normalization integral

$$\int_0^\infty d\nu\,|\psi_\beta(\nu)|^2=1\,.$$

In the limit $E\rightarrow E'$, the remaining terms give a derivative to yield

$$B_F=i\pi\sum_{\alpha,\beta}\int_{-\min(B_\alpha,B_\beta)}^{\infty}\left\{dE\,\tilde{F}_{\alpha\beta}(E+B_\alpha;E+B_\beta)\frac{\partial F_{\beta\alpha}}{\partial E}(E+B_\beta;E+B_\alpha)\right.$$
$$\left.-F_{\beta\alpha}(E+B_\beta;E+B_\alpha)\frac{\partial}{\partial E}\tilde{F}_{\alpha\beta}(E+B_\alpha;E+B_\beta)\right\}\,. \quad \text{(B13)}$$

This expression can be rewritten in terms of $T_{\alpha\beta}$ defined in Eq. (5.11),

$$B_F=i\pi\sum_{\alpha,\beta}\int_{-\min(B_\alpha,B_\beta)}^{\infty}dE\left\{T_{\beta\alpha}{}^\dagger\frac{\partial T_{\beta\alpha}}{\partial E}-T_{\beta\alpha}\frac{\partial}{\partial E}T_{\beta\alpha}{}^\dagger\right\}\,. \quad \text{(B14)}$$

Combining Eqs. (B14) and (B8), we have

$$\Lambda_B=i\pi\left[\sum_\alpha\int_0^\infty dE\,\mathrm{Tr}\left\{T_{0\alpha}{}^\dagger\frac{\partial}{\partial E}T_{0\alpha}-T_{0\alpha}\frac{\partial}{\partial E}T_{0\alpha}{}^\dagger\right\}+\sum_{\alpha,\beta}\int_{-\min(B_\alpha,B_\beta)}^{\infty}dE\left\{T_{\alpha\beta}{}^\dagger\frac{\partial}{\partial E}T_{\alpha\beta}-T_{\alpha\beta}\frac{\partial}{\partial E}T_{\alpha\beta}{}^\dagger\right\}\right]\,. \quad \text{(B15)}$$

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Restrictions Implied by Lorentz and Spin Invariance for Scattering Amplitudes

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A combination of Lorentz invariance and spin independence may restrict scattering amplitudes so severely that no interesting reaction can be described. This is demonstrated for the scattering of two spin-$\frac{1}{2}$ particles with spin independence defined as in a definition of $SU(6)$ for quarks.

APPLICATIONS of $SU(3)$ and $SU(6)$ symmetries to strongly interacting particles are achieved in coexistence with a number of theorems[1] delineating a growing class of situations in which a nontrivial combination of the symmetry group with the Poincaré group is impossible. Situations in which these group-theoretic theorems are not applicable are characterized most noticeably by commutators of generators of the symmetry group with generators of the Poincaré group failing to be linear combinations of generators of the two groups. Here a different kind of theorem is to be expected. If a generator of the symmetry group and a generator of the Poincaré group both commute with the scattering operator, then their commutator also commutes with the scattering operator. If this commutator is not a linear combination of the generators of the symmetry and Poincaré groups, it represents an additional symmetry which may put entirely unwanted restrictions on the scattering amplitudes. A particularly simple example of this is demonstrated in the following:

Consider two particles each with positive mass and spin $\frac{1}{2}$. We describe the nth particle ($n=1,2$) by Hermitian position and momentum operators $\mathbf{Q}^{(n)}$ and $\mathbf{P}^{(n)}$ (which satisfy canonical commutation relations) and Hermitian spin operators $\mathbf{S}^{(n)}$ (which commute with $\mathbf{Q}^{(n)}$ and $\mathbf{P}^{(n)}$ and satisfy angular-momentum commutation relations) in terms of which the generators of the Poincaré group for two noninteracting particles have

[1] W. D. McGlinn, Phys. Rev. Letters **12**, 467 (1964); F. Coester, M. Hamermesh, and W. D. McGlinn, Phys. Rev. **135**, B451 (1964); C. W. Gardiner, Phys. Letters **11**, 258 (1964); A. Beskow and U. Ottoson, Nuovo Cimento **34**, 248 (1964); O. W. Greenberg, Phys. Rev. **135**, B1447 (1964); M. E. Mayer, H. J. Schnitzer, E. C. G. Sudarshan, R. Acharya, and M. Y. Han, *ibid.* **136**, B888 (1964); W. Ruhl, Phys. Letters **13**, 349 (1964); L. Michel, Phys. Rev. **137**, B405 (1965); U. Ottoson, A. Kihlberg, and J. Nilsson, *ibid.* **137**, B658 (1965); L. O'Raifeartaigh, Phys. Rev. Letters **14**, 332 (1965); E. C. G. Sudarshan, J. Math. Phys. (to be published); M. Y. Han, Phys. Rev. **138**, B689 (1965); Y. Tomozawa, J. Math. Phys. **6**, 656 (1965); L. Michel and B. Sakita (unpublished); S. Coleman (unpublished); L. O'Raifeartaigh (unpublished).

the form[2]

$$H=\sum_{n=1}^{2}(\mathbf{P}^{(n)2}+m_n^2)^{1/2},$$

$$\mathbf{P}=\mathbf{P}^{(1)}+\mathbf{P}^{(2)},$$

$$\mathbf{J}=\sum_{n=1}^{2}(\mathbf{Q}^{(n)}\times\mathbf{P}^{(n)}+\mathbf{S}^{(n)}),$$

$$\mathbf{K}=\sum_{n=1}^{2}\{(\tfrac{1}{2})(\mathbf{P}^{(n)2}+m_n^2)^{1/2}\mathbf{Q}^{(n)}+(\tfrac{1}{2})\mathbf{Q}^{(n)}(\mathbf{P}^{(n)2}+m_n^2)^{1/2} +[m_n+(\mathbf{P}^{(n)2}+m_n^2)^{1/2}]^{-1}\mathbf{P}^{(n)}\times\mathbf{S}^{(n)}\},$$

with m_n being the mass of particle n. We define a total spin operator $\mathbf{S}=\mathbf{S}^{(1)}+\mathbf{S}^{(2)}$ and use the notation $\mathbf{V}^{(n)}=[m_n+(\mathbf{P}^{(n)2}+m_n^2)^{1/2}]^{-1}\mathbf{P}^{(n)}$ to evaluate the commutator

$$-i(S_3K_j-K_jS_3)=V_3^{(1)}S_j^{(1)}+V_3^{(2)}S_j^{(2)} \qquad (1)$$

for $j=1,2$.

Consider the amplitudes

$$\langle\mathbf{p}^{(3)},r_3,\mathbf{p}^{(4)},r_4|T|\mathbf{p}^{(1)},r_1,\mathbf{p}^{(2)},r_2\rangle \qquad (2)$$

for $1+2\rightarrow 3+4$, where particles 3 and 4 also each have positive mass and spin $\frac{1}{2}$. The initial and final states are eigenstates of the momentum and spin operators $\mathbf{P}^{(n)}$ and $S_3^{(n)}$ with eigenvalues $\mathbf{p}^{(n)}$ and r_n for each particle. Lorentz invariance means that the scattering operator T commutes with the generator $\mathbf{K}$ of Lorentz transformations. For spin invariance, we postulate that T commutes with the total spin operator $\mathbf{S}$. It follows that T commutes with the commutators (1). By taking linear combinations we get

$$\langle\mathbf{p}^{(3)},r_3,\mathbf{p}^{(4)},r_4|T(V_3^{(1)}S_\pm^{(1)}+V_3^{(2)}S_\pm^{(2)})|\mathbf{p}^{(1)},r_1,\mathbf{p}^{(2)},r_2\rangle =\langle\mathbf{p}^{(3)},r_3,\mathbf{p}^{(4)},r_4|(V_3^{(3)}S_\pm^{(3)}+V_3^{(4)}S_\pm^{(4)})T \times|\mathbf{p}^{(1)},r_1,\mathbf{p}^{(2)},r_2\rangle, \qquad (3)$$

where $S_\pm^{(n)}=S_1^{(n)}\pm iS_2^{(n)}$ are the raising and lowering operators for the spin values r_n. We denote the two values $\pm\frac{1}{2}$ of r_n by $\pm$ and suppress the dependence on the momentum variables. By using the properties of the $S_\pm^{(n)}$ operators we find

$$v_3^{(1)}\langle++|T|++\rangle=v_3^{(3)}\langle-+|T|-+\rangle+v_3^{(4)}\langle+-|T|-+\rangle,$$

$$v_3^{(2)}\langle++|T|++\rangle=v_3^{(3)}\langle-+|T|+-\rangle+v_3^{(4)}\langle+-|T|+-\rangle,$$

$$v_3^{(3)}\langle++|T|++\rangle=v_3^{(1)}\langle-+|T|-+\rangle+v_3^{(2)}\langle-+|T|+-\rangle,$$

$$v_3^{(4)}\langle++|T|++\rangle=v_3^{(1)}\langle+-|T|-+\rangle+v_3^{(2)}\langle+-|T|+-\rangle,$$

as specific cases of (3), where

$$\mathbf{v}^{(n)}=[m_n+(\mathbf{p}^{(n)2}+m_n^2)^{1/2}]^{-1}\mathbf{p}^{(n)}$$

are the eigenvalues of the operators $\mathbf{V}^{(n)}$.

From the commutation of the scattering operator T with the total spin operator $\mathbf{S}$ it follows that all of the amplitudes (2) which are not zero can be written as

$$\langle++|T|++\rangle=T^{(1)}=\langle--|T|--\rangle,$$

$$2\langle+-|T|+-\rangle=T^{(1)}+T^{(0)}=2\langle-+|T|-+\rangle,$$

$$2\langle+-|T|-+\rangle=T^{(1)}-T^{(0)}=2\langle-+|T|+-\rangle,$$

in terms of spin-one and spin-zero amplitudes $T^{(1)}$ and $T^{(0)}$. After substituting and adding and subtracting, we find that the four cases of (3) written out above are equivalent to

$$(v_3^{(1)}+v_3^{(2)}-v_3^{(3)}-v_3^{(4)})T^{(1)}=0,$$

$$(v_3^{(1)}-v_3^{(2)})T^{(1)}=(v_3^{(3)}-v_3^{(4)})T^{(0)},$$

$$(v_3^{(3)}-v_3^{(4)})T^{(1)}=(v_3^{(1)}-v_3^{(2)})T^{(0)}.$$

Now $T^{(1)}$ and $T^{(0)}$ are functions only of the momentum variables and do not depend on our choice of the three axes. Hence the above equations remain valid if each $v_3^{(n)}$ is replaced by $\mathbf{v}^{(n)}$. From these equations we can conclude that $T^{(1)}$ is nonzero only for values of the momentum variables such that either $\mathbf{v}^{(1)}=\mathbf{v}^{(3)}$ and $\mathbf{v}^{(2)}=\mathbf{v}^{(4)}$ or $\mathbf{v}^{(1)}=\mathbf{v}^{(4)}$ and $\mathbf{v}^{(2)}=\mathbf{v}^{(3)}$ and $T^{(0)}$ is nonzero only for these values of the momenta and for values such that $\mathbf{v}^{(1)}=\mathbf{v}^{(2)}$ and $\mathbf{v}^{(3)}=\mathbf{v}^{(4)}$.

This theorem applies at least to the scattering of two quarks with exact $SU(6)$ symmetry as defined by Mahanthappa and Sudarshan.[3]

The unwanted symmetry represented by the commutator of the spin and the Lorentz generator is excluded if the total spin operator $\mathbf{S}$ is replaced by

$$U^\dagger(L(p))\mathbf{S}U(L(p)),$$

where $U(L(p))$ operating on a momentum eigenstate is the unitary representative for the system of two noninteracting particles of a Lorentz transformation $L(p)$ which takes the total four-momentum eigenvalue p to the direction of the time axis.[4] One then gets conservation of total spin only in the center-of-mass frame.

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[4] F. Coester, Helv. Phys. Acta **38**, 7 (1965); B. Schroer, Proceedings of the Third Annual Eastern U. S. Theoretical Physics Conference, University of Maryland, 1964 (unpublished).