Broken Charge Symmetry in Static Strong Coupling*†

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Standard perturbation techniques are applied to the study of the energy level splitting in the nucleon isobar states which results from introducing charge-symmetry violation. Whereas the scalar interaction model shows a high sensitivity to the symmetry breaking, the corresponding asymmetries are typically suppressed in the model with pseudoscalar mesons, owing to the interdependence of spin and isospin in the strong-coupling theory.

I. INTRODUCTION

IN the framework of the static strong-coupling theory,[1] the isospin-symmetric meson-nucleon interaction is known to give a series of nucleon excited states (isobars) with energies

$$E_T=\epsilon T(T+1);\quad T=\tfrac{1}{2},\tfrac{3}{2}\cdots,\qquad T_3=\pm T,\ \pm(T-1)\cdots,\tag{1.1}$$

where T denotes the isospin of the multiplet. In the case of the scalar interactions, all the isobar states have their spin $J=\frac{1}{2}$, since in the static limit the scalar mesons are bound only in S states. For the pseudoscalar mesons only isobar states with equal spin (J) and isospin (T) quantum numbers are found to occur, associated with the fact that the spin and isospin variables play a symmetric role. In particular, the part of the Hamiltonian describing the isobar states has the same form as that of a spherical top, whose angular momenta along the "space-fixed" and the "body-fixed" axes are interpreted as the J_z and T_3 quantum numbers of the isobar state, while the total "angular momentum" is identified with both J and T.

The degeneracy among the different charge states of these isobar multiplets will be broken if the charge symmetry is broken. This can be achieved in essentially two inequivalent ways. The bare neutron-proton mass difference would produce no effect in the strong-coupling limit, since in all the stationary states equal mixtures of both would occur. The symmetry violation could then be introduced either in the mass spectrum of mesons (which we shall term the μ mechanism) or in their couplings with the nucleon source (which will be called the g mechanism). We will examine in this paper, using standard perturbation techniques, the magnitude of the level splitting thus resulting among the multiplets.

The first-order perturbation calculations for the scalar interactions are found to give a large splitting, particularly enhanced by a "large" g^2 factor. In contrast, in the case of pseudoscalar mesons the effect is considerably inhibited, the first-order effect entirely *vanishing* if the symmetry violation is through the μ mechanism[2] and very small (compared with the scalar model) in the g mechanism. This difference in sensitivity to the symmetry breaking may be attributed to the peculiar interdependence of spin and isospin in the pseudoscalar model. The anisotropy in charge space cannot be very large if the isotropy of ordinary space is maintained.

II. SCALAR INTERACTIONS

A. μ Mechanism

We write the Hamiltonian for a scalar field, omitting the bare nucleon energy:

$$H=H_0+H',\tag{2.1}$$

$$H_0=\sum_\rho\int d^3x[\pi_\rho^2(\mathbf{x})+\psi_\rho(\mathbf{x})(\mu_\rho^2-\Delta)\psi_\rho(\mathbf{x})],\tag{2.2}$$

$$H'=g_s\sum_\rho\int d^3x\,\delta_a(\mathbf{x})\tau_\rho\psi_\rho(\mathbf{x}),\tag{2.3}$$

where the symbols have their usual meaning. $\delta_a(\mathbf{x})$ specifies the form factor of the nucleon, normalized so that

$$\int d^3x\,\delta_a(\mathbf{x})=1\,,\quad \int d^3x\,\delta_a^2(\mathbf{x})=C^2.\tag{2.4}$$

g_s is the dimensionless coupling constant. Charge symmetry is strictly preserved in the interaction Hamiltonian H' (2.3) and the violation is carried entirely by H_0 (2.2), specified through

$$\mu_\rho^2=\mu^2+\alpha_\rho;\quad \alpha_1=\alpha_2=-\tfrac{1}{2}\alpha_3=\alpha(\ll\mu^2).\tag{2.5}$$

Equation (2.1) may be rewritten so that the weak symmetry-breaking terms are explicitly expressed as a perturbation:

$$H=\mathcal{H}+\delta H\,,$$

where $\mathcal{H}$ is given by simply writing μ^2 for μ_ρ^2 and

$$\delta H=\tfrac{1}{2}\sum_\rho\alpha_\rho\int d^3x\psi_\rho^2(\mathbf{x}).\tag{2.6}$$

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[1] G. Wentzel, Rev. Mod. Phys. 19, 1 (1947). Further references are quoted here.

[2] G. Wentzel, using a different approach, has observed this effect in the μ mechanism, Helv. Phys. Acta 38, 65 (1965). The extension to the g mechanism is one main task of this work.

To treat this perturbation, we may now recapitulate some usual techniques in the strong-coupling theory. The meson field operator $\psi_\rho(\mathbf{x})$ is normally split into a dominant part $\psi_\rho^{(0)}(\mathbf{x})$ that describes the mesons "bound" to form the isobar states, and a part orthogonal to it that corresponds to the quasifree mesons and interacts only weakly with the bound nucleon. Explicitly

$$\psi_\rho^{(0)}(\mathbf{x})=g_sC\frac{1}{(\mu^2-\Delta)}\delta_a(\mathbf{x})e_\rho\,, \tag{2.7}$$

where e_ρ is a unit vector in a 3-dimensional space and the corresponding polar angles θ and ϕ are the essential coordinates of the bound system. In particular, the rotational energy, with eigenvalues (1.1), is[3]

$$\begin{aligned}H_T&=\epsilon P^2\,,\\ P^2&=-\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\left(\frac{\partial^2}{\partial\phi^2}+i\cos\theta\frac{\partial}{\partial\phi}+\tfrac{1}{4}\right)\right],\\ P_3&=\frac{1}{i}\frac{\partial}{\partial\phi}=T_3\,,\\ \epsilon&=\tfrac{1}{2}g_s^{-2}\left[\int d^3x\delta_a(\mathbf{x})(\mu^2-\Delta)\delta_a(\mathbf{x})\right]^{-1}=\frac{4\pi\mu}{g_s^2}.\end{aligned} \tag{2.8}$$

Thus, writing only the dominant $\psi_\rho^{(0)}(\mathbf{x})$ for $\psi_\rho(\mathbf{x})$ in (2.6), we get

$$\begin{aligned}\delta H&=\tfrac{1}{2}g_s^2C^2z\sum_\rho\alpha_\rho e_\rho^2\,,\\ C^2z&=\int d^3x\,\delta_a(\mathbf{x})(\mu^2-\Delta)^{-2}\delta_a(\mathbf{x})=8\pi/\mu\,.\end{aligned} \tag{2.9}$$

The expectation value of δH for the rotational states gives the first-order perturbation to the energy of the isobar states. Accordingly, using (2.4), we get

$$\Delta E_{T,T_3}=\frac{4\pi g_s^2\alpha}{\mu}\langle T,T_3|1-3e_3^2|T,T_3\rangle. \tag{2.10}$$

The matrix elements are easily evaluated, giving

$$\Delta E_{T,T_3}=\frac{4\pi g_s^2\alpha}{\mu}\left[1-\frac{6T(T+1)-3}{(2T-1)(2T+3)}+\frac{6T_3^2}{(2T-1)(2T+3)}\right]. \tag{2.11}$$

It may be noticed that because of the factor $g_s^2(\gg 1)$, even for a small anisotropy in the meson masses we may have a large splitting among the isobar states. The ratio of this level splitting within a multiplet to the spacing ($\sim\epsilon$) between the multiplets is of the order of $3\alpha g_s^4/2\mu^2$, which should be small for the perturbation approximation to be valid. Thus

$$\alpha\ll\alpha_c=\frac{2\mu^2}{3g_s^4}\,,\quad\text{or}\quad\frac{|\mu_1-\mu_3|}{\mu}\ll g_s^{-4}. \tag{2.12}$$

This may be compared with Wentzel's[2] Eq. (22), where it has been shown that when α reaches the critical value $(4\pi)^2\alpha_c$, a strong violation of charge symmetry sets in and the isobar levels get drastically modified.

B. g Mechanism

Now we shall investigate the effects of introducing the primary symmetry violation in the coupling constant. The interaction Hamiltonian (2.3) is then replaced by

$$H'=g_\rho\sum_\rho\int d^3x\,\delta_a(\mathbf{x})\tau_\rho\psi_\rho(\mathbf{x})\,, \tag{2.13}$$

with $g_\rho=g_s+\beta_\rho$, where $\beta_1=\beta_2=-\frac{1}{2}\beta_3=\beta(\ll|g_s|)$.[4] It is most convenient to make a transformation defining a new set of field variables such that

$$\psi_\rho(\mathbf{x})\rightarrow(g_s/g_\rho)\psi_\rho(\mathbf{x})\,, \tag{2.14a}$$

$$\pi_\rho(\mathbf{x})\rightarrow(g_\rho/g_s)\pi_\rho(\mathbf{x})\,, \tag{2.14b}$$

the transformation preserving canonical commutativity. This restores isotropy in H' and the violation of the symmetry is once again carried in H_0.

$$\begin{aligned}H&=\tfrac{1}{2}\sum_\rho\int d^3x[(1+\beta_\rho/g_s)^2\pi_\rho^2(\mathbf{x})\\ &\qquad+(1+\beta_\rho/g_s)^{-2}\psi_\rho(\mathbf{x})(\mu^2-\Delta)\psi_\rho(\mathbf{x})]\\ &\qquad\qquad+g_s\sum_\rho\int d^3x\,\delta_a(\mathbf{x})\tau_\rho\psi_\rho(\mathbf{x})\\ &=\mathcal{H}+\delta H\,.\end{aligned} \tag{2.15}$$

To first order in β

$$\delta H=g_s^{-1}\sum_\rho\beta_\rho\int d^3x[\pi_\rho^2(\mathbf{x})-\psi_\rho(\mathbf{x})(\mu^2-\Delta)\psi_\rho(\mathbf{x})]. \tag{2.16}$$

As before, the field variables $\psi_\rho(\mathbf{x})$ and $\pi_\rho(\mathbf{x})$ have to be split into a "bound" part and "free" terms. The contributions from $\pi_\rho^2(\mathbf{x})$ are negligible for $g_s\gg 1$. On substituting for $\psi_\rho^{(0)}(\mathbf{x})$, the second term gives

$$\delta H=-g_s^{-1}\sum_\rho\beta_\rho g_s^2C^2Ye_\rho^2\,, \tag{2.17}$$

[3] In the scalar model, ϵ^{-1} is convergent and we may let the source radius tend to zero. Thus $\delta_a(\mathbf{x})\rightarrow\delta(\mathbf{x})$, the Dirac delta function.

[4] When the charge symmetry is broken, we can also have an additional interaction term of the type $g'\int d^3x\,\delta_a(\mathbf{x})\psi_3(\mathbf{x})$, indicating different coupling strengths for neutral mesons with protons and neutrons. Their first-order effect, however, will be vanishing for any unperturbed isobar state.

where[5]

$$C^2Y=\int d^3x\,\delta_a(\mathbf{x})(\mu^2-\Delta)^{-1}\delta_a(\mathbf{x})=\frac{1}{4\pi a}. \quad (2.18)$$

Hence, the first-order correction to the energy of the isobar states is given by

$$\Delta E_{T,T_3}=-\frac{\beta}{g_s}\frac{g_s^2}{4\pi a}\langle T,T_3|1-3e_3^2|T,T_3\rangle$$
$$=-\frac{\beta g_s}{4\pi a}\left[1-\frac{6T(T+1)-3}{(2T-1)(2T+3)}+\frac{6T_3^2}{(2T-1)(2T+3)}\right]. \quad (2.19)$$

Again, because of the factor g_s and the factor $1/a$, the symmetry breaking in isobar multiplets will be large. The validity of the perturbation approximation requires, since $\epsilon\sim 4\pi\mu/g_s^2$,

$$\beta\ll\beta_c=\frac{(4\pi)^2a\mu}{g_s^3} \quad\text{or}\quad \frac{|g_1-g_3|}{g_s}\ll\frac{(4\pi)^2a\mu}{g_s^4}. \quad (2.20)$$

III. PSEUDOSCALAR MESONS

A. μ Mechanism

Instead of Eq. (2.3), the interaction part now reads

$$H'=g_p\sum_\rho\int d^3x\,\sigma_i\tau_\rho\psi_\rho(\mathbf{x})\nabla_i\delta_a(\mathbf{x}) \quad (3.1)$$

which, as in the scalar model, remains strictly charge-symmetric. g_p now has the dimensions of length, and the strong-coupling condition requires $g_p\gg a$, the source radius a being as defined by Eq. (2.18), supposing again $a\mu\ll 1$. Then we once again have Eq. (2.6) giving the energy perturbation:

$$\delta H=\tfrac{1}{2}\sum_\rho\alpha_\rho\int d^3x\psi_\rho^2(\mathbf{x}). \quad (2.6)$$

$\psi_\rho(\mathbf{x})$, when split into the relevant bound meson part and the "free" meson part, takes the form

$$\psi_\rho(\mathbf{x})=\psi_\rho^{(0)}(\mathbf{x})+\psi_\rho'(\mathbf{x}), \quad (3.2)$$

$$\psi_\rho^{(0)}(\mathbf{x})=g_p\sum_i S_{i\rho}\frac{\partial\xi}{\partial x_i};\quad \xi(\mathbf{x})=(\mu^2-\Delta)^{-1}\delta_a(\mathbf{x}), \quad (3.3)$$

where $S_{i\rho}$ is an orthogonal transformation matrix, expressible in terms of the 3 Eulerian angles Θ, Φ, Ψ, which are now the essential coordinates of the bound system. In particular, similar to the scalar model, the rotational energy with eigenvalues (1.1) is

$$H_T=\epsilon P^2,$$

$$P^2=-\left\{\frac{1}{\sin\Theta}\frac{\partial}{\partial\Theta}\sin\Theta\frac{\partial}{\partial\Theta}+\frac{1}{\sin^2\Theta}\left(\frac{\partial^2}{\partial\Phi^2}+2\cos\Theta\frac{\partial^2}{\partial\Psi\partial\Phi}+\frac{\partial^2}{\partial\Psi^2}\right)\right\}, \quad (3.4)$$

$$P_3=\frac{1}{i}\frac{\partial}{\partial\Psi}=T_3$$

with

$$\epsilon=\tfrac{3}{4}g_p^{-2}\left[\int d^3x\delta_a(\mathbf{x})\frac{-\Delta}{(\mu^2-\Delta)^2}\delta_a(\mathbf{x})\right]^{-1}\simeq 3\pi a/g_p^2 \quad (3.5)$$

when $a\mu\ll 1$. In contrast to the scalar theory, we shall also need ψ_ρ', describing the quasifree mesons, but it will be sufficient to approximate them as "free" mesons:

$$\psi_\rho'(\mathbf{x})=\frac{1}{(2\pi)^{3/2}}\int d^3k\frac{\exp(i\mathbf{k}\cdot\mathbf{x})}{(2\omega_\mathbf{k})^{1/2}}(a_{\mathbf{k},\rho}+a^\dagger_{-\mathbf{k},\rho});$$
$$\omega_\mathbf{k}=(k^2+\mu^2)^{1/2}, \quad (3.6)$$

where $a^\dagger_{\mathbf{k},\rho}$ and $a_{\mathbf{k},\rho}$ act as creation and destruction operators and obey the usual commutation rules.

Now, substituting (3.2) and (3.3) in (2.6), we obtain

$$\delta H=\tfrac{1}{2}\sum_\rho\alpha_\rho\int d^3x\left[g_p\sum_i S_{i\rho}\frac{\partial\xi}{\partial x_i}+\psi_\rho'(\mathbf{x})\right]^2. \quad (3.7)$$

Since

$$\int d^3x\frac{\partial\xi}{\partial x_i}\frac{\partial\xi}{\partial x_j}=Z\delta_{ij}, \quad (3.8)$$

Eq. (3.7) becomes

$$\delta H=\tfrac{1}{2}\sum_\rho\alpha_\rho\left\{g_p^2Z\sum_i S_{i\rho}^2+2g_p\sum_i S_{i\rho}\int d^3x\psi_\rho'(\mathbf{x})\frac{\partial\xi}{\partial x_i}+\int d^3x\,\psi_\rho'(\mathbf{x})^2\right\}. \quad (3.9)$$

The first term corresponds to Eq. (2.9), but here it vanishes identically in consequence of the orthogonality relations $\sum_i S_{i\rho}^2=1$ (and $\sum_\rho\alpha_\rho=0$). This is not surprising in view of the argument made in the Introduction. While the third term can have no effect on the bound system, we find with regard to the contributions from the second term, since the expectation value of $\psi_\rho'(x)$ is zero for any unperturbed isobar state, that the *first-order* energy correction δE vanishes. So we have to calculate the second-order effect of the pertinent term

[5] Equation (2.18) corresponds to the conventional definition for the nucleon size a in the strong coupling theories if $a\mu\ll 1$. We will indeed be primarily interested in the limit $a\mu\ll 1$.

in (3.9):

$$\Delta E_{T,T_3} = \sum_{|I\rangle} \frac{\left|\langle I|g_p\sum_\rho \alpha_\rho \sum_i S_{i\rho}\int d^3x\psi_\rho'(x)\frac{\partial\xi}{\partial x_i}|T,T_3,J_z\rangle\right|^2}{(E_T-E_I)}, \tag{3.10}$$

where $|T,T_3,J_z\rangle$ is an isobar eigenstate $(J=T)$, with no free mesons. Only intermediate states with one free meson will contribute to (3.10); thus $|I\rangle$ assumes the form $|T', T_3',J_z'; k\rho\rangle$. Substituting (3.6) for $\psi_\rho'(x)$ in Eq. (3.10), we easily obtain

$$\Delta E_{T,T_3,J_z} = \sum_{T',T_3',J_z'} g_p^2 \int \frac{4\pi}{6}\frac{k^4dk}{\omega_k^5}v^2(k) \times \sum_{i\rho} \frac{\alpha_\rho^2|\langle T',T_3',J_z'|S_{i\rho}|T,T_3,J_z\rangle|^2}{E_T-E_{T'}-\omega_k}. \tag{3.11}$$

(vk) is the Fourier transform of $\delta_a(x)$ and the normalization (2.4) implies $v(0)=1$. $S_{i\rho}$ has nonvanishing matrix elements only for states with[6] $T'-T=0, \pm1$. They are evaluated to give

$$\sum_{i\rho T_3',J_z'} \alpha_\rho^2|\langle T',T_3',J_z'|S_{i\rho}|T,T_3,J_z\rangle|^2$$

$$=\frac{\alpha^2}{T(T+1)}[T(T+1)+3T_3^2], \quad \text{when } T'=T; \tag{3.12a}$$

$$=\alpha^2\left[\frac{5T+6}{2T+1}-\frac{3T_3^2}{(T+1)(2T+1)}\right], \quad \text{when } T'=T+1; \tag{3.12b}$$

$$=\alpha^2\left[\frac{5T-1}{2T+1}-\frac{3T_3^2}{T(2T+1)}\right], \quad \text{when } T'=T-1. \tag{3.12c}$$

Substituting in (3.11), we get

$$\Delta E_{T,T_3,J_z} = -g_p^2\alpha^2\left[\mathcal{A}+\frac{5T+6}{2T+1}\mathcal{B}(T)+\frac{5T-1}{2T+1}\mathcal{C}(T)\right] +3T_3^2g_p^2\alpha^2\left[-\frac{\mathcal{A}}{T(T+1)} +\frac{\mathcal{B}}{(T+1)(2T+1)}+\frac{\mathcal{C}}{T(2T+1)}\right], \tag{3.13}$$

[6] M. Fierz, Helv. Phys. Acta **17**, 181 (1944); **18**, 158 (1945).

where

$$\mathcal{A}=\frac{4\pi}{6}\int\frac{k^4dk}{\omega_k^6}=\frac{4\pi^2}{6\mu}\times\frac{3}{16},$$

$$\mathcal{B}(T)=\frac{4\pi}{6}\int\frac{k^4dk}{\omega_k^5[2\epsilon(T+1)+\omega_k]}=\frac{4\pi^2}{6\mu}f_B\left(\frac{2\epsilon(T+1)}{\mu}\right)$$

$$\mathcal{C}(T)=\frac{4\pi}{6}\int\frac{k^4dk}{\omega_k^5[\omega_k-2\epsilon T]}=\frac{4\pi^2}{6\mu}f_C\left(\frac{2\epsilon T}{\mu}\right).$$

Since the level spacings of the rotational states $[2\epsilon(T+1)]$ are of the dimension of the pion mass (in the real situation) $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ are of comparable magnitudes. For example, with the help of the position of the observed (3,3) resonance, ϵ can be determined, and using this value we have evaluated the integrals for the $T=\frac{3}{2}$ state: With $\epsilon\sim0.7\,\mu$,

$$\mathcal{A}=\frac{4\pi^2}{6\mu}(0.19),\quad \mathcal{B}(\tfrac{3}{2})=\frac{4\pi^2}{6\mu}(0.10),$$

and

$$\mathcal{C}(\tfrac{3}{2})=\frac{4\pi^2}{6\mu}(0.45).$$

Hence,

$$\Delta E_{3/2,T_3,J_z}=\frac{4\pi^2g_p^2\alpha^2}{6\mu}[-1.26+0.102T_3^2]. \tag{3.14}$$

Note the factor α^2, as compared with α in (2.11).

Again, for the validity of the perturbation, the level splitting in a given multiplet must be small compared to the level spacing ($\sim\epsilon$) between the multiplets. This requires

$$\frac{g_p^2\alpha^2}{\mu}\Big/\frac{a}{g_p^2}\ll1 \quad\text{or}\quad \frac{|\mu_1-\mu_3|}{\mu}\ll\frac{3}{2}\frac{(a\mu)^{1/2}}{\mu^2g_p^2}. \tag{3.15}$$

On substituting the nucleon Compton wavelength for the cutoff parameter a, and taking g_p^2 from $\epsilon=3\pi a/g_p^2$, the right-hand side of (3.15) amounts to 0.29, in comparison with which the actual mass difference between charged and neutral mesons, 0.033 in units of μ, is indeed small. The resulting contribution to the mass difference between the $|T_3|=\frac{3}{2}$ and $|T_3|=\frac{1}{2}$ states of the (3,3) isobar is also very small (of the order of 1.6 MeV).[7]

B. g Mechanism

Charge symmetry broken by the g mechanism introduces the anisotropy in the interaction Hamiltonian.[8]

[7] This, of course, omits possible electromagnetic corrections like that of the neutron-proton mass difference, which is also of the same order of magnitude.

[8] Here again, as in the scalar model, the additional term in the interaction Hamiltonian $g'\int d^3x\,\sigma_i[\nabla_i\delta_a(\mathbf{x})]\psi_3(\mathbf{x})$, which denotes asymmetrical $\bar{p}p\pi^0$ and $\bar{n}n\pi^0$ couplings, gives vanishing contributions, since it has only off-diagonal terms which mix states that are *widely* separated in the strong-coupling limit. Further, the second-order (in g') corrections, resulting from such mixing, are much weaker than the corresponding effect (which involves mixing between adjacent isobar multiplets) in the scalar model. (Ref. 4.)

As in the scalar model, the transformations on $\psi_\rho(\mathbf{x})$ and $\pi_\rho(\mathbf{x})$, defined by (2.14 a, b), restore isotropy in H'; and corresponding to (2.16), but considering the symmetry violation to all orders in β, we have

$$\delta H=\sum_\rho (\beta_\rho/g_p+\tfrac{1}{2}\beta_\rho^2/g_p^2)\int d^3x\pi_\rho^2(\mathbf{x})+\tfrac{1}{2}\sum_\rho[(1+\beta_\rho/g_p)^{-2}-1]\times\int d^3x\,\psi_\rho(\mathbf{x})(\mu^2-\Delta)\psi_\rho(\mathbf{x}). \quad (3.16)$$

First let us examine the ψ_ρ-dependent terms. After splitting $\psi_\rho(\mathbf{x})$ into $\psi_\rho^{(0)}(\mathbf{x})$ and $\psi_\rho'(\mathbf{x})$, we obtain with (3.4)

$$\int d^3x\,\psi_\rho^{(0)}(\mathbf{x})(\mu^2-\Delta)\psi_\rho^{(0)}(\mathbf{x})=g_p^2\sum_{ij}S_{i\rho}S_{j\rho}\int d^3x\frac{\partial\xi}{\partial x_i}\frac{\partial\delta_a}{\partial x_j}=g_p^2\sum_{ij}S_{i\rho}S_{j\rho}\gamma\delta_{ij}, \quad (3.17)$$

where we have performed the integration over d^3x. Now, since $\sum_i S_{i\rho}^2=1$, the terms in Eq. (3.17), the analogs of which gave the major contribution in the scalar model, turn out to be independent of the isobar variables. Next, the weaker term

$$2\int d^3x\,\psi_\rho'(\mathbf{x})(\mu^2-\Delta)\psi_\rho^{(0)}(\mathbf{x})=2g_p\sum_i S_{i\rho}\int d^3x\,\psi_\rho'(\mathbf{x})\frac{\partial\delta_a}{\partial x_i} \quad (3.18)$$

also does not contribute, since the part of $\psi_\rho'(\mathbf{x})$ projected out by $\partial\delta_a(\mathbf{x})/\partial x_i$, together with $S_{i\rho}$, defines only the p-wave part of the "free" mesons which have negligible interaction with the bound system.[9] These correspond to the variables $\xi_{\rho\sigma}$, defined by Houriet.[10] Thus, in contrast to the scalar model, the ψ-dependent term in (3.16) plays no part in the energy splitting.

The symmetry-breaking effect is then given by a "smaller" $\pi_\rho^2(\mathbf{x})$ term, which must therefore be split into the relevant bound part and the free-meson part.

$$\pi_\rho(\mathbf{x})=\pi_\rho^{(0)}(\mathbf{x})+\pi_\rho'(\mathbf{x}), \quad (3.19)$$

$$\pi_\rho^{(0)}(x)=\frac{1}{C}\sum_i p_{i\rho}\frac{\partial\delta_a}{\partial x_i};\quad C^2=\int d^3x\left(\frac{\partial\delta_a}{\partial x_i}\right)^2. \quad (3.20)$$

$p_{i\rho}$ are the conjugates of $S_{i\rho}$ and obey the commutation relations

$$[p_{i\rho},S_{j\tau}]=\frac{1}{2\Gamma}(\delta_{ij}\delta_{\rho\tau}-S_{i\tau}S_{j\rho});\quad \Gamma\sim\epsilon^{1/2}.$$

The bound-field part of $\int d^3x\,\pi_\rho^2(\mathbf{x})$ is essentially proportional to $\sum_i p_{i\rho}^2$. These terms, omitting all free-field parts, are diagonal in the T, T_3, J_z representation.

$$\sum_{i\rho}p_{i\rho}^2=2\epsilon P^2+\cdots,\quad \sum_i p_{i3}^2=(P^2-P_3^2)+\cdots. \quad (3.21)$$

Accordingly, the pertinent part of (3.16) can be written as

$$\delta H=\frac{\beta}{g_p}\epsilon[2P^2-3(P^2-P_3^2)]+\frac{1}{2}\left(\frac{\beta}{g_p}\right)^2[2P^2+3(P^2-P_3^2)]. \quad (3.22)$$

Hence, we have the energy correction

$$\Delta E_{T,T_3,J_z}=\langle T,T_3,J_z|\delta H|T,T_3,J_z\rangle=-\frac{\beta}{g_p}\epsilon[T(T+1)-3T_3^2]+\frac{1}{2}\left(\frac{\beta}{g_p}\right)^2\epsilon[5T(T+1)-3T_3^2]. \quad (3.23)$$

Notice that with $\epsilon\sim a/g_p^2$ the anisotropy among the charge states of the isobar is weaker for the pseudoscalar mesons, when compared with the corresponding g mechanism in the scalar model. This confirms our expectation based on the spin-isospin interdependence.

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[9] In contrast, it might be noted that in the "μ mechanism" we had $\partial\xi/\partial x_i$ in place of $\partial\delta_a/\partial x_i$, which gave a nonvanishing second-order contribution involving the bound-system variables.

[10] A. Houriet, Helv. Phys. Acta **18**, 473 (1945), Eq. (2.32). In the notation of W. Pauli and S. M. Dancoff [Phys. Rev. **62**, 85 (1942)], this corresponds to $q_{\alpha\beta}^{(1)}$. See their Eq. (70).