Determination of the Nucleon-Nucleon Elastic Scattering Matrix. III. Phase-Shift Analyses of Experiments Near 25 and 50 MeV

H. PIERRE NOYES
Stanford Linear Accelerator Center, Stanford University, Stanford, California

AND

DAVID S. BAILEY, RICHARD A. ARNDT, AND MALCOLM H. MACGREGOR
Lawrence Radiation Laboratory, University of California, Livermore, California
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Phase-shift analyses have been carried out for (p,p) and (n,p) experiments near 25 and 50 MeV. Analyses of both (p,p) data and of combined (p,p) plus (n,p) data were made at these energies. The solutions thus obtained extrapolate to the Stapp type 1 solution found at higher energies. Error matrices were calculated for all solutions. The $T=1$ phase shifts from the (p,p) and combined (p,p) plus (n,p) analyses were in good agreement. It was not possible to obtain determinations of the pion-nucleon coupling constant g^2.

I. INTRODUCTION

FAIRLY complete collections of (p,p) and (n,p) nucleon-nucleon scattering data are now available at six energy bands spanning the elastic scattering energy region. In Paper I of this series,[1] we gave the results of phase shift analyses near 142 MeV. Paper II[2] included analyses near 95, 142, 210, and 310 MeV. Paper III, the present article, completes the series by giving the results of analyses near 25 and 50 MeV.

The notation and the method of analysis in the present paper are identical with those of the previous papers, and the reader is referred to them for a more detailed discussion of many points that are mentioned only briefly here.

Section II describes the analyses near 25 MeV. Section III describes the analyses near 50 MeV. In Sec. IV, we compare these results with the results from I and II and show that a unique and consistent set of $T=0$ and $T=1$ phase shifts is obtained that covers the elastic scattering region from 25 to 310 MeV.

As in our previous analyses, we have used each set of data at the correct experimental energy and have given the phase shifts an energy dependence that is obtained from the results of analyses at all energies (Figs. 1 and 2 of II).

II. ANALYSIS NEAR 25 MeV

The data used in the analysis near 25 MeV[3–9] are listed in Table I. The (p,p) data include a complete differential cross section,[3] two polarization points,[4,5] and three values each for $R(\theta)$ and $A(\theta)$.[6] The differential cross-section data[3] are absolute measurements and do not need to be normalized to match total cross-section values. The polarization data[4,5] serve mainly to show that the maximum polarization at these energies is very small (less than 0.5%). While the (p,p) cross-section data together with a total of eight pieces of P, R, and A data do not constitute a "complete set," this fact is partially offset by the consideration that at 25 MeV only a very few partial waves enter significantly into the scattering. The (p,p) scattering matrix that we obtained from an analysis of these data is quite well determined.

The (n,p) data include only differential cross-section[7,8] and polarization[9] measurements. Again, we are aided in the analysis of these data by the fact that only a few partial waves contribute significantly. The (n,p) differential cross section[7,8] is almost isotropic, showing the dominance of S-wave scattering. For the first combined analyses, only the 27.5-MeV cross-section data[8] were included. In the final searches, both the 22.5[7] and the 27.5-MeV[8] (n,p) differential cross-section data were included, so as to span the entire angular scattering range. The 27.5-MeV data[8] carry an absolute normalization error, as shown in Table I. The 22.5-MeV measurements[7] are relative data and were allowed to float freely.

The results of analyzing the (p,p) data are illustrated in the first three data columns of Table II. With 32 pieces of data, a five-parameter search problem has an expected χ^2 of 27. Five free phases gave a χ^2 of 16, showing that S, P, and D waves give sufficient freedom for an accurate fit to the data. When ϵ_2 was included as a sixth free parameter, the error matrix still gave small statistical errors. However, when $^3F_2$ was also included as a free parameter, the χ^2 decrease was small,

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[2] M. H. MacGregor and R. A. Arndt, preceding paper, Phys. Rev. **139**, B362 (1965).
[3] T. H. Jeong, L. H. Johnston, D. E. Young, and C. N. Waddell, Phys. Rev. **118**, 1080 (1960). The 90.00° value is from C. J. Batty, G. H. Stafford, and R. S. Gilmore, Nucl. Phys. **51**, 225 (1964).
[4] P. Christman and A. E. Taylor, Nucl. Phys. **41**, 388 (1963). O. N. Jarvis and B. Rose (Harwell preprint) state that these data should be multiplied by a factor 0.89. This correction came in after our analysis was completed. However, the experimental errors are so large that this change would make no difference in our results.
[5] C. J. Batty, R. S. Gilmore, and G. H. Stafford, Nucl. Phys. **45**, 481 (1963).
[6] A. Ashmore, M. Devine, S. J. Hoey, J. Litt, M. E. Shephard, R. C. Hanna, L. P. Robertson, and B. W. Davies (private communication). The 27.6 MeV R and A values should be regarded as preliminary data.
[7] E. R. Flynn and P. J. Bendt, Phys. Rev. **128**, 1268 (1962).
[8] J. P. Scanlon, G. H. Stafford, J. J. Thresher, P. H. Bowen, and A. Langsford, Nucl. Phys. **41**, 401 (1963). [See Ref. 16.]
[9] R. B. Perkins and J. E. Simmons, Phys. Rev. **130**, 272 (1963).

Table I. Data used in final (p,p) (set A) and combined (p,p) plus (n,p) (set D) phase-shift analyses near 25 MeV.

Energy (MeV)	Type of data	C. M. angle (deg)	Datum	Experimental error	Normalization error	Renormalized value Set A	Set D	Ref.
25.63	$\sigma(\theta)$ (p,p)	10.07	109.6	2.925	0.0093	1.000	0.986	3
		12.08	56.31	0.918				
		14.09	33.20	0.295				
		16.11	23.76	0.176				
		18.12	19.90	0.147				
		19.13	18.70	0.139				
		20.13	17.98	0.133				
		22.15	17.33	0.128				
		24.16	17.09	0.126				
		25.16	17.16	0.127				
		26.17	17.17	0.127				
		28.18	17.30	0.128				
		30.19	17.43	0.129				
		32.21	17.68	0.131				
		34.22	17.80	0.133				
		36.23	17.93	0.133				
		40.25	18.20	0.135				
		44.27	18.33	0.136				
		50.30	18.52	0.137				
		60.34	18.56	0.137				
		70.37	18.65	0.138				
		80.38	18.60	0.138				
		90.39	18.59	0.138				
25.62		90.00	18.30	0.11				3
27.4	$P(\theta)$ (p,p)	45	0.0031	0.0046				4
30	$P(\theta)$ (p,p)	45	−0.0004	0.0033	0.04	1.001	1.001	5
27.6	$R(\theta)$ (p,p)	23.2	−0.324	0.054	0.03	0.997	1.001	6
		39.0	−0.187	0.030				
		54.6	−0.243	0.032				
27.6	$A(\theta)$ (p,p)	23.2	0.012	0.030	0.03	1.001	1.000	6
		39.0	0.037	0.025				
		54.6	0.090	0.022				
22.5	$\sigma(\theta)$ (n,p)	65	33.3	1.9	∞		0.969	7
		75	32.8	1.5				
		85	32.7	1.2				
		95	33.2	1.2				
		105	32.2	1.2				
		115	32.4	1.3				
		125	34.7	1.4				
		135	35.6	1.5				
		145	37.0	1.7				
		155	34.0	1.9				
		165	34.3	2.6				
		175	35.9	4.3				
27.5	$\sigma(\theta)$ (n,p)	7	28.5	1.05	0.03		0.998	8
		14	28.5	1.34		[See note added in proof.]		
		21	29.6	1.54				
		31	28.3	0.99				
		41	27.5	1.18				
		51	27.3	0.87				
		62	26.5	1.22				
		72	27.0	0.97				
		159	25.3	0.81				
		166	26.8	0.75				
		173	29.9	0.93				
23.1	$P(\theta)$ (n,p)	50	0.0492	0.0140	0.017		0.999	9
		70	0.0529	0.0100				
		90	0.0522	0.0071				
		110	0.0310	0.0071				
		130	0.0247	0.0090				
		150	−0.0036	0.0090				

and the error limits on several of the phases increased considerably. Hence, for the (p,p) analysis, the addition of $^3F_2$ represents too much freedom in the searched parameters. As we will see below, when the (n,p) data are also included in the problem (with the validity of charge independence assumed), then $^3F_2$ seems to be more accurately determined.

Our attention in this paper was focused only on the Stapp type 1 solution. As a check, a (p,p) problem was tried in which the starting point was far removed from

TABLE II. Phase-shift solutions at 25 MeV.

(p,p) data	32	33[a]	32	32	32	32	32	32	
(n,p) data				29	29	29	29	29	
χ^2	16.44	17.19	16.23	40.09	40.37	38.78	39.43	40.02	Slope
g^2	13	*CR* 21	13	13	13	11	13	15	(°/MeV)
$^1S_0$	50.24±0.40	49.04±0.30	50.09±0.61	49.97±0.35	49.97±0.35	50.14±0.45	50.16±0.45	50.12±0.44	−0.5
$^1D_2$	0.63±0.04	0.74±0.03	0.69±0.23	0.65±0.03	0.65±0.03	0.61±0.17	0.60±0.17	0.62±0.18	0.032
$^3P_0$	6.23±0.73	7.14±0.56	6.94±1.78	6.01±0.71	6.01±0.71	5.57±0.85	5.56±0.86	5.60±0.87	0.4
$^3P_1$	−3.12±0.58	−3.56±0.43	−3.12±0.58	−3.65±0.37	−3.66±0.38	−3.52±0.42	−3.48±0.43	−3.48±0.43	−0.16
$^3P_2$	1.81±0.29	2.17±0.22	1.84±0.29	2.14±0.10	2.15±0.11	2.01±0.15	2.03±0.15	2.05±0.16	0.16
ϵ_2			−0.87±0.43			−0.72±0.43	−0.70±0.43	−0.76±0.47	−0.03
$^3F_2$			−0.14±0.53			0.25±0.16	0.26±0.16	0.27±0.17	0.01
$^1P_1$				0.52±0.87	0.64±1.28	0.50±0.90	0.66±0.87	0.82±0.86	−0.08
$^3S_1$				78.40±5.11	78.18±6.06	79.12±5.35	78.70±5.22	78.29±5.04	−1.00
ϵ_1				6.39±1.62	5.79±6.03	6.58±1.54	6.28±1.65	5.91±1.78	0.01
$^3D_1$				−2.77±0.26	−3.17±3.06	−2.40±0.28	−2.68±0.27	−2.97±0.27	−0.12
$^3D_2$					3.79±5.51				

[a] Solution obtained by Signell (see text).

the S-parameter solution minimum, and the phase shift search converged on the correct solution (shown in Table II). Hence the (p,p) data are complete enough to give a good determination of the $T=1$ scattering matrix.

When the combined (p,p) plus (n,p) analysis was carried out, it was found that 5, 6, or 7 free $T=1$ phase shifts together with four free $T=0$ phase shifts gave a well-determined solution. Data column 4 of Table II gives an example. However, when a fifth $T=0$ phase shift $^3D_2$ was included in the analysis, the $T=0$ scattering matrix became almost indeterminate, as shown in the fifth data column of Table II. Hence five free $T=0$ phase shifts represent too much freedom in the combined analysis. Freeing six $T=0$ phases gave even larger experimental uncertainties. This result seems somewhat at variance with a recent Dubna analysis,[10] in which a more restricted data selection gave moderate error limits (less than 8°) when six free $T=0$ phase shifts were included in the search.

The Dubna 25-MeV solution[10] is clearly of the same general type as our solution; however, the large coupling constant ($g^2\approx 36$) used for the one-pion-exchange contribution (OPEC) phases in the Dubna analysis makes a quantitative comparison of the results difficult. The Dubna workers[10] treated g^2 as a free parameter in the search. Our analyses are based on fixed values for g^2. We obtained $\chi^2(g^2)$ parabolas for many combinations of free $T=1$ and $T=0$ phases, but the parabolas gave indeterminate results. At this low an energy, the bulk of the OPEC amplitudes are contained in the phases we have treated as free parameters. Thus, we must rely on results from higher energies in our selection of a value for g^2. From Table XI of II, $g^2=13$ is a reasonable value. As in I and II, we quote our best 25-MeV solution (data columns 6–8 of Table II) for three values of g^2.

We did not make a detailed study to check on the uniqueness of the $T=0$ solution. However, a problem using six free $T=1$ and 4 free $T=0$ phases was started at a point far from the $T=0$ solution, and the search converged back to the expected solution. Hence the data are sufficient to give at least a well-defined local minimum in the $\chi^2(\delta)$ surface. A comparison of the 25-MeV phase shifts with the phases obtained at higher energies, given at the end of this paper, indicates that the solution we have obtained is in fact the continuation of the unique solution that was obtained at 142 MeV.

Another phase shift analysis near 25 MeV has been done by Signell [P. Signell, this issue, Phys. Rev. **139**, B315 (1965)]. The Signell analysis, like ours, uses an energy dependence for the phase shifts. However, Signell does not use OPEC to represent all of the higher l phases, but instead uses phases labeled "CR 21" (which closely resemble OPEC) that are taken from a Signell energy-dependent phase-shift representation. Hence his solution should differ slightly from ours, even though the data selections are essentially identical. The second column of Table II gives the Signell solution that corresponds to our five-phase solution.

III. ANALYSIS NEAR 50 MeV

The data[4–6,8,11–17] used for the analysis near 50 MeV are listed in Table III. The (p,p) data include differ-

[10] Yu. M. Kazarinov, V. S. Kiselev, and V. I. Satarov, Zh. Eksperim. i Teor. Fiz. **46**, 920 (1964) [English transl.: Soviet Phys.—JETP **19**, 627 (1964)].

[11] K. Nisimura, J. Sanada, I. Hayashi, S. Kobayashi, D. C. Worth, H. Imada, N. Ryu, K. Fukunaga, H. Hasai, Sung Baiknung, and Y. Haradate, Institute for Nuclear Study, University of Tokyo INST-45, 1961 (unpublished). We have used the data in the form listed in Ref. 12.

[12] P. Signell, N. R. Yoder, and N. M. Miskovsky, Phys. Rev. **133**, B1490 (1964).

[13] C. J. Batty, R. S. Gilmore, and G. Stafford, Nucl. Phys. **51**, 225 (1964).

[14] T. C. Griffith, D. C. Imrie, G. S. Lush, and A. J. Methringham, Phys. Rev. Letters **10**, 444 (1963).

[15] K. Nisimura, J. Sanada, P. Catillon, K. Fukunaga, T. Hasegawa, H. Hasai, N. Ruy, D. C. Worth, and H. Imada, Progr. Theoret. Phys. (Kyoto) **30**, 719 (1963).

[16] P. H. Bowen, G. C. Cox, G. B. Huxtable, A. H. Langsford, J. P. Scanlon, and J. J. Thresher, Phys. Rev. Letters **7**, 248 (1961). We have used the data in the form given in Table A-7 of Wilson's book (Ref. 17). [See note added in proof.]

[17] R. Wilson, *The Nucleon-Nucleon Interaction* (Interscience Publishers, Inc., New York, 1963).

TABLE III. Data used in final (p,p) (set A) and combined (p,p) plus (n,p) (set D) phase shift analyses near 50 MeV.

Energy, (MeV)	Type of data	C. m. angle, deg	Datum	Exptl. error	Normalization error	Renormalized value Set A	Set D	Ref.
51.50	$\sigma(\theta)$	12.2	8.2	1.1	0.045	1.019	1.020	11
	(p,p)	16.2	6.7	0.47				
		17.2	6.4	0.29				
		18.2	6.4	0.25				
		20.3	6.5	0.26				
		22.3	6.6	0.27				
		24.3	7.0	0.27				
		26.3	7.1	0.28				
		30.4	7.7	0.15				
		35.5	7.7	0.15				
51.8	$\sigma(\theta)$	35.5	7.7	0.15	0.025	1.024	1.024	11
	(p,p)	40.5	7.9	0.16				
		45.5	7.6	0.15				
		50.6	7.9	0.16				
		55.6	7.7	0.15				
		60.7	7.8	0.16				
		70.7	7.6	0.15				
		80.8	8.0	0.16				
		90.8	8.0	0.16				
50.0	$\sigma(\theta)$ (p,p)	90.0	8.34	0.05				13
50.0	$P(\theta)$ (p,p)	45	0.0316	0.0017				5
51.7	$P(\theta)$ (p,p)	60.0	0.0409	0.0102				4
53.2	$P(\theta)$ (p,p)	75	0.0084	0.0086				4
50	$D(\theta)$ (p,p)	70	−0.249	0.075				14
47.8	$R(\theta)$	23.5	−0.318	0.046	0.05	1.004	0.998	6
	(p,p)	39.0	−0.327	0.044				
		54.7	−0.367	0.031				
		71.3	−0.435	0.032				
		87.1	−0.488	0.033				
47.8	$A(\theta)$	23.5	0.017	0.044	0.05	0.993	0.998	16
	(p,p)	39.0	−0.001	0.038				
		54.6	−0.000	0.033				
		71.3	0.084	0.029				
		87.1	0.223	0.028				
52	$C_{NN}(\theta)$ (p,p)	90	−0.034	0.095				15
52	$C_{KP}(\theta)$ (p,p)	90	0.130	0.11				15
52.5	$\sigma(\theta)$	7	18.36	0.70	0.02		1.018	8
	(n,p)	14	18.02	0.81				
		21	18.18	0.93		[See note added in proof.]		
		31	14.89	0.54				
		41	14.16	0.65				
		51	12.91	0.44				
		62	12.36	0.62				
		72	10.98	0.43				
		82	10.88	0.66				
		92	10.36	0.47				
		102	9.5	0.71				
		112	11.12	0.88				
		78	11.60	1.12	0.02		0.982	8
		88	12.61	1.07				
		98	11.38	0.96				
		108	11.64	0.44				
		118	12.20	0.41				
		129	12.04	0.48				
		139	12.91	0.52				
		149	14.83	0.73				
		159	14.99	0.78				
		166	17.80	0.77				
		173	17.01	0.90				
50.0	$P(\theta)$	20.0	0.106	0.032	0.05		0.975	16, 17
	(n,p)	30	0.142	0.028				
		40	0.167	0.026				
		50	0.213	0.029				
		60	0.244	0.035				
		70	0.201	0.035				
		120	0.097	0.020				
		140	0.030	0.017				

ential cross sections,[11–13] polarization,[4,5] $D(\theta)$,[14] $R(\theta)$,[6] $A(\theta)$,[6] and 90° measurements of C_{NN} and C_{KP}.[15] For the differential cross-section data,[11] we have used the relative errors as listed by Signell *et al.*[12] These data,[11] which are relative data only, are in almost exact agreement with the absolute measurement of Batty *et al.*[13] when the difference in energy is taken into account. The (n,p) data include differential cross-section[8] and polarization[16] measurements.

An early analysis of (p,p) data near 50 MeV was carried out by the Japanese group at Kyoto.[18] An analysis by Batty and Perring[19] included both (p,p) and (n,p) data. Signell and co-workers[12,20] have made recent analyses of the (p,p) data. These do not include the $R(\theta)$ data.[6] The Kyoto group has made a very recent analysis of the (p,p) data in which the $R(\theta)$ data are included.[21]

When the (p,p) data listed in Table III, but *excluding* the $R(\theta)$ data, were analyzed, it was found[12,20] that the $^3F_3$ phase shift should be treated as a free parameter in order to minimize χ^2. When this was done, the $^3F_3$ phase shift searched to a positive value, in disagreement with the results of analyses at other energies [Fig. 1(i) of II]. We repeated this analysis of Signell[12,20] and found essentially identical results for $^3F_3$, whether we did a (p,p) or a combined (p,p) plus (n,p) analysis.[22] However, the addition of the $R(\theta)$ data[6] has reduced this anomalous behavior of the $^3F_3$ phase shift, as we shall see below. This indicates the importance of having a complete data selection to eliminate false minima in the χ^2 surface.

In Table IV, we have summarized the least-squares values χ^2 for several combinations of phenomenological phase shifts. The seven-parameter fits to the (p,p) data give statistically better fits than do any of the six-parameter fits. The eight-parameter fit is even better, but the addition of a ninth free parameter gives little improvement. The inclusion of $^3F_3$ is helpful in minimizing χ^2, but the eight-parameter fit is not drastically better than the seven-parameter fit that does not include $^3F_3$. This is in marked contrast to the results of analyses[12,20,22] in which $R(\theta)$ was not included. For the combined analysis, five free $T=0$ phases together with six or seven $T=1$ phases give a good fit to the (p,p) and (n,p) data. The addition of a sixth $T=0$ phase, $^3D_3$ gives a slight reduction in χ^2, but the errors in the $T=0$ phases become much larger, showing that there is too much freedom in the search parameters. From these results, we selected eight $T=1$ and five $T=0$ phase shifts as the proper number of free parameters in the 50-MeV analysis.

Table V lists the results of our analysis of the 50-MeV (p,p) data. Also included are solutions by Signell[20] and by Hoshizaki and Watari.[21] The effect of the $R(\theta)$ data is illustrated by a comparison of the Signell solution with our corresponding solution. [In Ref. 22, Table III, we show our solution for the analysis with the $R(\theta)$ data excluded. It is essentially identical with the Signell solution shown here, especially as regards the low value for $^3P_0$ and the positive value for $^3F_3$.] The Kyoto solution is based on a data selection similar to ours, and the two sets of phase shifts are in excellent agreement.

Table VI lists the results of the combined (p,p) plus (n,p) analyses. A comparison of the $T=1$ phases in Tables V and VI shows that they are almost identical, as we expect from charge independence. When five $T=0$ phases are used in the combined analysis, the error matrix gives small statistical uncertainties. However, when a sixth phase $^3D_3$ is added, the $T=0$ error limits greatly increase, as shown in data column 5 of Table VI. Hence, we conclude that five is the proper number of $T=0$ phases to include in the search. However, a strong word of caution should be inserted at this point. The value of ϵ_1 as given by our "best" solution ($\epsilon_1=12.8\pm2.5$ deg) does not appear to be consistent with the values for ϵ_1 we obtained at other energies (see Sec. IV). We saw above that 6 kinds of (p,p) experiments at 50 MeV gave an anomalous value for $^3F_3$, and that the addition of a seventh kind of data, $R(\theta)$, reduced this difficulty. We are basing our $T=0$ phase shift results on only two kinds of (n,p) data. While the $T=0$, $T=1$ interference terms in effect give a double weighting to the (n,p) data, the data are in

TABLE IV. χ^2 values for various choices of the phenomenological phase shifts at 50 MeV. The $^1S_0$, $^1D_2$, $^3P_0$, $^3P_1$, and $^3P_2$ phases for $T=1$ and the $^1P_1$, $^3S_1$, ϵ_1, $^3D_1$, and $^3D_2$ phases for $T=0$ were included in addition to the ones listed below. The values are for $g^2=13$.

No. of free phases	Free phases	χ^2
	(p,p) analysis, 36 data	
6	$^3F_2$	30.00
6	ϵ_2	28.00
6	$^3F_3$	26.06
7	$^3F_2$, $^3F_3$	25.57
7	ϵ_2, $^3F_2$	25.18
7	ϵ_2, $^3F_3$	24.51
8	ϵ_2, $^3F_2$, $^3F_3$	23.91
9	ϵ_2, $^3F_2$, $^3F_3$, $^3F_4$	23.84
	(p,p) plus (n,p) analysis, 68 data	
6+5	$^3F_2$	53.32
6+5	ϵ_2	50.50
6+5	$^3F_3$	47.41
7+5	ϵ_2, $^3F_3$	46.01
7+6	ϵ_2, $^3F_3$, $^3D_3$	45.76
8+5	ϵ_2, $^3F_2$, $^3F_3$	45.63
8+6	ϵ_2, $^3F_2$, $^3F_3$, $^3D_3$	45.50

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TABLE V. (p,p) phase-shift solutions at 50 MeV.[a]

(p,p) data	36	36	36	36	36	Signell (Ref. 20)[b]	Kyoto (Ref. 21)
χ^2	28.00	25.18	23.91	23.84	24.51		
g^2	13	13	13	13	13	*CR* 21	14.4
$^1S_0$	37.41±0.51	36.94±0.61	37.54±0.80	37.48±0.85	37.93±0.53	38.07±0.47	36.85±0.60
$^1D_2$	2.27±0.24	2.25±0.25	2.16±0.26	2.14±0.26	2.13±0.24	2.32±0.22	2.28±0.25
$^3P_0$	12.79±0.62	12.99±0.64	12.40±0.91	12.42±0.92	12.08±0.76	10.26±0.71	12.48±0.66
$^3P_1$	−7.61±0.29	−7.89±0.33	−7.93±0.34	−7.95±0.35	−7.85±0.34	−8.04±0.37	−7.93±0.30
$^3P_2$	6.00±0.16	6.22±0.20	6.11±0.23	6.12±0.23	5.99±0.16	6.26±0.17	6.02±0.21
ϵ_2	−2.49±0.24	−2.35±0.26	−2.13±0.33	−2.17±0.36	−2.09±0.32	−2.15±0.25	−2.59±0.28
$^3F_2$		0.80±0.28	0.60±0.36	0.67±0.42			0.61±0.28
$^3F_3$			−0.33±0.40	−0.42±0.50	−0.17±0.32	0.24±0.34	
$^3F_4$				0.15±0.23			

[a] The energy derivatives for the phases of the present analysis are listed in Table VI.
[b] This analysis did not include the $R(\theta)$ data.

no sense complete, and it might be that the anomalous value for ϵ_1 obtained here is an indication of a systematic error due to the incompleteness. The (n,p) data at 50 MeV actually impose very little restriction on the value for ϵ_1. For example, the Dubna workers[10] list three solutions at 52 MeV, with the following values for ϵ_1: $-2.4\pm29.4°$, $9.8\pm4.9°$, $53.8\pm0.7°$. As an experiment, we inserted the hypothetical datum $C_{nn}(55°)$ for (n,p) scattering$=0.37\pm0.037$ into our data selection, and we then obtained the value $\epsilon_1\sim1°$, with the other phases remaining relatively unchanged. We should regard the error matrix value of $\pm2.5°$ for ϵ_1 to be clearly misleading, as it does not reflect systematic errors due to the incompleteness of the data selection. However, we cannot simply choose the six-phase $T=0$ solution as being more nearly correct, since if we free a seventh phase, the error limits become very much larger. Too much freedom leads to a breakdown in the error limit determination.

At 142 MeV, there is a fairly complete (n,p) data collection. At all other energies, only differential cross section and polarization measurements exist. While we have made an analysis at each energy and obtained $T=0$ solutions that appear to be reasonable extrapolations from the solution at 142 MeV, there may be systematic errors at some of these energies, due to the data incompleteness, that are not reflected in the error matrix limits. The value of ϵ_1 at 50 MeV may be an example.

In Table VI we have listed what we consider to be the best solution for the 50-MeV analysis for g^2 values of 11, 13, and 15. A $\chi^2(g^2)$ parabola for this solution gives the value $g^2=12.4\pm8.6$. This shows that at 50 MeV, as we also found at 25 MeV, the modified analysis does not permit a determination of g^2, as too much of the OPEC amplitudes are contained in the free phase shifts.

The Batty and Perring solution[19] is also shown in Table VI for comparison with our solution. Their analysis did not include the recent $R(\theta)$ data.[6] The $T=0$ phases from their solution are quite similar to our $T=0$ phases. Signell[20] has tried to reproduce their error matrix results, however, and was unable to obtain agreement. (Note that the value we obtain for $^3F_3$ still looks anomalously low compared to results at higher energies, as shown in Table VII.)

TABLE VI. (p,p) plus (n,p) phase-shift solutions at 50 MeV.

(p,p) data	36	36	36	36	36	Batty and Perring	
(n,p) data	32	32	32	32	32	(Ref. 19)	
χ^2	50.50	45.65	45.63	45.72	45.50		Slope
g^2	13	11	13	15	13	14	(°/MeV)
$^1S_0$	37.41±0.51	37.71±0.72	37.67±0.73	37.64±0.73	37.69±0.75	38.45±0.8	−0.35
$^1D_2$	2.26±0.24	2.15±0.25	2.15±0.25	2.14±0.25	2.15±0.25	1.71±0.09	0.043
$^3P_0$	12.50±0.58	11.95±0.82	11.98±0.82	12.01±0.82	11.97±0.83	12.0 ±0.7	0
$^3P_1$	−7.73±0.27	−8.03±0.32	−8.03±0.32	−8.03±0.32	−8.02±0.32	−7.7 ±0.4	−0.22
$^3P_2$	6.04±0.15	6.08±0.22	6.10±0.22	6.12±0.22	6.10±0.22	6.0 ±0.6	0.12
ϵ_2	−2.45±0.24	−2.05±0.32	−2.07±0.32	−2.09±0.32	−2.07±0.32		−0.05
$^3F_2$		0.49±0.31	0.52±0.31	0.54±0.31	0.50±0.33		0.09
$^3F_3$		−0.20±0.38	−0.22±0.38	−0.24±0.38	−0.22±0.39		−0.01
$^1P_1$	−3.57±1.50	−4.47±1.68	−4.22±1.67	−3.97±1.64	−4.20±1.76	−2.0 ±2.0	−0.11
$^3S_1$	60.49±2.32	61.00±2.39	60.82±2.47	60.63±2.56	62.60±6.30	60.8 ±4.3	−0.54
ϵ_1	12.65±2.50	12.84±2.45	12.79±2.49	12.74±2.51	10.78±8.37	9.3 ±6.7	0.02
$^3D_1$	−8.56±0.89	−8.30±0.98	−8.46±1.00	−8.61±1.02	−7.34±4.52	−7.4 ±2.9	−0.18
$^3D_2$	12.78±2.28	12.29±2.49	12.31±2.61	12.31±2.72	11.24±4.40	11.8 ±3.8	0.21
$^3D_3$					0.08±3.04	−0.4 ±1.7	

TABLE VII. Final phase-shift values from the combined (p,p) plus (n,p) phase-shift analyses.

	25 MeV[a]	50 MeV[a]	95 MeV[b]	142 MeV[b]	210 MeV[b]	310 MeV[b]
$^1S_0$	50.16±0.45	37.67±0.73	25.08±2.41	16.63±0.73	5.14±0.56	−6.94±1.63
$^1D_2$	0.60±0.17	2.15±0.25	3.48±0.38	5.17±0.22	7.05±0.32	8.98±0.72
$^1G_4$	(0.04)[c]	(0.14)	(0.34)	0.58±0.12	1.03±0.15	1.41±0.33
$^3P_0$	5.56±0.86	11.98±0.82	12.83±1.88	6.32±0.57	−0.68±0.60	−11.34±1.66
$^3P_1$	−3.48±0.43	−8.03±0.32	−12.95±0.50	−17.13±0.41	−21.59±0.61	−28.48±1.27
$^3P_2$	2.03±0.15	6.10±0.22	10.55±0.53	13.74±0.22	15.88±0.27	16.37±0.67
ϵ_2	−0.70±0.43	−2.07±0.32	−2.75±0.33	−2.89±0.15	−2.75±0.19	−3.04±0.50
$^3F_2$	0.26±0.16	0.52±0.31	1.29±0.74	0.75±0.32	1.55±0.35	0.88±0.69
$^3F_3$	(−0.23)	−0.22±0.38	−1.62±0.59	−2.08±0.21	−2.57±0.22	−3.00±0.67
$^3F_4$	(0.02)	(0.07)	0.62±0.24	0.92±0.17	2.32±0.20	2.67±0.40
ϵ_4	(−0.05)	(−0.18)	(−0.48)	−0.64±0.07	−0.94±0.09	−0.99±0.32
$^3H_4$	(0.00)	(0.02)	(0.09)	0.16±0.08	0.43±0.34	1.28±0.39
$^3H_5$	(−0.01)	(−0.08)	(−0.27)	−0.48±0.14	−0.62±0.21	−1.46±0.50
$^3H_6$	(0.00)	(0.01)	(0.03)	(0.07)	0.38±0.25	0.65±0.28
$^1P_1$	0.66±0.87	−4.22±1.67	−16.53±3.34	−13.45±2.43	−21.83±4.00	−30.07±3.19
$^1F_3$	(−0.42)	(−1.12)	(−2.13)	−0.93±0.70	−6.51±1.85	−3.82±1.18
H_5	(−0.03)	(−0.16)	(−0.49)	(−0.81)	(−1.19)	(−1.59)
$^3S_1$	78.70±5.22	60.82±2.47	44.54±1.71	29.65±0.90	17.60±2.49	−1.01±5.19
ϵ_1	6.28±1.65	12.79±2.49	−0.61±1.74	1.40±0.74	3.72±2.27	21.67±4.29
$^3D_1$	−2.68±0.27	−8.46±1.00	−11.35±0.81	−14.94±0.70	−22.13±3.10	−23.19±3.16
$^3D_2$	(3.13)	12.31±2.61	13.59±2.62	24.42±1.02	24.63±2.37	22.01±2.32
$^3D_3$	(−0.21)	(−0.71)	1.93±0.63	0.91±0.91	3.07±1.41	3.08±1.41
ϵ_3	(0.55)	(1.59)	(3.34)	2.56±0.57	6.90±0.71	8.12±0.95
$^3G_3$	(−0.06)	(−0.25)	(−0.77)	−2.54±0.54	−0.68±1.38	−5.28±1.86
$^3G_4$	(0.17)	(0.69)	(1.88)	4.64±0.93	(4.67)	(6.60)

[a] Present paper.
[b] Paper II, Ref. 2.
[c] OPEC phases with $g^2=13$ and $M_\pi=136.5$ MeV.

IV. SUMMARY OF ENERGY-INDEPENDENT ANALYSES

In Table VII we have listed the final phase shift values from the present analyses and from those given in Paper II, Ref. 2. We have in each case listed the solution corresponding to the combined (p,p)-plus-(n,p) analysis, as this appears to us to give the most reliable values for the $T=1$ phase shifts. We have included as many free phase shifts as the data permitted at each energy.

The $T=1$ phase shifts form a unique and reasonably consistent set of phases spanning the elastic energy region. The $T=0$ phase shifts are naturally not as well-determined, although the results shown in Table VII are surprisingly good in view of the fact that, except at 142 MeV, they are based solely on differential cross-section and polarization measurements. However, the reader is advised to heed the words of caution contained in Sec. III of this paper with regard to the $T=0$ phase shifts.

Note added in proof. The data of Refs. 8 and 16 were taken from Wilson's book.[17] The errors for Ref. 8 should be increased somewhat, and newer data are available to replace Ref. 16. These changes, along with others, have been incorporated in paper IV of this series. These changes have only a slight effect on the results of the analyses. Paper IV (to be submitted to Phys. Rev.) includes energy-dependent and energy-independent analyses of the data included in I–III.

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