Analysis of the Proton-Proton Scattering Data Near 27 MeV*

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Phase-shift analyses were made of 33 proton-proton single-, double-, and triple-scattering data in the energy range 25–30 MeV. The resulting good definition of the phase shifts was marred somewhat by an undesirable sensitivity to the single datum $R(39°)$. The phases were in moderate agreement with those of recent nucleon-nucleon models.

I. INTRODUCTION

In previous communications,[1–4] extensive phase-shift analyses of the proton-proton scattering data near 50, 96, 142, 213, and 310 MeV have been shown to result in very good definition of most of the phases at those energies. Although rather good cross-section measurements had long been available at lower energies, it was felt that the lack of double- and triple-scattering experiments precluded useful analyses there. The triple-scattering parameters R and A have now been measured at 27.6 MeV by Ashmore *et al.*,[5] at the Rutherford Laboratory: In combination with the previously mentioned cross-section data, they are here shown to fix the phase shifts near 27 MeV to approximately the same precision as that already found at 50 MeV.[4]

II. DATA SELECTION AND TREATMENT

There are 34 proton-proton scattering data available in the energy range 21–31 MeV. One of these data, a polarization measurement[6] nominally at 27 MeV, had such a large uncertainty in energy that it was not possible to use it properly. The remaining 33, ranging in energy from 25.62 to 30 MeV, are listed in Table I.

In a previous analysis[4] at 50 MeV, an energy-dependent phase-shift representation labeled CR21 was used to interpolate the data to a single energy. A much simpler, but roughly equivalent procedure was used here. The data were used at the experimental energies, with the phase shifts now having the CR21 energy dependence. More precisely, each phase was equally shifted from its CR21 values at the several energies. Thus each energy-dependent phase shift was varied in order to produce a least-squares fit to the data, and to determine the phase-shift standard deviations.

The R and A data contained a possible systematic error of $\pm 3\%$ in addition to that shown in Table I. This was considered insignificant in comparison to the errors shown in Table I so it was disregarded.

III. ANALYSIS RESULTS

The method of analysis followed was the same as that used previously[4] for the 50-MeV data. The higher angular-momentum (L) phases were fixed at their CR21

TABLE I. Data available in the energy range 21–30 MeV. N_σ indicates (absolute) normalization for the relative σ's which follow it. The cross section values are in mb/sr.

Lab system energy (MeV)	C.m. angle (degrees)	Type	Value	Error	Reference
25.62	90.00	σ_{abs}	18.30	0.11	a
25.63		N_σ	1.000	0.008	b
	10.07	σ	109.60	2.97	b
	12.08		56.31	0.89	
	14.09		33.20	0.30	
	16.11		23.76	0.18	
	18.12		19.90	0.15	
	19.13		18.70	0.13	
	20.13		17.98	0.13	
	22.15		17.33	0.13	
	24.16		17.09	0.13	
	25.16		17.16	0.13	
	26.17		17.17	0.13	
	28.18		17.30	0.13	
	30.19		17.43	0.13	
	32.21		17.68	0.13	
	34.22		17.80	0.13	
	36.23		17.93	0.13	
	40.25		18.20	0.13	
	44.27		18.33	0.13	
	50.30		18.52	0.13	
	60.34		18.56	0.13	
	70.37		18.65	0.13	
	80.38		18.60	0.13	
	90.39		18.59	0.13	
27.6	23.2	R	−0.324	0.054	c
	39.0		−0.187	0.030	
	54.6		−0.243	0.032	
27.6	23.2	A	0.012	0.030	c
	39.0		0.037	0.025	
	54.6		0.090	0.022	
28.16	90.0	σ_{abs}	16.27	0.31	d
30.00	45.0	P	−0.0004	0.0033	e

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TABLE II. Results of the phase-shift analyses of the 33 data, with the higher L phases fixed at the CR21 values (see Table IV and text). The number of free, searched-upon, phases is denoted by N. "Phase" indicates the phase shift just released from its CR21 value. χ^2 is the least-squares error sum, M is the number of degrees of freedom (equal to the expected value of χ^2), and the χ^2 ratio is χ^2/M. The χ^2 and F probabilities are labeled P_q and P_f.

N	Phase	χ^2	M	χ^2 ratio	P_q	P_f
0		63.50	33	1.92		
1	$^1S_0$	43.84	32	1.37		
4	$^3P_{0,1,2}$	27.69	29	0.95	0.54	
5	$^1D_2$	17.19	28	0.61	0.94	<0.01
6	ϵ_2	17.14	27	0.63	0.92	
6	^3F_f	17.08	27	0.63	0.92	
6	$^3F_2$	16.55	27	0.61	0.94	
6	$^3F_4$	15.72	27	0.58	0.95	
6	$^1G_4$	14.74	27	0.55	0.97	0.03

values at the several energies, and the lower L phases were shifted as described in Sec. II until a least-squares fit to the data was obtained. The method used, described in a previous report,[3] guaranteed that the least-squares error sum had indeed been minimized with respect to the low-L phases. Only the solutions of the type commonly called No. 1 were investigated. All other solution types have been rejected by analyses at other energies.[4]

The "comparison representation" CR21 which supplied the higher L phases was designed to simulate the phases resulting from potential models. It predicts, as do potential models, that $\bar{\epsilon}_2$ and all phases with $L>2$ should be very close to their one-pion-exchange (OPE) values. Accordingly, the $^1S_0$, $^3P_{0,1,2}$, and $^1D_2$ were the first phases released from their CR21 values. The result is shown in Table II. The precipitous drop in χ^2 as the phases were released is examined more closely in Table III. The three data displayed in the first three columns contributed over 70% of the χ^2 for CR21, yet amounted to less than 10% of the data. It is apparent from the table that the $^1S_0$ is highly correlated with the absolute cross section, but that changes in all five low-L phases were necessary in order to fit the $R(39°)$ datum.

The phase shifts corresponding to several lines of Table II are listed in Table IV. The preferred sixth phase, $^1G_4$, is seen to move two standard deviations from the OPE value. Since any reasonable model gives the OPE value for $^1G_4$ at this energy, the tendency is to favor the $N=5$ solution.

IV. COMPARISON OF MODELS

The least-squares error sums, for the fit of several recent models to the present data, are compared in Table V. The data predictions were made directly from the model parameters. The Hamada-Johnston (HJ)[7]

TABLE III. Contributions to χ^2 from various members of the 33-piece data set. "Sum" and "remainder" refer to the three columns to the left. Notation as in Tables I and II.

N	Phase	$R(39°)$	$\sigma(90°)$	N_σ	Sum	Remainder	$\sigma_{\rm rel}$	R	A	P
0		12.7	8.7	20.8	42.2	21.3	8.5	14.8	4.8	5.9
1	$^1S_0$	11.2	0.4	4.8	16.4	27.4	15.6	12.6	4.7	5.7
4	$^3P_{1,2,3}$	5.0	0.1	2.1	7.2	20.5	14.1	5.0	4.5	1.9
5	$^1D_2$	1.2	0.4	2.9	4.5	12.7	7.1	3.2	3.3	0.3
6	$^1G_4$	1.3	0.4	1.5	3.2	11.5	6.1	3.4	3.2	0.2

TABLE IV. The 27.6-MeV nuclear-bar phase shifts, in degrees, for two of the $N=5$ phase-shift analyses and for several models (see text). The first line is for the 32-piece data set which omits $R(39°)$. The second line corresponds to the fourth line of Table II. The entries in the line labeled (6,33) are from separate analyses; they correspond to the last five lines of Table II.

Model or (N,No. data)	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$
(5,32)	48.64±0.38	7.62±0.59	−4.12±0.50	2.38±0.24	0.76±0.03
(5,33)	49.04±0.30	7.14±0.56	−3.56±0.43	2.17±0.22	0.74±0.03
CR21	46.64	8.74	−5.66	3.18	0.86
HJ	47.54	8.66	−5.22	2.37	0.81
Yale	46.00	10.02	−6.27	2.87	0.75
FLT-II	47.10	10.65	−5.60	2.60	0.88
OPE ($g^2=14.4$)					0.67
	ϵ_2	$^3F_2$	$^3F_3$	$^3F_4$	$^1G_4$
(6,33)	−0.83±0.11	−0.22±0.32	−0.43±0.32	0.49±0.31	0.01±0.02
CR21	−0.90	0.12	−0.30	0.02	0.50
HJ	−0.90	0.13	−0.26	0.02	0.05
Yale	−1.10	0.13	−0.31	0.04	0.05
FLT-II	−1.01	0.14	−0.30	0.03	0.05
OPE	−0.98	0.13	−0.31	0.02	0.05

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TABLE V. Goodness of fit of recent models (see text) to the 33-piece data set.

Model	χ^2	$\chi^2/\chi^2[N=5]$	Comments
$N=5$	17.2	1.0	Good fit to all data.
FLT-II	51.0	2.9	$R(39°)$ much too low, $A(\theta)$ too high.
CR21	63.5	3.7	$R(39°)$ and $\sigma(90°)$ much too low.
Yale	78.7	4.6	$R(39°)$ much too low.
HJ	77.0	4.7	$R(39°)$ too low, $\sigma(\theta)$ wrong shape.

and Yale[8] models are hard-core-type potentials, while the latest Feshbach-Lomon-Tubis (FLT-II)[9] model is of the boundary-condition type. The FLT-II model is considerably better than the best previous model of its kind, that due to Saylor, Bryan, and Marshak.[10] The Yale energy-dependent solutions[11] YLAM and YRB1 are not listed in Table V, since they have been superseded by the superior[12] energy-dependent solution CR21.

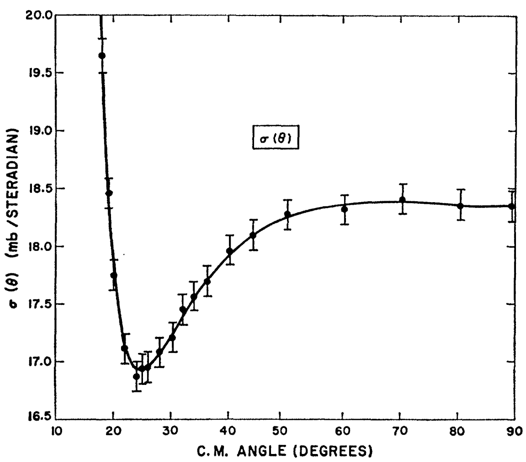

FIG. 1. The unpolarized cross-section data, normalized by $N_\sigma=0.964$. The curve is the prediction of the analysis with five free phases, 33 data. Note the too-good fit to the data.

TABLE VI. Predictions for experimental quantities not shown in the graphs. Notation as in Tables I and IV.

Model	$\sigma(90°)$ 28.16 MeV	$\sigma(90°)$ 25.62 MeV	N_σ	$P(45°)$
CR21	16.12	17.98	0.964	0.008
(5,33)	16.37	18.38	0.987	0.002
(6,33)	16.35	18.36	0.990	0.002
Exp'l	16.27±0.11	18.30±0.11	1.000±0.008	0.0004±0.0033

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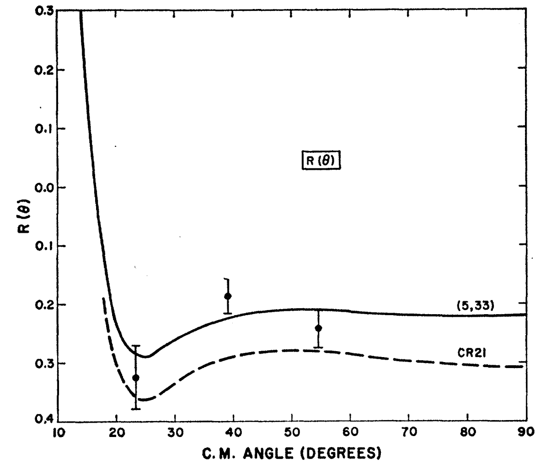

FIG. 2. The rotation data. The solid line is the prediction of the same analysis as in Fig. 1. The dashed curve is the prediction of CR21: it is close to the predictions of the potential models.

Note should be taken that CR21 was adjusted to fit the same data set as was used for YLAM and YRB1, and for the HJ and Yale potential models. In that sense, CR21 should be considered to be of the same vintage as those models.

The better fit of FLT-II at this energy is in marked contrast to its relatively poor fit at higher energies.[3]

Comparison of the model phase shifts with those of the analyses in Table IV shows that all models had too low $^1S_0$ and $^3P_1$ phases. An attempt was made to fit seven $^1S_0$ phases from 1–310 MeV with hard-core and momentum-dependent type potentials. Even then, the predicted 27-MeV phase was always lower than that given by the present analyses.

Another result, shown in Table V, was that the model predictions for $R(39°)$ were much lower than the measured value. Such model unanimity in rejection of a datum has occurred in analyses of higher energy data. Generally, such a datum was then also rejected when Chauvenet's criterion was applied to the phase-shift analyses. Here, however, the $R(39°)$ was fit by the phase-shift analyses to well within Chauvenet's criterion (see Table III). Thus the $R(39°)$ must be retained as a valid datum. The effect of nonetheless

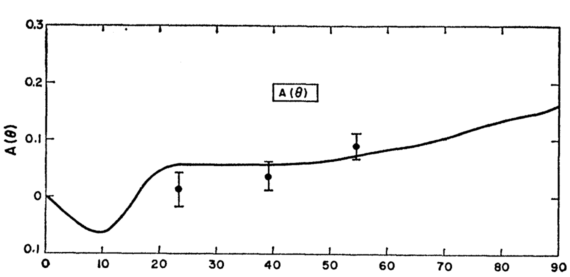

FIG. 3. The $A(\theta)$ data and the prediction of the same analysis as in Fig. 1.

omitting the $R(39°)$ is shown in the first line of Table IV. Comparing to the line below it for the full data set, it is obvious that most of the phases are very sensitive to this datum, both in their values and standard deviations. Such a situation is rather undesirable: One would prefer that the results not depend on a single datum.

V. DETAILED FIT TO THE DATA

The experimental data and the corresponding preditions are displayed in Table VI and Figs. 1–3.

The too-good fit of the relative cross-section data, Fig. 1, is presumably accidental. The $\sigma(90°)$ prediction is within both of the experimental errors, so it is in agreement with both measurements. Removal of either or both of the Minnesota $\sigma(90°)$ and Rutherford $P(45°)$ data had no significant effect on the analyses.

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Analysis of Single Excitations in Inelastic Deuteron Scattering from Ni^{60}, Zr^{92}, and Sn^{120} Nuclei*

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Inelastic scattering of 15-MeV deuterons from Ni^{60}, Zr^{92}, and Sn^{120} nuclei has been studied with adequate resolution to enable identification of almost all states of known spin and parity. Detailed angular distributions of deuteron groups corresponding to well-resolved states of these nuclei have been measured and compared with distorted-wave Born approximation calculations for single excitations using a deformed optical-model potential. The theoretical predictions, including Coulomb excitation and for a complex coupling, are found to be quite successful for strongly excited states. The status of the Blair phase rule is discussed in the context of the aforesaid comparison. Spin and parity assignments are made for several new levels. Excitation energies, differential cross sections, and reduced transition probabilities have been tabulated and compared with previously known values.

I. INTRODUCTION

In a previous paper[1] angular distributions of inelastically scattered deuterons from Ni were reported and the results along with those for Sn and Zr taken from an earlier work of Cohen and Price[2] were examined to check the validity of the Blair phase rule. Blair[3] has shown that, under certain approximations, the angular distribution of inelastically scattered particles is oscillatory, and for angles $\lesssim 60°$ the phase of these oscillations, relative to that of the oscillations in the angular distribution for the elastically scattered particles, depends on the angular momentum transferred in the inelastic process. The angular distributions for the inelastic groups are in or out of phase with that for the elastic group, depending on whether the angular momentum transferred is odd or even, respectively. This result, if true, offers a very convenient tool for parity assignments to nuclear levels, and therefore a detailed investigation of the validity of this rule was undertaken as reported in Ref. 1. The conclusions of the above study were that the phase rule holds for the strongly excited states and that there are important differences between the angular distributions for the strongly excited and weakly excited levels of the same parity. It was also noticed that members of the two-phonon triplet showed angular distributions somewhat like those of negative parity states. On account of its partial success, the phase rule was used for parity assignments only with caution.

However, recent theoretical and experimental work on inelastic scattering of α particles[4] and protons[5] has proved inelastic scattering to be a dependable tool for nuclear spectroscopy, particularly of the strongly excited states. Distorted-wave Born approximation (DWBA) calculations have successfully reproduced the angular distributions for single quadrupole and octupole oscillations using a nonspherical optical-model poten-

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