PHYSICAL REVIEW VOLUME 139, NUMBER 5B 6 SEPTEMBER 1965

Poles and Resonances in the Process $\pi^+ + p \rightarrow \Sigma^+ + K^+$ †

W. G. HOLLADAY
Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee
(Received 12 April 1965)

Several models considered in successively greater degrees of complexity are discussed for the $T=\frac{3}{2}$ process $\pi^+ + p \rightarrow \Sigma^+ + K^+$. The ingredients of the model are Λ and K^* poles, the $p_{3/2}$, 1238-MeV resonance and an $f_{7/2}$ resonance at 1925 MeV. Although various combinations of these terms yield generally good results for the total and differential cross sections from threshold (beam momentum of 1021 MeV/c) to 1.76 BeV/c, calculated Σ^+ polarizations are too small. Much better agreement is found with the polarization if the pole terms are multiplied by $e^{i\delta}$ where δ is taken to be a constant of magnitude 40–50°. Such a term may arise from the presence of other open channels. The quantum numbers $f_{7/2}$ for the 1925-MeV resonance are consistent with the notion that this resonance is a Regge recurrence of the $p_{3/2}$, 1238-MeV state.

I. INTRODUCTION

SEVERAL experiments[1–13] on the total and differential cross section and the polarization of the Σ^+ have been performed for the process $\pi^+ + p \rightarrow \Sigma^+ + K^+$ from just above threshold (laboratory pion kinetic energy T of 891 MeV) to 1630 MeV. The total cross section rises steadily from threshold and peaks at about 1350 MeV. The differential cross section shows some forward peaking of the Σ^+ at 920 and 980 MeV, with some interesting oscillations beginning to appear at 1040 MeV and becoming quite dramatic at 1260 MeV.[7,11] At the latter energy the differential cross section reaches a maximum of over 60 μb/sr at $\cos\theta_\Sigma = 0.5$, drops to less than 10 μb/sr at $\cos\theta_\Sigma = -0.5$ and begins to climb rapidly as $\cos\theta_\Sigma$ approaches -1. (The quantity θ_Σ refers to the angle in the center-of-mass system of the produced Σ with respect to the pion beam.)

The polarization of the Σ^+ (or rather $\alpha^0\bar{P}$) has reached approximately 60% when T is 920 MeV, which is only 30 MeV above threshold. As the energy increases, there are some indications that the polarization decreases, but the experimental situation is not very clear. The partial-wave analyses which have been made of these data show that s and p waves are needed[9] at 920 MeV; s, p, and d waves[7] at 980 MeV; s, p, d, and possibly f waves[10] at 1040 MeV; and s, p, d, and f waves at 1090 and 1260 MeV.

When the work reported in this paper was started,[14] very little in the way of explicit dynamical considerations had been attempted for the process under discussion. In the meantime, a paper by Evans and Knight[15] has appeared with a viewpoint very similar to that presented herein. However, there are some differences in details and conclusions; furthermore, some new data[13] have appeared with which to compare the computations. The approach in this paper is to compute the Σ-K production amplitudes using what is known about the resonances in the direct channel together with certain pole terms in the crossed channels. The constants that appear in these contributions are either known or are fixed to give as good agreement as possible with the existing data on cross sections and Σ^+ polarizations. Several models consisting of various combinations of these poles and resonances are discussed.

The aim is to discover to what extent simple resonance and pole contributions constitute an adequate description of two-particle reactions involving strongly interacting particles. Many reactions can be studied on this basis. In addition to the one discussed here, some of these are $\pi^- + p \rightarrow \Lambda + K^0$,[16–22] $\pi^- + p \rightarrow \Sigma^- + K^+$,[16] $\pi^+ + p \rightarrow p + \rho^+$, $\pi^- + p \rightarrow n + \eta$, $\gamma + p \rightarrow \Lambda + K^+$,[23–28]

† Supported in part by the National Science Foundation.

[1] J. L. Brown, D. A. Glaser, D. I. Meyer, M. L. Perl, and J. Vander Velde, Phys. Rev. **107**, 906 (1957).

[2] R. L. Cool, B. Cork, J. W. Cronin, and W. A. Wenzel, Phys. Rev. **114**, 912 (1959).

[3] A. R. Erwin, Jr., J. K. Kopp, and A. M. Shapiro, Phys. Rev. **115**, 669 (1959).

[4] W. H. Hannum, H. Courant, E. C. Fowler, H. L. Kraybill, J. Sandweiss, and J. Sanford, Phys. Rev. **118**, 577 (1960).

[5] B. Cork, L. Kerth, W. A. Wenzel, J. W. Cronin, and R. L. Cool, Phys. Rev. **120**, 1000 (1960).

[6] A. Berthelot, A. Daudin, O. Goussu, F. Grard, M.-A. Jabiol, F. Levy, G. Lewin, A. Rogozinski, J. Laberrigue-Frolow, C. Ouannes, and L. Vigneron, Nuovo Cimento **21**, 693 (1961).

[7] C. Baltay, H. Courant, W. J. Fickinger, E. C. Fowler, H. L. Kraybill, J. Sandweiss, J. R. Sanford, D. L. Stonehill, and H. D. Taft, Rev. Mod. Phys. **33**, 374 (1961).

[8] E. F. Beall, B. Cork, D. Keefe, P. G. Murphy, and W. A. Wenzel, Phys. Rev. Letters **7**, 285 (1961); **8**, 75 (1962).

[9] F. Grard and G. A. Smith, Phys. Rev. **127**, 607 (1962).

[10] F. S. Crawford, F. Grard, and G. A. Smith, Phys. Rev. **128**, 368 (1962).

[11] H. W. J. Foelsche, A. Lopez-Cepero, C.-Y. Chien, and H. L. Kraybill (unpublished).

[12] E. C. Fowler, L. R. Fortney, J. W. Chapman, P. L. Connolly, E. L. Hart, P. V. C. Hough, and R. C. Strand, Bull. Am. Phys. Soc. **9**, 420 (1963).

[13] N. L. Carayannopoulos, G. W. Tautfest, and R. B. Willman, Bull. Am. Phys. Soc. **10**, 115 (1965).

[14] W. G. Holladay, Bull. Am. Phys. Soc. **9**, 346 (1963); **10**, 258 (1965).

[15] L. E. Evans and J. M. Knight, Phys. Rev. **137**, B1232 (1965).

[16] M. Gourdin and M. Rimpault, Nuovo Cimento **20**, 1166 (1961); **24**, 414 (1962).

[17] A. Kanazawa, Phys. Rev. **123**, 997 (1961).

[18] T. Tsuchida, T. Sakuma, and S. Furui, Progr. Theoret. Phys. (Kyoto) **26**, 1005 (1961).

[19] G. T. Hoff, Phys. Rev. **131**, 1302 (1963).

[20] G. T. Hoff, Phys. Rev. Letters **12**, 652 (1964).

[21] J. E. Rush and W. G. Holladay, Bull. Am. Phys. Soc. **9**, 385 (1964).

[22] M. Rimpault, Nuovo Cimento **31**, 56 (1964).

[23] M. Gourdin, Nuovo Cimento **20**, 1035 (1961); M. Gourdin and J. Dufour, *ibid.* **27**, 1410 (1963).

[24] S. Hatsukade and H. J. Schnitzer, Phys. Rev. **128**, 468 (1962); **132**, 1301 (1963).

[25] T. K. Kuo, Phys. Rev. **129**, 2264 (1963).

[26] N. Beauchamp and W. G. Holladay, Phys. Rev. **131**, 2719 (1963).

[27] J. Dufour, Nuovo Cimento **34**, 645 (1964).

[28] Fayyazuddin, Phys. Rev. **134**, B182 (1964).

$\gamma+p \to \Sigma^+ + K^0$,[28–30] and others. The two reactions $\pi^- + p \to \Lambda + K^0$ and $\pi^+ + p \to \Sigma^+ + K^+$ have three properties which are especially meritorious within the framework of our discussion. First, they take place in a pure state of isospin $T=\frac{1}{2}$ and $T=\frac{3}{2}$, respectively, a fact which simplifies the computational effort. Second, because of the nonconservation of parity in the decay of the Λ and Σ^+, the polarization of these particles is relatively easy to measure. These measured polarizations provide a very sensitive test of any method of computing the production amplitudes. Third, cross section and polarization data exist at various energies for these two reactions. For these three reasons we have concentrated our effort on these two reactions.[14,21] The hope is that the insight gained concerning the mechanism of these two processes can be used to study some of the more complicated reactions listed above.

II. KINEMATICS

Since Σ production by the bombardment of nucleons by pions involves two spin-$\frac{1}{2}$ baryons and two pseudoscalar mesons, the kinematical structure is very similar to that in pion-nucleon elastic scattering.[31] However, inasmuch as the various conventions and normalizations are not completely standardized, we define our notation in this section. The four-momenta (three-momenta and energy) of the π, p, Σ, and K are, respectively, $k(\mathbf{k},\omega_1)$, $p_1(\mathbf{p}_1,E_1)$, $p_2(\mathbf{p}_2,E_2)$, and $q(\mathbf{q},\omega_2)$. The total center-of-mass energy is W. We write the S matrix as

$$S_{fi}=\frac{-i}{(2\pi)^2}\delta^4(k+p_1-q-p_2)\left(\frac{N\Sigma}{4E_1E_2\omega_1\omega_2}\right)^{1/2} \times(\bar{u}_\Sigma(p_2)Tu_N(p_1)), \quad (1)$$

where N and Σ represent the mass of the nucleon and sigma, respectively, and where the Dirac spinors satisfy the equation

$$(M+i\gamma\cdot p)u=0$$

in which $\gamma\cdot p=\boldsymbol{\gamma}\cdot\mathbf{p}-\gamma_0\cdot p_0$. The γ's are Hermitian and $\gamma_0=-i\gamma_4$ is anti-Hermitian. Here, $p_0=(p^2+M^2)^{1/2}$. The positive energy spinors are normalized by the condition

$$\bar{u}u=1$$

where $u=u^\dagger\gamma_4$. They can be written

$$u=\frac{M-i\gamma\cdot p}{[2M(E+M)]^{1/2}}\binom{\chi}{0},$$

where the χ is a Pauli two-component spinor. The quantity T in Eq. 1 can be written in terms of two invariant amplitudes A and B in the form.

$$T=A+\tfrac{1}{2}i\gamma\cdot(k+q)B. \quad (2)$$

Center-of-mass amplitudes f_1 and f_2 can be defined by the relation

$$\bar{u}_\Sigma(p_2)Tu_N(p_1)=\frac{4\pi W}{(N\Sigma)^{1/2}}\left(\phi_f,\left[f_1+i\frac{\boldsymbol{\sigma}\cdot\mathbf{q}\times\mathbf{k}}{qk}f_2\right]\chi_i\right), \quad (3)$$

where χ_i and ϕ_f are initial- and final-state Pauli spinors, and W is the total energy in the barycentric system. It then follows that f_1 and f_2 are related to A and B by

$$f_1=(8\pi W)^{-1}[(E_1+N)(E_2+\Sigma)]^{1/2} \times[A+(\tfrac{1}{2}(N+\Sigma)-W)B]+f_2\cos\theta, \quad (4)$$

$$f_2=-(qk/8\pi W)[(E_1+N)(E_2+\Sigma)]^{-1/2} \times[A+(\tfrac{1}{2}(N+\Sigma)+W)B], \quad (5)$$

where $\mathbf{q}\cdot\mathbf{k}=qk\cos\theta$.

In terms of f_1 and f_2 the differential cross section and polarization of the Σ^+ can be written as

$$d\sigma/d\Omega=(q/k)[|f_1|^2+\sin^2\theta|f_2|^2], \quad (6)$$

$$P=\frac{2\sin\theta\,\mathrm{Im}(f_1f_2^*)}{|f_1|^2+|f_2|^2\sin^2\theta}, \quad (7)$$

where the polarization is defined with respect to the direction $\mathbf{q}\times\mathbf{k}$.

The functions f_1 and f_2 admit the following partial-wave expansions:

$$f_1=\sum_l[f_l^+(l+1)+lf_l^-]P_l(\cos\theta), \quad (8)$$

$$f_2=\sum_l[f_l^- - f_l^+]P_l', \quad (9)$$

where $f_l^\pm$, which are functions of the total center-of-mass energy W, are reaction amplitudes in the state of total angular momentum $j=l\pm\frac{1}{2}$.

III. POLES AND RESONANCES

Several pole terms can contribute to Σ production. Among these are the K^*, Λ, and Σ^0 poles. In addition, in the crossed channels where these poles occur, other singularities, in particular branch cuts, appear. We assume that the contribution of one of these crossed channels to the production amplitude can be represented by the K^* pole and from the other channel by the Λ pole. It is expected that in the energy region under consideration the energy and angular dependence of the amplitudes thus obtained will be fairly accurately represented, but because other singularities are being ignored, it is doubtful that much faith should be placed in the proposition that the residue at these poles accurately represents products of coupling constants.

The contribution of the Λ pole to the invariant amplitudes A and B of Eq. 2 can be written

$$A_\Lambda=\frac{G\sqrt{2}\,4\pi(\tfrac{1}{2}(N+\Sigma)-\Lambda)}{\Lambda^2-u}, \quad (10)$$

$$B_\Lambda=\frac{G\sqrt{2}\,4\pi}{\Lambda^2-u}, \quad (11)$$

[29] T. K. Kuo, Phys. Rev. **130**, 1537 (1963).
[30] J. Dufour, Nuovo Cimento **35**, 860 (1965).
[31] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).

where

$$G=\frac{g_{\Lambda NK}g_{\Sigma\Lambda\pi}}{4\pi\hbar c},$$

and

$$u=-(p_1-q)^2=K^2+N^2-2\omega_2E_1-2\mathbf{q}\cdot\mathbf{k},$$

with the last equality holding in the barycentric system. The contributions to A and B of the K^* pole are given by[32]:

$$A_{K^*}=\frac{8\pi}{K^{*2}-t}\left[G'\frac{(\Sigma-N)}{K^{*2}}(K^2-\pi^2)+\mu(u-s)+\mu'(K^2-\pi^2)\frac{(K^{*2}-t)}{K^{*2}}\right], \quad (12)$$

$$B_{K^*}=(-16\pi/(K^{*2}-t))[G'+\mu(\Sigma+N)], \quad (13)$$

where $G'=g_{K^*K\pi}g_{\Sigma NK^*}/4\pi\hbar c$ and where the terms involving the constants μ and μ' arise as a consequence of K^* being a massive particle with spin 1. In these expressions

$$t=-(p_1-p_2)^2=-(q-k)^2=K^2+\pi^2-2\omega_1\omega_2+2\mathbf{q}\cdot\mathbf{k},$$
$$s=-(p_1+p_2)^2=W^2,$$

where the last equality in each of these two relations holds in the barycentric system. The μ' term in A_{K^*} is a constant; furthermore, the μ term does not have drastic energy and angular dependence. Consequently, the effect of these two terms is not likely to be too pronounced and they are dropped in the remainder of the discussion.

The resonance terms to be included in the model will be taken to have the Breit-Wigner form.[33] More explicitly, $f_l^{\pm}$ of Eqs. (8), (9) is taken to be

$$f_l^{\pm}=\frac{(\Gamma_l^{\pm}(i)\Gamma_l^{\pm}(f)/kq)^{1/2}}{2(W-W_l+i\Gamma_l^{\pm}/2)}, \quad (14)$$

where W_l is the position of the resonance, $\Gamma_l^{\pm}$ is the total width of the resonance and $\Gamma_l^{\pm}$ (i or f) is the partial width of the entrance or exit channel, respectively. The partial widths are written in the form

$$\Gamma_l^{\pm}(i)=2kR\gamma_l^{\pm}(i)v_l^{\pm}(kR), \quad (15a)$$

$$\Gamma_l^{\pm}(f)=2qR\gamma_l^{\pm}(f)v_l^{\pm}(qR), \quad (15b)$$

where R is a distance associated with the radius of the interaction, γ is the reduced width, and the penetration factors v_l can be found in Ref. 33. The quantity $G_l=R[\gamma_l^{\pm}(i)\gamma_l^{\pm}(f)]^{1/2}/\hbar c$ is a measure of the strength of a resonance. In one of the models discussed below, the total width of each resonance is taken to be constant. In the other models, $\Gamma_l^{\pm}$ is taken to have the form

$$\Gamma_l^{\pm}=2kR\gamma_l^{\pm}v_l^{\pm}(kR), \quad (16)$$

where the reduced width $\gamma^{\pm}$ is chosen to yield a certain total width at the position of the resonance.

The resonances to be considered are the excited $T=\frac{3}{2}$ states at 1238[34] and 1925 MeV.[35] The characteristics that have been used for these two resonances are listed in Table I. The width and position of the 1238 resonance are taken from Gell-Mann and Watson.[34] The position of the 1925 resonance is taken from Refs. 35 and 36, which indicate that the width is in the vicinity of 150 to 175 MeV. We have reported results for $\Gamma_3=150$ MeV. See, however, the discussion in Sec. V. The results of Helland *et al.*[37] on the angular dependence of π^+-p elastic scattering show that the total angular momentum $j=\frac{7}{2}$ for this resonance. Studies by Auvil *et al.*[38] and Donnachie *et al.*[39] further suggest that the state has even parity ($l=3$). All the results reported here are for $R=1.0\hbar/(m_\pi c)$.

TABLE I. Some properties of the resonances at 1238 and 1925 MeV.

l	j	W_l (MeV)	Γ_l (MeV)	$P_l(z)$	$v_l(x)$
1	$\frac{3}{2}$	1238	120	z	$x^2/(1+x^2)$
3	$\frac{7}{2}$	1925	150	$\frac{1}{2}(5z^3-3z)$	$x^6/(x^6+6x^4+45x^2+225)$

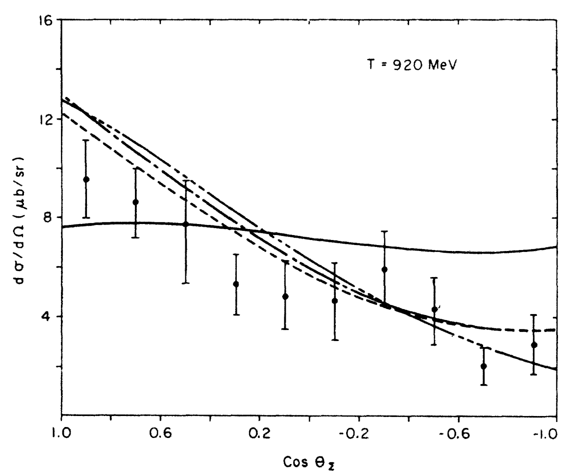

FIG. 1. Angular distribution of the produced Σ^+ at pion laboratory momentum of approximately 1050 MeV/c. The data are from Fig. 2(a), Ref. 9. The solid curve is calculated with Model I; the curve — - - - - - with Model III; the broken curve —— — from Model IV, and the dashed curve - - - - from Model V.

[32] W. G. Wagner and D. H. Sharp, Phys. Rev. **128**, 2899 (1962).
[33] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952); R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1953).

[34] M. Gell-Mann and K. M. Watson, Ann. Rev. Nucl. Sci. 4, 219 (1954). See also M. G. Olsson, Phys. Rev. Letters **14**, 118 (1965).
[35] R. Cool, O. Piccioni, and D. Clark, Phys. Rev. **103**, 1082 (1956).
[36] T. J. Devlin, B. J. Moyer, and V. Perez-Mendez, Phys. Rev. **125**, 690 (1962).
[37] J. A. Helland, T. J. Devlin, D. E. Hagge, M. J. Longo, B. J. Moyer, and C. D. Wood, Phys. Rev. **134**, B1062 (1964).
[38] P. Auvil, C. Lovelace, A. Donnachie, and A. T. Lea, Phys. Letters **12**, 76 (1964).
[39] A. Donnachie, J. Hamilton, and A. T. Lea, Phys. Rev. **135**, B515 (1964).

IV. MODELS

Several models involving various combinations of the poles and resonances discussed above will now be considered, beginning with Model I, which is simpler (and also yields less satisfactory agreement with the data) than the others.

Model I

In this model only the Λ pole and the $f_{7/2}$ resonance at 1925 MeV are used. Apart from a scale factor, there is only one adjustable parameter in the model, namely the ratio of the strengths of the pole and the resonance, G/G_3. The values of G and G_3 given in Table II are

TABLE II. The values of the parameters in the various models.

Model	G	G'	G_1	G_3	δ
I	0.620	0	0	0.024	0
II	0.547	0	0.066	0.023	0
III	0.712	0.050	0.052	0.023	0
IV	0.779	0.063	0.084	0.021	40°
V	0.512	0	0.082	0.021	50°

chosen to give an acceptable fit to $d\sigma/d\Omega$ at $T=1090$ MeV and yield the solid curves drawn in Figs. 1–14. This model yields tolerable fits to the total cross section through the highest energy ($T=1630$ MeV) considered here. The general structure of the differential cross section is also reproduced from $T=1040$ to 1350 MeV. It has two serious defects: The differential cross section is much too flat at the energies just above threshold, and the angular dependence of the Σ^+ polarization as well as the magnitude of the average polarization P are in gross disagreement with the available data.

Models II and III

In an attempt to remove the defects of Model I, the $p_{3/2}$ resonance at 1238 MeV was included. The three

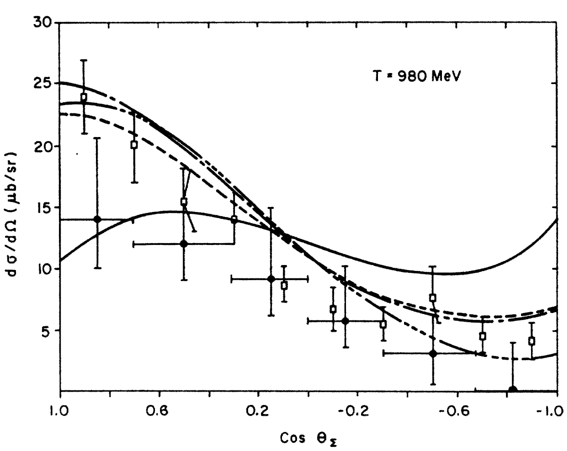

FIG. 2. Angular distribution of the produced Σ^+ at pion laboratory momentum of 1111 MeV/c. The data given by the solid circles are from Ref. 3 and by the open squares from Ref. 13. The caption of Fig. 1 gives the meaning of the curves.

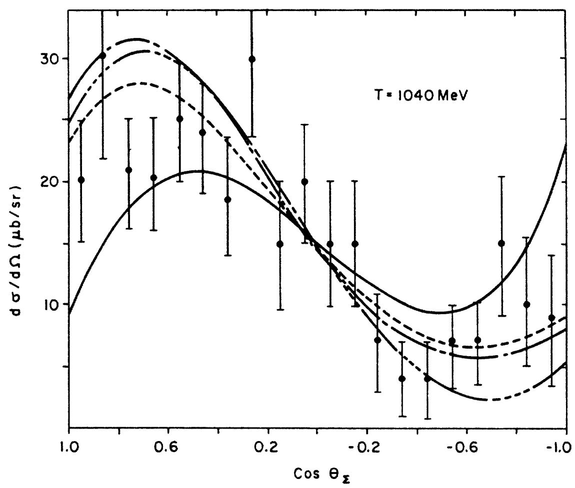

FIG. 3. Angular distribution of the produced Σ^+ at pion laboratory momentum of 1170 MeV/c. The data are from Ref. 10. The caption of Fig. 1 gives the meaning of the curves.

parameters which characterize the strength of the three contributions in this model were chosen to give good fits to $d\sigma/d\Omega$ at $T=920$ and 1090 MeV and are listed in Table II. The results of this model are substantially the same as those of Model III, which gives the curves in Figs. 1–14 of the form ——– – – –. The presence of the p wave improves (compared to Model I) the angular dependence of $d\sigma/d\Omega$ just above threshold and does no violence to the agreement attained in Model I with the total and differential cross sections. Furthermore, the angular and energy dependence of the Σ^+ polarization are in qualitative agreement with the measured values. Quantitatively, however, the calculated values of the Σ^+ polarization are much too small.

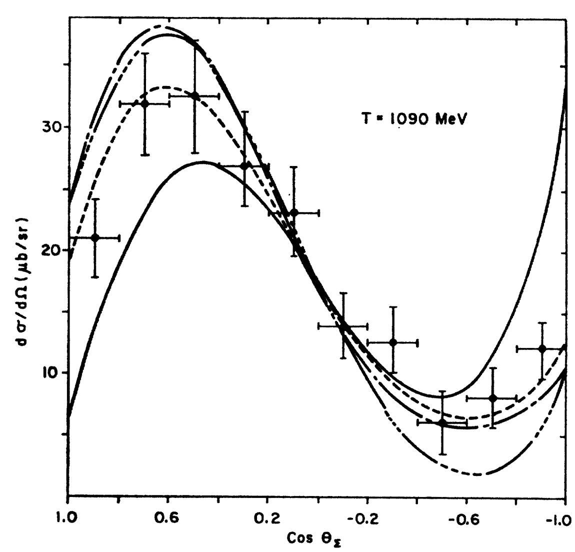

FIG. 4. Angular distribution of the produced Σ^+ at 1220 MeV/c. The data are from Ref. 7. Similar fits exist for the data at 1206 MeV/c from Ref. 13. The caption of Fig. 1 gives the meaning of the curves.

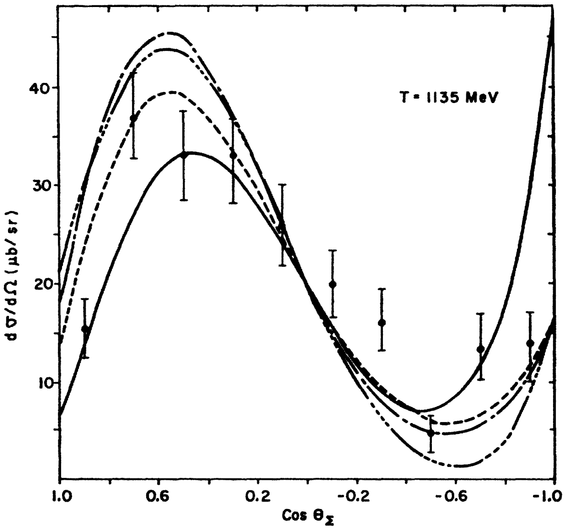

FIG. 5. Angular distribution of the produced Σ^+ at 1265 MeV/c. The data are from Ref. 13. The caption of Fig. 1 gives the meaning of the curves.

The addition of K^* exchange with parameters as given in Table II constitutes Model III and offers little, if any, improvement over Model II.

Model IV

In the previous models, the s-wave contribution to the production amplitudes comes entirely from the pole terms, which are real. Because other channels are open, it is not reasonable to suppose that the s-wave amplitude is real. We have, therefore, simply multiplied the pole contributions by the factor $e^{i\delta}$. In Models IV and V, δ is taken to be a constant. It may bear some relation to the s-wave π^+p elastic-scattering phase shift. With the

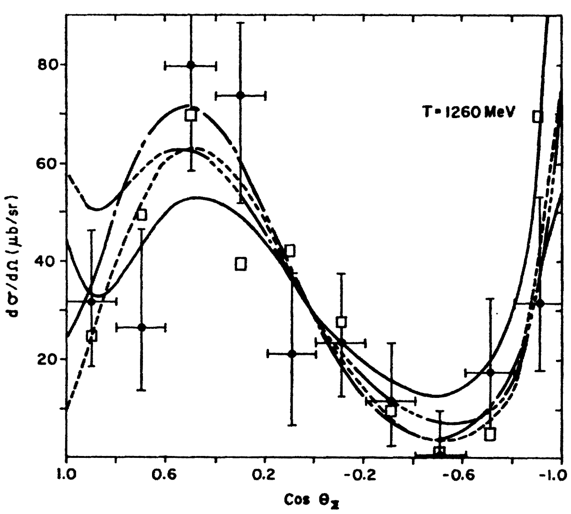

FIG. 6. Angular distribution of the produced Σ^+ at 1390 MeV/c. The data given by the solid circle are from Ref. 7; the open squares have comparable deviations and are from Ref. 11. The caption of Fig. 1 gives the meaning of the curves.

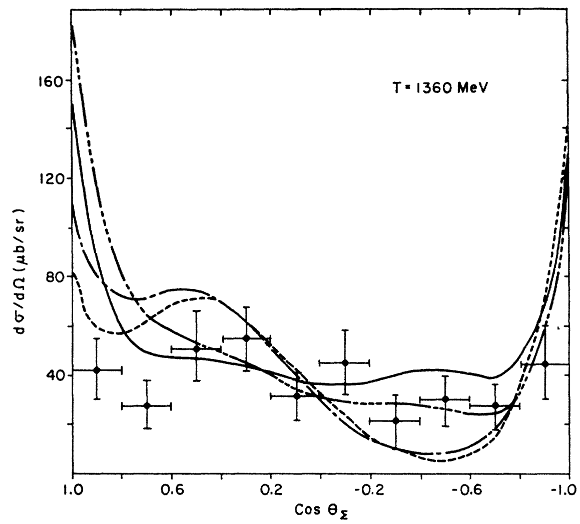

FIG. 7. Angular distribution of the produced Σ^+ at 1494 MeV/c. The data are from Ref. 12. The caption of Fig. 1 gives the meaning of the curves.

parameters chosen as in Table II, the curves in Figs. 1–14 with the form ——— — are calculated. The total and differential cross sections generally remain in acceptable agreement with the data and the magnitude of the Σ^+ polarization increases substantially over the results of the models discussed above. Even so, these calculated values of Σ^+ polarization appear, at least at some energies, not to be large enough. However, the experimental values have large statistical deviations and do not completely agree among themselves. Some experimental clarification of this matter would be beneficial.

Model V

The primary difference between Model V and Model IV is that in the former the total widths of the two

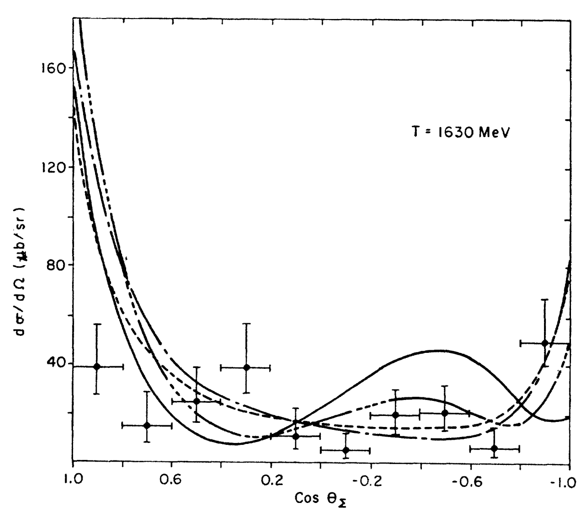

FIG. 8. Angular distribution of the produced Σ^+ at 1760 MeV/c. The data are from Ref. 11. The caption of Fig. 1 gives the meaning of the curves.

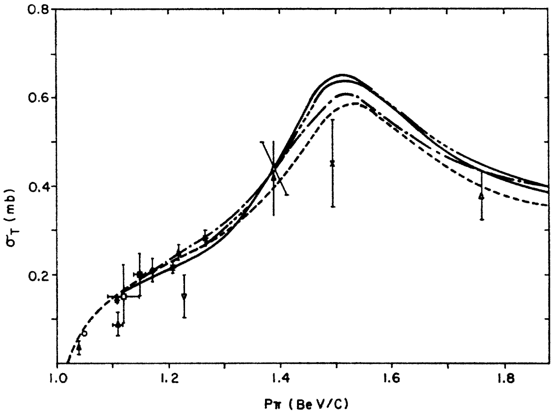

FIG. 9. The total cross-section for the process $\pi^+ + p \rightarrow \Sigma^+ + K^+$ as a function of pion laboratory momentum. The data represented by the closed triangle ▲ are from Ref. 7; the open circle ○, Ref. 9; the asterisk *, Ref. 13; the open square □, Ref. 3; the closed square ■, Ref. 6; the closed circle ●, Ref. 10; the open del ▽, Ref. 1; the open triangle △, Ref. 11; the cross ×, Ref. 12. Figure 1 gives the meaning of the curves.

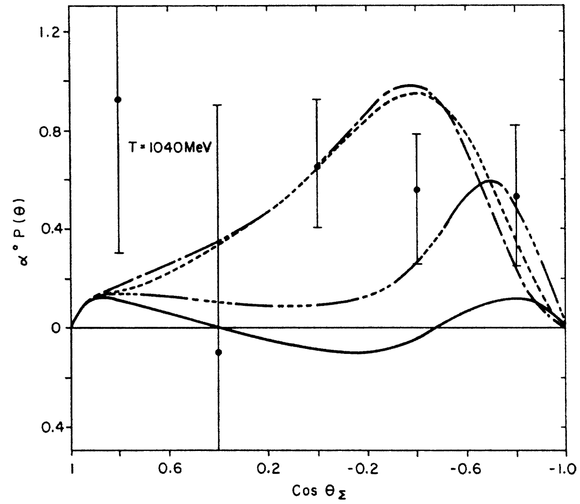

FIG. 11. The polarization of the Σ^+ as a function of the production angle in the center of mass at pion laboratory momentum of 1170 MeV/c. The data on $\alpha^0 P(\theta)$ are from Ref. 10. The caption of Fig. 1 gives the meaning of the curves, which are the computed polarization (plotted here as if $\alpha^0 = 1$).

resonances are taken to be constants as opposed to the energy dependence given in Eq. (16). Results for Model V with parameters given in Table II are plotted in Figs. 1–14 as the curves – – –. It is apparent that no very distinctive differences exist between the results of Model V and Model IV.

Note that in Model V the K^* pole is not used. This is not particularly significant except insofar as it points up the lack of sensitivity of the results on the presence or absence of the K^* pole (compare also Models II and III). Because of this lack of sensitivity, very little weight should be attached to the values of G' in Table II.

V. DISCUSSION AND CONCLUSIONS

It is apparent that a relatively simple model yields fair agreement with the available data on the total and differential cross sections for the process $\pi^+ + p \rightarrow \Sigma^+ + K^+$ through the 1925-MeV resonance. It has been difficult to obtain a large enough value of the Σ^+ polarization. Considerably better results have been obtained for this quantity through the introduction of the purely phenomenological parameter δ. It will be interesting to see whether this parameter has any utility in the description of other similar processes. Experiments on the Σ^+ polarization almost anywhere in the energy region under consideration would be helpful in assessing the value of the models discussed above. Even so, the generally favorable agreement with the data provided by these models lends strong support to an $l = 3$ assignment for the 1925-MeV resonance, which is in

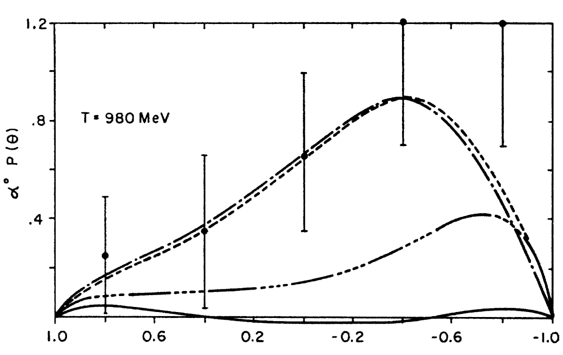

FIG. 10. The polarization of the Σ^+ as a function of the production angle in the center of mass at pion laboratory momentum of 1111 MeV/c. The data on $\alpha^0 P(\theta)$ are from Ref. 13. The caption of Fig. 1 gives the meaning of the curves, which are the computed polarization (plotted here as if $\alpha^0 = 1$).

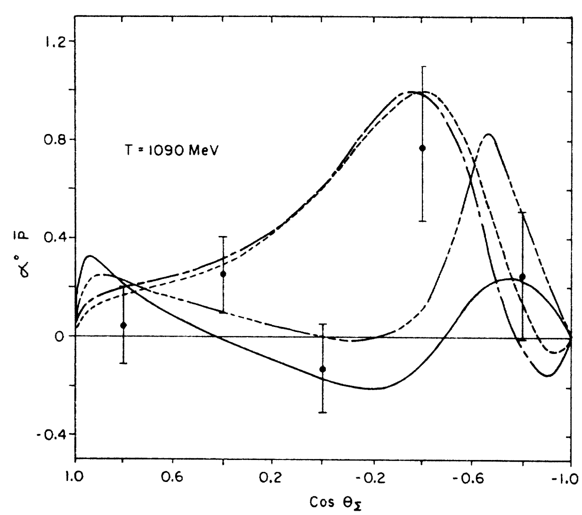

FIG. 12. The polarization of the Σ^+ as a function of the production angle in the center of mass at pion laboratory momentum of 1220 MeV/c. The data on $\alpha^0 P(\theta)$ are from Ref. 7, Table V. Similar fits exist for the data at 1206 MeV/c from Ref. 13. The caption of Fig. 1 gives the meaning of the curves, which are the computed polarizations (plotted here as if $\alpha^0 = 1$).

accord with the hypothesis that this resonance is a Regge recurrence of the $p_{3/2}$ 1238-MeV resonance.

None of the models considered above fits very well the angular distribution of Σ^+ production at beam momentum 1760 MeV/c. This result indicates that some other contribution to Σ^+ production is beginning to appear at this energy.

Choices of the width and position of the 1925 resonance other than those which appear in Table II have been considered. The angular distribution at 1260 MeV is fairly sensitive to the position of this resonance, and results at this energy were not quite as good when W_3 is taken to be 1900 and 1940 MeV as for $W_3=1925$. The poor statistics in the data prevents this conclusion from being too cogent. Additional data at this energy would be useful in this regard. Similarly, the data are not really able to distinguish between choices of Γ_3 ranging from 125 to 175 MeV.

Several values for the range parameter R were also considered. No particular difficulty is encountered if R is chosen to be greater than one-pion Compton wave length. However, values of R less than this cause the total cross section to be too large at and above the position of the 1925-MeV resonance.

It should be noted that (as Evans and Knight[15] also found) the introduction of the $p_{3/2}$ resonance is necessary in order to obtain the correct angular distribution at beam momentum within 100 MeV/c of threshold. In addition, the success of the models at higher energy has been attained with quantum numbers of $f_{7/2}$ for the 1925-MeV resonance. Studies are under way with quantum numbers of $g_{7/2}$ for this resonance, and preliminary results, which need further attention, indicate that the $f_{7/2}$ is preferable to $g_{7/2}$, in agreement also with Evans and Knight.

Some additional comments of comparison between the results of this paper and those of Evans and Knight[15]

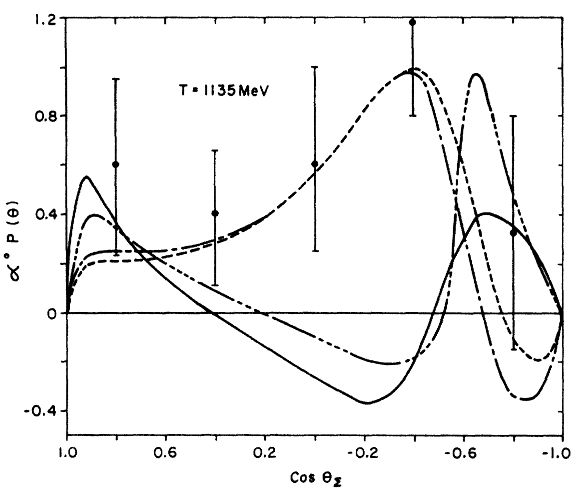

FIG. 13. The polarization of the Σ^+ as a function of the production angle in the center of mass at pion laboratory momentum of 1265 MeV/c. The data on $\alpha^0 P(\theta)$ are from Ref. 13. The caption of Fig. 1 gives the meaning of the curves, which are the computed polarization (plotted here as if $\alpha^0=1$).

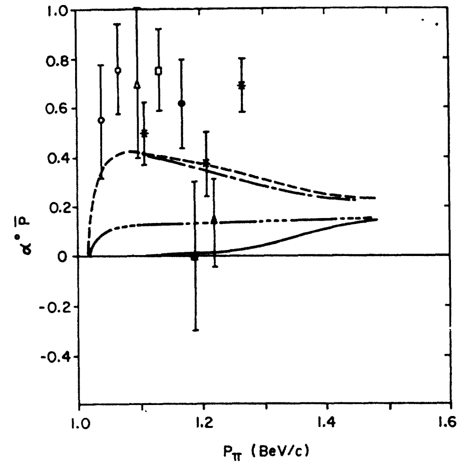

FIG. 14. The average polarization of the Σ^+ as a function of the pion laboratory momentum. The data on $\alpha^0\bar{P}$ are represented by the open circles ○, Ref. 9; the open triangle △, Ref. 2; the asterisk *, Ref. 13; the open square □, Ref. 5; the solid circle ●, Ref. 10; the closed square ■, Ref. 8; the closed triangle ▲, Ref. 7. The caption of Fig. 1 gives the meaning of the curves, which are the computed polarization (plotted here as if $\alpha^0=1$).

may be in order. First, it is clear that the general content and conclusions of the two models are similar. The difference originates in that they have used a barrier factor in their expression for the total and partial widths as given by Glashow and Rosenfeld[40] with a range parameter corresponding to $0.4\hbar/(m_\pi c)$. With such a choice the total widths of the resonances, which determine the imaginary parts of the production amplitudes, increase rapidly with energy, and lead to fairly sizeable polarization. On the other hand, the older form of the barrier factors[32] with a larger range parameter[34] R of $1.0\hbar/m_\pi c$ has been used here. The total cross section computed with such an R does not peak at a value quite as large as that of Evans and Knight, but neither is the computed polarization as large as theirs (Model III resembles most closely their model). To increase the polarization, the parameter δ (Models IV and V) has been introduced. With $\delta\neq 0$, equally good results can be obtained whether or not the total widths of the resonances are made energy-dependent (compare Models IV and V).

ACKNOWLEDGMENTS

The dedicated efforts of Ralph F. Penland in writing computer programs have been essential in obtaining the results reported in this paper. Additional assistance was provided by H. L. Crowell. Professor L. E. Evans of Duke was most helpful in providing information on his computations with Dr. Knight, and, especially, in emphasizing the importance of the energy dependence of the total width of a resonance. Professor Earle C. Fowler has provided beneficial written and oral information on the experimental situation.

[40] S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters 10, 192 (1963).