Double-Charge-Exchange Scattering of Low-Energy Pions by Nuclei*

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The double-charge-exchange cross section for the scattering of low-energy pions by nuclei is calculated in second Born approximation, considering only s-wave scattering of the pions by individual nucleons. The differential cross section for pions of energy 20 to 40 MeV, and a light target with two neutrons outside a closed shell, is calculated to be $\sim$7 μb/sr, with considerable enhancement for targets with larger neutron excess. The scattering to the isobaric analog of the target ground state is strongly favored.

I. INTRODUCTION

THE possibility of studying nuclear structure by means of the double-charge-exchange reaction

$$\pi^{\pm}+[Z,N]\rightarrow\pi^{\mp}+[Z\pm2,\,N\mp2]$$

has been discussed by several authors.[1–5] Kerman and Logan,[2] using an impulse approximation, have estimated the total cross section for this process, with a variety of targets, at about one microbarn ($1\mu\text{b}=10^{-30}$ cm^2) for low-energy ($\sim$40-MeV) pions. Kohmura[4] has treated the scattering in perturbation theory to fourth order in the pseudovector pion-nucleon interaction, obtaining a cross section for the reaction

$$\pi^{+}+\text{O}^{18}\rightarrow\pi^{-}+\text{Ne}^{18}\ \text{(ground state)} \qquad (1)$$

of about 15 μb at 10 MeV. Parsons, Trefil, and Drell[3] have computed the cross section in the impulse approximation, using the Chew-Low approximation for (3,3) resonant scattering of the pion by each of two nucleons in succession. For the reaction (1) the forward differential cross section peaks above 100-MeV pion energy, reaching an order of magnitude of 10 μb/sr. At 30 MeV the cross section has decreased by several orders of magnitude. A calculation similar in spirit, but with a different treatment of the pion-nucleon resonant scattering, has been reported by Barshay and Brown,[5] estimating a lower forward differential cross section than that of Parsons, Trefil, and Drell for high-energy pions (100–200 MeV).

In the present paper we report a calculation of the double-charge-exchange cross section for low-energy pions (20 to 40 MeV), considering the scattering on nucleon pairs in the target, in second Born approximation, using a semiphenomenological s-wave pion-nucleon interaction.[6] It is perhaps surprising that for a ground-state reaction such as (1), the s-wave cross section at low energy is predicted to be of the same order of magnitude (10 μb/sr) as the cross section calculated in Ref. 3 at 100 MeV, in spite of the fact that the elastic pion-nucleon cross sections at the two energies differ by more than an order of magnitude. The reason for this, as will be shown below, is that the low-energy double scattering is largely diagonal in all quantum numbers other than charge, favoring scattering between members of an isospin multiplet, as in (1). The high-energy scattering is sensitive to short-range nucleon pair correlations, and therefore scatters largely inelastically, that is, to states other than isobaric analogs. Thus the total cross section to all final states should be larger at high than at low pion energy.

The fact that the low-energy reaction goes through a monopole in everything but isospin has some other interesting features. For nuclei with large neutron excess in shells just above the last *proton*-filled shell (e.g., Ca^{48}, Zr^{90}) large enhancements of the cross section may be expected, since all pairs of "valence" neutrons may participate, irrespective of angular-momentum coupling.

The other feature of the insensitivity to short-range pair correlations is that the appropriate nuclear matrix elements for the reaction are largely model-independent, and may be calculated directly by use of tables of nuclear masses.

Two experiments on double charge exchange have been reported, but at energies above the range of our calculations. The first[7] yielded total cross sections of 500 μb for 80-MeV π^+ in emulsions. The second,[8] which was performed at 195 MeV, using various light nuclear targets, produced differential cross sections at zero degrees ranging from 1 to 100 μb/sr.

II. THEORY

An s-wave pion-nucleon interaction of the form

$$H(x)=4\pi\lambda\mu^{-2}\boldsymbol{\tau}\cdot\boldsymbol{\phi}(x)\times\boldsymbol{\pi}(x) \qquad (2)$$

has been suggested[6] to explain the low-energy charge-exchange scattering of pions by nucleons. The quantities $\boldsymbol{\tau}$, $\boldsymbol{\phi}$, and $\boldsymbol{\pi}$ are vectors in isospin space; $\boldsymbol{\phi}(x)$ is the pion

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[1] T. Ericson, in *Proceedings of the* 1963 *Annual International Conference on High Energy Physics and Nuclear Structure at CERN*, edited by J. Prentki (CERN, Geneva, 1963).
[2] A. K. Kerman and R. K. Logan, Argonne National Laboratory Report No. ANL-6848, 1964 (unpublished).
[3] R. G. Parsons, J. S. Trefil, and S. D. Drell, Phys. Rev. **138**, B847 (1965).
[4] T. Kohmura (unpublished report).
[5] S. Barshay and G. E. Brown (unpublished report).
[6] S. D. Drell, M. H. Friedman, and F. Zachariasen, Phys. Rev. **104**, 236 (1956). A. Klein, *ibid.* **99**, 998 (1955).
[7] Yu. A. Batusov *et al.*, Zh. Eksperim. i Teor. Fiz. **46**, 817 (1964) [English transl.: Soviet Phys.—JETP **19**, 557 (1964)].
[8] L. Gilly *et al.*, Phys. Letters **11**, 244 (1964).

field operator at the position of the nucleon, $\boldsymbol{\pi}(x)=\dot{\boldsymbol{\phi}}(x)$, and $\boldsymbol{\tau}$ is the isospin vector of the nucleon and μ is the mass of the pion.

We consider the double charge exchange from state i to state f of a nucleus of A nucleons, where the incident pion has positive charge. The transition amplitude in the lowest (second-order) Born approximation, is given by the expression

$$\langle f,\mathbf{q}'|T|i,\mathbf{q}\rangle=\langle\mathbf{q}'|\sum_{i\neq j}^{A}\int d^3x_i d^3x_j \Psi_f^*(x_1\cdots x_A)H(x_i) \times(E-H_0+i\epsilon)^{-1}H(x_j)\Psi_i(x_1\cdots x_A)|\mathbf{q}\rangle, \quad (3)$$

where $\mathbf{q}$ and $\mathbf{q}'$ are the initial and final pion momenta, Ψ_i and Ψ_f are the nuclear wave functions, E the initial energy, H_0 the Hamiltonian for the nucleus and free pion. Spin and isospin coordinates are implicit.

We evaluate the mesonic part of the matrix element (3) by introducing a set of virtual intermediate meson states:

$$\langle\mathbf{q}'|H(x_i)(E-H_0+i\epsilon)^{-1}H(x_j)|\mathbf{q}\rangle =\exp i(\mathbf{q}\cdot\mathbf{x}_j-\mathbf{q}'\cdot\mathbf{x}_i)\int\frac{d^3k}{(2\pi)^3}\exp-i\mathbf{k}\cdot(\mathbf{x}_j-\mathbf{x}_i) \times\left\{\sum_{\pm}\frac{\langle\mathbf{q}'|H(i)|\mathbf{k}(\pm)\rangle\langle\mathbf{k}(\pm)|H(j)|\mathbf{q}\rangle}{\pm(\omega_q-\frac{1}{2}\Delta)-\omega_k+i\epsilon}\right\}, \quad (4)$$

where the $(\pm)$ refers to propagation of the virtual meson forward or backward in time. We use $\omega_k=(k^2+\mu^2)^{1/2}$, and $\Delta=\omega_q-\omega_{q'}$ is the energy transferred to the nucleon pair. We have arbitrarily divided this energy evenly between the nucleons.

From (2) we can evaluate

$$\langle\mathbf{k}(\pm)|H(j)|\mathbf{q}\rangle=4\pi\lambda\mu^{-2}\tau_+(j)(\omega_q\pm\omega_k)\tfrac{1}{2}(\omega_q\omega_k)^{-1/2}, \quad (5)$$

where $\tau_+=2^{-1/2}(\tau_1+i\tau_2)$.

The sum in curly brackets in (4) is

$$\{\sum_{\pm}\}=(4\pi\lambda\mu^{-2})^2\tau_+(i)\tau_+(j)\left[\frac{2\mu}{q^2-\Delta\mu-k^2+i\epsilon}-\frac{1}{2\mu}\right], \quad (6)$$

where we have kept only the lowest powers of q and Δ, so that $\omega_q=\omega_{q'}=\mu$ except in the denominator. On performing the k integral in (4), using (6), Eq. (3) becomes

$$\langle f,\mathbf{q}'|T|i,\mathbf{q}\rangle=4\pi(\lambda\mu^{-2})^2\times 2\mu\sum_{i\neq j}^{A}\int d^3r_{ij}d^3R_{ij}\Psi_f^* \times e^{-i\mathbf{P}\cdot\mathbf{R}_{ij}}e^{i\mathbf{Q}\cdot\mathbf{r}_{ij}}e^{ipr_{ij}}r_{ij}^{-1}\tau_+(i)\tau_+(j)\Psi_i, \quad (7)$$

where we use the usual center-of-mass and relative coordinates

$$\mathbf{R}_{ij}=\tfrac{1}{2}(\mathbf{x}_i+\mathbf{x}_j);\quad \mathbf{r}_{ij}=\mathbf{x}_i-\mathbf{x}_j,$$

and where

$$p=(q^2-\Delta\mu)^{1/2}$$
$$\mathbf{P}=\mathbf{q}-\mathbf{q}',\quad \mathbf{Q}=\tfrac{1}{2}(\mathbf{q}+\mathbf{q}').$$

We note that Eq. (7) should contain a second term, with $\delta(\mathbf{r}_{ij})$ replacing $\exp(ipr_{ij})/r_{ij}$, which comes from the Fourier transform of the second term in (6). However, for nuclear states reflecting the short-range repulsion of nuclear forces, this term will not contribute.

A first estimate of (7) may be made in the zero-energy limit ($q\sim q'\sim 0$). Then

$$\langle f|T|i\rangle\simeq 8\pi\lambda^2\mu^{-2}\sum_{i,j}\langle j|\frac{\tau_+(i)\tau_+(j)}{\mu r_{ij}}|i\rangle. \quad (8)$$

We note that the terms of (8) are proportional to the *Coulomb interaction* of the pairs of protons which can be obtained from the target state by the operation $\tau_+(i)\tau_+(j)$. The Coulomb interaction is mostly diagonal in shell-model configurations, since it is of long range, so that the important two-body matrix elements in (8) involve only particles in unfilled shells. For these particles, the Coulomb pair-interaction is also largely independent of the angular-momentum coupling of the pair, so that the isobaric and radial parts of the matrix element may be approximately factored:

$$\sum_{i,j}\langle f|\frac{\tau_+(i)\tau_+(j)}{\mu r_{ij}}|i\rangle =4\left\langle\frac{1}{\mu r}\right\rangle_{\text{av}}\langle T,T_3{}^f|T_+T_+|T,T_3{}^i\rangle, \quad (9)$$

where

$$T_+=\tfrac{1}{2}\sum_{i=1}^{A}\tau_+(i)$$

and

$$\langle T,T_3+1|T_+|T,T_3\rangle=[\tfrac{1}{2}(T-T_3)(T+T_3+1)]^{1/2}. \quad (10)$$

That is, the nuclear transition is largely "monopole" in all quantities but T; the final state differs from the initial only in T_3.

The average matrix element in (9) is proportional to the average Coulomb interaction of pairs of nucleons in the unfilled shells, and we have

$$C\equiv\langle 1/\mu r\rangle_{\text{av}}=\mathcal{E}_{\text{coul}}{}^{(2)}/\mu e^2, \quad (11)$$

where $\mu e^2=1.02$ MeV. For light nuclei ($A<40$) the pair Coulomb energy can be evaluated by comparing the binding energies[9] of isobars with $A=4m+2$ and $A=4m+1$ ($m=$integer) in the unfilled shell:

$$\mathcal{E}_{\text{Coul}}{}^{(2)}=E_B(4m+2n)-E_B(4m+2p) +2[E_B(4m+p)-E_B(4m+n)], \quad (12)$$

($p=$proton, $n=$neutron). Some typical values of the ratio C are

$$A=14,\quad C=0.55;$$
$$A=18,\quad C=0.58;$$
$$A=22,\quad C=0.87;\quad (13)$$
$$A=32,\quad C=0.42.$$

[9] See, e.g., L. A. Konig *et al.*, Nucl. Phys. 31, 18 (1962).

Finally, we can evaluate the low-energy cross section using (8), (9), (10),

$$\frac{d\sigma}{d\Omega}=\frac{\mu^2}{(2\pi)^2}\frac{q'}{q}|\langle f|T|i\rangle|^2$$
$$=(16)^2\lambda^4\mu^{-2}C^2\frac{q'}{q}\langle f|T_+T_+|i\rangle^2\,, \tag{14}$$

where

$$\langle f|T_+T_+|i\rangle^2=\tfrac{1}{4}(T-T_3{}^i-1)(T+T_3{}^i+2)\times(T-T_3{}^i)(T+T_3{}^i+1)\,, \tag{15}$$

which reduces to the excess neutron pair number $\frac{1}{2}N(N-1)$ when there are only neutrons in the unfilled (target) shells: $T=-T_3{}^i=\frac{1}{2}N$.

We obtain λ from the first Born term for *single* charge exchange, using (5). In terms of the s-wave phase shifts δ_3 and δ_1 we have

$$\lambda=(-\mu/6q)(\delta_3-\delta_1)\,. \tag{16}$$

This gives[10]

$$\lambda=0.045\,.$$

Then

$$\frac{d\sigma}{d\Omega}\simeq 7.6\frac{q'}{q}\langle f|T_+T_+|i\rangle^2\mu\text{b/sr}\,. \tag{17}$$

If we ignore inelasticity (q'/q), this gives an estimate of 7.6 μb/sr for the reaction (1). For nuclei with large neutron excess the cross section will be much larger, as can be seen from Eq. (15). The enhancement factor is thus 28 for Ca^{48} and 45 for Zr^{90}.

However, several corrections must be applied to the "zero-energy" result Eq. (17) to get a useful prediction. Firstly, the positive pion must have sufficient energy to penetrate the Coulomb repulsion of the nucleus. We estimate[11] that the probability of penetration for 10-MeV pions is more than one-half for targets with $Z\leq 40$. A more serious lower limit on the energy is the inelasticity; for a light target, the pion must lose $\sim$10–12 MeV to excite the isobaric state, owing to the increase of Coulomb energy in the target. Thus the factor $(q/q')<\frac{1}{2}$ for $E<15$ MeV. But Eq. (17) clearly cannot be used for $E\lesssim 12$ MeV.

At finite pion energy, however, the momenta $\mathbf{q}$, $\mathbf{q}'$ must be retained in the complete expression (7) for the transition matrix element. This leads to a correction to Eq. (8), which is analogous to the introduction of form factors in relative and center-of-mass motion. For moderately low pion energies, and for light nuclei which are well represented by harmonic oscillator wave functions, the correction to (8) is of the order of

$$\exp(-\tfrac{1}{2}q^2\beta^2)\,, \tag{18}$$

where β is the length parameter of the oscillator:

$$\rho(r)\propto\exp(-r^2/\beta^2)\,,\quad r\to\infty\,.$$

Thus the cross-section estimate (17) should be valid for energies such that $q^2\beta^2<1$. For a light nucleus,[12] $\beta^2\simeq 3$ F^2, so that the limit $q^2\beta^2\sim 1$ is reached for pion energies $\sim$40 MeV.

In summary, the range of validity of the approximations involved in Eq. (17) is given by $Z<40$, $12<E<40$ (MeV).

III. DISCUSSION

We conclude that the cross section for double-charge-exchange scattering of low-energy pions may be as large as that of high-energy pions, when the final nuclear state in either case is the isobaric analog of the target ground state. The low-energy pion has a smaller probability for each single scattering, but the two scatterings are largely in phase, as can be seen from Eq. (7). This enhances the *diagonal* nuclear matrix element. Conversely, the rapid phase change for higher energy pions between scatterings means that non-analog states may be reached.

We wish to point out a similarity between our analysis and that of Kerman and Logan.[2] Our derivation of Eq. (7) is equivalent in principle to treating their optical potential for single charge exchange in second Born approximation. However, we then separate the integral of Eq. (7) in center-of-mass and relative coordinates, because we can then evaluate the matrix element (8) in an approximately model-independent manner, using Coulomb energies. This avoids the approximate steps used by Kerman and Logan in constructing the optical potential, which requires an assumption of the form factor for the charge-exchange potential. It is not clear, however, that this difference is sufficient to explain the fact that we predict larger cross sections at low energy than they.

Finally, we note that multiple-scattering corrections to the second Born approximation should not be important, since the s-wave pion-nucleon scattering length is considerably smaller than the mean distance between scattering nucleons.[13]

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[10] J. M. McKinley, Rev. Mod. Phys. **35**, 788 (1963).
[11] W. T. Sharp, H. E. Gove, and E. B. Paul, At. Energy Can. Ltd., AECL TP1–70, 1953.
[12] See, e.g., R. Hofstadter, Ann. Rev. Nucl. Sci. **7**, 276 (1957).
[13] S. D. Drell and L. Verlet, Phys. Rev. **99**, 849 (1955).