and N.[11] This type of mass-splitting is impossible if the baryon representation is any of the other three-quark representations of $SU(6)$, i.e., $(1,1,0,0,0)^{70}$, $(0,0,1,0,0)^{20}$, $(2,0,0,0,1)^{120}$ or $(0,1,0,0,1)^{84}$, for in these cases the number of $SU(2)\otimes SU(3)$ multiplets in the $SU(6)$ representation is greater than the number of $SU(2)\otimes SU(2)$ multiplets in the corresponding representation of $SU(4)$.

In conclusion, the requirement that the baryon multiplet correspond to a single irreducible representation favors strongly the group $SU(6)$ to the direct product group $SU(2)\otimes SU(3)$ as the basic symmetry group. The 56-fold representation is favored over other simple representations for the baryons, but not very strongly in some cases. However, the bootstrap hypothesis does provide a possible reason for the nonexistence of the fundamental multiplet of $SU(6)$.

[11] Since the mesons are emitted in P waves in the static limit, it is convenient to identify the spin-1 singlet with the pseudoscalar X_0 particle, rather than with a vector meson. This point is discussed in detail in Refs. 2 and 3.

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Dynamical Basis for η-Baryon Interactions*

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The recent experimental evidence for a rapid rise in η production near threshold in the reactions $\pi^-+p\rightarrow\eta^0+n$ and $K^-+p\rightarrow\eta^0+\Lambda$ is analyzed on the basis of a constant-K-matrix formalism. Representative scattering lengths for the η-baryon systems are introduced to illustrate pertinent features of the presently known experimental data. A dynamical basis for the physical situation is discussed in terms of possible new baryon states due to virtual or bound states of the η-baryon systems. Comparison of the results with expectations from a conjectured η octet of baryon ($\frac{1}{2}$-) states for $SU(3)$ symmetry is briefly referred to.

1. INTRODUCTION

It is well known that because of the lack of orbital angular momentum to give centrifugal barrier containment, S-wave pseudoscalar meson-baryon interactions do not in general form favorable situations for resonances or peak enhancements. Some years back Dalitz and Tuan[1] showed that for a coupled two-channel problem, owing to strong attractive forces in the closed channel in the S state, a quasi-bound state can be formed which will *manifest* itself as an S-wave resonance in the open channel. Likewise, several papers have discussed threshold effects or cusps[2] in strong interactions involving particle reactions. It has been noted[3,4] that both classes of phenomena are attributable to poles of the S matrix in appropriate unphysical sheets of the two-channel problem. To establish an unambiguous terminology we call the former the quasi-bound-state problem with a corresponding *bound*-state pole, the latter, a virtual-state problem with a *virtual*-state pole. Threshold effects involving only particles stable in strong interactions do produce large cusp effects, when there exists a virtual-state pole in the S matrix *close* to an S-wave threshold on the unphysical sheet reached by passing through the branch cut associated with the threshold. Note that in both classes of phenomena, the existence of an $S_{1/2}$ resonance or cusp effect for an open meson-baryon channel is due to the dynamical influence from an additional second channel present. A simple illustration of the physics involved is to take the analogous situation of the ^3S (bound-state deuteron) and ^1S (virtual state of deuteron) of the n-p system; here only one channel, corresponding to the second of the above channels, is present.

Recently, Berley *et al.*[5] have raised the question whether the rapid rise and subsequent sharp drop in η^0 production (over a small range of energy) from $K^-+p\rightarrow\eta^0+\Lambda^0$ is a possible dynamical manifestation of a quasi-bound-state of the η-Λ system. Bulos *et al.*[6]

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[1] R. H. Dalitz and S. F. Tuan, Phys. Rev. Letters 2, 425 (1959). See also R. H. Dalitz and S. F. Tuan, Ann. Phys. (N. Y.) 8, 100 (1959).

[2] M. Nauenberg and A. Pais, Phys. Rev. **126**, 360 (1962); J. S. Ball and W. R. Frazer, Phys. Rev. Letters **7**, 204 (1961).

[3] R. H. Dalitz and S. F. Tuan, Ann. Phys. (N. Y.) 3, 307 (1960).

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[5] D. Berley *et al., Proceedings of the International Conference on High Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965).

[6] F. Bulos *et al.*, Phys. Rev. Letters **13**, 486 (1964).

and Peterson *et al.*[7] have found a similar rapid rise in inelasticity due to threshold production of η^0 from $\pi^-+p\rightarrow\eta^0+n$, though here the subsequent drop in production cross section is less marked. The existence of an $\eta+N$ cusp effect was suggested some time ago[8] to account for the anomalously sharp peak in $\gamma+p\rightarrow\pi^++n$ at $E_\gamma=700$ MeV[9]; there has been confirmatory evidence for this type of behavior from the elegant phase-shift analysis of Auvil *et al.*[10] which lends support to the idea of a postulated virtual baryon state in a neighborhood of the $\eta+N$ threshold.[11] The appropriate quantum numbers for the baryon states in the two cases are of course ($T=0$, $J=\frac{1}{2}^-$) for the $\eta+\Lambda$ system, and ($T=\frac{1}{2}$, $J=\frac{1}{2}^-$) for the $\eta+N$ system. An important question to ask at this stage is (a) whether these phenomena are associated with well-defined poles (corresponding to bound or virtual states of the η-baryon systems) of the S matrix and (b) their role as input baryon states in an $SU(3)$ classification of particle states. The importance of the latter consideration has recently been underlined by Pais[12] in connection with the possible existence of the **70** dimensional representation of $SU(6)$ symmetry, where, among other things, an octet of $(1/2^-)$ baryon states is called for.

It is evident that to obtain definitive answers to the above question, considerable improvement in the experimental information is needed. What we can hope for at the present moment is some qualitative guide as to the nature of these interactions, in terms of a K-matrix formulation for two-channel processes. In Sec. 2 we summarize the relevant properties of the K-matrix formalism for convenience of reference; the emphasis here is on the scattering-length approximation. In Sec. 3 this formalism is specifically applied to the η production problems $K^-+p\rightarrow\eta+\Lambda^0$ and $\pi^-+p\rightarrow\eta+n$; representative scattering lengths for the η-baryon systems are obtained (together with their corresponding virtual- or bound-state poles) to illustrate salient features of the physical picture. The concluding section is devoted to a discussion of the theoretical analysis and its implications for the existence of an η octet of $(1/2^-)$ states. Particular attention is also made, especially in relation to the reaction $\pi^-+p\rightarrow\eta^0+n$, of those aspects of the experimental situation which will bear most decisively on the theory.

2. FORMALISM

It has been usual to characterize the relationship between the T matrix and the K matrix as

$$T-i\pi K\rho T=K\,, \tag{1}$$

where ρ denotes the matrix of phase-space densities.

For a two-channel problem, let m_1 and M_2 be the initial meson and baryon in channel I and m_3 and M_4 be the final meson-baryon system (channel II). We are interested in the following scattering and reaction processes:

$$\begin{aligned} &m_1+M_2\rightarrow m_1+M_2\,, \quad m_1+M_2\rightarrow m_3+M_4\,,\\ &m_3+M_4\rightarrow m_1+M_2\,, \quad m_3+M_4\rightarrow m_3+M_4\,. \end{aligned} \tag{2}$$

From the Hermiticity of the Hamiltonian and the requirement of *effective* time-reversal invariance in strong interactions,[13] the elements of the K matrix

$$K=\begin{pmatrix}\alpha & \beta\\ \tilde{\beta} & \gamma\end{pmatrix}$$

are real, where

$$\begin{aligned} \alpha&=\langle m_1M_2|K|m_1M_2\rangle\,, \quad \beta=\langle m_1M_2|K|m_3M_4\rangle\,,\\ \tilde{\beta}&=\langle m_3M_4|K|m_1M_2\rangle\,, \quad \gamma=\langle m_3M_4|K|m_3M_4\rangle\,, \end{aligned} \tag{3}$$

and for the two-channel problem the transpose $\tilde{\beta}=\beta$. Likewise we may introduce two-by-two matrix representations for T with $T(ij|kl)=\langle m_iM_j|T|m_kM_j\rangle=\tilde{T}(kl|ij)$ under effective time-reversal invariance, and a diagonal representation for the phase-space density ρ where

$$\rho_{12}=\langle 12|\rho|12\rangle\,, \quad \rho_{34}=\langle 34|\rho|34\rangle\,.$$

Using relativistic normalization for the particle wave functions, the diagonal elements of ρ for c.m. energy E in this representation are of the form

$$\int\delta(\omega_i+E_j-E)(2\pi/\omega_i)(M_j/E_j)4\pi k^2dk/(2\pi)^3 =\rho_{ij}=(M_j/\pi E)k\,, \tag{4}$$

where $\omega_i=(m_i^2+k^2)^{1/2}$ and $E_j=(M_j^2+k^2)^{1/2}$ are the meson and baryon total energies for the channel under consideration and E is the total c.m. energy of system.

Since we deal with S-wave meson-baryon interactions in both initial and final states, it is convenient to introduce a scattering-length approximation for one of the two sets of channels, say the (1,2) channel which we presume to have the higher energy threshold; hence

$$k_{12}\cot\delta_{12}=1/A=1/(a+ib)\,. \tag{5}$$

Again it was shown some years back,[3] that this type of parametrization led to the following simple closed-form expressions for elastic and production cross sections for the two-channel processes:

$$\sigma(12|12)=\frac{4\pi(a^2+b^2)}{(1+bk_{12})^2+a^2k_{12}^2} \tag{6}$$

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[7] V. Z. Peterson *et al.*, University of California Radiation Laboratory Report No. UCRL-11576 (unpublished).

[8] J. J. Sakurai, Phys. Rev. Letters **7**, 355 (1961).

[9] L. Hand and C. Schaerf, Phys. Rev. Letters **6**, 229 (1961).

[10] P. Auvil, A. Donnachie, A. T. Lea, and C. Lovelace, Phys. Letters **12**, 76 (1964). See also P. Auvil and C. Lovelace, Nuovo Cimento **33**, 473 (1964).

[11] I. P. Gyuk and S. F. Tuan, Phys. Rev. Letters **14**, 121 (1965).

[12] A. Pais, Phys. Rev. Letters **13**, 175 (1964); M. A. B. Bég and V. Singh, Phys. Rev. Letters **13**, 509 (1964). See also Ref 11

[13] Recently, Professor T. D. Lee (private communication) has raised the question of possible time-reversal noninvariance involving reciprocity relations in strong interactions. Such corrections to the amplitudes are, however, expected to be only of order 10^{-3} of the T-invariant amplitudes; hence we assume *effective* time-reversal invariance in the present discussion. For a detailed presentation of the underlying theory, see T. D. Lee and L. Wolfenstein, Phys. Rev. **138**, B1490 (1965).

$$\sigma(34|12)=4\pi(k_{12}/k_{34})(M_2M_4/E^2)\times\frac{\beta^2}{[1+(M_4^2/E^2)k_{34}^2\gamma^2][(1+bk_{12})^2+a^2k_{12}^2]},\qquad(7)$$

$$\sigma(34|34)=4\pi(M_4/E)^2\frac{|Z|^2}{|1-i(M_4/E)k_{34}Z|^2},\qquad(8)$$

where

$$Z=\gamma+i\beta^2(M_2/E)k_{12}/1-i(M_2/E)k_{12}\alpha\qquad(9)$$

in terms of a scattering length approximation for the (1,2) system

$$\alpha=(E/M_2)[a+b\gamma(M_4/E)k_{34}].\qquad(10)$$

In the derivation of these results on cross sections, it was tacitly assumed that the c.m. energy E was such that all of the channels considered were energetically permissible. However, since it is known as a consequence of causality that the scattering amplitudes are the limiting values, as E approaches the real axis, of functions analytic in the upper half of complex E plane, we can conclude that the expressions for the T-matrix elements will remain valid as the energy E decreases below the (m_1,M_2) threshold, despite the fact that some elements of ρ must then be taken imaginary. The correct continuation for these elements of ρ is settled by appealing to the analytic property just mentioned:—For a two-particle channel (i,j), the value of ρ_{ij} below its threshold is to be taken as $+i|\rho_{ij}|$.

To take an example, the expression for the (3,4) elastic cross section below the (3,4) threshold will become

$$\sigma(34|34)=4\pi(M_4/E)^2\frac{Z^2}{1+(M_4/E)^2k_{34}^2Z^2},\qquad(8')$$

$$Z=\gamma-\frac{\beta^2(M_2/E)|k_{12}|}{1+[a+b\gamma(M_4/E)k_{34}]|k_{12}|}.\qquad(9')$$

Note that (9′) is now a real quantity.

The following remarks deserve emphasis:

(1) *Analyticity of K-Matrix Elements.* The expressions presented above were derived on the basis of the mixed-boundary-condition approach[3] where each of the two channels under discussion is at least partially open in the energy range under consideration. This procedure ensures that the K-matrix elements $\alpha(E)$, $\beta(E)$, and $\gamma(E)$ are free of threshold singularities due to, say, the opening of the second channel at m_1+M_2, in fact these matrix elements become analytic functions.[14] Such analytic properties of the K matrix make it possible to approximate α, β, and γ by constant parameters (leading to a constant K matrix) in an immediate neighborhood of the m_1+M_2 threshold.

It must be remarked however that for the physical examples under discussion in Sec. 3, the energies are such that several other open channels are available in addition to the coupled two-channel problem we seek to emphasize. For example, in the neighborhood of the $\eta+n$ threshold, the two two-body channels of importance are π^-p and ηn, though the multiparticle open channel $\pi^-+p\rightarrow N+\pi+\pi$ is in competition throughout this energy range. Since specification in detail of the features of this competing process is not of obvious interest to the discussion at hand—namely, analysis of possible cusp and bound-state manifestations at the vicinity of the $\eta+n$ threshold, we have adopted, essentially, the "equivalent reaction K-matrix" formulation[15] here.

(2) *Use of Scattering Length Approximation.* Jackson, Ravenhall, and Wyld[16] first emphasized the importance of the zero-range approximation in connection with the phenomenological analysis of data. The basic premise here is that the range of interaction R of a meson m_1 and baryon M_2 is generally of order $\hbar/m_1c$ or at most $\hbar/2m_\pi c$; hence the effective-range term $\frac{1}{2}Rk^2$ can be considered in the context of the general formula

$$k_{12}\cot\delta_{12}=1/(a+ib)+\tfrac{1}{2}Rk_{12}^2,\qquad(11)$$

where k_{12} is the c.m. momentum of the (m_1,M_2) system. For c.m. energy of the order of a 50-MeV neighborhood of the threshold, it has generally been appropriate to neglect the second term of (11)—if the S-wave scattering lengths involved are of the order of a fermi (10^{-13} cm). We shall assume the scattering length approximation in the phenomenological analysis of Sec. 3.

(3) *Cusp Behavior in σ(34|34) at m_1+M_2 Threshold.* The cross section $\sigma(34|34)$ given by Eqs. (8)–(10) *above* m_1+M_2 threshold, and by (8′), and (9′) *below* m_1+M_2, is expected to manifest cusp properties due to different analytic forms for the kinematical variable k_{12} in a neighborhood of this new channel threshold. As a function of c.m. energy E, $d\sigma(34|34)/dE$ assumes the following forms:

$$d\sigma(34|34)/dE\approx C(E)\gamma(0)[(m_1+M_2)-E]^{-1/2},\quad(E\lesssim m_1+M_2)$$
$$d\sigma(34|34)/dE\approx -D(E)\beta^2(0)[E-(m_1+M_2)]^{-1/2},\quad(E\gtrsim m_1+M_2),\qquad(12)$$

where $C(E)$ and $D(E)$ are slowly varying positive definite functions of E in an immediate neighborhood of the threshold and $\gamma(0)$, $\beta(0)$ are threshold values of γ,β. It is evident from Eq. (12) that the cusp has upward (downward) curvature for positive (negative) $\gamma(0)$—below m_1+M_2, and upward curvature only above m_1+M_2. These points are illustrated (cf. Figs. 3 and 5) for typical cases encountered in practice in Sec. 3.

[14] A. particularly lucid account of the analyticity properties of the K matrix in a relativistic context is given by R. Oehme, Lectures on High Energy Physics, Sixth Summer Meeting of Nuclear Physicists in Hercegnovi, Yugoslavia, 1961 (unpublished).

[15] See, for instance, R. H. Dalitz, Rev. Mod. Phys. **33**, 471 (1961); also R. H. Dalitz, Ann. Rev. Nucl. Sci. **13**, 338 (1963).

[16] J. D. Jackson, D. G. Ravenhall and H. W. Wyld, Jr., Nuovo Cimento **9**, 834 (1958).

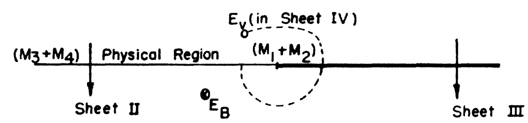

FIG. 1. Schematic diagram of the positions of bound- and virtual-state poles in the complex E plane. Sheet II is the unphysical sheet reached by continuing from the physical sheet (sheet I) across the cut in interval (m_3,M_4) to (m_1,M_2); sheet III is the unphysical sheet reached by continuing from sheet I across the cut in interval above (m_3,M_4). The bound-state pole E_b is in sheet II; the virtual-state pole E_v (in sheet IV) is reached from the physical region by continuing first across cut below (m_1,M_2) into sheet II and then across cut above (m_1,M_2) into sheet IV. A typical path is drawn for illustration in the diagram.

(4) *Bound- and Virtual-State Poles of* (m_1,M_2) *Channel.* Quasi-bound and virtual states will correspond to appropriate poles of the scattering amplitude in sheets II and IV (Fig. 1), respectively. These poles are determined from:

$$\text{For bound state: } 1+\alpha'|k_{12}|=0,$$

$$\text{For virtual state: } 1-\alpha'|k_{12}|=0; \tag{13}$$

$$E_r \cong (m_1^2-|k_{12}|^2)^{1/2}+(M_2^2-|k_{12}|^2)^{1/2}.$$

Here $\alpha'=a+b\gamma(M_4/E)k_{34}=(M_2/E)\alpha$, and E_r is an approximate expression for the real part of the S-matrix pole in the energy plane. The two physical situations of bound state and virtual state correspond, respectively, to $\alpha'<0$ and $\alpha'>0$. Actually the complete expression for the pole in the complex E plane is to be determined from the following implicit relation[3]:

$$[1-i(M_2/E)k_{12}\alpha][1-i(M_4/E)k_{34}\gamma]+(M_2M_4/E^2)\beta^2=0. \tag{14}$$

Note in connection with Fig. 1, that the virtual-state pole E_v "appears" to lie in the upper E-plane, the so called sheet I or physical sheet. The fact is that E_v is really in sheet IV and is reached from the physical region by analytic continuation first across the cut between (m_3,M_4) and (m_1,M_2) into sheet II and then back across the real axis above (m_1,M_2) into sheet IV. Because of its substantial distance from the region of physical energies, such virtual state poles are expected in general to have only marked cusp manifestations on scattering processes[4] when they are in the vicinity of the (m_1,M_2) threshold. It is of course well known that the bound state pole E_b is in the unphysical sheet II and hence directly accessible to the physical plane.

(5) *Limitations to the Real K-Matrix Formulation.* The scattering and reaction matrices, as a function of E, should exhibit an analytic structure reflecting the nature of dynamical interactions which give rise to the processes observed. For m_1+M_2, the dynamical singularity lying closest to the m_1+M_2 threshold is due to the process

$$m_1+\bar{m}_1\rightarrow m_\pi+m_\pi,$$

where $\bar{m}_1$ is the antiparticle of m_1. This leads to a branch cut ($E\leq E_{2\pi}$) beginning at the energy value

$$E_{2\pi}=(M_2^2-m_\pi^2)^{1/2}+(m_1^2-m_\pi^2)^{1/2}. \tag{15}$$

This branch point is present for $T(12|12)$ but not for $T(12|34)$ and $T(34|34)$, at least in the energy range of interest. However, the indirect effect on coupled channels will be felt since the existence of the singularity will render α, β, γ complex for $E\leq E_{2\pi}$. It is also known[17] that the threshold behavior of $T(E)$ near $E_{2\pi}$ is governed by

$$\mathrm{Im}T(12|12)\cong f(E_{2\pi}-E)^{3/2}, \tag{16}$$

where, in lowest approximation, f is proportional to the phenomenological coupling constant for $(m_1\bar{m}_1m_\pi m_\pi)$ interaction. In the absence of low-lying $\pi\pi$ resonances, f is expected to be small and this singularity may be expected to have a relatively weak effect (reflecting this smooth behavior near branch point) on elements of $T(E)$, even for energies $E<E_{2\pi}$.

For the specific problem of η-baryon interaction, with a central mass appropriate for a baryon octet used, $E_{2\pi}$ is about 30 MeV below the corresponding $\eta+B$ threshold (taken as m_1+M_2). The $\eta+\eta$ system is not coupled in strong interaction to any of the better established meson resonances below 1 BeV in mass. Hence, taken in conjunction with the limitations imposed by the use of the scattering length approximation, the constant K-matrix formulation is expected to be reliable at least for an energy range of about ± 50 MeV of the m_1+M_2 threshold.

3. TWO CHANNEL η-PRODUCTION PROBLEM

We discuss here two concrete cases for which limited experimental data are currently available. The paucity of data does not warrant a detailed computer search for over-all best fits to the scattering lengths involved for either case; rather we seek to illustrate the general trend by presenting representative solutions not inconsistent with the data.

Case (A). $m_1=m_\eta$, $M_2=M_\Lambda$; $m_3=m_{K^-}$, $M_4=M_p$. The two pertinent channels are

$$K^-+p\rightarrow K^-+p,\quad K^-+p\rightarrow \eta+\Lambda^0. \tag{17}$$

The available experimental information[5,18] is consistent with a sharp rise in η production, followed by a sharp drop, in a limited energy interval (just above $\eta+\Lambda^0$ threshold) for the reaction $K^-+p\rightarrow\eta+\Lambda^0$. As emphasized earlier,[5] this type of behavior can be reproduced in the K-matrix formalism by assuming a large

[17] See, for instance, S. F. Tuan, Phys. Letters **2**, 62 (1962).
[18] P. L. Bastien *et al.*, Phys. Rev. Letters **8**, 114 (1962); P. L. Bastien, University of California Radiation Laboratory Report No. UCRL 10779 (unpublished).

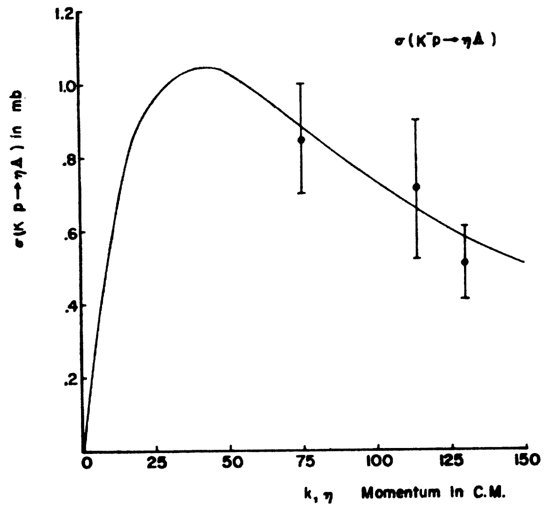

FIG. 2. A fit to the $K^-+p \to \eta+\Lambda$ production cross section (in mb) is plotted versus k (η momentum in c.m.). A scattering length $A=(\pm4.75+i0.5)$F and $|\gamma|=0.5$F, $\beta^2=0.6564\text{F}^2$ are employed. The experimental points are taken from Berley *et al.* (Ref. 5).

scattering length $A=a+ib$ for the $\eta-\Lambda^0$ system, with a large real component and negligible imaginary part. An examination of Eq. (7) shows a dependence on a^2 only, and thus the production cross section will not yield information on the *sign of real part* a[19]; likewise, there is no dependence on the sign of γ. For physical processes such that $|a|\gg b$, Eq. (10) shows that generally sign $\alpha=$ sign a, and $+a$ corresponds to a virtual state and $-a$ corresponds to a bound state. These points are illustrated in Figs. 2 and 3, where the production cross section[20] and the $K^-+p\to K^-+p$ S-wave cross section [computed from Eqs. (8) to (10) and (8′), (9′)] are plotted with the fit ($\beta^2=0.6564\text{F}^2$)

$$\begin{aligned}&a=+4.75\text{F},\quad b=0.5\text{F},\quad \gamma=+0.5\text{F},\\&\qquad\qquad\qquad\qquad(\text{virtual-state solution})\\&a=-4.75\text{F},\quad b=0.5\text{F},\quad \gamma=-0.5\text{F},\\&\qquad\qquad\qquad\qquad(\text{bound-state solution})\,.\end{aligned}\tag{18}$$

Here F is one fermi (10^{-13} cm). The choice of magnitude of γ is arbitrary; in principle it can be determined from the S-wave K^--p cross section, say, at the $\eta+\Lambda^0$ threshold, or alternatively from an accurate fit of the $K^-+p\to\eta+\Lambda^0$ production cross section above threshold (assuming S-wave interaction only). The presently available data do not warrant such a detailed analysis and, since the over-all energy variation under consideration is less than 20 MeV (Fig. 3), the exact magnitude of γ will not drastically affect the over-all picture. The choice of sign of γ is made in (18) to accentuate the illustration of possible cusp manifestations for both types of solutions at the $\eta+\Lambda^0$ threshold, as has been discussed generally in the preceding section.

The real part of the virtual-state pole E_v and bound-state pole E_B (see Fig. 1) can be evaluated approximately by Eqs. (13) and (18) to give

$$E_r=1661\text{ MeV}.\tag{19}$$

Note that this value is consistent with the maximum in S-wave K^--p scattering cross section for the bound-state solution (Fig. 3); for the virtual-state pole E_v we have the pronounced "inverted V" cusp effect at $\eta^0+\Lambda^0$ threshold as shown by the virtual-state solution. The exact expression for E_v and E_B in the complex E plane can be determined from Eq. (14).

Case (B). $m_1=m_\eta$, $M_2=M_n$; $m_3=m_{\pi^-}$, $M_4=M_p$. The two channels of interest are

$$\pi^-+p\to\pi^-+p,\quad \pi^-+p\to\eta^0+n.\tag{20}$$

Both sets of experimental data[6,7] on η^0 production

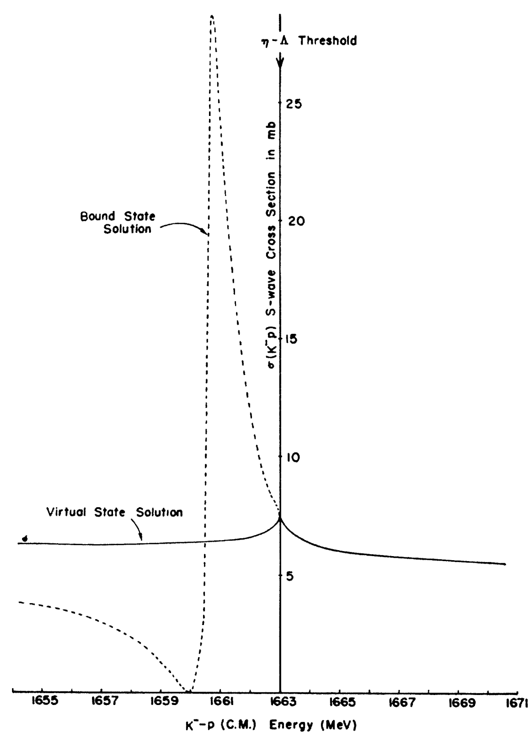

FIG. 3. The S-wave K^--p scattering cross section (in mb), calculated on the basis of the constant K-matrix formalism, is plotted against the total c.m. energy E in a neighborhood of the η-Λ threshold. The bound-state solution (dashed line) is for $A=(-4.75+0.5i)$F, $\gamma=-0.5$F; the virtual state solution (unbroken line) is for $A=(+4.75+0.5i)$F, $\gamma=+0.5$F. The choice of magnitude of γ [which in turn determines the value of $\sigma(K^--p)$ at threshold via Eqs. (8) and (9)] is arbitrary; the choice of sign of γ is made to accentuate the illustration of the cusp effects at the opening of the $\eta+\Lambda$ channel.

[19] Unlike the low-energy K^--p problem (see Ref. 1), there does not appear here the simple method of differentiating between ($\pm a$) solutions via Coulomb-nuclear interference, because of the neutral charges involved in the $\eta-\Lambda^0$ system.

[20] M. Nauenberg [Nuovo Cimento **27**, 1168 (1963)] has pointed out that, for a m_3-M_4 resonance energy lying quite close to the second threshold (m_1,M_2), there can be additional factors which depend on $|k_{12}|$ and so have quite rapid variation for the production amplitude. Since additional parameters are introduced as well in this more general form, we shall confine our attention to the simplest possibility for the purpose of illustration.

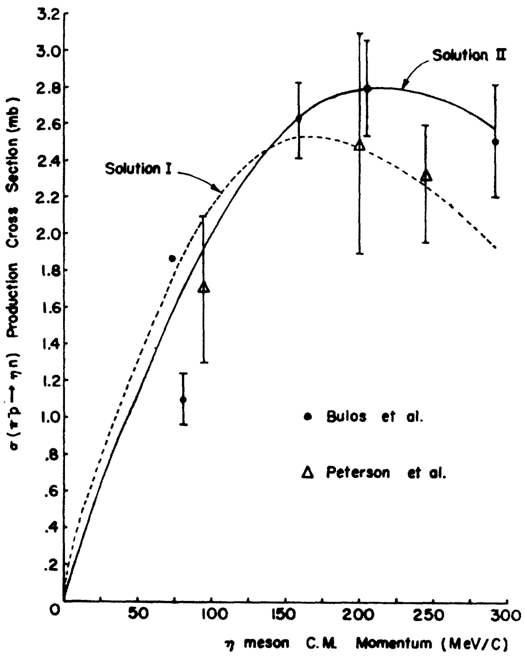

FIG. 4. Two fits to the $\pi^- + p \to \eta^0 + n$ production cross section (in mb) are plotted versus η meson c.m. momentum (MeV/c). Solution I corresponds to scattering parameters ($A = a + ib$) with $a = 0.8304$F, $b = 0.05$F, $\gamma = +1.2526$F, $4\pi\beta^2 = 4.122$F^2; Solution II corresponds to scattering parameters $a = 0.52$F, $b = 0$, $\gamma = +1.2526$F, and $4\pi\beta^2 = 3.469$F^2. The sign and magnitude of γ are chosen in rough agreement with the phase shift analysis of Auvil *et al.* (Ref. 10). The experimental points are taken from Refs. 6 and 7, with the multiplicative branching-ratio factor $\eta^0 \to (\text{all})/2\gamma$ (~ 2.83) incorporated to obtain the complete production cross section for η^0.

from threshold to a maximum at about 60 MeV above $\eta^0 + n$ threshold in the barycentric system, and a gradual tapering off above this energy. In Fig. 4 we present two sets of solutions, I and II, as a rough fit to the $\pi^- + p \to \eta^0 + n$ production data where

$$\text{Solution I:}\quad a = +0.8304\text{ F},\quad b = 0.05\text{ F}\quad \gamma = +1.2526\text{ F},\quad 4\pi\beta^2 = 4.122\text{ F}^2; \tag{21}$$

$$\text{Solution II:}\quad a = +0.52\text{ F},\quad b = 0\quad \gamma = +1.2526\text{ F},\quad 4\pi\beta^2 = 3.469\text{ F}^2. \tag{22}$$

The sign and magnitude of γ as well as the sign of a are chosen to be in crude agreement with the phase-shift analysis of Auvil *et al.*[10] which suggests a cusp effect (virtual state solution) in $\pi^- p$ scattering at the energy of the $\eta^0 + n$ threshold. In Fig. 5, we have plotted the S-wave $\pi^- p$ scattering cross section as a function of c.m. energy E for solution I (solution II is not markedly different in this case), again emphasizing the cusp effect at threshold.

It is evident from Fig. 4 that neither solution represents a completely adequate fit of the data for the range of η c.m. momentum from 0 to 250 MeV/c, where the use of the scattering length approximation might be expected to be reasonable; in particular the fit for low momentum is poor. The corresponding virtual-state poles are—

$$\text{Solution I:}\quad E_r \cong 1418\text{ MeV},\qquad \text{Solution II:}\quad E_r \cong 1255\text{ MeV}. \tag{23}$$

4. DISCUSSION

The *qualitatively* similar behavior of η^0 production from $\pi^- + p \to \eta^0 + n$ and $K^- + p \to \eta^0 + \Lambda^0$ close to their respective thresholds, has given rise to a natural speculation about the possible existence of an η-baryon octet of $\frac{1}{2}^-$ states with the remaining members associated with the $\eta + \Sigma$ and $\eta + \Xi$ thresholds.[11] Such an "η octet" of baryon states $\tilde{B}$ is expected to satisfy an octet-type mass formula in $SU(3)$ symmetry

$$\tfrac{1}{2}(\tilde{N} + \tilde{\Xi}) = \tfrac{1}{4}(3\tilde{\Lambda} + \tilde{\Sigma}). \tag{24}$$

Indeed a cursory examination of the simpler Feynman graphs for η-baryon production from meson-nucleon initial states shows a marked similarity in production mechanisms; in particular, η production does not neces-

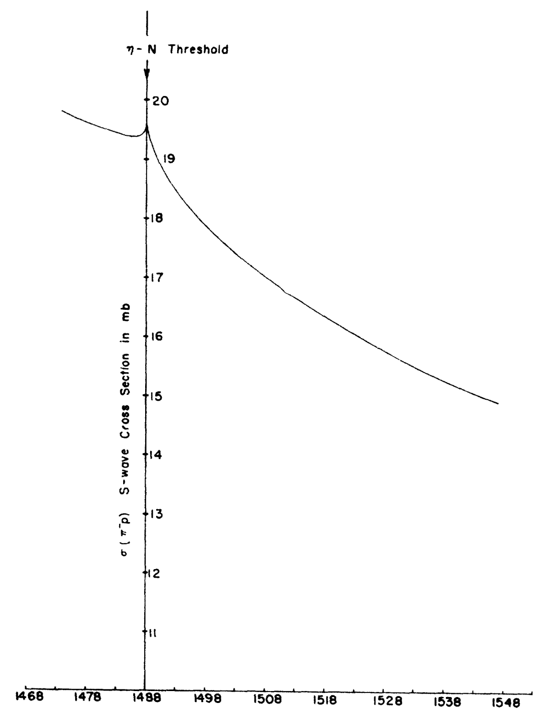

FIG. 5. A plot of the S-wave $\pi^- - p$ cross section (in mb) versus c.m. energy of the ($\pi^- - p$) system. The parameters of solution I are used to illustrate this example. The choice of a positive real part for the scattering length of the ($\eta^0 - n$) system is made so as to be consistent with indications (Refs. 8–10) of a sharp cusp effect due to the opening of the $\eta^0 + n$ channel. We must emphasize however, that the cross section here plotted is meant as a *qualitative* indication of the possible shape form to be expected. The scattering parameters are insufficiently well determined from empirical data to warrant a quantitative statement of the S-wave $\pi^- - p$ scattering cross section.

sarily test strangeness of incoming meson[21] (we expect of course the strength of interaction to be *different* from member to member, since this does depend sensitively on the strangeness and form factors involved). Dynamically we have shown in the preceding sections, that these phenomena can be understood in terms of bound or virtual states of the η-baryon system with well-defined S-matrix poles. This circumvents the usual difficulty about visualizing S-wave resonances because of the lack of centrifugal barrier containment. The experimental data on production does not enable one to infer whether the virtual $(+a)$ or the quasi-bound $(-a)$ solution is appropriate to the physical situation.[22] We are aware that the phase-shift analysis of Auvil *et al.*[10] on π^-p scattering is more consistent with a substantial cusp effect due to the opening of the $\eta+n$ channel. Based on the argument of "dynamical similarity" for the various members of the η-octet, we are tempted to infer by analogy that *all* members of the octet correspond to virtual states with possible cusp manifestations at the $\eta+B$ thresholds. However, no firm conclusions are warranted at the present stage.

In a recent study of $SU(6)$ mass formulas, Gyuk and Tuan[11] postulated the existence of an "η-octet" of baryon states with the $\tilde{N}(T=\frac{1}{2},\ J=\frac{1}{2}^-)$ and $\tilde{\Lambda}(T=0,\ J=\frac{1}{2}^-)$ members associated with the η^0+n (1488 MeV) and $\eta^0+\Lambda^0$(1663 MeV) thresholds. This is a somewhat novel procedure of assigning baryon states as "input" into symmetry mass formulas, since, whereas the physical phenomena in the form of cusps, shoulders[6–10] etc. *are* indeed associated closely with the positions of these thresholds, the positions of the corresponding S-matrix poles are likely to reflect the *different* strengths of interaction of the various members of the octet. We see from Eqs. (18), (21), and (22) that the scattering length for the $\eta^0-\Lambda^0$ system is substantially larger than that for the η^0-n system; this in turn yields pole positions which can be near [cf. Eq. (19)] the former threshold and relatively distant from the latter threshold [Eq. (23)]. A conventional viewpoint would be to apply group-theoretic mass formulas only to the positions of the S-matrix poles; this will introduce some ambiguity into a discussion of the η octet, at least if current experimental data hold up. As emphasized by Rajasekaran,[23] even for the decuplet $p_{3/2}$ states of $SU(3)$, the range of interaction is not small enough to derive group-theoretic results such as the sum rules for masses and amplitudes if these are defined as the poles of the $T'(=k^{-l}Tk^{-l})$ matrix—though such sum rules are valid for the poles of corresponding K' matrix. For S-wave interactions, the combined requirement of smooth motion of the resonances as the thresholds move and the purity of the appropriate $SU(3)$ wave functions[24] can, in principle, be satisfied. However, the lack of orbital momentum barrier here, among other points,[25] can cause the S-matrix poles to be substantially shifted by changes in thresholds as we move from perfect symmetry to broken $SU(3)$. Our analysis gives qualitative support to this picture and hence the logical foundations of S-wave mass formulas in terms of "poles" are less clear than the corresponding formulas for $P, D,\cdots$ waves. On the other hand, especially in view of the growing empirical evidence for possible cusp (or bound state) phenomena at the vicinity of $(\eta^0\Lambda^0)$[5] and (η^0n)[10] thresholds, it is certainly not ruled out that an *effective* S-wave η-baryon mass formula of the octet type is present in nature. The study of η-$\Sigma^{\pm,0}$ and η-$\Xi^{-,0}$ interactions near their respective thresholds (1740 and 1866 MeV) is thus of very great interest.

We conclude this work with some brief comments about features of the $\pi^-+p\to\eta^0+n$ interaction without prejudice to a specific model of interaction. It is known from the data of Bulos *et al.*[6] that the angular distribution for production remains isotropic for a wide range of energy above threshold. This result is consistent with $S_{1/2}$ or $P_{1/2}$ production but is inconsistent with a large $P_{3/2}$ or $D_{3/2}$ component throughout the energy region measured. We are aware of the possible existence of a $T=\frac{1}{2}$ $N^*(1512)$ with spin-parity $(\frac{3}{2}^-)$ which is energetically allowed to decay into $\eta+N$; in addition, there is evidence of a large P_{11} phase shift in π^-p scattering at $E_{\text{c.m.}}\sim 1410$ MeV—though perhaps not resonating.[26] It is at least consistent for the tail from such a large P_{11} interaction to play a role at energy $E\gtrsim 1488$ MeV ($\eta+n$ threshold). A sensible phenomenological analysis in terms of partial waves,[27] should proceed in terms of three possibly important complex reaction amplitudes for production, S_1, P_1, and D_3. The appropriate expressions for the differential cross section and polarization are

$$k^2d\sigma/d\Omega=(|S_1-D_3|^2+|P_1|^2)+2\,\text{Re}(S+2D_3)^*\times(P_1)\cos\theta+3(\text{Re}(SD_3^*)+|D_3|^2)\cos^2\theta\,,\quad(25)$$

$$(k^2d\sigma/d\Omega)\mathbf{P}=\mathbf{n}\sin\theta[2\,\text{Im}(S-D_3)^*P_1+6\,\text{Im}(S^*D_3)\cos\theta]\,,\quad(26)$$

[21] I am indebted to Professor Joseph Sucher for calling this point to my attention.

[22] The existence of a $T=0$, $J=\frac{1}{2}^-$ quasi-bound state of the $\eta-\Lambda^0$ system at about 1661 MeV need not be in conflict with experimental data in K^--p. It is well known that a $Y_1^*(1660)$ is present at this energy and has a substantial width $\Gamma/2=20$ MeV. The proximity of this latter resonance will have a "masking" effect on the very narrow $(\frac{1}{2}^-)$ state (Fig. 3). A set of degenerate baryon states is likely to raise interesting problems for the determination of the parity quantum number for either state.

[23] G. Rajasekaran, Ph.D. thesis, University of Chicago, 1964 (unpublished).

[24] C. N. Yang, Proceedings of the Argonne User's Group, 1963 (unpublished); R. J. Oakes and C. N. Yang, Phys. Rev. Letters **11**, 174 (1963).

[25] In particular (see Ref. 23), S-wave quasi-bound-state resonances raise special problems of interpretation if they belong to an $SU(3)$ multiplet, rather than as unitary singlets [like the $Y_0^*(1405)$].

[26] R. H. Dalitz and R. G. Moorhouse, Phys. Letters **14**, 159 (1965).

[27] Some of these results on the partial wave analysis have also been obtained by S. Minami (unpublished).

where $\mathbf{k}$ is the incident c.m. momentum and $\mathbf{n}=\mathbf{k}_i\times\mathbf{k}_f/|\mathbf{k}_i\times\mathbf{k}_f|$ is the normal to the scattering plane.

An isotropic distribution for production then imposes the following restrictions among the three complex amplitudes S_1, P_1, and D_3:

$$2\,\mathrm{Re}(S_1+2D_3)^*(P_1)=0\,,\qquad (|S_1|/|D_3|)\cos(\varphi_S-\varphi_D)=-1(\text{if }|D_3|\neq 0)\,, \tag{27}$$

where

$$S_1=|S_1|e^{i\varphi_S}\quad\text{and}\quad D_3=|D_3|e^{i\varphi_D}.$$

The simplest way to understand the isotropic production angular distribution is to assume a dominant S_1 amplitude [which in the constant K-matrix formalism is simply related to Eq. (7) for the production cross section] with negligible P_1 and D_3 amplitudes.[28] This will require that the known large P_{11} phase shift at the lower energy must be suppressed or attenuated in the η^0+n channel at energies greater than 1488 MeV. Despite the presence of $N^*(1512)$, a small D_3 amplitude can perhaps be understood qualitatively in terms of the small amount of phase space available for decay into η^0+n.

Models which analyze the data in terms of two dominant reaction amplitudes, like (i) S_1 and P_1 present but no D_3 and (ii) S_1 and D_3 present but no P_1[29] can fit the isotropic production angular distribution, but require special phase relationships among the amplitudes like

$$\varphi_P-\varphi_S=(2r+1)\pi/2,\ r\text{ an integer, for case (i)} \tag{27'}$$

and the second condition of (27) for case (ii); φ_P is the phase angle for the reaction amplitude $P_1=|P_1|e^{i\varphi_P}$. A general discussion in terms of all three amplitudes present will of course impose much more stringent interference and phase conditions [Eq. (27)] to obtain the necessary isotropic angular distribution.

We have remarked in Sec. 3 that neither of the solutions I and II presented, is a good fit to the low-momentum region of $\pi^-+p\to\eta^0+n$ production cross section. Indeed, extrapolation of solution I in Fig. 4 to higher momentum (and hence beyond the normally expected range of validity of the scattering length approximation) will give a poor fit to the higher energy points of Bulos *et al.*[6]—though the experimental angular distribution remains isotropic until an η c.m. momentum of order 443 MeV/c. A speculative solution is possible here, if we accept for instance case (ii) that both S_1 and D_3 are present in the production mechanism for η^0+n at low and moderate η^0 momentum. According to this picture, the S_1 amplitude will give rise to a sharp enhancement and drop in cross section akin to the $\eta^0+\Lambda^0$ system (Fig. 2); the less marked drop actually noted experimentally[6,7] can perhaps be attributed to D-wave contributions [say from $N^*(1512)$] filling in. The advantage of this heuristic interpretation is that it allows then for the possibility of a virtual state pole of the S-wave η^0-n interaction closer to threshold—for consistency with the η octet concept. Some $\cos^2\theta$ terms in the angular distribution are generally expected here, unless the phase condition of Eq. (27) is rigidly satisfied throughout the low η-momentum range.

It is evident that further experimental information on the general shape of the $\pi^-+p\to\eta^0+n$ total production cross section, the angular distribution (especially for the possible presence of a $\cos^2\theta$ term), and the polarization of the recoil nucleon, in the low and moderate η-momentum range, will be of the greatest value.

Note added in proof. Professor Peter Dobson (private communication) has emphasized to me, that using the η-N phase shifts from 300–700 MeV of the University of Hawaii High Energy Physics Group (see, for instance, Robert J. Cence, UH Report HEPG-3-65, to be published) as input data, quite adequate fits to the η-nucleon S_{11} threshold interaction can be obtained on the basis of a two-channel constant K-matrix formulation *without* appreciable background contributions. The position of the *virtual* state pole in Dobson's analysis (to be published) is around 1465 MeV. This is then in quite satisfactory agreement with the input assumptions of Gyuk and Tuan [Phys. Rev. (to be published) see especially solutions (a) and (b) therein] about the possible existence of a *70*$^-$ multiplet in $SU(6)$ theory.

ACKNOWLEDGMENTS

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[28] An equally simple, but less attractive hypothesis, is to assume $S_1=D_3=0$, and $P_{11}\neq 0$. This will give an isotropic angular distribution as well as zero polarization of the recoil nucleon from Eqs. (25) and (26). The conflict of this assumption with the large S_{11} cusp effect noted by Auvil *et al.* (Ref. 10) is then somewhat disturbing.

[29] We discard the case of P_1 and D_3 but no S_1 since this will contradict both the S-wave pion-nucleon phase shift (Ref. 10) and the isotropic angular distribution [cf. Eq. (25)] of Bulos *et al.* (Ref. 6).