Sec. II, the simple $SU(6)$ symmetry is lost for diagrams involving virtual mesons, even in the static limit. This may be seen from the example that V exchange, but not P exchange, contributes to the S-wave $B_{1/2}\bar{B}_{1/2}$ amplitudes at the $B\bar{B}$ threshold.

In order to discuss the bootstrapping of the mesons, we must assume a form for the $\mu\mu\mu$ interactions. One of the attractive features of the $SU(6)$-invariant μBB interaction is the fact that certain of the mesons are coupled to conserved or nearly conserved currents. It was pointed out in Sec. II that the observed octet-decuplet baryon mass splitting implies that the diagonal generators of $SU(6)$ associated with the η and π_0 particles are not coupled to nearly conserved quantum numbers. This suggests the hypothesis that the coupling of the ρ_0, φ, and X_0 to the I_z, Y, and J_z currents, and the other couplings necessitated by $SU(3)$- and $SU(2)$-invariance, are universal, and that these rules describe the dominant $\mu\mu\mu$ interactions. The only novel feature of this hypothesis is the coupling of the X_0 to the "spin current." The singlet vector meson (the ω) is assumed coupled to the baryons as an $SU(6)$ singlet, with an interaction of the type $\gamma \cdot e$. The ω is then coupled to the baryon number.

The above assumptions require that the dominant $\mu\mu\mu$ interactions are of three types—the well-known $P_8P_8V_8$ interaction, the completely antisymmetric $V_8V_8V_8$ interaction considered by Cutkosky[9] and a $V_8V_8P_1$ interaction, (where P_1 is the X_0). The absence of a $V_8V_8P_8$ interaction in this scheme is consistent with the observed small upper limit of the $\varphi \to \pi + \rho$ decay

[9] R. E. Cutkosky, Phys. Rev. **131**, 1888 (1963).

amplitude,[10] and with the approximate conservation of the A quantum number postulated by Bronzan and Low.[11]

We discuss briefly the possible origin of the mesons in this picture. Since the ω meson is coupled to the baryon number current, $B\bar{B}$ states should be dominant for the ω bootstrap. Physically, the ω meson is nearly degenerate with the vector octet and is lighter than the X_0. Thus, for consistency, the forces in the $B\bar{B}$ states should be more attractive in the vector-singlet state than in any other state with the possible exception of the pseudoscalar octet. We have not made detailed calculations of the effects of meson exchange in the $B\bar{B}$ channels. However, Hara has considered such forces in the $B_{1/2}\bar{B}_{1/2}$ states.[12] The results of Hara are that the lightest multiplets are a pseudoscalar octet and vector singlet. Therefore, this preliminary calculation is consistent with our assumptions concerning the ω meson.

We next turn our attention to the P and V octets and the X_0. It is easy to verify that the $SU(3)$ crossing matrix associated with the $P_8P_8V_8$ interaction is such that the exchanges associated with this interaction may be regarded consistently as being responsible for bootstrapping the P and V octets.[13] In addition, the $B\bar{B}$ states may be important for the P octet,[12] and the V_8V_8 states may be important for bootstrapping the V octet.[9]

Finally, we turn to the X_0 bootstrap; we consider only the contribution of $\mu\mu$ states. Our interaction hypothesis is such that these must be of the V_8V_8 type. The forces in these states must come from V_8 and from X_0 exchange. The exchange of the singlet X_0 particle cannot be responsible for separating the singlet state from those of the other $SU(3)$ representations, so we consider V_8 exchange. The $V_8V_8V_8$ interaction is of the totally antisymmetric type discussed in Ref. 9. If the convention is adopted that the three V mesons a, b, and c are all absorbed at the vertex, the interaction may be written,

$$I\sim e_a\cdot e_b(p_a-p_b)\cdot e_c+e_b\cdot e_c(p_b-p_c)\cdot e_a+e_c\cdot e_a(p_c-p_a)\cdot e_b. \quad (9)$$

The symbols p_i and e_i denote the four-momentum and polarization of the meson i.

Since the V masses are all of the same order, we use the full relativistic interaction to calculate the V exchange force (amplitude in Born approximation). Attention is limited to the pseudoscalar VV states, since it is difficult to compare forces in states of different spin or parity when no static approximation is applicable. Pseudoscalar VV states exist only for the symmetric representations of $SU(3)$. A calculation shows that the force is attractive in the $SU(3)$ singlet state. The ratios of forces F_i corresponding to the different symmetric representations of $SU(3)$ are proportional to the appropriate ratios of crossing matrix elements, i.e.,[14]

$$F_1:F_8:F_{27}=6:3:-2. \quad (10)$$

It is seen that our model of the X_0 is consistent. The experimental fact that the 4π decay model of the X_0 is weak cannot be taken as strong evidence against the hypothesis that the $X_0\rho\rho$ interaction is strong, since the X_0 mass is much smaller than twice the ρ mass.

We do not discuss here many of the interesting questions that exist in connection with bootstrapping the mesons. One such question is that of whether or not the assumed universal coupling of the ρ_0 and φ to the I_z and Y currents is consistent with a bootstrap treatment of the coupling of the various two-particle states to the V-meson poles. The main purpose of this section is to show that it is not difficult to construct a bootstrap model in which the lowest meson states are a P singlet and octet and a V singlet and octet. The basic reason that these states are easy to produce can be seen from the crossing matrix elements associated with the various one-particle exchange mechanisms; those elements associated with forces in states of large multiplicity generally are not large.[15] For example, it may be seen from an examination of $SU(3)$ crossing matrices that forces in states of the 27-fold representation, that result from interacting octets and decuplets, are usually small.[14]

IV. BARYON RESONANCES OF ODD PARITY

A. Experimental Predictions of the Model

Several baryon resonances have been observed that are most likely of odd parity. Notable examples are the N^{**}(1512 MeV) and the Y_0^{**}(1520 MeV).[16] The most easily identified decay modes of the $J^P=\frac{3}{2}^-$ resonances are $D_{3/2}$ states of the $PB_{1/2}$ type, since these states are the strongly coupled states with the lowest thresholds. However, the successful application of $SU(3)$ and $SU(6)$ symmetry schemes to nondegenerate multiplets suggests that the position of a channel threshold is not the most important criterion for the importance of the channel to the production of a resonance. In this section we make the simple hypothesis that the S-wave μB channels are dominant for the lightest odd-parity resonances, in the sense that the existence of the resonances may be understood from the forces in these channels. Forces caused by exchange of the baryon supermultiplet are not particularly effective in the odd-parity baryon states, so we consider only meson exchange forces.

[10] P. L. Connolly *et al.*, Phys. Rev. Letters **10**, 371 (1963).

[11] J. B. Bronzan and F. E. Low, Phys. Rev. Letters **12**, 522 (1964).

[12] Yasuo Hara, Phys. Rev. **133**, B1565 (1964).

[13] Such a model of the P and V octets is discussed by R. H. Capps, Phys. Rev. **137**, B125 (1965).

[14] Various crossing matrices involving octets and decuplets are listed by R. E. Cutkosky, Ann. Phys. **23**, 415 (1963), and by D. E. Neville, Phys. Rev. **132**, 844 (1963).

[15] This has been pointed out and discussed by R. H. Capps, Nuovo Cimento **34**, 932 (1964).

[16] A compilation of experimental data concerning baryon resonances is given by A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **36**, 977 (1964).

Meson exchange forces involve $\mu\mu\mu$ interactions, so the predicted results depend on the form assumed for these interactions. In this paper we limit consideration to forces in the $P_8B_{1/2}$ and $P_8B_{3/2}$ states. It is likely that resonances in such states will be among the lightest odd-parity resonances, because the P_8 mesons are the lightest mesons. In this simple model there is no ambiguity associated with the $\mu\mu\mu$ interactions, because PPP and $V_1P_8P_8$ interactions are forbidden by parity conservation and $SU(3)$ symmetry, and the approximate strength of the $V_8P_8P_8$ interaction is known experimentally. $SU(6)$ symmetry need not be assumed in full; the ratios of the constants at all the vertices are given by $SU(3)$ symmetry, together with the condition that the ρ and φ are coupled universally to the I_z and hypercharge currents. The principal new feature of the model is that the $B_{3/2}$ are treated on the same basis as the $B_{1/2}$ baryons. Deviations from degeneracy within each of the P, V, $B_{1/2}$, and $B_{3/2}$ multiplets are neglected.

The determinantal method is used as a basis for predicting the quantum numbers of the states that may resonate. The Born approximation for the amplitudes resulting from V meson exchange is given in the static approximation by the expression,

$$T^B=-\frac{F_BF_P}{4\pi}\frac{2\omega}{m^2+2k^2(1-\cos\theta)}, \qquad (11)$$

where m, ω, and k are the V mass, P energy, and P momentum, and F_B and F_P are the VBB and VPP interaction constants. The S-wave Born amplitudes, defined in terms of the appropriate elements of the unitary S matrix by the equation $T_0{}^B=(S-1)/(2ik)$, are then given by

$$T_0{}^B=-(F_BF_P/4\pi)(\tfrac{1}{2}\omega/k^2)\ln(1+4k^2/m^2). \qquad (12)$$

A positive $T_0{}^B$ in the physical region $k>0$ corresponds to an attractive force. The quantity F_BF_P may be regarded as a matrix in the representation of the different PB states. It is necessary only to find the eigenvalues of this matrix in order to see where resonances may be expected. Since there are no $VB_{1/2}B_{3/2}$ interactions in the static limit of $SU(6)$, $PB_{1/2}$ and $PB_{3/2}$ states are not coupled. [The V-meson interactions are of the electric-type in the $SU(6)$ model used here.] The eigenvalues Λ_i of the F_BF_P matrix may be written $\Lambda_i=\lambda_iF_\rho{}^2$, where $F_\rho{}^2$ is the ρ_0pp interaction constant defined in Sec. II. The constants λ_i, computed from the crossing matrices of Ref. 14, are

$$\lambda_{35,4}=6,\ \lambda_{27,2}=4,\ \lambda_{10,2}=\lambda_{10^*,2}=0,\ \lambda_{27,4}=-2,$$
$$\lambda_{8s,2}=\lambda_{8a,2}=\lambda_{10,4}=-6,\ \lambda_{1,2}=\lambda_{8,4}=-12. \qquad (13)$$

The subscripts refer to the $SU(3)$ representation and the spin multiplicity; $8s$ and $8a$ denote symmetric and antisymmetric octet states.

Since the coefficient of F_BF_P in Eq. (12) is negative, a negative λ_i corresponds to attraction. It is seen that the most likely candidates for odd-parity baryon resonances are the spin $\frac{1}{2}$ singlet and the spin-$\frac{3}{2}$ octet. The $J=\frac{1}{2}$ singlet may be the 1405-MeV $Y_0{}^*$, and seven members of the $J=\frac{3}{2}$ octet may be the N^{**}(1512 MeV), Σ^{**}(1660 MeV), and Ξ^{**}(1810 MeV). We will discuss later the possible identity of the isoscalar member of this octet. Since the values of λ_i are the same for these two multiplets, one would expect the difference between the mass of the $\frac{1}{2}^-$ singlet and the average mass of the $\frac{3}{2}^-$ octet to be equal approximately to that between the average masses of the $\frac{1}{2}^+$ octet and the $\frac{3}{2}^+$ decuplet. This is in accordance with the experimental masses. The assumption that the $PB_{3/2}$ states are crucial for the existence of the $\frac{3}{2}^-$ resonances is consistent with the observation of strong $\Xi^{**}\to\pi\Xi^*$ and $N^{**}\to\pi N^*$ decay modes.[17,18]

There is no reason to believe that the exchange forces considered here are the only important forces. Brehm and Freedman have shown that inelastic amplitudes of the type $VB_{1/2}\to PB_{1/2}$ may be important for the $\frac{3}{2}^-$ resonances.[19] Martin and Wali have predicted the existence of the $Y_0{}^*$ from a calculation involving both meson and baryon exchange forces.[20] They have estimated that the $VB_{1/2}B_{1/2}$ magnetic interaction leads to a repulsion that cancels partially the effect of the electric interaction on the V exchange contribution to the $Y_0{}^*$. There is no way to estimate the relative strengths of these forces accurately; for example, the dispersion integral associated with the magnetic interaction diverges in the static model. Our main reason for considering only the one mechanism in the static limit is simplicity.

We use the determinantal method to make a rough estimate of the magnitude of the V interaction necessary to produce resonances near threshold in our model. The dispersion relation for the denominator function corresponding to the eigenstate i of the scattering is

$$D_i(\omega)=1-\frac{\omega-\omega_s}{\pi}\int_0^\infty\frac{d\omega'k'N_i(\omega')}{(\omega'-\omega)(\omega'-\omega_s)}, \qquad (14)$$

where ω_s is a subtraction energy, and the numerator N_i is to be set equal to the Born-approximation amplitude. Rather arbitrarily, we choose $\omega_s=0$, (the value $\omega^2=\mu^2-\frac{1}{4}m^2$ corresponds to the end of the left-hand cut.) If the factor ω in the term $(\omega'-\omega)^{-1}$ is neglected, evaluation of the integral leads to the result,

$$D_i(\omega)=1+(\lambda_iF_\rho{}^2/4\pi)(\tfrac{1}{2}\omega/\mu)\ln(1+2\mu/m). \qquad (15)$$

[17] Some recent data concerning the Ξ^{**} is given by G. A. Smith, J. S. Lindsey, J. Button-Shafer, and J. J. Murray, Phys. Rev. Letters **14**, 25 (1965). See also Ref. 16.

[18] Evidence for $N^{**}\to\pi+N^*$ decay is given by J. Kirz, J. Schwartz, and R. D. Tripp, Phys. Rev. **130**, 2481 (1963), and by C. N. Vittitoe, B. R. Riley, W. J. Fickinger, V. P. Kenney, J. G. Mowat, and W. D. Shephard, Phys. Rev. **135**, B232 (1964).

[19] J. J. Brehm, Phys. Rev. **136**, B216 (1964); D. Z. Freedman, Phys. Rev. **134**, B652 (1964).

[20] A. W. Martin and K. C. Wali, Nuovo Cimento **31**, 1324 (1964).

If (μ/m) is set equal to $\frac{1}{2}$, it is seen from Eq. (15) that the value of $F_\rho^2/(4\pi)$ that corresponds to resonances right at threshold in the (1,2) and (8,4) states is about 0.25. This is comparable to the range of values predicted by the $SU(6)$ model in Sec. II; we conclude that the V exchange force may be strong enough to produce odd-parity resonances in the neighborhood of the various $PB_{3/2}$ thresholds.

We next consider the question of whether or not the 1520 MeV Y_0^{**} may be the isoscalar member of the $\frac{3}{2}^-$ octet. A strong argument against this assignment is that the mass predicted for the isoscalar resonance from the Gell-Mann–Okubo formula and the other octet assignments is about 1660 MeV. We wish to point out, however, that there exists a simple dynamical mechanism that may lead to violations of the mass formula for S-wave resonances that occur in the neighborhood of the thresholds of the strongly coupled channels. At an energy appreciably above the threshold of a particular channel, there is a large negative contribution to a dispersion integral such as that of Eq. (14), resulting from the rapid rise of the S-wave phase space factor above threshold. For this reason, a particular channel loses much of its effectiveness in producing an S-wave resonance, if the resonance is appreciably above threshold. [When deviations from degeneracy are included, the dispersion relations may not be replaced by simple one-channel relations, as was done in Eqs. (11) through (15).]

In order to find the possible effect of this mechanism on the $\frac{3}{2}^-$ resonances, we write the wave functions ψ that relate these particles to $PB_{3/2}$ states,[3]

$$\begin{aligned}
\psi(\Xi^{**}) &= (\tfrac{1}{5})^{1/2}(\Xi^*\pi)+(\tfrac{1}{5})^{1/2}(\Xi^*\eta) \\
&\quad +(\tfrac{1}{5})^{1/2}(Y^*\bar{K})+(\tfrac{2}{5})^{1/2}(\Omega K)\,, \\
\psi(\Sigma^{**}) &= (2/15)^{1/2}(Y^*\pi)+(\tfrac{1}{5})^{1/2}(Y^*\eta) \\
&\quad +(8/15)^{1/2}(N^*\bar{K})+(2/15)^{1/2}(\Xi^*K)\,, \\
\psi(\Lambda^{**}) &= (\tfrac{3}{5})^{1/2}(Y^*\pi)+(\tfrac{2}{5})^{1/2}(\Xi^*K)\,, \\
\psi(N^{**}) &= (\tfrac{4}{5})^{1/2}(N^*\pi)+(\tfrac{1}{5})^{1/2}(Y^*K)\,.
\end{aligned} \tag{16}$$

If the Y_0^{**}(1520 MeV) is an octet member, its rest mass is not appreciably greater than that of any of the coupled $PB_{3/2}$ channels. On the other hand, the masses of the other $\frac{3}{2}^-$ particles are greater than those of the coupled pion-$B_{3/2}$ channels. Furthermore, the $SU(3)$ coefficients of Eq. (16) are such that the N^{**} is much more strongly coupled to the pionic channel than are the Σ^{**} and Ξ^{**}. Thus, one would expect the threshold mechanism to contribute a positive effect to the N^{**}, Σ^{**}, and Ξ^{**} masses that might be predicted from a dynamical calculation, the effect on the N^{**} mass being much the largest. This is just the type of effect needed to account for the deviation of the masses of the four $\frac{3}{2}^-$ resonances from the general ordering found in the $B_{1/2}$ octet, an ordering that does satisfy the mass formula.

On the other hand, Martin has shown that the experimental decay amplitudes of the $\frac{3}{2}^-$ particles favor an $SU(3)$ singlet assignment for the Y_0^{**}.[21] This point is discussed further in the next section.

We do not consider here the possible existence of heavier members of the odd-parity baryon supermultiplet. It has been proposed that the Y_0^*(1405 MeV) and $J^P=\frac{3}{2}^-$ octet are members of a 70-fold $SU(6)$ supermultiplet.[22] It is amusing to note that if the 1520-MeV Y_0^* is an $SU(3)$ singlet, and if it is part of a completely realized $SU(6)$ multiplet that is coupled to the μB states in a manner allowed by $SU(6)$-invariance, then 1130 other baryon states are predicted. This follows because the representation 1134 is the only $SU(6)$ representation in the direct product $35\otimes 56$ that contains a spin $\frac{3}{2}$ singlet.[23] This argument should not be taken very seriously since, as mentioned in Sec. III, many of the forces that may be important for producing the baryon resonances are not $SU(6)$-symmetric.

B. Decay Amplitudes of the $\frac{3}{2}^-$ Resonances

In this section we examine the experimental data concerning the various decay widths of the B^{**} resonances. These data do not test the dynamical model of Sec. IVA, but do test $SU(3)$ symmetry and the octet character of these particles. Since the $B^{**}\to PB_{3/2}$ decays may occur in S states, we assume than an appropriate phase space factor for such a decay is $(M^{**}-M^*-\mu)^{1/2}$, where M^{**}, M^*, and μ are the masses of the B^{**}, $B_{3/2}$, and P particles involved. The phase space factors for the $N^{**}\to\pi N^*$, $\Sigma^{**}\to\pi Y^*$, and $\Xi^{**}\to\pi\Xi^*$ decays differ from one another by less than 13%. From these factors, and the Clebsch-Gordan coefficients of Eq. (16), one finds the following predictions for the ratios of partial widths,

$$\Gamma(\pi N^*):\Gamma(\pi Y^*):\Gamma(\pi\Xi^*)\approx 36:6:10. \tag{17}$$

The experimental data are not yet sufficiently accurate for a test of Eq. (17).[16] However, the large predicted $N^{**}\to\pi+N^*$ width is consistent with the measurements of Ref. 18, and with the estimate of Auvil *et al.*, that the N^{**} decays into $N+\pi+\pi$ states approximately 30% of the time.[24]

We next consider the $PB_{1/2}$ decays. The experimental data have changed greatly since the analyses of Glashow and Rosenfeld,[25] and of Martin[21]; in particular, the measurements of Huwe have led to revision of the Σ^{**} branching ratios.[26] The various decay amplitudes are linear functions of the two $B^{**}PB$ coupling constants F

[21] A. W. Martin, Nuovo Cimento **32**, 1645 (1964).

[22] A. Pais, Phys. Rev. Letters **13**, 175 (1964); I. P. Gyuk and S. F. Tuan, Phys. Rev. Letters **14**, 121 (1965).

[23] The $SU(2)$ and $SU(3)$ structures of many of the representations of $SU(6)$ are listed by C. R. Hagen and A. J. Macfarlane, Phys. Rev. **135**, B432 (1964).

[24] P. Auvil, A. Donnachie, A. T. Lea, and C. Lovelace, Phys. Letters **12**, 76 (1964).

[25] S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963).

[26] Darrell O. Huwe, University of California Lawrence Radiation Laboratory Report LRL-11 291 (unpublished).

and D. [The ratio F/D is equal to the ratio $f'/(1-f')$ defined in Ref. (21).] The squares of the amplitudes are listed in Table II, together with the decay momenta of the various modes. For these considerations, we assume the Y_0^{**}(1520 MeV) to be the octet member Λ^{**}.

One does not know the most appropriate form for the D-state phase-space factors, except at energies close to threshold, where the factors are proportional to k^5. Fortunately, the momenta involved in the various decay modes of the same B^{**} resonance are similar. We use only these branching ratios for estimating the F/D ratio. The experimental branching ratios for the $PB_{1/2}$ decay modes of the Σ^{**} and Λ^{**} are listed in column 2 of Table III. The ratios of column 3 are corrected by simple k^5 phase space factors. Columns 4 and 5 contain the two solutions for $\theta'=\tan^{-1}[(9/5)^{1/2}(F/D)]$ that correspond to the corrected ratios. It is seen that the Y_0^{**} branching ratio is consistent with the octet assignment, and that there are two rough fits to the data, $\theta'\approx 40°$ and $\theta'\approx -70°$. The positive solution is close to that obtained from the earlier data.[21,25]

The two solutions may be tested with the Ξ^{**} decay data. There is considerable uncertaintly concerning the $\pi\Xi$ mode, but the $\bar{K}\Sigma/\bar{K}\Lambda$ ratio appears to be quite small.[16,17] If a k^5 phase-space factor is used, the predicted branching ratios corresponding to the positive and negative solutions for θ' are

$$(\bar{K}\Sigma/\bar{K}\Lambda)>1.8\,, \quad \text{if} \quad 28°<\theta'<60°\,,$$
$$(\bar{K}\Sigma/\bar{K}\Lambda)<0.18\,, \quad \text{if} \quad -81°<\theta'<-55°\,.$$

The data favor the negative value of θ'. This conclusion would be strengthened if a more gentle phase space factor were used. The solution $\theta'\approx -70°$ is in disagreement with the value of $\approx 45°$ that follows from the model of Freedman and Brehm.[19]

TABLE II. Squares of $SU(3)$ amplitudes, and momenta for the various $B^{**}\to PB_{1/2}$ decays.

Decay	Probability	k (MeV/c)
$N^{**}\to\pi N$	$(D+F)^2$	454
$\Sigma^{**}\to\bar{K}N$	$\frac{2}{3}(D-F)^2$	406
$\Sigma^{**}\to\pi\Lambda$	$4/9\,D^2$	439
$\Sigma^{**}\to\pi\Sigma$	$8/3\,F^2$	383
$\Xi^{**}\to\pi\Xi$	$(D-F)^2$	406
$\Xi^{**}\to\bar{K}\Lambda$	$(\frac{1}{3}D-F)^2$	386
$\Xi^{**}\to\bar{K}\Sigma$	$(D+F)^2$	307
$\Lambda^{**}\to\pi\Sigma$	$\frac{4}{3}D^2$	266
$\Lambda^{**}\to\bar{K}N$	$2(\frac{1}{3}D+F)^2$	243

TABLE III. Interaction angle θ' resulting from experimental Σ^{**} and Λ^{**} branching ratios.

1. Branching ratio	2. Measured value	3. Corrected value	4. Solution one	5. Solution two
$\pi\Lambda/\pi\Sigma$ (Σ^{**})	0.19	0.095	60°	−60°
$\bar{K}N/\pi\Sigma$ (Σ^{**})	0.50	0.37	31°	−81°
$\bar{K}N/\pi\Sigma$ (Λ^{**})	0.53	0.82	28°	−55°

Finally we turn to the comparison of the decay widths for the different resonances. It is seen from Table II that if the appropriate modes are chosen, predictions may be made that are independent of θ'. These predictions are

$$\Gamma(\Xi^{**}\pi\Xi)/\Gamma(\Sigma^{**}\bar{K}N)=\tfrac{3}{2}\rho_1\,, \tag{18}$$

$$\Gamma(N^{**}\pi N)/\Gamma(\Xi^{**}\bar{K}\Sigma)=\rho_2\,, \tag{19}$$

$$\Gamma(\Sigma^{**}\pi\Lambda)/\Gamma(\Lambda^{**}\pi\Sigma)=\tfrac{1}{3}\rho_3\,, \tag{20}$$

where the ρ_i are phase space ratios. The $\Xi^{**}\pi\Xi$ and $\Sigma^{**}\bar{K}N$ decay momenta are almost exactly the same, so that ρ_1 is approximately one and Eq. (18) leads to a simple prediction. If the phase space factor for a decay is taken to be k^5/W^2 (essentially the factor used in Ref. 21), then both ρ_2 and ρ_3 are approximately 10. Therefore, Eq. (19) is consistent with experiment, despite the fact that only an upper limit exists for $\Gamma(\bar{K}\Sigma)$.[17] Experimentally, $\Gamma(\Lambda^{**}\pi\Sigma)$ and $3\Gamma(\Sigma^{**}\pi\Lambda)$ are comparable; hence, the Y_0^{**} decay amplitudes are too large for the octet assignment. Martin has pointed out this fact previously, and has suggested that the Y_0^{**} is a unitary singlet.[21] This argument is certainly reasonable, but since it depends on the strong energy dependence of the phase-space factors, the unitary spin of the 1520-MeV Y_0^{**} must be regarded as undetermined.

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