π^- Capture by He^3 and the Two-Nucleon Capture Model*

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The rates of capture of π^- mesons by He^3 leading to the final states $d+n$ and $p+n+n$ are calculated using a phenomenological two-nucleon Hamiltonian for the capture interaction $\pi NN \rightarrow NN$, obtained previously by Eckstein from an analysis of the experimental data on the inverse process of one-pion production in nucleon-nucleon collisions. Both S- and P-orbit captures are considered and it is shown that P-orbit capture does not compete with the $P \rightarrow S$ radiative transition. The radiative capture rate W_γ (for final state $H^3+\gamma$) is computed in terms of the photoproduction amplitude of Chew, Goldberger, Low, and Nambu and is used to deduce the nucleonic absorption rate $W_{abs}=W_d+W_p$ from the ratio W_{abs}/W_γ measured experimentally. The agreement between calculation and experiment for W_{abs} is satisfactory. Our conclusion is that the form of close pair correlations in nuclei is essentially the same as that for the "free" nucleon-nucleon interaction and is comparatively insensitive to the presence of other nucleons.

1. INTRODUCTION

THERE has recently been a renewed interest, both experimental[1–6] and theoretical,[7–12] in the reactions resulting from the capture of stopped π^- mesons by nuclei, motivated partly by the hope that from the study of such quantities as branching ratios, momentum spectra and angular correlations of ejected particles, one may get reliable information on properties like nucleon momentum distributions and short-range pair correlations within the nucleus. However, a clear understanding of the π^--capture mechanism is a prerequisite to any attempt to interpret data on capture reactions in terms of nuclear properties. We do know that the basic πNN interaction $\pi+N \leftrightarrows N$ has the form $G\bar{\psi}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\phi}\psi$ and, *in principle*, it is possible to calculate the matrix element for the nuclear capture process from this interaction just as it is possible to calculate the nucleon-nucleon force. But such a program has not yet been carried through successfully even for the N-N problem. In the case of the capture process, momentum and energy conservation alone requires that at least two nucleons should be involved; the mutual interactions of the nucleons inside the nucleus are therefore of great importance. Obviously, the pion physics involved in any attempt to calculate the capture matrix element from first principles is very complicated. Further, because the entire rest mass of the pion is converted into the kinetic energy of the two nucleons, even if we were to describe the nucleon-nucleon interaction by a phenomenological wave function, it is the wave function for very small separations which is most important. We do not at present have an accurate idea of the wave function for a pair of nucleons very close together in a nucleus.

An alternative way of approaching the pion-capture problem was indicated sometime ago by Brueckner, Serber, and Watson.[13] On the basis of the fact that the capture intimately involves two nucleons (at least), they assume that these two nucleons are to be treated on an equal footing and that the nuclear capture proceeds through the basic reactions

$$\begin{aligned}\pi^-+n+p &\rightarrow n+n\,,\\ \pi^-+p+p &\rightarrow p+n\,.\end{aligned} \tag{1.1}$$

Capture by clusters of three or more particles is neglected on the grounds that the probability of finding more than two particles very close together is much smaller than the corresponding probability for two particles. Thus, the capture process is considered to be

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12 R. I. Jibuti and T. I. Kopaleishvili, Nucl. Phys. **55**, 337 (1964). References to earlier work will be found in Refs. 7–12.

13 K. A. Brueckner, R. Serber, and K. M. Watson, Phys. Rev. **84**, 258 (1951).

the exact inverse of one-pion production in N-N collisions. The matrix element for capture by a complex nucleus is then expressible in terms of the matrix elements for capture by a two-nucleon system or, equivalently, in terms of the pion-production matrix elements. The justification for this procedure is that the wave function for a closely correlated pair of nucleons, which is, as we have seen above, most important in the capture problem, is expected to be largely independent of the presence of neighboring nucleons, essentially because the average interparticle separation in nuclei is considerably larger than the separation of the two correlated capturing nucleons. With this picture, the effect of the other nucleons is felt only through the long-range nuclear wave function.

Although this picture (generally called the "two-nucleon capture model") is in qualitative accord with the experimental data on capture by complex nuclei,[4–6,14] there has been only one strictly quantitative test so far. This depends on Eckstein's calculation[7] of the rates of the three absorption modes for π^- capture in He^4 (the final states being H^3+n, $d+2n$, and $p+3n$). The only available experimental information bearing on this calculation is the ratio of the triton rate to the total capture rate.[1,3] Eckstein uses a general phenomenological Hamiltonian appropriate to the two-nucleon model [the same as used in the present calculation, Eqs. (2.1) and (2.2)], and gets two possible values for this quantity, depending on the relative phase (known, independently, only to be either 0° or 180°) of the two amplitudes occurring in the matrix elements. The choice of 0° for this phase gives agreement with the large triton rate found experimentally (the other choice, 180°, leads to a negligible triton rate). It has, however, been argued[10] that branching ratios of two rates, both of which are sensitive to the presence of correlations, are not necessarily a good check on the validity of the model, since the effect of correlations may, conceivably, cancel out from the two rates compared. A comparison with experiment of the absolute capture rates predicted by the two-nucleon model is therefore desirable.

The aim of this calculation is to do this for the nucleus He^3, since in this case there exist experimental data[2] bearing on the total capture rates. The reactions resulting from π^- capture in He^3 are

$$\pi^-+He^3 \to d+n \tag{1.2}$$

$$\to p+n+n \tag{1.3}$$

$$\to H^3+\gamma \tag{1.4}$$

$$\to H^3+\pi^0 \tag{1.5}$$

$$\to d+n+\gamma \tag{1.6}$$

$$\to p+n+n+\gamma. \tag{1.7}$$

[14] N. Metropolis, R. Bivins, M. Storm, J. M. Miller, G. Friedlander, and A. Turkevitch, Phys. Rev. **110**, 204 (1958).

Of these, the pure absorption processes (1.2) and (1.3) are what we are interested in. The other four reactions can all proceed through basic processes which involve only one nucleon:

$$\pi^-+p \to n+\gamma, \tag{1.8}$$

$$\pi^-+p \to n+\pi^0, \tag{1.9}$$

and can therefore be reliably calculated independently of the validity of the two-nucleon model. In the diffusion-chamber experiment of Falomkin *et al.*,[2] which provides us with the data for comparison, the number of π^- captures resulting in the final states $H^3+\gamma$ and $H^3+\pi^0$ are separately counted. Neglecting the contribution of the rare modes (1.6) and (1.7) to the total capture rate, and knowing the total number of capture stars, we may then obtain the ratio of the total pure absorption rate to, say, the rate for the radiative capture (1.4). From the remarks above regarding the nature of the radiative capture process, it is clear that this ratio W_{abs}/W_γ provides just as good a test of the two-nucleon model as would the total absorption rate W_{abs}.

We may note that the agreement of the measured[2] value of the Panofsky ratio in He^3, $P_{He^3}=W_{\pi^0}/W_\gamma$, with earlier calculations[15,16] is of no relevance to the question considered in this paper. The fact that charge exchange and radiative capture can be adequately described by one-nucleon Hamiltonians may indeed be interpreted to mean that capture reactions do proceed dominantly through basic processes which involve the minimum number of nucleons necessary to provide energy and momentum balance, and gives another reason for neglecting capture by three (or more) correlated particles in the pure absorption modes.

The main sections of this paper are devoted to calculations of the nucleonic absorption rates and the radiative capture rate. The method of calculating the absorption rate is a straightforward adaptation of the methods used by Eckstein.[7] Since the effects of close correlations are assumed to be the same for π^- capture in He^3 as for the processes (1.1) or their inverses, and since the rates are expressed in terms of the *measured* pion-production matrix elements, smooth wave functions are used which do not describe correctly the short-range correlations but which are chosen to give the *long-range* behavior accurately (by fitting the radius of He^3). The radiative-capture matrix element is expressed in terms of the matrix element for the basic photoproduction reaction

$$\gamma+p \to \pi^++n \tag{1.10}$$

at threshold, which is obtained from the dispersion relations of Chew, Goldberger, Low, and Nambu.[17]

In calculating these rates it is, of course, necessary to

[15] A. M. L. Messiah, Phys. Rev. **87**, 639 (1952).

[16] B. V. Struminsky, in *Proceedings of the 1962 International Conference on High Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 17.

[17] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

know the particular Bohr orbit from which the nuclear interaction of the meson takes place. In liquid *hydrogen*, the capture of both π^- and K^- mesons is now known to occur from S orbits of large principal quantum number through a mechanism the nature of which was elucidated by Day, Snow, and Sucher[18]: The neutral π^--p (or K^--p) atom experiences the strong electric field of the protons of neighboring hydrogen atoms, and the resultant Stark mixing of the orbital angular momentum causes preferential capture from the $l=0$ state because of the greater overlap of the $l=0$ wave function with the nuclear wave function. Calculations[19–21] of the cascade time of pions in liquid hydrogen made on the basis of this picture are in agreement with experiment.[22,23] In the case of capture by *helium*, however, estimates of the cascade time ($\approx 10^{-15}$ sec for $n=16$) based on analogous arguments[24–26] are orders of magnitude smaller than the experimentally observed value,[3,27] which is about 3×10^{-10} sec. The experimental number is in fact what one would expect[27] if the de-excitation proceeded through external Auger and radiative transitions. It is therefore reasonable to conclude that the meson reaches the lowest atomic orbits before nuclear capture takes place.[28] The proportion of captures which occurs from the $1S$ orbit is then determined by the competition between $2P\rightarrow 1S$ radiative transitions and direct nuclear capture from the $2P$ level. In the case of π^- capture in He^3, we have estimated (Sec. 4) the capture rate from the $2P$ orbit and find that it is about 4% of $W_{\rm rad}(2P\rightarrow 1S)$. Accordingly, effects of capture from the $2P$ orbit are neglected in our discussion.

[18] T. B. Day, G. A. Snow, and J. Sucher, Phys. Rev. Letters **3**, 61 (1959).
[19] J. Russel and G. Shaw, Phys. Rev. Letters **4**, 369 (1960).
[20] T. B. Day, G. A. Snow, and J. Sucher, Phys. Rev. **118**, 864 (1960).
[21] M. Leon and H. A. Bethe, Phys. Rev. **127**, 636 (1962).
[22] T. Fields, G. Yodh, M. Derrick, and J. Fetkovitch, Phys. Rev. Letters **5**, 69 (1960).
[23] J. H. Doede, R. H. Hildebrand, M. H. Israel, and M. R. Pyka, Phys. Rev. **129**, 2808 (1963).
[24] T. B. Day, Nuovo Cimento **18**, 381 (1960).
[25] T. B. Day and G. A. Snow, Phys. Rev. Letters **5**, 112 (1960).
[26] G. A. Snow, in *Proceedings of the 1960 International Conference on High-Energy Physics at Rochester*, edited by E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissinos (Interscience Publishers, Inc., New York, 1960), p. 407.
[27] J. G. Fetkovitch and E. G. Pewitt, Phys. Rev. Letters **11**, 290 (1963).
[28] See, however, a recent paper by G. T. Condo, Phys. Letters **9**, 65 (1964), in which it is pointed out that the external Auger effect and the Stark effect of Day and Snow (Refs. 24–26) are similar processes, and that any cause which inhibits the Stark effect should equally well inhibit the Auger effect. Condo observes further that if a meson is caught initially in a circular orbit, the ejection of the other electron through the internal Auger effect requires $\Delta l\approx 3$. Such transitions cannot compete with radiative dipole transitions, and a considerable fraction of pions originally caught in circular orbits will decay before capture by the nucleus. Condo estimates that if approximately 2% of the pions are caught in circular orbits and the rest absorbed from S orbits via the Stark mixing, the calculated number of pion decays into the backward hemisphere (experimentally, the moderation time is obtained by measuring this number and using the pion lifetime) is in agreement with experiment. If this is indeed the correct mechanism, the considerations of Sec. 4 of this paper are unnecessary.

An earlier calculation of the various capture modes in He^3 was performed by Messiah[15] quite some time ago using an s-wave Hamiltonian of the form $\psi^\dagger\boldsymbol{\sigma}\cdot(\mathbf{p}_i+\mathbf{p}_f)\boldsymbol{\tau}\cdot\boldsymbol{\phi}\psi$ (where $\mathbf{p}_i$ and $\mathbf{p}_f$ are the initial and final nucleon momenta) which arises from the requirement of Galilean invariance on the usual p-wave πNN coupling having the form $\psi^\dagger(\boldsymbol{\sigma}\cdot\mathbf{p}_\pi)(\boldsymbol{\tau}\cdot\boldsymbol{\phi})\psi$. The wave functions used by Messiah do not attempt to describe close correlations of pairs of nucleons. The large deuteron rate (27% of absorption) found by Messiah using the single-nucleon s-wave πNN interaction given above and smooth wave functions means that the triton wave function used corresponds to an average kinetic energy much higher than that given by a wave function fitted to the observed rms radius of He^3. The extreme sensitivity of the absorption rate to the short-range behavior of the wave function can already be seen in the case of π^- capture by deuterons; the matrix element is essentially the Fourier transform of the deuteron wave function for a momentum equal to that of the fast outgoing nucleons, which is about 1 F^{-1}.

So far as the present calculation is concerned, we find that the experimental value of $W_{\rm abs}/W_\gamma$ is in accord with the value calculated here. We obtain $W_{\rm abs}/W_\gamma=8.1\pm3.8$, to be compared with the number 13.0 ± 1.8 obtained from the experiment of Falomkin *et al.*[2]

2. THE TWO-NUCLEON HAMILTONIAN

In contrast to field-theoretic approaches in which one attempts the (exceedingly difficult) task of describing the complex capture process in terms of a basic πNN interaction vertex, the two-nucleon model simply assumes that the capture involves the simultaneous creation and annihilation of a pair of nucleons.[29] The appropriate effective Hamiltonian is therefore of the form

$$H=\int\psi^\dagger(\mathbf{x}_1)\psi^\dagger(\mathbf{x}_2)\mathfrak{M}_{12}\psi(\mathbf{x}_1)\psi(\mathbf{x}_2)d\mathbf{x}_1d\mathbf{x}_2\,,\qquad(2.1)$$

where $\psi^\dagger(\mathbf{x})$ and $\psi(\mathbf{x})$ create and annihilate a nucleon at the point $\mathbf{x}$ and $\mathfrak{M}_{12}$ is a transition operator which involves the coordinates (momenta, spins, and isospins) of both particles (labeled 1 and 2). One important requirement on $\mathfrak{M}_{12}$ is that it should not be a sum of terms each of which involves only one of the labels, since in that case the Hamiltonian (2.1) will reduce effectively to a one-particle operator. Also, for capture from the S orbit (P-orbit capture is discussed in Sec. 4) the nucleon pair must change parity; conservation of angular momentum, along with charge independence, then leads to the following expression as the most general

[29] In addition to the remarks of Brueckner, Serber, and Watson for pion capture, essentially similar remarks were made, at about the same time, by J. S. Levinger, Phys. Rev. **84**, 43 (1951), in discussing a similar problem, the nuclear absorption of high-energy photons. Apart from the atomic physics involved in the capture of stopped pions, the two problems differ basically only in the matrix elements which enter in them.

form of $\mathfrak{M}_{12}$ when the pion is in a relative s wave with respect to the two nucleons:

$$\mathfrak{M}_{12}{}^s=[\tfrac{1}{2}(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2)\cdot\boldsymbol{\phi}(\mathbf{x}_1)\tfrac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{k}(g_0{}^{(t)}T_{12}{}^\tau+g_0{}^{(s)}S_{12}{}^\tau) +\tfrac{1}{2}(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2)\cdot\boldsymbol{\phi}(\mathbf{x}_1)\tfrac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{k} \times(g_1{}^{(t)}T_{12}{}^\sigma+g_1{}^{(s)}S_{12}{}^\sigma)]\delta(\mathbf{x}_1-\mathbf{x}_2). \quad (2.2)$$

Here ϕ is the pion field operator, $\boldsymbol{\tau}_1$, $\boldsymbol{\tau}_2$ and $\boldsymbol{\sigma}_1$, $\boldsymbol{\sigma}_2$ are respectively the isospin and spin operators of the two nucleons, $\mathbf{k}$ is the relative momentum of the final nucleons, $T_{12}{}^\sigma$, $T_{12}{}^\tau$ are the spin and isospin triplet projection operators, respectively, and $S_{12}{}^\sigma$, $S_{12}{}^\tau$ are the corresponding singlet projection operators. Strictly speaking, $\mathfrak{M}_{12}{}^s$ should be a nonlocal expression, i.e., the δ function in Eq. (2.2) should be replaced by a form factor corresponding to a nonzero radius of influence. As a rough estimate, we may take the radius to be given by the interaction distance corresponding to the momentum transfer involved. This is about 0.4 F, to be compared with the average internucleon distance in He^3 of about 2 F. Thus, the use of the δ function is quite a reasonable approximation.

In expression (2.2), four independent amplitudes $g_0{}^{(s)}$, $g_1{}^{(s)}$, $g_0{}^{(t)}$, and $g_1{}^{(t)}$ are needed to describe the s-wave capture process. These are not known *a priori*, nor can they be calculated reliably from first principles; they are to be determined *empirically*. In this calculation we use the values obtained by Eckstein[7] by studying one-pion production in nucleon-nucleon collisions and comparing the results with available experimental data. The important point is that such an approach ensures that the short-range correlations between two nucleons are not neglected; their effect is included in the empirically determined g's, on the basis of the assumptions that these short-range correlations have the same form as for the free NN interaction and that, for the capturing nucleon pair, these short-range correlations are little affected by the presence of neighboring nucleons in the nucleus. With this situation we may then use simple wave functions in our calculations which do not specifically describe the short-range correlations, but give an accurate picture of long-range nuclear properties. It is on this basis that, in this calculation, we use a simple Gaussian wave function for the He^3 nucleus, the falloff parameter of the Gaussian being adjusted to give a nuclear rms radius in agreement with the electron-scattering measurements.

When the initial nuclear state is such that every pair of nucleons is in a spatially symmetric state (which is essentially the case for He^3), the transition operator (2.2) simplifies considerably. In particular, the amplitudes $g_0{}^{(t)}$ and $g_1{}^{(t)}$ are zero, and $\mathfrak{M}_{12}{}^s$ can be rewritten in the form

$$\mathfrak{M}_{12}{}^s=T_{12}{}^\tau T_{12}{}^\sigma[g_0\tfrac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{k}\tfrac{1}{2}(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2)\cdot\boldsymbol{\phi} +g_1\tfrac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{k}\tfrac{1}{2}(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2)\cdot\boldsymbol{\phi}]\delta(\mathbf{x}_1-\mathbf{x}_2), \quad (2.3)$$

where we have dropped the superscripts from $g_0{}^{(s)}$ and $g_1{}^{(s)}$. The amplitudes g_0 and g_1 are, in general, functions of the magnitude of $\mathbf{k}$; this momentum dependence will be assumed to be negligible in this calculation. It is clear from Eq. (2.3) that the amplitude g_0 describes the transition from the $^3S_1$ $(I=0)$ state of the two nucleons to the $^3P_1$ $(I=1)$ state, while the amplitude g_1 describes the transition $(^1S_0, I=1)\rightarrow(^3P_0, I=1)$.

It is clear from our discussion that the criticism of the Hamiltonian (2.1) and (2.2) on the grounds that it "is not a simple sum of one-body operators[10]" is not relevant. On the other hand, calculations which employ both a single-particle πNN Hamiltonian and simple (e.g., shell-model) wave functions do ignore the most important feature of the problem, for it is well known that the shell-model wave function makes no claim to describe the short-range NN correlations correctly.

Perhaps it is appropriate to emphasize here that the two-nucleon model *does not* predict that pion capture will necessarily lead to the ejection of two fast nucleons or that "the two ejected nucleons" will have equal and opposite momenta. Such details as the distribution in momentum and angle of the emitted particles are determined mainly by the center-of-mass motion of the capturing pair. They are, in general, strongly affected by the over-all nuclear wave function; in particular, the momentum and angular distribution can be quite different from what they would be for two isolated nucleons. As an example, we quote that the result of the present calculation for the deuteron production ("one-nucleon ejection") rate is about $\frac{1}{5}$ of the total absorption rate; small, but not negligible. In the case of capture by He^4, one-nucleon ejection is even more prominent. Eckstein[7] finds that a triton production rate of about 22% is possible, and this in fact is the result of the experiment of Schiff, Hildebrand, and Giese[1] and that of Block *et al.*[3]

3. S-ORBIT ABSORPTION IN He^3

We now specialize to the case of negative pion capture and write the Hamiltonian, Eqs. (2.1) and (2.3), as the sum of two terms describing capture by a p-n pair and a p-p pair, respectively:

$$H^s=H_1{}^s+H_2{}^s,$$

$$H_1{}^s=-\sqrt{2}\int\delta(\mathbf{x}_1-\mathbf{x}_2)\phi^-(\mathbf{x}_1)\left[-\frac{i}{2}(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2)\psi_n{}^\dagger(\mathbf{x}_1)\psi_n{}^\dagger(\mathbf{x}_2)\right]T_{12}{}^\sigma\cdot[g_0\tfrac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)+g_1\tfrac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)]\psi_p(\mathbf{x}_1)\psi_n(\mathbf{x}_2)d\mathbf{x}_1d\mathbf{x}_2, \quad (3.1)$$

$$H_2{}^s=-\sqrt{2}\int\delta(\mathbf{x}_1-\mathbf{x}_2)\phi^-(\mathbf{x}_1)\left[-\frac{i}{2}(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2)\psi_p{}^\dagger(\mathbf{x}_1)\psi_n{}^\dagger(\mathbf{x}_2)\right]T_{12}{}^\sigma\cdot g_1\tfrac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\psi_p(\mathbf{x}_1)\psi_p(\mathbf{x}_2)d\mathbf{x}_1d\mathbf{x}_2. \quad (3.2)$$

We need to calculate the matrix elements of H_1^s and H_2^s between an initial state consisting of two protons and one neutron bound in a He^3 nucleus and a π^- meson, and a final state consisting of one proton and two neutrons. This can be done conveniently by using the Fock representation:

$$|He^3\rangle=\frac{1}{\sqrt{2}}\int \varphi_i(\xi_1,\xi_2;\xi_3)\psi_p^\dagger(\xi_1)\psi_p^\dagger(\xi_2)\psi_n^\dagger(\xi_3)|0\rangle d\xi_1 d\xi_2 d\xi_3,$$

where φ_i is the initial (He^3) nuclear wave function. Similarly, for the final state,

$$|1p;2n\rangle=\frac{1}{\sqrt{2}}\int \varphi_f(\xi_1';\xi_2',\xi_3')\psi_p^\dagger(\xi_1')\psi_n^\dagger(\xi_2')\psi_n^\dagger(\xi_3')|0\rangle d\xi_1' d\xi_2' d\xi_3'.$$

The matrix elements of H_1^s and H_2^s are then found to be

$$\langle f|H_1^s|i\rangle=-\frac{2}{\sqrt{\mu}}\int \delta(\mathbf{x}_2-\mathbf{x}_3)\varphi_\pi^S(\mathbf{x}_2) \times\left[-\frac{i}{2}(\nabla_2-\nabla_3)\varphi_f^*(\mathbf{x}_1;\mathbf{x}_2,\mathbf{x}_3)\right]\cdot T_{23}^\sigma[g_0\tfrac{1}{2}(\boldsymbol{\sigma}_2+\boldsymbol{\sigma}_3)+g_1\tfrac{1}{2}(\boldsymbol{\sigma}_2-\boldsymbol{\sigma}_3)]\varphi_i(\mathbf{x}_1,\mathbf{x}_2;\mathbf{x}_3)d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3 \quad (3.3)$$

and

$$\langle f|H_2^s|i\rangle=-\frac{2}{\sqrt{\mu}}\int \delta(\mathbf{x}_1-\mathbf{x}_2)\varphi_\pi^S(\mathbf{x}_1)\left[-\frac{i}{2}(\nabla_1-\nabla_2)\varphi_f^*(\mathbf{x}_1;\mathbf{x}_2,\mathbf{x}_3)\right]\cdot T_{12}^\sigma[g_1\tfrac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)]\varphi_i(\mathbf{x}_1,\mathbf{x}_2;\mathbf{x}_3)d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3, \quad (3.4)$$

μ being the pion mass and $\varphi_\pi^S(\mathbf{x})$ the S-orbit pion wave function. φ_i and φ_f may be written in terms of the space and spin wave functions as

$$\varphi_i=F_i(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)\chi^{(S)}(12)\chi(3),\quad \varphi_f=(1/\sqrt{2})[F_f(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)\chi(123)-F_f(\mathbf{x}_1,\mathbf{x}_3,\mathbf{x}_2)\chi(132)]. \quad (3.5)$$

The spatial function $F_i(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$ is symmetric with respect to interchange of any pair of coordinates; $\chi^{(S)}$ denotes the singlet spin function and χ an arbitrary spin function. Substituting (3.5) in (3.3) and (3.4) we obtain

$$\langle f|H_1^s|i\rangle=\frac{1}{(8\mu)^{1/2}}\chi^\dagger(123)(\mathbf{I}_1^S-P_{23}^\sigma\mathbf{I}_2^S)\cdot T_{23}^\sigma[g_0(\boldsymbol{\sigma}_2+\boldsymbol{\sigma}_3)+g_1(\boldsymbol{\sigma}_2-\boldsymbol{\sigma}_3)]\chi^{(S)}(12)\chi(3) \quad (3.6)$$

and

$$\langle f|H_2^s|i\rangle=\frac{1}{(8\mu)^{1/2}}\chi^\dagger(123)(\mathbf{I}_3^S-P_{23}^\sigma\mathbf{I}_4^S)\cdot T_{12}^\sigma g_1(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\chi^{(S)}(12)\chi(3), \quad (3.7)$$

where $P_{23}^\sigma=\frac{1}{2}(1+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)$ is the spin-exchange operator for nucleons 2 and 3 and the integrals $\mathbf{I}_1^S$, $\mathbf{I}_2^S$, $\mathbf{I}_3^S$, and $\mathbf{I}_4^S$ are given by

$$\mathbf{I}_1^S=\int \delta(\mathbf{x}_2-\mathbf{x}_3)\varphi_\pi^S(\mathbf{x}_2)[i(\nabla_2-\nabla_3)F_f^*(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)] \times F_i(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3, \quad (3.8)$$

$$\mathbf{I}_2^S=\int \delta(\mathbf{x}_2-\mathbf{x}_3)\varphi_\pi^S(\mathbf{x}_2)[i(\nabla_2-\nabla_3)F_f^*(\mathbf{x}_1,\mathbf{x}_3,\mathbf{x}_2)] \times F_i(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3, \quad (3.9)$$

$$\mathbf{I}_3^S=\int \delta(\mathbf{x}_1-\mathbf{x}_2)\varphi_\pi^S(\mathbf{x}_1)[i(\nabla_1-\nabla_2)F_f^*(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)] \times F_i(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3, \quad (3.10)$$

$$\mathbf{I}_4^S=\int \delta(\mathbf{x}_1-\mathbf{x}_2)\varphi_\pi^S(\mathbf{x}_1)[i(\nabla_1-\nabla_2)F_f^*(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)] \times F_i(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3. \quad (3.11)$$

In accordance with the ideas explained in Sec. 2, we may evaluate these integrals using for F_i and F_f simple wave functions which do not specifically reflect the close correlations of the nucleons. For the initial He^3 nuclear wave function, we therefore choose a Gaussian form, since this gives He^3 and H^3 form factors which are in agreement with the electron-scattering data over a wide range of momentum transfers,

$$F_i(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)=N_i\exp\{-\tfrac{1}{2}\lambda[(\mathbf{x}_1-\mathbf{x}_2)^2+(\mathbf{x}_2-\mathbf{x}_3)^2+(\mathbf{x}_3-\mathbf{x}_1)^2]\}. \quad (3.12)$$

The normalization constant N_i is given by

$$N_i^2=(1/V)(\lambda/\pi)^3 3^{3/2}, \quad (3.13)$$

where V is the normalization volume. The parameter λ is directly related to the rms radius R of the nucleon distribution for He^3 and H^3; thus,

$$\lambda=1/3R^2. \quad (3.14)$$

In terms of the rms charge radii of He^3 and H^3 and the proton rms radius R_p, this rms radius R is given by

$$R^2=\tfrac{2}{3}R_{\mathrm{He}}^2+\tfrac{1}{3}R_{\mathrm{H}}^2-R_p^2.$$

R_{He} and R_{H} have recently been measured experimentally[30]; their values are $R_{\mathrm{He}}=1.97\pm0.1$ F and $R_{\mathrm{H}}=1.68$ F (experimental uncertainty less than 10%), leading to $R=1.60$ F. Then, from Eq. (3.14), $\lambda=0.13$ F^{-2}.

So far as the pion wave function is concerned, the correct S-orbit atomic wave function may be replaced by its value at the center of the nucleus in view of the extremely small variation of this wave function over the nuclear dimensions.

The Deuteron Mode

The final deuteron and neutron have a relative momentum of about 400 MeV/c. This large relative momentum means that we may safely ignore final-state n-d interactions and use plane waves for the two particles. If $\mathbf{p}$ and $\mathbf{q}$ are the neutron and deuteron momenta and ϕ_d the deuteron wave function, the final-state wave function is

$$F_d(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)=V^{-1}\phi_d(\mathbf{x}_1-\mathbf{x}_2)\times\exp[i\mathbf{q}\cdot(\mathbf{x}_1+\mathbf{x}_2)/2+i\mathbf{p}\cdot\mathbf{x}_3]. \quad (3.15)$$

Substitution of (3.12) and (3.15) in (3.8) then leads to the following expression for $\mathbf{I}_{1,d}{}^S$ (the details are given in the Appendix):

$$\mathbf{I}_{1,d}{}^S=-64\pi^4N_iV^{-1}\varphi_\pi{}^S(0)\delta(\mathbf{p}+\mathbf{q})\times\left[\left(\frac{4\lambda}{p^2}+1\right)J(p)-\frac{4\lambda}{p}J'(p)\right]\hat{p}, \quad (3.16)$$

where we use $\hat{p}$ to denote the unit vector along $\mathbf{p}$. The integrals $J(p)$ and $J'(p)$ are defined by

$$J(p)=\int_0^\infty \exp(-\lambda u^2)\sin(\tfrac{1}{2}pu)\phi_d(u)udu \quad (3.17)$$

and

$$J'(p)=\frac{dJ(p)}{dp}=\frac{1}{2}\int_0^\infty \exp(-\lambda u^2)\cos(\tfrac{1}{2}pu)\phi_d(u)u^2du. \quad (3.18)$$

It is also easily found (see Appendix) that $\mathbf{I}_{2,d}{}^S=-\mathbf{I}_{1,d}{}^S$, $\mathbf{I}_{3,d}{}^S=0$, and $\mathbf{I}_{4,d}{}^S=\mathbf{I}_{1,d}{}^S$. The final spin function is

$$\chi_d(123)=\chi^{(T)}(12)\chi(3)=T_{12}{}^\sigma\chi(123). \quad (3.19)$$

The matrix element for the deuteron mode is therefore

$$\langle f|H^s|i\rangle_d=\frac{1}{4(2\mu)^{1/2}}\chi^\dagger(123)T_{12}{}^\sigma \mathbf{I}_{1,d}{}^S\cdot\{g_1\boldsymbol{\sigma}_1+(3g_0+4g_1)\boldsymbol{\sigma}_2+3(g_0-g_1)\boldsymbol{\sigma}_3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3[-g_1\boldsymbol{\sigma}_1+(g_0+2g_1)\boldsymbol{\sigma}_2+(g_0-g_1)\boldsymbol{\sigma}_3]\}\times S_{12}{}^\sigma\chi(123). \quad (3.20)$$

After the necessary trace calculations, we get for the deuteron rate for S-orbit capture,

$$W_d{}^S=0.0294(3|g_0|^2+7|g_1|^2+6\,\mathrm{Re}g_0{}^*g_1)\times|\varphi_\pi{}^S(0)|^2(2MQ_d)^{1/2}\left[\left(\frac{2\lambda}{MQ_d}+1\right)J((2MQ_d)^{1/2})-2\lambda\left(\frac{2}{MQ_d}\right)^{1/2}J'((2MQ_d)^{1/2})\right]^2, \quad (3.21)$$

where the mass M is $M=M_dM_n/(M_d+M_n)=3.174$ F^{-1} and $Q_d=0.674$ F^{-1} is the energy released in the reaction. The functions J and J' depend on the choice of ϕ_d and are defined by Eqs. (3.17) and (3.18). We use simple square-well wave functions for ϕ_d, in accordance with our remarks in Sec. 2. Triplet n-p scattering data[31] determine the depth and range of the potential to be $V=36.5$ MeV and $a=2$ F. The corresponding deuteron wave function is

$$\begin{aligned}\phi_d(u)&=A(\sin\alpha u/u)\,, && u\leq 2\text{ F}\\ &=B[\exp(-\beta u)/u]\,, && u\geq 2\text{ F}\end{aligned} \quad (3.22)$$

where $\alpha=0.910$ F^{-1}, $\beta=0.232$ F^{-1}, and the normalization constants are $A^2=0.0243$ F^{-1}, $B^2=0.0575$ F^{-1}. For this choice of ϕ_d, the integrals (3.18) and (3.19) have to be evaluated numerically, leading to

$$W_d{}^S=0.00258(3|g_0|^2+7|g_1|^2+6\,\mathrm{Re}g_0{}^*g_1)|\varphi_\pi{}^S(0)|^2\times3\times10^{23}\text{ sec}^{-1}. \quad (3.23)$$

In this equation, $|\varphi_\pi{}^S(0)|^2$ is measured in F^{-3} and the conversion factor 3×10^{23} has its origin in the system of units we use: With $c=\hbar=1$ and with the Fermi as the unit of length, the unit of time is $(3\times10^{23})^{-1}$ sec. For 1S-orbit capture

$$W_d{}^S=2.89(3|g_0|^2+7|g_1|^2+6\,\mathrm{Re}g_0{}^*g_1)\times10^{14}\text{ sec}^{-1}. \quad (3.24)$$

The Proton Mode

We begin by calculating the proton momentum spectrum for the 3-body mode (1.4) assuming, again, that

[30] H. Collard, R. Hofstadter, A. Johansson, R. Parks, M. Ryneveld, A. Walker, M. R. Yearian, R. B. Day, and R. T. Wagner, Phys. Rev. Letters **11**, 132 (1963).

[31] M. R. Moravcsik, Ann. Rev. Nucl. Sci. **10**, 324 (1960).

final-state interactions can be disregarded. This assumption needs justification since it is now possible for a pair of nucleons to have a small relative momentum. We discuss this point at the end of this subsection, where we reach the conclusion that as far as the total rates (in which we are primarily interested) are concerned the error introduced by neglecting the final-state interactions is quite small.

If $\mathbf{p}_1$ is the proton momentum and $\mathbf{p}_2$, $\mathbf{p}_3$ the momenta of the two neutrons, the final wave function with the outgoing nucleons represented by plane waves is

$$F_p(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)=V^{-3/2}\exp[i(\mathbf{p}_1\cdot\mathbf{x}_1+\mathbf{p}_2\cdot\mathbf{x}_2+\mathbf{p}_3\cdot\mathbf{x}_3)]. \quad (3.25)$$

With this choice of F_f, the integrals $\mathbf{I}_1$ to $\mathbf{I}_4$ can all be evaluated analytically. The results are (see Appendix)

$$\mathbf{I}_{1,p}{}^S(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)=2(2\pi)^3V^{-3/2}N_i\varphi_\pi{}^S(0)(\pi/\lambda)^{3/2}(\mathbf{p}_2-\mathbf{p}_3)\times\exp[-(\mathbf{p}_1-\mathbf{p}_2-\mathbf{p}_3)^2/16\lambda]\delta(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3), \quad (3.26)$$

$$\mathbf{I}_{2,p}{}^S(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)=-\mathbf{I}_{1,p}{}^S(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3), \quad (3.27)$$

$$\mathbf{I}_{3,p}{}^S(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)=\mathbf{I}_{1,p}{}^S(\mathbf{p}_3,\mathbf{p}_1,\mathbf{p}_2), \quad (3.28)$$

and

$$\mathbf{I}_{4,p}{}^S(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)=\mathbf{I}_{1,p}{}^S(\mathbf{p}_2,\mathbf{p}_1,\mathbf{p}_3). \quad (3.29)$$

There are no spin correlations among the final nucleons, and the matrix element has the form

$$\langle f|H^s|i\rangle_p=\frac{1}{4(2\mu)^{1/2}}\chi^\dagger(123)\times\{\sum_{i=1}^{3}\boldsymbol{\sigma}_i\cdot\mathbf{A}_i+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3\sum_{i=1}^{3}\boldsymbol{\sigma}_i\cdot\mathbf{B}_i\}S_{12}{}^\sigma\chi(123), \quad (3.30)$$

where

$$\begin{aligned}\mathbf{A}_1&=g_1(2\mathbf{I}_{3,p}{}^S-\mathbf{I}_{4,p}{}^S),\\ \mathbf{A}_2&=3g_0\mathbf{I}_{1,p}{}^S+g_1(3\mathbf{I}_{1,p}{}^S-2\mathbf{I}_{3,p}{}^S+\mathbf{I}_{4,p}{}^S),\\ \mathbf{B}_1&=-g_1\mathbf{I}_{4,p}{}^S,\quad \mathbf{B}_2=g_0\mathbf{I}_{1,p}{}^S+g_1(\mathbf{I}_{1,p}{}^S+\mathbf{I}_{4,p}{}^S),\\ \mathbf{A}_3&=3\mathbf{B}_3=3(g_0-g_1)\mathbf{I}_{1,p}{}^S.\end{aligned} \quad (3.31)$$

If $W_p{}^S(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)$ is the transition rate into a final state with the proton having momentum $\mathbf{p}_1$ and the neutrons having momenta $\mathbf{p}_2$ and $\mathbf{p}_3$, then

$$W_p{}^S(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)d\mathbf{p}_1d\mathbf{p}_2d\mathbf{p}_3=\frac{2\pi}{8}\sum|\langle f|H^s|i\rangle_p|^2\delta(E_i-E_f)V^3\frac{d\mathbf{p}_1d\mathbf{p}_2d\mathbf{p}_3}{(2\pi)^9}. \quad (3.32)$$

The sum is over the spin directions of all particles (all particles unpolarized). Integration of (3.32) over $\mathbf{p}_2$ and $\mathbf{p}_3$ and the directions of $\mathbf{p}_1$ gives the proton momentum spectrum:

$$W_p{}^S(p_1)=\int W_p{}^S(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)d\mathbf{p}_2d\mathbf{p}_3d\Omega_1. \quad (3.33)$$

$W_p{}^S(p_1)$ is most conveniently written as a sum of direct and cross terms in $\mathbf{I}_{1,p}{}^S$, $\mathbf{I}_{3,p}{}^S$, and $\mathbf{I}_{4,p}{}^S$ coming from $|\langle f|H^s|i\rangle_p|^2$:

$$\begin{aligned}W_p{}^S(p_1)=&(2|g_0|^2+|g_1|^2)W_{11}(p_1)+|g_1|^2W_{33}(p_1)\\&+|g_1|^2W_{44}(p_1)+(|g_1|^2+2\,\mathrm{Re}g_0{}^*g_1)W_{13}(p_1)\\&+(|g_1|^2+2\,\mathrm{Re}g_0{}^*g_1)W_{14}(p_1)+|g_1|^2W_{34}(p_1).\end{aligned} \quad (3.34)$$

As is clear from the notation, $W_{11}(p_1)$, $W_{33}(p_1)$, and $W_{44}(p_1)$ are the direct terms; $W_{13}(p_1)$ and $W_{14}(p_1)$ are cross terms arising from interference of the amplitudes for capture by the p-p pair and a p-n pair; and $W_{34}(p_1)$ represents antisymmetrization in the capture by the p-p pair.

The integrations required in the evaluation of $W_p{}^S(p_1)$, Eq. (3.33), can be performed analytically. One obtains

$$W_{11}(p_1)=8KP^3p_1^2\exp(-p_1^2/2\lambda)\theta(P^2), \quad (3.35)$$

$$W_{33}(p_1)=W_{44}(p_1)=KPp_1^2\exp[-(p_1^2+P^2)/8\lambda]\times\left\{\frac{4\lambda}{p_1P}\sinh\left(\frac{p_1P}{4\lambda}\right)(18p_1^2+2P^2-48\lambda)+48\lambda\cosh\left(\frac{p_1P}{4\lambda}\right)\right\}\theta(P^2), \quad (3.36)$$

$$W_{13}(p_1)=W_{14}(p_1)=2KPp_1^2\exp[-(5p_1^2+P^2)/16\lambda]\times\left\{\frac{8\lambda}{p_1P}\sinh\left(\frac{p_1P}{8\lambda}\right)(2P^2-48\lambda)+48\lambda\cosh\left(\frac{p_1P}{8\lambda}\right)\right\}\theta(P^2), \quad (3.37)$$

and

$$W_{34}(p_1)=2KPp_1^2(P^2-9p_1^2)\times\exp[-(p_1^2+P^2)/8\lambda]\theta(P^2). \quad (3.38)$$

In these equations, θ is the usual step function $[\theta(x)=0$ for $x<0$, $\theta(x)=1$ for $x\geqslant 0]$, $P^2=4M_nQ_p-3p_1^2$ ($Q_p=0.664$ F^{-1} being the energy released) and

$$K=2^{-9}3^{3/2}\pi^{-3}(M_n/\mu)|\varphi_\pi{}^S(0)|^2=0.0022|\varphi_\pi{}^S(0)|^2. \quad (3.39)$$

The various W functions are shown graphically in Fig. 1 and the complete proton momentum spectrum in Fig. 2.

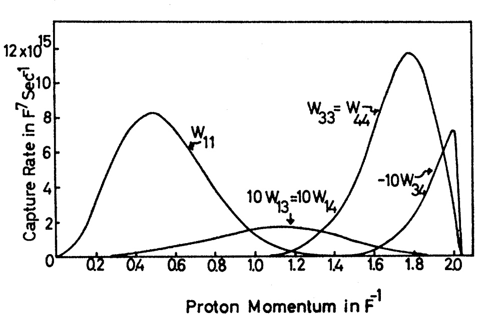

FIG. 1. The functions $W_{ij}(p_1)$, Eqs. (3.35)–(3.38), which contribute to the proton momentum spectrum.

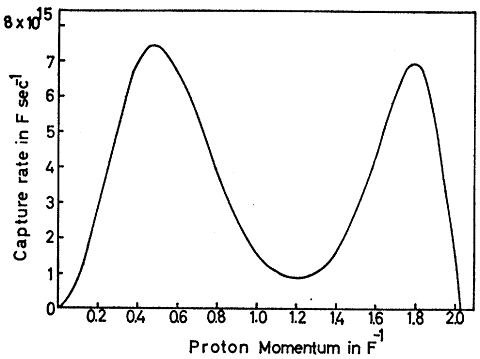

FIG. 2. The proton momentum spectrum for $|g_0|^2=0.32$ F^8, $|g_1|^2=0.29$ F^8, $\arg(g_0/g_1)=0$.

(For the values of g_0 and g_1 used in drawing Fig. 2, see the next subsection.) As would be expected, the spectrum has two fairly broad peaks—the lower momentum peak corresponds to spectator protons, and the higher momentum peak corresponds to participant protons.

Actually, the shape of the spectrum as shown in Fig. 2 will be distorted because of n-p and n-n final-state interactions, owing to the existence of the deuteron and the low-energy $^1S_0$ n-n virtual resonance. The latter will arise for configurations in which the relative n-n momentum is small, i.e., it will affect the shape of the spectrum at the maximum-proton-momentum end. The effect of the n-p interactions will be strongest around a proton momentum of about 200 MeV/$c \approx 1$ F^{-1}, when the p-n pair has zero relative momentum. However, configurations with small relative momenta between any pair of particles form only a small fraction of the total phase space available. This fact and, even more strongly, the existence of a completeness relation for the absorption rate $W_{\text{abs}}=W_d+W_p$, ensure that W_{abs} will not be affected to any appreciable extent by these final-state interactions.

To see this, let us first consider the effect of the low-energy n-p interaction. This is important in configurations of the type $|f\rangle=|(np),n\rangle$ where the notation (np) indicates that the pair has a small relative momentum. The effects of any interactions between (np) and the (fast) n may be disregarded. If $\Psi_{(np)}(E,\mathbf{r})$ is the wave function of the interacting (np) pair having a center-of-mass energy E, the matrix element for the final-state $|(np),n\rangle$ is

$$M(E)=\int \Psi_{(np)}^*(E,\mathbf{r}) \exp[-i\mathbf{r}_n\cdot\mathbf{p}_n(E)]O\Psi_i d\mathbf{R}, \quad (3.40)$$

where we have suppressed the spin dependence of $M(E)$ and have used O to denote the spatial part of the Hamiltonian operator and $\mathbf{R}$ to denote the set of coordinates of all three particles. The total absorption rate is then proportional to

$$P_{\text{abs}}=\sum_E|M(E)|^2\rho_n(E)$$
$$=\int d\mathbf{R}d\mathbf{R}'\sum_E\{(\Psi_i^*O^\dagger\Psi_{(np)}(E,\mathbf{r})\exp[i\mathbf{r}_n\cdot\mathbf{p}_n(E)])$$
$$\times(\Psi_{(np)}^*(E,\mathbf{r}')\exp[-i\mathbf{r}_n'\cdot\mathbf{p}_n(E)]O\Psi_i)\rho_n(E)\}. \quad (3.41)$$

Here $\rho_n(E)$ denotes the final density of states of the fast neutron, and the sum over E includes the deuteron state. For small (np) energy, only for which $\Psi_{(np)}(E,\mathbf{r})$ differs from the corresponding free-particle wave function, $\mathbf{p}_n$ and ρ_n are practically constant; so we may write

$$P_{\text{abs}}=\int \Psi_i^*O^\dagger\exp[i\mathbf{r}_n\cdot\mathbf{p}_n]$$
$$\times[\sum_E\Psi_{(np)}(E,\mathbf{r})\Psi_{(np)}^*(E,\mathbf{r}')]$$
$$\times\exp[-i\mathbf{r}_n'\cdot\mathbf{p}_n]O\Psi_i\rho_n d\mathbf{R}d\mathbf{R}'. \quad (3.42)$$

We now use the completeness relation in the form that, with the sum over E extended to all n-p states including energetically inaccessible ones,

$$\sum_E\Psi_{(np)}(E,\mathbf{r})\Psi_{(np)}^*(E,\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'),$$

independent of the n-p forces, to conclude that no error is made in P_{abs} if $\Psi_{(np)}(E,\mathbf{r})$ is replaced by the corresponding free-particle wave function. The essential point is that the deuteron state is implicitly included in the sum over all plane-wave states on account of their completeness property. Similar remarks, of course, hold true for the effects of the n-n forces.

Integration of the proton momentum spectrum over p_1 therefore leads directly to the absorption rate W_{abs}^S:

$$W_{\text{abs}}^S=\int W_p^S(p_1)dp_1, \quad (3.43)$$

and the rate of transitions to three free particles, the proton rate, is given by

$$W_p^S=W_{\text{abs}}^S-W_d^S, \quad (3.44)$$

where W_d^S is the deuteron rate which we have already calculated.

Determination of g_0 and g_1

The values of $|g_0|^2$ and $|g_1|^2$ may most conveniently be determined by using the two-nucleon Hamiltonian to study single-pion production in N-N collisions. In particular, $|g_0|^2$ can be obtained by calculating the cross section for the reaction

$$p+p\rightarrow\pi^++d \quad (3.45)$$

as a function of the center-of-mass momentum in the final state and comparing with available experimental data.[32] Similarly, a study of the process[33]

$$p+p\rightarrow\pi^0+p+p \tag{3.46}$$

yields a value for $|g_1|^2$. This has already been carried out by Eckstein (Appendix B of Ref. 7), using plane waves for the fast initial nucleons and square-well potential wave functions (the appropriate potential parameters being determined from the triplet and singlet scattering lengths and effective ranges) for the final nucleons. Eckstein obtains the values

$$|g_0|^2=0.32\pm0.04\text{ F}^8,\quad |g_1|^2=0.29\pm0.15\text{ F}^8, \tag{3.47}$$

where the errors quoted come from uncertainties in the experimental cross sections. Assuming time-reversal invariance, the phases of g_0 and g_1 are identical with the nucleon-nucleon scattering phase-shifts at an initial center-of-mass energy of 140 MeV and in the appropriate angular-momentum states

$$\begin{aligned} g_0&=\pm|g_0|\exp[i\delta(^3P_1)],\\ g_1&=\pm|g_1|\exp[i\delta(^3P_0)]. \end{aligned} \tag{3.48}$$

Using the results of N-N phase-shift analyses[31] around 140 MeV, Eckstein concludes that the phase of g_0/g_1 is either 0° or 180° to within about 20°. A choice between these two possibilities can only be made on the basis of other, independent, data; Eckstein,[34] for example, finds that the two-nucleon model prediction for the triton rate for capture by He^4 is in agreement with experiment only for $\arg(g_0/g_1)=0$. In comparing our results with experiment we adopt the values given by Eqs. (3.47) and also take $\arg(g_0/g_1)=0$.

4. ESTIMATE OF THE P-ORBIT ABSORPTION RATE

In discussing the magnitude of the P-orbit absorption rate, we distinguish between the following two modes of capture:

(a) The pion is in a P orbit with respect to the He^3 nucleus but in an s state with respect to the capturing pair of nucleons.

(b) The pion is in a P orbit, and is in a p state with respect to the capturing nucleon pair.

The amplitudes for these two cases will not interfere with the S-orbit capture amplitude because of the orthogonality between the atomic $1S$ and $2P$ wave functions.

A. s Wave

The calculation of the absorption rate in this case is analogous to what we have already done for capture from the S orbit—the only difference is that in evaluating the integrals $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, $\mathbf{I}_4$ we now have to use the $2P$ orbital wave function instead of the $1S$ orbital wave function. The transition matrix is the same as before, Eq. (2.2), and the process is described as before in terms of the amplitudes g_0 and g_1. Using free-particle wave functions for the final nucleons, we write again the expression for $\mathbf{I}_{1,p}$:

$$\begin{aligned}\mathbf{I}_{1,p}=V^{-3/2}N_i(\mathbf{p}_2-\mathbf{p}_3)\int d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3\\ \times\delta(\mathbf{x}_2-\mathbf{x}_3)\varphi_\pi(\mathbf{x}_2-\tfrac{1}{3}(\mathbf{x}_1+\mathbf{x}_2+\mathbf{x}_3))\\ \times\exp[-i(\mathbf{p}_1\cdot\mathbf{x}_1+\mathbf{p}_2\cdot\mathbf{x}_2+\mathbf{p}_3\cdot\mathbf{x}_3)]\\ \times\exp\{-\tfrac{1}{2}\lambda[(\mathbf{x}_1-\mathbf{x}_2)^2+(\mathbf{x}_2-\mathbf{x}_3)^2+(\mathbf{x}_3-\mathbf{x}_1)^2]\},\end{aligned} \tag{4.1}$$

where we measure the pion coordinate from the center of the nucleus. The pion wave function to be used now is the $2P$ orbital wave function:

$$\begin{aligned}\varphi_\pi{}^P(\mathbf{x})&=R_{21}(x)Y_1{}^m(\vartheta,\chi)\\ &=\frac{1}{(4\pi)^{1/2}}R_{21}(x)P_1{}^m(\cos\vartheta)\exp(im\chi),\end{aligned} \tag{4.2}$$

where R_{21} is a normalized radial wave function. If we denote the first Bohr radius of the pion around protons by a, R_{21} is given by

$$R_{21}(x)=(2x/3^{1/2}a^{5/2})\exp(-x/a). \tag{4.3}$$

Since $a\gg R$, the radius of the nucleus, we may replace the exponential in (4.3) by unity. This permits the integrations in (4.1) to be done analytically, and the result (see Appendix) is

$$\begin{aligned}\mathbf{I}_{1,p}{}^P=\frac{(2\pi)^3\pi N_i}{2i3^{3/2}V^{3/2}a^{5/2}\lambda^{5/2}}\delta(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3)(\mathbf{p}_2-\mathbf{p}_3)p_1\\ \times\exp[-(\mathbf{p}_1-\mathbf{p}_2-\mathbf{p}_3)^2/16\lambda].\end{aligned} \tag{4.4}$$

Also, Eqs. (3.27), (3.28), and (3.29) hold for the P-orbit case as well. Comparing these integrals with the corresponding S-orbit integrals, we get

$$\begin{aligned}\mathbf{I}_{1,p}{}^P&=(p_1/6^{3/2}ia\lambda)\mathbf{I}_{1,p}{}^S,\\ \mathbf{I}_{2,p}{}^P&=(p_1/6^{3/2}ia\lambda)\mathbf{I}_{2,p}{}^S,\\ \mathbf{I}_{3,p}{}^P&=(p_3/6^{3/2}ia\lambda)\mathbf{I}_{3,p}{}^S,\\ \mathbf{I}_{4,p}{}^P&=(p_2/6^{3/2}ia\lambda)\mathbf{I}_{4,p}{}^S.\end{aligned} \tag{4.5}$$

These equations are exact. We now make the approximation of replacing p_1, p_2, and p_3 by their average value $\langle p_1\rangle=\langle p_2\rangle=\langle p_3\rangle\equiv\langle p\rangle$. The rate for absorption from the $2P$ orbit when the pion is a relative s wave with respect to the nucleons is then expressible in terms of the corresponding $1S$-orbit rate

$$W_{\text{abs}}{}^P(s\text{-wave }\pi)\approx(\langle p\rangle^2/216a^2\lambda^2)W_{\text{abs}}{}^S. \tag{4.6}$$

[32] F. S. Crawford and M. L. Stevenson, Phys. Rev. **97**, 1305 (1955).

[33] R. A. Stallwood, R. B. Sutton, T. H. Fields, J. G. Fox, and J. A. Kane, Phys. Rev. **109**, 1716 (1958).

[34] It may also be mentioned here that Eckstein (Ref. 7) has shown theoretically that, to the extent the N-N correlations arise from p-wave emission and absorption of pions, $g_0=g_1$ as is actually found to be the case.

TABLE I. Transition amplitudes for p-wave pion.

Initial state		Final state		
Angular momentum	Isotopic spin	Angular momentum	Isotopic spin	Transition matrix $\mathfrak{M}_{12}^{p}$
$^1S_0$	1	$^3S_0$	0	$\gamma_1 T_{12}^{\sigma} S_{12}^{\tau} (\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{p}_\pi (\boldsymbol{\tau}_1-\boldsymbol{\tau}_2)\cdot\boldsymbol{\phi}$
$^1S_0$	1	$^3D_1$	0	$\gamma_2 T_{12}^{\sigma} S_{12}^{\tau} (\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\{(\mathbf{k}\cdot\mathbf{p}_\pi)\mathbf{k}-\frac{1}{3}k^2\mathbf{p}_\pi\}(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2)\cdot\boldsymbol{\phi}$
$^3S_1$	0	$^1S_0$	1	$\gamma_3 S_{12}^{\sigma} T_{12}^{\tau} (\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{p}_\pi (\boldsymbol{\tau}_1-\boldsymbol{\tau}_2)\cdot\boldsymbol{\phi}$
$^3S_1$	0	$^1D_2$	1	$\gamma_4 S_{12}^{\sigma} T_{12}^{\tau} (\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\{(\mathbf{k}\cdot\mathbf{p}_\pi)\mathbf{k}-\frac{1}{3}k^2\mathbf{p}_\pi\}(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2)\cdot\boldsymbol{\phi}$

With $\langle p\rangle = 1\ \mathrm{F}^{-1}$, we get

$$W_{\mathrm{abs}}{}^{P}(s\text{-wave }\pi)\approx 7.5\times10^{-6}W_{\mathrm{abs}}{}^{S}. \quad (4.7)$$

B. p Wave

When the pion is in a p wave with respect to the capturing pair of nucleons, the nucleon pair undergoes no change of parity. The final pair of nucleons are therefore either in a relative S state or a D state. Conservation of angular momentum and isospin then restricts the total number of amplitudes contributing to p-wave capture to four. The initial and final states corresponding to these four amplitudes and the appropriate transition matiices are shown in Table I. The dominant amplitudes are those with an N^* intermediate state:

$$\pi+N+N\rightarrow N+N^*\rightarrow N+N.$$

We assume that all other amplitudes are negligible and so need consider only final states of the type (2,2), (1,2), (2,1), and (1,1), where (I,J) stands for a final state with isospin I and angular momentum J. Of the four amplitudes given in the table, we therefore confine attention to the fourth one, namely the amplitude γ_4 describing the transition $(^3S_1, I=0)\rightarrow(^1D_2, I=1)$. It is clear that this amplitude contributes also to the P-orbit pion capture by deuterons and, by time-reversal invariance, to the p-wave pion-production reaction

$$p+p\rightarrow\pi^++d. \quad (4.8)$$

It is therefore possible to express $W_{\mathrm{abs}}{}^{P}(p\text{-wave }\pi)$, the rate for p-wave absorption in He^3, in terms of $\sigma(pp\rightarrow\pi^+d,\ p\text{-wave }\pi)$ and use the experimentally measured value of this cross section to estimate $W_{\mathrm{abs}}{}^{P}(p\text{-wave }\pi)$.

First, the relation between $W^P(\mathrm{deut})$, the rate for P-orbit absorption in deuterium, and $\sigma(pp\rightarrow\pi^+d,\ p\text{-wave }\pi)$ is established through the sequence of reactions

$$\pi^-+d\rightarrow n+n \text{ (from } P \text{ orbit)} \quad (4.9)$$
$$\downarrow$$
$$\pi^-+d\rightarrow n+n\ (p\text{-wave }\pi\text{, in flight}) \quad (4.10)$$
$$\downarrow \text{ (detailed balance)}$$
$$n+n\rightarrow\pi^-+d\ (p\text{-wave }\pi) \quad (4.11)$$
$$\downarrow \text{ (charge symmetry)}$$
$$p+p\rightarrow\pi^++d\ (p\text{-wave }\pi). \quad (4.12)$$

If we denote the rate for (4.10) by $W_{fl}{}^{P}(\mathrm{deut})$, for small pion momenta we have the relation[35]

$$\frac{W^P(\mathrm{deut})}{W_{fl}{}^{P}(\mathrm{deut})}=\frac{V|(d/dr)\varphi_\pi{}^{P}(r)|_{r=0}{}^2}{p_\pi{}^2}=\frac{V}{96\pi a^5 p_\pi{}^2}. \quad (4.13)$$

Further,

$$\frac{\sigma(\pi^-d\rightarrow nn,\ p\text{-wave }\pi)}{\sigma(nn\rightarrow\pi^-d,\ p\text{-wave }\pi)}=\frac{2}{3}\frac{p_n{}^2}{p_\pi{}^2} \quad (4.14)$$

and

$$\sigma(pp\rightarrow\pi^+d,\ p\text{-wave }\pi)=\sigma(nn\rightarrow\pi^-d,\ p\text{-wave }\pi). \quad (4.15)$$

Combining Eqs. (4.13)–(4.15), and the relation

$$\sigma(\pi^-d\rightarrow nn,\ p\text{-wave }\pi)=(\mu V/p_\pi)W_{fl}{}^{P}(\mathrm{deut}), \quad (4.16)$$

we get

$$W^P(\mathrm{deut})=\frac{1}{144\pi a^5\mu}\frac{p_n{}^2}{p_\pi{}^3}\sigma(pp\rightarrow\pi^+d,\ p\text{-wave }\pi)$$
$$=\beta p_n{}^2/144\pi a^5\mu^4, \quad (4.17)$$

where we have written the cross section for p-wave pion production (4.12) as $\beta p_\pi{}^3/\mu^3$. The corresponding rate in He^3 may be written as

$$W_{\mathrm{abs}}{}^{P}(p\text{-wave }\pi)=32\xi W^P(\mathrm{deut}). \quad (4.18)$$

The factor 32 takes account of the fact that the Bohr radius in He^3 is half that in deuterium and ξ is a correction factor for the different probabilities of finding a pair of correlated nucleons in He^3 and the deuteron. We may eliminate this factor by using the relation corresponding to Eq. (4.18) for S-orbit capture. Rembering that only the amplitude g_0 (for the transition $^3S_1\rightarrow{}^3P_1$) contributes to the S-orbit capture in deuterons and noting that in He^3, $W_{\mathrm{abs}}{}^{S}(^3S_1\rightarrow{}^3P_1)\approx\frac{2}{5}W_{\mathrm{abs}}{}^{S}$, we obtain

$$\tfrac{2}{5}W_{\mathrm{abs}}{}^{S}=8\xi W^S(\mathrm{deut}). \quad (4.19)$$

In exactly the same way as the P-orbit deuteron capture rate [Eq. (4.17)] was obtained, we have

$$W_{\mathrm{abs}}{}^{S}(\mathrm{deut})=\tfrac{2}{3}(\alpha p_n{}^2/a^3\mu^2), \quad (4.20)$$

where

$$\alpha=(\mu/p_\pi)\sigma(pp\rightarrow\pi^+d,\ s\text{-wave }\pi).$$

[35] J. M. Cassels, Nuovo Cimento Suppl. **14**, 259 (1959).

Equations (4.17)–(4.20) lead to

$$W_{\mathrm{abs}}{}^P(p\text{-wave }\pi)/W_{\mathrm{abs}}{}^S=(1/60a^2\mu^2)(\beta/\alpha). \quad (4.21)$$

Using the experimental values[32] $\alpha=0.14$ mb, $\beta=1.0$ mb, we finally obtain

$$W_{\mathrm{abs}}{}^P(p\text{-wave }\pi)=6.9\times10^{-6}W_{\mathrm{abs}}{}^S. \quad (4.22)$$

In general, the amplitudes for p-wave and s-wave absorption from the P orbit can interfere even though the initial and final angular-momentum states of the two capturing nucleons are different for the two cases. The reason for this is that there are more than two nucleons in the capturing nucleus; thus, for example, interference is possible between the amplitude for the pair (1,3) undergoing a ${}^3S_1\rightarrow{}^3P_1$ transition with the nucleon 3 as a spectator with the amplitude for the pair (2,3) undergoing a ${}^3S_1\rightarrow{}^1D_2$ transition with the nucleon 1 as a spectator.[36] For a first estimate, we neglect such interference effects and take the complete P-orbit absorption rate to be given by the sum of Eqs. (4.7) and (4.22):

$$\begin{aligned}W_{\mathrm{abs}}{}^P&\approx W_{\mathrm{abs}}{}^P(s\text{-wave }\pi)+W_{\mathrm{abs}}{}^P(p\text{-wave }\pi)\\&=1.1\times10^{11}\ \mathrm{sec}^{-1},\end{aligned} \quad (4.23)$$

taking the value of $W_{\mathrm{abs}}{}^S$ given in Sec. 6. This is to be compared with $W_{\mathrm{rad}}(2P\rightarrow1S)$ for a He^3-π^- atom, which is[37] 3×10^{12} sec^{-1}. So

$$W_{\mathrm{abs}}{}^P/W_{\mathrm{rad}}(2P\rightarrow1S)\approx3.7\%. \quad (4.24)$$

We therefore conclude that capture from the $2P$ orbit may be ignored.

5. RADIATIVE CAPTURE

In accordance with our earlier remarks on the nature of the radiative capture, we take the general form of the effective Hamiltonian to be

$$H_\gamma=g_-\int\psi_n{}^\dagger(\mathbf{x})\boldsymbol{\sigma}\cdot\mathbf{A}(\mathbf{x})\psi_p(\mathbf{x})\phi^-(\mathbf{x})d\mathbf{x}, \quad (5.1)$$

where $\psi_n{}^\dagger(\mathbf{x})$ creates a neutron at the point $\mathbf{x}$, $\psi_p(\mathbf{x})$ annihilates a proton at the same point, $\mathbf{A}(\mathbf{x})$ and $\phi^-(\mathbf{x})$ are the photon and π^- meson field operators, $\boldsymbol{\sigma}$ is the nucleon spin operator, and g_- is the effective coupling constant. It may be recalled that H_γ is identical with the so-called gauge invariance term which contributes to the s-wave photoproduction of charged pions.

The matrix element of H_γ between an initial state consisting of a He^3 nucleus and a π^- meson and a final state consisting of a triton and a photon is calculated in exactly the same way as before by using the Fock representation for the nucleon states. The result is

$$\begin{aligned}\langle f|H_\gamma|i\rangle=&\frac{g_-}{(\mu\omega)^{1/2}}\int\varphi_f{}^*(\mathbf{x}_1;\mathbf{x}_2,\mathbf{x}_3)\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}_2\\&\times\varphi_i(\mathbf{x}_1,\mathbf{x}_2;\mathbf{x}_3)\varphi_\pi{}^S(\mathbf{x}_2)\exp(-i\mathbf{k}\cdot\mathbf{x}_2)d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3,\end{aligned} \quad (5.2)$$

where $\boldsymbol{\varepsilon}$ is the polarization vector of the photon, $\mathbf{k}$ is its momentum, and $\omega=|\mathbf{k}|$, its energy. Also, if F_i and F_f are the initial and final spatial nuclear wave functions, then

$$\varphi_i(\mathbf{x}_1,\mathbf{x}_2;\mathbf{x}_3)=F_i(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)\chi^{(S)}(12)\chi(3)$$

and

$$\varphi_f(\mathbf{x}_1;\mathbf{x}_2,\mathbf{x}_3)=F_f(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)\chi(1)\chi^{(S)}(23)$$

so that

$$\begin{aligned}\langle f|H_\gamma|i\rangle=&[g_-/(\mu\omega)^{1/2}]\chi^\dagger(123)\\&\times S_{23}{}^\sigma\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}_2S_{12}{}^\sigma\chi(123)K,\end{aligned} \quad (5.3)$$

where K is the integral

$$\begin{aligned}K=\int&F_f{}^*(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)F_i(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)\\&\times\exp(-i\mathbf{k}\cdot\mathbf{x}_2)\varphi_\pi{}^S(\mathbf{x}_2)d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3.\end{aligned} \quad (5.4)$$

If we use the same nuclear wave function for H^3 as for He^3 then F_f is given by

$$\begin{aligned}F_f(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)=&V^{-1/2}\exp[i\mathbf{p}\cdot(\mathbf{x}_1+\mathbf{x}_2+\mathbf{x}_3)/3]\\&\times F_i(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)\end{aligned} \quad (5.5)$$

and F_i is given by (3.12), $\mathbf{p}$ being the triton momentum. The integral (5.4) is then easily evaluated:

$$K=V^{-3/2}(2\pi)^3|\varphi_\pi{}^S(0)|^2\exp(-k^2/18\lambda)\delta(\mathbf{p}+\mathbf{k}), \quad (5.6)$$

where again $\varphi_\pi{}^S(\mathbf{x}_2)$ has been replaced by $\varphi_\pi{}^S(0)$. The rate for unpolarized initial and final particles is then

$$W_\gamma=\frac{|g_-|^2|\varphi_\pi{}^S(0)|^2}{8\pi\mu}\frac{k_0}{(1+k_0/M_t)}\exp(-k_0{}^2/9\lambda), \quad (5.7)$$

$k_0=M_t[(1+2\mu/M_t)^{1/2}-1]$ being the center-of-mass momentum of the final-state particles and M_t the triton mass. Numerically

$$W_\gamma=0.0247|g_-|^2|\varphi_\pi{}^S(0)|^2. \quad (5.8)$$

The coupling constant $|g_-|^2$ may be determined using the available experimental data on charged pion photoproduction. In order to isolate the s-wave contribution to the total photoproduction cross section it is necessary to extrapolate the data to threshold. Since π^+ photoproduction experiments are more complete and more accurate than the limited experiments on π^- photoproduction, we use the former, along with the π^-/π^+ ratio, for which the threshold value is rather unambiguously predicted by theory.

Thus we calculate the cross section for the reaction

$$\gamma+p\rightarrow n+\pi^+ \quad (5.9)$$

[36] It is for the same reason that terms in $\mathrm{Re}g_0{}^*g_1$ appear in the S-orbit absorption rates also, even though g_0 and g_1 are amplitudes for different angular-momentum channels.

[37] G. R. Burbidge and A. H. de Borde, Phys. Rev. **89**, 189 (1953).

using the Hamiltonian

$$H_+ = g_+ \int \psi_n^\dagger(\mathbf{x}) \boldsymbol{\sigma} \cdot \mathbf{A}(\mathbf{x}) \psi_p(\mathbf{x}) \phi^-(\mathbf{x}) d\mathbf{x}, \quad (5.10)$$

where g_+ is the π^+ amplitude corresponding to g_- in Eq. (5.1). One obtains easily

$$\sigma_+ = (|g_+|^2/4\pi)(p/k_0)(1+k_0/M_n)^{-1}. \quad (5.11)$$

Here k_0 and p are, respectively, the initial and final center-of-mass momenta. To facilitate the comparison of Eq. (5.11) with experiment, it is convenient to write the total s-wave π^+-photoproduction cross section in the form

$$\sigma_+ = 4\pi C a_+, \quad (5.12)$$

C being the dimensionless kinematical factor

$$C = \frac{p(\mu^2+p^2)^{1/2}}{\mu^2(1+k_0/M_n)^2}. \quad (5.13)$$

Comparing Eqs. (5.11) and (5.12) we see that the Hamiltonian (5.10) leads to the following expression for a_+:

$$a_+ = \frac{|g_+|^2}{16\pi^2} \frac{\mu^2(1+k_0/M_n)}{k_0(\mu^2+p^2)^{1/2}}. \quad (5.14)$$

At threshold,

$$a_+^0 = (|g_+|^2\mu/16\pi^2 k_0)(1+k_0/M_n) \quad (5.15)$$

with k_0 taking the value $k_0 = 0.67$ F^{-1}.

The values of a_+^0 and a_-^0 (defined similarly for π^- photoproduction) have both been determined by Hamilton and Woolcock[38] using the experimental data available (rather limited in the case of a_-; further, experiments to measure a_- have to be done in deuterium and there are difficulties associated with the corrections to be made in extracting information on photoproduction on free neutrons). They use the dispersion relations of Chew, Goldberger, Low, and Nambu[17] (CGLN) to obtain a_+ and a_- as functions of incident photon energy, these functions being normalized by fitting the experimental points available in the neighborhood of the pion threshold. For our purposes it is sufficient to know that for a certain choice of the parameters involved in the CGLN amplitude, and for energies close to the threshold the agreement is satisfactory for a_+. In the case of a_- there are fewer points available and it is difficult to judge how good the agreement is. The value of a_+^0 obtained in this way is $a_+^0 = 20.2 \times 10^{-4}$ F^2 with an uncertainty of about 8%, leading to $|g_+|^2 = 0.264 \pm 0.02$ F^2.

To obtain $|g_-|^2$, we note that

$$|g_-|^2/|g_+|^2 = (\sigma_-/\sigma_+)_{\text{thresh}}.$$

A simple theoretical argument[39] based on the fact that s-wave photoproduction is entirely an electric-dipole transition leads to $(\sigma_-/\sigma_+)_{\text{thresh}} = (1+\mu/M)^2 = 1.32$, so that we finally obtain

$$|g_-|^2 = 0.35 \pm 0.03 \text{ F}^2. \quad (5.16)$$

6. RESULTS AND COMPARISON WITH EXPERIMENT

With $|g_0|^2$ and $|g_1|^2$ taking the values given by Eq. (3.47) and assuming that g_0 and g_1 have the same phase, the 1S-orbit capture rates are

$$W_d = 1.40 \pm 0.51 \times 10^{15} \text{ sec}^{-1}, \quad (6.1)$$

$$W_{\text{abs}} = 7.85 \pm 2.80 \times 10^{15} \text{ sec}^{-1}, \quad (6.2)$$

so that the rate for the final state $p+2n$ is

$$W_p = 6.45 \pm 3.31 \times 10^{15} \text{ sec}^{-1}. \quad (6.3)$$

The value of W_γ, with $|g_-|^2 = 0.35 \pm 0.03$ F^2, is

$$W_\gamma = 0.97 \pm 0.08 \times 10^{15} \text{ sec}^{-1}. \quad (6.4)$$

The branching rates of interest to us are predicted to be

$$W_d/W_\gamma = 1.44 \pm 0.68, \quad (6.5)$$

$$W_{\text{abs}}/W_\gamma = 8.1 \pm 3.8, \quad (6.6)$$

$$W_p/W_d = 4.6 \pm 4.1. \quad (6.7)$$

Of these, the only quantity for which an experimental number is available is W_{abs}/W_γ. This comes from the experiment of Falomkin *et al.*[2] Pions were stopped in a He^3 diffusion chamber and the two-body radiative-capture events (1.5) and the charge-exchange events (1.6) were identified through the unique ranges of the recoil tritons. The deuteron and proton absorption modes could not be distinguished and counted separately, as all of the deuterons and some of the protons escaped from the chamber. The three and four body radiative modes are expected to be very small (Messiah[15] estimates them to be $\sim$2% of total captures). If we neglect their contribution to the total number of capture stars, we obtain, from the data of Falomkin *et al.*,[2] the number

$$(W_{\text{abs}}/W_\gamma)_{\text{exp}} = 13.0 \pm 1.8. \quad (6.8)$$

Thus, within their respective errors, the theoretical and experimental values of W_{abs}/W_γ are in agreement. It would be very desirable to have a more precise knowledge of $|g_1|^2$, since the uncertainty quoted in Eq. (6.7) is determined mainly by the error in $|g_1|^2$. It would be especially desirable to have an experimental value of (W_d/W_γ) since it is independent of the completeness approximation used in the calculation of W_{abs}. The ratios (6.5) and (6.6) may be considered independent predictions of the two-nucleon model, to be compared with experimental data when they become available.

An examination of Eq. (3.34) and Fig. 1 shows that the two terms in the proton-momentum spectrum which depend on $g_0^* g_1$ namely, W_{13} and W_{14}, are both very small. Thus the calculated value of W_{abs}/W_γ, Eq. (6.8), is almost completely independent of whether

[38] J. Hamilton and W. S. Woolcock, Phys. Rev. **118**, 291 (1960).
[39] M. Gell-Mann and K. M. Watson, Ann. Rev. Nucl. Sci. **4**, 219 (1954).

$\arg(g_0/g_1)=0$ or 180°. The deuteron rate W_d is, on the other hand, less by a factor of 4 for $\arg(g_0/g_1)=180°$ as compared to its value for $\arg(g_0/g_1)=0$. The ratio $W_{\rm abs}/W_d$ therefore provides an independent way of determining the relative sign of g_0 and g_1. We would, of course, expect to find that $\arg(g_0/g_1)=0$ as it is in the capture by He^4, in accordance with the idea that g_0 and g_1 are essentially independent of the particular nucleus in which the capture takes place. Once the phase of g_0/g_1 is fixed at either 0 or 180°, the ratio $W_{\rm abs}/W_d$ is very insensitive to the value of $g_0/g_1(=x)$, for x having a value near unity. In fact, if we take $\int W_{11}(p_1)dp_1 =\int W_{33}(p_1)dp_1$ and neglect the contribution of the other terms (they are all small, and cancel each other out), then $(d/dx)[W_{\rm abs}/W_d(x)]=0$ for $x=1$. It is therefore not possible to determine with any accuracy the value of g_0/g_1 from a measurement of $W_{\rm abs}/W_d$.

In the comparison above, we have ignored the effects of P-orbit capture on the basis of the result of Sec. 4 that $W_{\rm abs}{}^P/W_{\rm rad}(2P\rightarrow 1S)\approx 4\%$. However, the fact that this ratio is as large as 4% in He^3 means that already in He^4, a considerable fraction of the capture processes may go from the P orbit, since the nuclear capture rate in He^4 is greater than that in He^3 by more than an order of magnitude, while the $2P\rightarrow 1S$ radiative rate is essentially unchanged. Of course, if the mechanism suggested by Condo is indeed the correct one for π^- capture in helium, there is no need to consider P-orbit capture at all.

We finally conclude that the process of pion absorption can indeed be satisfactorily described by using nuclear wave functions which are known to be correct for large internucleon separations, provided that we use a two-nucleon Hamiltonian which incorporates the effects of close correlations. It is therefore difficult to learn much more from the study of pion absorption in a complex nucleus than one can already from studying the inverse process for the two-nucleon system. The idea that the form of the close correlations is to a large extent insensitive to the presence of other nucleons, and is determined essentially by the nucleon-nucleon force has been used in nuclear physics calculations before. Two typical examples are its use in the construction of variational wave functions by Austern and Iano[40] and in the study of nuclear matter by Moszkowski and Scott.[41] The results of this calculation tend to support this idea.

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APPENDIX: EVALUATION OF $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, AND $\mathbf{I}_4$

(1) Deuteron Mode: S Orbit

With the wave functions given by Eqs. (3.12) and (3.15), $\mathbf{I}_{1,d}{}^S$ is

$$\begin{aligned}\mathbf{I}_{1,d}{}^S&=V^{-1}N_i\varphi_\pi{}^S(0)\int d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3\delta(\mathbf{x}_2-\mathbf{x}_3)\{i(\boldsymbol{\nabla}_2-\boldsymbol{\nabla}_3)\phi_d(|\mathbf{x}_1-\mathbf{x}_2|)\exp[-i\{\tfrac{1}{2}\mathbf{q}\cdot(\mathbf{x}_1+\mathbf{x}_2)+\mathbf{p}\cdot\mathbf{x}_3\}]\}\\&\qquad\qquad\times\exp[-\tfrac{1}{2}\lambda\{(\mathbf{x}_1-\mathbf{x}_2)^2+(\mathbf{x}_2-\mathbf{x}_3)^2+(\mathbf{x}_3-\mathbf{x}_1)^2\}]\\&=V^{-1}N_i\varphi_\pi{}^S(0)\int d\mathbf{x}_1d\mathbf{x}_2\exp[-i\{\tfrac{1}{2}\mathbf{q}\cdot(\mathbf{x}_1+\mathbf{x}_2)+\mathbf{p}\cdot\mathbf{x}_2\}]\exp[-\lambda(\mathbf{x}_1-\mathbf{x}_2)^2]\\&\qquad\qquad\times\{(\tfrac{1}{2}\mathbf{q}-\mathbf{p})\phi_d(|\mathbf{x}_1-\mathbf{x}_2|)+i(\hat{x}_1-\hat{x}_2)\phi_d'(|\mathbf{x}_1-\mathbf{x}_2|)\}\,,\end{aligned}\tag{A1}$$

where the prime on ϕ_d denotes differentiation with respect to $|\mathbf{x}_1-\mathbf{x}_2|$ and $\hat{x}$ is the unit vector along $\mathbf{x}$. In terms of the new variables

$$\mathbf{u}=\mathbf{x}_1-\mathbf{x}_2\,,\quad \mathbf{v}=\tfrac{1}{2}(\mathbf{x}_1+\mathbf{x}_2)\,,\tag{A2}$$

(A1) may be rewritten as

$$\mathbf{I}_{1,d}{}^S=V^{-1}N_i\varphi_\pi{}^S(0)\int d\mathbf{u}d\mathbf{v}\exp[-i\mathbf{v}\cdot(\mathbf{q}+\mathbf{p})]\exp(\tfrac{1}{2}i\mathbf{u}\cdot\mathbf{p}-\lambda u^2)\{(\tfrac{1}{2}\mathbf{q}-\mathbf{p})\phi_d(u)+i\hat{u}\phi_d'(u)\}\,.\tag{A3}$$

Integration over $\mathbf{v}$ now gives the momentum-conservation δ function. Integration over the directions of $\mathbf{u}$ and a further integration by parts over u then lead to the form given in Eqs. (3.16), (3.17), and (3.18).

By interchanging $\mathbf{x}_2$ and $\mathbf{x}_3$ in the integrand of Eq. (3.8) and noting that $F_i(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$ is symmetric under this interchange, it is immediately seen that $\mathbf{I}_{2,d}{}^S=-\mathbf{I}_{1,d}{}^S$. $\mathbf{I}_{3,d}{}^S$ is zero as a consequence of the symmetry of the final-state wave function in $\mathbf{x}_1$ and $\mathbf{x}_2$; because of this symmetry, the integrand changes sign under the change of variables $\mathbf{x}_1\leftrightarrow\mathbf{x}_2$. The substitutions $\mathbf{x}_1\rightarrow\mathbf{x}_2$, $\mathbf{x}_2\rightarrow\mathbf{x}_3$, $\mathbf{x}_3\rightarrow\mathbf{x}_1$ show that $\mathbf{I}_{4,d}{}^S=\mathbf{I}_{1,d}{}^S$.

[40] N. Austern and P. Iano, Nucl. Phys. **18**, 672 (1960).
[41] S. A. Moszkowski and B. L. Scott, Ann. Phys. (N. Y.) **11**, 65 (1960).

(2) Proton Mode: *S* Orbit

With the wave functions given by (3.12) and (3.25), we have

$$\mathbf{I}_{1,p}{}^{S}=V^{-3/2}N_i\varphi_\pi{}^S(0)\int d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3\delta(\mathbf{x}_2-\mathbf{x}_3)\{i(\boldsymbol{\nabla}_2-\boldsymbol{\nabla}_3)\exp[-i\sum_1^3\mathbf{p}_i\cdot\mathbf{x}_i]\}$$
$$\times\exp[-\tfrac{1}{2}\lambda\{(\mathbf{x}_1-\mathbf{x}_2)^2+(\mathbf{x}_2-\mathbf{x}_3)^2+(\mathbf{x}_3-\mathbf{x}_1)^2\}]$$
$$=V^{-3/2}N_i\varphi_\pi{}^S(0)(\mathbf{p}_2-\mathbf{p}_3)\int d\mathbf{x}_1d\mathbf{x}_2\exp[-\lambda(\mathbf{x}_1-\mathbf{x}_2)^2]\exp[-i\{\mathbf{p}_1\cdot\mathbf{x}_1+(\mathbf{p}_2+\mathbf{p}_3)\cdot\mathbf{x}_2\}]. \qquad \text{(A4)}$$

The change of variables (A2) and integration over $\mathbf{v}$ again give the momentum-conservation δ function:

$$\mathbf{I}_{1,p}{}^{S}=V^{-3/2}(2\pi)^3N_i\varphi_\pi{}^S(0)\delta(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3)\int d\mathbf{u}\exp[\tfrac{1}{2}i\mathbf{u}\cdot(\mathbf{p}_2+\mathbf{p}_3-\mathbf{p}_1)-\lambda u^2]. \qquad \text{(A5)}$$

The integration over $\mathbf{u}$ can now be done analytically, leading to Eq. (3.26) for $\mathbf{I}_{1,p}{}^S(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)$.

As in the case of $\mathbf{I}_{2,d}{}^S$, $\mathbf{I}_{2,p}{}^S(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)=\mathbf{I}_{1,p}{}^S(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)$. Similar interchanges of $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ also lead to Eqs. (3.28) and (3.29).

(3) Proton Mode: *P* Orbit

After integration over $\mathbf{x}_3$ using the δ function, Eq. (4.1) is

$$\mathbf{I}_{1,p}{}^{P}=V^{-3/2}N_i(\mathbf{p}_2-\mathbf{p}_3)\int\varphi_\pi{}^P(\tfrac{1}{3}(\mathbf{x}_2-\mathbf{x}_1))\exp[-i\{\mathbf{p}_1\cdot\mathbf{x}_1+(\mathbf{p}_2+\mathbf{p}_3)\cdot\mathbf{x}_2\}]\exp[-\lambda(\mathbf{x}_1-\mathbf{x}_2)^2]d\mathbf{x}_1d\mathbf{x}_2. \qquad \text{(A6)}$$

The change of variables (A2) and integration over $\mathbf{v}$ lead to

$$\mathbf{I}_{1,p}{}^{P}=V^{-3/2}N_i(2\pi)^3\delta(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3)(\mathbf{p}_2-\mathbf{p}_3)\int d\mathbf{u}\;\varphi_\pi{}^P(-\mathbf{u}/3)\exp[-\tfrac{1}{2}i\mathbf{u}\cdot\mathbf{p}'-\lambda u^2],\quad(\mathbf{p}'=\mathbf{p}_1-\mathbf{p}_2-\mathbf{p}_3)$$
$$\equiv V^{-3/2}N_i(2\pi)^3\delta(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3)(\mathbf{p}_2-\mathbf{p}_3)A\,,\quad\text{say.} \qquad \text{(A7)}$$

Substituting for $\varphi_\pi{}^P$ from Eq. (4.2), the integral A is given by

$$A=\frac{1}{(4\pi)^{1/2}}\int du\,d(\cos\vartheta)d\chi\,u^2\exp[-\tfrac{1}{2}i\mathbf{u}\cdot\mathbf{p}'-\lambda u^2]R_{21}(u/3)P_1{}^m(\cos\vartheta)\exp(im\chi)$$
$$=\frac{1}{\sqrt{\pi}}\int_0^\infty du\int_{-1}^{+1}dy\exp(-\tfrac{1}{2}iup'y)yu^2\exp(-\lambda u^2)R_{21}(u/3)$$
$$=i(4\pi)^{1/2}\int du\,u^2\exp(-\lambda u^2)R_{21}(u/3)\left\{\frac{\cos(\frac{1}{2}up')}{(\frac{1}{2}up')}-\frac{\sin(\frac{1}{2}up')}{(\frac{1}{2}up')^2}\right\}. \qquad \text{(A8)}$$

Approximating R_{21} by

$$R_{21}(u/3)=2u/3^{3/2}a^{5/2},$$

we get

$$A=(8\pi^{1/2}i/3^{3/2}a^{5/2})(A_1-A_2), \qquad \text{(A9)}$$

where

$$A_1=\frac{1}{p'}\int_0^\infty\exp(-\lambda u^2)u^2\cos(\tfrac{1}{2}up')du=\frac{\pi^{1/2}}{4p'\lambda^{3/2}}\left(1-\frac{p'^2}{8\lambda}\right)\exp\left(\frac{-p'^2}{16\lambda}\right),$$

$$A_2=\frac{2}{p'^2}\int_0^\infty\exp(-\lambda u^2)u\sin(\tfrac{1}{2}up')du=\frac{\pi^{1/2}}{4p'\lambda^{3/2}}\exp\left(\frac{-p'^2}{16\lambda}\right).$$

So

$$A=(-i\pi p'/3^{3/2}4a^{5/2}\lambda^{5/2})\exp(-p'^2/16\lambda), \qquad \text{(A10)}$$

leading to Eq. (4.4) of the text.