Positron-Proton Scattering*

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The importance of two-photon exchange in elastic electron-proton scattering was investigated by measuring the ratio of positron-proton to electron-proton scattering. Four-momentum transfers as large as 0.756 $(\mathrm{BeV}/c)^2$ (19.5 F^{-2}) were used. The data indicate that two-photon effects are $(4.0\pm1.5)\%$ larger than those predicted by the radiative corrections at the highest momentum transfers attained in these experiments. The two-photon corrections predicted using a static charge distribution fit the data well at lower momentum transfers and forward angles, but appear to be small at higher momentum transfers and backward angles.

I. INTRODUCTION

THE charge and magnetic-moment form factors of the proton are determined by analyzing electron-proton scattering data with formulas derived using the first Born approximation. This approximation does not include many-photon exchanges between the proton and electron, but since the neglected terms all involve higher powers of the expansion parameter ($\alpha\cong1/137$) they are expected to give corrections of order 1% to the Born-approximation analysis. The experiments described here were continuations of the work of Yount and Pine[1] in the investigation of two-photon effects in elastic electron-proton scattering by measuring the ratio of positron-proton to electron-proton scattering.

The two-photon corrections come from the radiative corrections, proton polarizability effects, and the electric and magnetic fields of the proton. The radiative corrections for positron and electron scattering are different when proton recoil effects are considered.[2] The difference in the scattering cross sections predicted by the radiative corrections ranges from 0.5 to 4.5% for the measurements reported here.[3,4] The effects of proton polarizability (e.g., virtual meson production near a resonance) have been estimated to influence the elastic-scattering cross section $\lesssim1\%$ in the energy range of these experiments.[5,6] The corrections arising from the electric and magnetic fields of the proton are model-dependent and have not been calculated for high-energy scattering. In the approximation that magnetic scattering can be neglected, the two-photon corrections have been calculated by several authors.[7–10] The differences in the positron and electron-scattering cross sections predicted by these theories for the measurements reported here are several percent, although the predictions are not expected to be quantitatively accurate at high momentum transfer since they are derived for a static charge distribution.

These experiments were motivated by the possibility that anomalously large two-photon corrections might be found at larger momentum transfers. With present available incident electron energies these corrections might be difficult to detect by simply plotting electron-proton scattering cross sections as a function of $\tan^2(\theta/2)$, since the two-photon exchange contribution is proportional to $\tan^2(\theta/2)$ at large angles.[11] A knowledge of their size is therefore desirable because of the interest in the electron-proton scattering results.

II. APPARATUS

The positron and electron beams from the Stanford Mark III Linear Accelerator were momentum-analyzed in the usual fashion.[1] The positron radiator and focusing coils were located 40 ft from the electron gun and produced 3×10^9 positrons per second in an energy spread of 2% with a maximum energy of 850 MeV. Field reversibility in the momentum analyzing magnets was checked using both nuclear magnetic resonance and rotating coil probes. The tracking of these monitors was better than 0.1% for all changes of the magnetic fields, and it was believed that the magnetic fields were reversed to within this limit. Identical momentum settings were used for the electron and positron beams and the mean energies of the beams were very closely equal.

The target was one foot of liquid hydrogen contained in a thin-walled stainless steel cylinder. Beam centering at the target was continuously monitored using a split-plate ionization chamber. The centers of the beams and the beam sizes were checked using fluorescent crystals or by taking x-ray pictures. The beam spots were about $\frac{3}{8}$ in. in diameter and similar for both the positron and electron beams. The primary beam current monitor was

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[1] D. Yount and J. Pine, Phys. Rev. **128**, 1842 (1962).
[2] Y. Tsai, Phys. Rev. **122**, 1898 (1961).
[3] N. Meister and D. Yennie, Phys. Rev. **130**, 1210 (1963).
[4] R. Atkinson, doctoral dissertation, Stanford, 1964 (unpublished).
[5] S. Drell and S. Fubini, Phys. Rev. **113**, 741 (1959).
[6] N. Werthamer and M. Ruderman, Phys. Rev. **123**, 1005 (1961).
[7] S. Drell and R. Pratt, Phys. Rev. **125**, 1394 (1962).
[8] G. Bisiacchi and G. Furlan, Phys. Letters **3**, 186 (1963).
[9] R. Lewis, Phys. Rev. **102**, 537 (1956).
[10] W. McKinley and H. Feshbach, Phys. Rev. **74**, 1759 (1948).
[11] M. Gourdin (private communication). For a more detailed discussion of the possible angular-momentum states contributing to the two-photon channel, see also D. Flamm and W. Kummer, Nuovo Cimento **28**, 33 (1963).

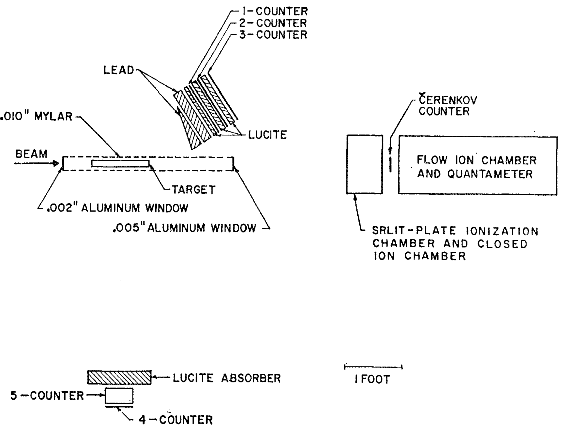

FIG. 1. Experimental arrangement for the first experiment.

the 300-MeV Faraday cup,[12] modified as described by Yount and Pine.[1] Hydrogen-filled ionization chambers were used as secondary monitors to guard against reading errors. Comparison of the Faraday cup with another, larger Faraday cup showed that the two monitors measured the same charge to an accuracy of 0.2% for energies as high as 850 MeV. The ratios of the readings obtained from the ion chambers and Faraday cup when both electrons and positrons were incident showed that the efficiencies of the two types of monitors were the same for electrons and positrons to within the measuring accuracy of 0.3%. Fluctuations in the ratios of the charge obtained from different monitors were of order 0.3% in any one reading. The charge collected was integrated with feedback-type integrators which were stable and linear to 0.1%.

The first experiment performed counted the recoiling proton and electron in coincidence using "open" counters. The experimental setup is shown in Fig. 1. The beam was measured by the two ion chambers (which were calibrated absolutely using the Faraday cup); the quantameter provided a check on the energy of the incident beams; and the Čerenkov counter in the beam line was used for counting rate corrections. An "event" was defined as two particles in coincidence, one of which had a range greater than the absorber in front of the proton telescope (and which could produce a count in counters 1, 2, and 3 which were biased so that a single minimum ionizing particle had an estimated efficiency of 20%), and the other had a range greater than 6 in. of Lucite and a velocity greater than 0.8 c. These criteria were not sufficient to reject all unwanted processes. In particular, the electroproduction and photoproduction of mesons produced events which had a fairly large probability of being recorded. The data obtained at the highest momentum transfer point in this experiment was badly contaminated by these background processes (almost half of the total events recorded at this point came from backgrounds), so this point was remeasured in a second experiment.

[12] K. Brown and G. Tautfest, Rev. Sci. Instr. **27**, 696 (1956).

The second experiment detected the recoiling electrons or positrons in a counter telescope located at the focal plane of a single-focusing, 90-deg-bend, 44-in.-radius-of-curvature magnetic spectrometer designed by Professor D. Ritson. The experimental arrangement for this measurement is shown in Fig. 2. The beam-position-sensing ion chamber (next to the Faraday cup) was moved into the beam line periodically to measure the angle of the beams in the experimental area. The momentum and angle requirements imposed on the recoil particles were not sufficient to completely separate the elastically scattered positrons from π^+ mesons (produced in the hydrogen) and protons (produced mainly in the walls of the target). The complete separation of these processes was accomplished in the counter telescope shown in Fig. 3, by the use of absorbers and a shower counter. The Lucite absorber was sufficient to completely stop the protons, which made the separation of the pions and positrons easier. The lead-scintillator "sandwich" formed a shower counter which was used to separate the pions from the positrons by detecting the shower produced by the positrons in the lead. The six ladder elements were used to determine the center (and thus the central momentum) of the elastic recoil peak.

III. DATA, CORRECTIONS AND UNCERTAINTIES

A. First Experiment

The ratio $R=\sigma^+/\sigma^-$ was measured at five points in this experiment: 700 MeV, 50 deg; 600 MeV, 100 deg; 850 MeV, 68.5 deg; 800 MeV, 80 deg; and 850 MeV, 90 deg, where the incident-energy and recoil-electron angles are given in the laboratory system. Each point consisted of several complete determinations of R taken over a period of six months. A single measurement of R included the following steps:

(1) beam position and size check using x-ray film with electrons incident,
(2) full-target data with electrons incident,

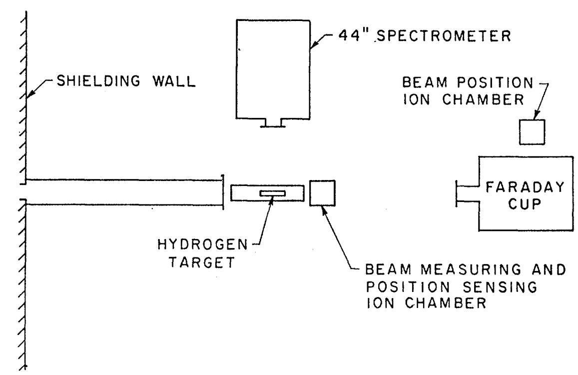

FIG. 2. Experimental arrangement for the second experiment.

TABLE I. Results, corrections, and errors for the first experiment.[a] $R_{\rm raw}$ is the measured ratio, σ^+/σ^-, with statistical errors. $R_{\rm meas}$ is this ratio corrected for counting rate losses, energy shifts, monitor errors, etc. The b_1 and ξ_1 are the backgrounds and the errors caused by the backgrounds in the final ratio. The b_i are expressed as a ratio of the number of counts recorded from process i to the expected number of counts from e^-+p elastic scattering. ξ_i gives the change in $R_{\rm true}$ if the background b_i were in error by 1 standard deviation. $R_{\rm true}$ is the final corrected ratio with the estimated error.

Point	$R_{\rm raw}$	$R_{\rm meas}$	b_1	ξ_1	b_2	ξ_2	b_4	ξ_4	b_5	ξ_5	$R_{\rm true}$
850 MeV, 90°	1.215 ±0.066	1.219 ±0.068	1.014 ±0.006	±0.006	0.840 ±0.177	±0.035	1.015 ±0.008	±0.008	9.194 ±0.026	±0.028	1.207 ±0.088
800 MeV, 80°	1.088 ±0.082	1.086 ±0.083	0.958 ±0.006	±0.006	0.241 ±0.047	±0.004	0.958 ±0.008	±0.008	0.108 ±0.015	±0.016	0.981 ±0.084
850 MeV, 70°	1.050 ±0.061	1.058 ±0.062	1.048 ±0.006	±0.006	0.360 ±0.085	±0.018	1.054 ±0.008	±0.008	0.042 ±0.005	±0.042	1.038 ±0.065
600 MeV, 100°	1.101 ±0.062	1.113 ±0.063	0.936 ±0.006	±0.006	0.044 ±0.009	±0.008	0.927 ±0.008	±0.008	0.026 ±0.003	±0.026	1.091 ±0.067
700 MeV, 50°	1.048 ±0.026	1.020 ±0.031	0.923 ±0.006	±0.006	0.097 ±0.015	±0.001	0.927 ±0.008	±0.008	0.009 ±0.001	±0.001	1.010 ±0.031

[a] b_3 was assumed to be negligible.

(3) empty-target data with electrons incident,
(4) full-target data with positrons incident,
(5) empty-target data with positrons incident,
(6) beam position and size check using x-ray film with positrons incident. The order of these procedures was changed about half the time, with positrons being measured first.

The solid angle of the detectors was determined by the solid angle of the electron telescope and was about 0.03 sr. The proton telescope was large enough to detect the recoiling protons and had enough extra size to allow for beam energy, position, or angle changes.

The empty-target rate, corrected for chance coincidences, were $<1\%$ of the full-target rate, except at the 700-MeV point where it was about 5% of the full target rate. The experimental uncertainties arising from beam-angle shifts, energy differences, monitor errors, and beam position and size differences were all very small ($<1\%$) compared with the statistical accuracy obtained ($\sim\pm5\%$). The only troublesome correction (with the exception of the background processes mentioned earlier) was the counting rate correction. This correction was minimized in the ratio by counting at the same rate with electrons and positrons, and in addition the methods used to make the corrections were verified to be accurate to 2% in the ratio with corrections three times as large as those actually used.

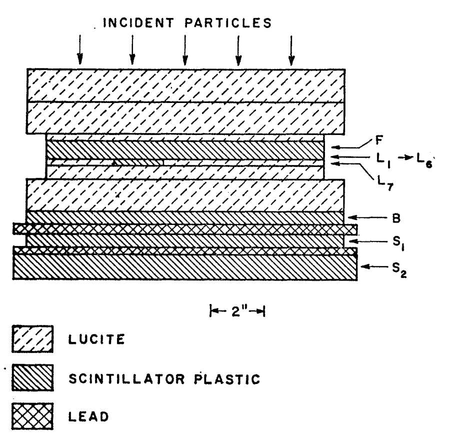

FIG. 3. Counter telescope for detecting electrons or positrons.

The following formulas were used to determine the effects of the various types of backgrounds on the ratio.

$$\sigma_m^- = S(b_1\sigma_T^- + b_2\sigma_T^- + b_3\sigma_T^-)\,,$$
$$\sigma_m^+ = S(b_4\sigma_T^+ + b_2\sigma_T^- + b_5\sigma_T^-)\,.$$

Here σ_m^- and σ_m^+ represent the measured counts recorded with electrons and positrons incident, respectively, and σ_T^- and σ_T^+ stand for the actual elastic-scattering cross sections for electrons and positrons. S represents those quantities which were the same for all processes (e.g., target length), b_1 and b_4 the fraction of the true electron (or positron) scattering events which were recorded, b_2 those backgrounds which contributed the same number of events to both the electron and the positron data, and b_3 and b_5 those background processes which occurred only with electrons or positrons incident. The desired quantity was $R_t \equiv \sigma_T^+/\sigma_T^-$ which was obtained from the formula

$$R_T = \frac{b_1}{b_4}R_m - \frac{b_5}{b_4} + (R_m - 1)\frac{b_2}{b_4} + \frac{b_3}{b_4}R_m\,.$$

It was assumed that b_3 was negligible in this experiment.

The b's were calculated by integrating the known[13–16] cross sections over counter dimensions, target length, etc. It was assumed that the cross sections involved were the same for positrons and electrons, except for the added photoproduction reactions arising from annihilation gamma rays. The backgrounds calculated included

$$\gamma + p \to \pi^+ + n\,, \quad \gamma + p \to \pi^0 + p\,, \quad \gamma + p \to \gamma + p\,,$$

[13] K. Berkelman and J. Waggoner, Phys. Rev. **117**, 1364 (1960).
[14] A. Browman and J. Pine, Nuovo Cimento **27**, 850 (1963).
[15] L. Hand, Phys. Rev. **129**, 1834 (1963).
[16] W. Panofsky, W. Woodward, and G. Yodh, Phys. Rev. **102**, 1392 (1956).

$$e^{\pm}+p \rightarrow e^{\pm}+\pi^{+}+n, \quad e^{\pm}+p \rightarrow e^{\pm}+\pi^{0}+p$$

$e^{+}+e^{-} \rightarrow \gamma+\gamma \rightarrow$ either photon interacts via one of the first three processes.

It has been shown[17] theoretically that the bremsstrahlung spectrum for electrons and positrons is the same to very good accuracy (the slight difference calculated near the tip of the spectrum[18] was negligible in this experiment), so the photoproduction cross sections were expected to give the same contribution to the counting rate on electrons and positrons. The electroproduction cross sections are not nearly so well known at these energies[15] and the assumption of the equality of positroproduction and electroproduction processes has not been verified. For these reasons an uncertainty of 50% was assigned to the calculated electroproduction backgrounds.

Table I gives the results of this experiment. The column labeled R_{raw} gives the measured value of the ratio with statistical errors. The column labeled R_{meas} gives the value of the ratio corrected for the various experimental corrections mentioned before, such as counting-rate corrections, beam-energy-shift corrections, monitor errors, etc. This column also includes the effects of the radiative corrections which were calculated for this experiment by Atkinson.[4] The next columns give the calculated values of the b's. The ξ_i give the change that would result in the final ratio if the value used for the background b_i was in error by one standard deviation. The last column gives the corrected value of the ratio with the final errors.

B. Second Experiment

The momentum analysis of the recoil particles in this experiment greatly reduced the number of background processes. The only significant contamination, that produced by π^+ mesons, contributed less than 20% of the observed counts at the highest momentum transfer point. The ratio $R=\sigma^+/\sigma^-$ was measured at two points in this experiment; 518 MeV, 45 deg; and 850 MeV, 90 deg. The 850-MeV point was the same as the highest momentum transfer point measured in the first experiment, while the 518-MeV point was used as a check (the measurements of Yount and Pine[1] indicated that at this point the ratio would be one). The exact angle and energy of this check point were determined by the requirement that the recoil particles should have the same momenta as the recoil particles at the 850 MeV point. A value of one for the ratio at the 518 MeV point would thus demonstrate that the detection apparatus was unbiased.

The data were obtained from a series of runs in which electron and positron yields were both measured several times. The 518- and 850-MeV data were alternated to guard against drifts. A measurement of either electron or positron yield included the following:

(1) setting the incident energy,
(2) setting the recoil energy and angle,
(3) centering the beam spot and checking its size and shape,
(4) measuring the beam direction,
(5) measuring full- and empty-target yields.

During the measurements the position of the beam and the fields in the magnets were continuously monitored. The measurement of the beam direction, combined with the measurement of the recoil particles' momenta allowed the relative energy of the incoming beams to be determined with very high accuracy. At various times during the data runs all possible combinations of magnet polarity and beam charge were used to check that there were no unexpected backgrounds. Cosmic-ray background was measured before and after the runs.

Each of the six ladder elements in the counter telescope accepted a momentum spread of about 1.8% (see Fig. 3 for a diagram of the telescope used). The solid angle of the spectrometer was 0.01 sr, and the effective target length was 10 in. at the 518-MeV point and 7 in. at the 850-MeV point. The empty target rate was about 1% of the full target rate for both electrons and positrons. The cosmic-ray background was 0.5% of the full target rate with positrons incident and negligible with electrons incident (the electron data was taken at a higher rate, hence, for shorter times). The charge collected by the Faraday cup and the ion chamber was recorded on all runs, but since the ratio of the charge collected by the two monitors showed no unexplained variations the Faraday cup reading was used in the analysis.

Two independent methods of data analysis were used. The first used the data from the shower counter in addition to the ladder data to separate the pions from the positrons. The second used the assumption that the photoproduction and electroproduction of π^+ mesons from hydrogen was the same for electrons and positrons.

The first method of data analysis, the shower counter method, assumed that the efficiency of the shower counter was the same for electrons and positrons. This assumption was checked by measuring the ratio at the 518-MeV point. The counts in the $F+B+L_i$ coincidences (see Fig. 3) were called T_i, and the counts in the $F+B+L_i+S$ coincidences were called Q_i. The following relations

$$T_i=e_i+\pi_i, \quad Q_i=\alpha_i e_i+\beta_i\pi_i$$

were solved for the number of electrons and pions in each ladder channel giving

$$e_i=\frac{1}{(\alpha_i-\beta_i)}(Q_i-\beta_i T_i), \quad \pi_i=\frac{1}{(\alpha_i-\beta_i)}(\alpha_i T_i-Q_i).$$

[17] H. Bethe and L. Maximon, Phys. Rev. **93**, 768 (1954).
[18] R. Jabbur and R. Pratt, Phys. Rev. **129**, 184 (1963).

TABLE II. Results for the second experiment. R_{raw} is the measured ratio, σ^+/σ^-, with statistical errors. The other columns give the change in R_{raw}, and the estimated error, for each of the effects listed. The last column gives the final corrected ratio with the estimated error.

Point	R_{raw}	Count corr.	Empty target and cosmic ray	Monitor	Peak shift	Rad. corr.	Angle	Energy	Annih.	Analysis	Shower	Subtr.	Final results
850 MeV,	1.117	−0.002	−0.002	0.000	+0.002	−0.045	+0.003	−0.013	+0.009				1.069
90°	±0.030	±0.000	±0.001	±0.003	±0.001	±0.005	±0.002	±0.007	±0.001	±0.015	±0.014	±0.023	±0.040
518 MeV,	0.986	0.000	0.000	0.000	+0.001	−0.012	+0.016	−0.002	+0.009				0.998
45°	±0.012	±0.003	±0.000	±0.003	±0.000	±0.000	±0.008	±0.001	±0.001	±0.015	±0.011	±0.005	±0.023

In these formulas α_i is the efficiency of the shower counter for electrons or positrons, and β_i is the efficiency of the shower counter for pions (i runs from 1 to 6). The triples efficiency T_i was taken as 100% for both electrons and pions.

The β_i were determined by counting with a pure beam of pions, and the α_i were determined by counting a beam of electrons. Once the values of these constants were determined, the separation of the data into electrons and pions was achieved by using the formulas given above. Measurements were taken throughout the course of the experiment with electrons incident and with the spectrometer set to detect negative recoil particles (to determine α_i) and with the spectrometer set to detect positive recoil particles (to determine β_i). These measurements also showed that the constants were time stable during the course of the experiment. The weighted average of these measurements was taken as the value of α_i and β_i. The statistical errors obtained in the experiment were increased in this method of data analysis both by possible fluctuations in the values of the constants, and by a possible incorrect determination of β_i (note that the values of α_i used cancelled when the ratio was taken).

The second method of data analysis, the pion-subtraction method, used the measurement of pions obtained when electrons were incident but the spectrometer was set to count positive recoil particles. The measured pions were subtracted, channel by channel, from the data obtained when positrons were incident. The remaining particles were elastically scattered positrons and the pions produced by the annihilation gamma rays. The annihilation cross section is known[14] to within 5% in this energy region and the photoproduction cross section is known to about 10% accuracy.[19,20] The pions produced by the annihilation gamma rays were subtracted using the known cross sections by normalizing to the observed electron scattering counts. This procedure cancelled uncertainties in solid angle and target length.

Figure 4 shows the data obtained with positrons incident. The top curve is the triples data (the data from the T_i). The middle curve is the triples data with the measured pions subtracted and the bottom curve shows the results after subtraction of the calculated number of pions produced by annihilation gamma rays. Figure 5 shows the final positron curve (the bottom curve in Fig. 4) compared with the electron data from which no subtractions were made. The results obtained by the shower counter method of data analysis were similar. Note that the method successfully removed the pions even where they were many times more numerous than the positrons. The "cut" shown is the location of the momentum acceptance limit used. Only data to the

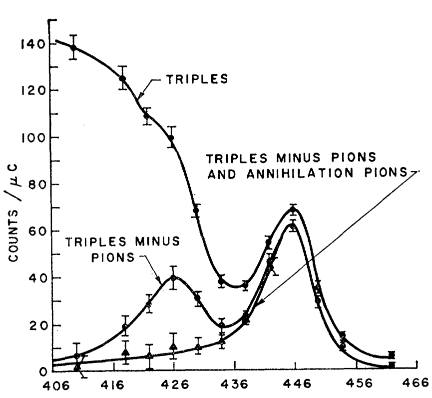

FIG. 4. Scattered positron momentum spectrum at $q^2 = 19.5\ \text{F}^{-2}$.

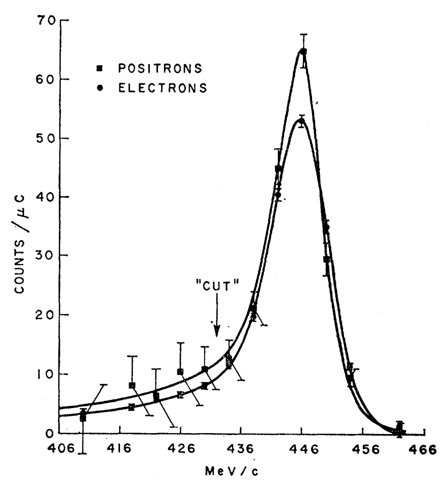

FIG. 5. Scattered positron and electron momentum spectra at $q^2 = 19.5\ \text{F}^{-2}$.

[19] M. Heinberg, W. McClelland, F. Turkot, W. Woodward, R. Wilson, and D. Zipoy, Phys. Rev. 110, 1211 (1958).

[20] R. Alvarez, Z Bar-Yam, W. Kern, D. Luckey, L. S. Osborne, and S. Tazzari and R. Fessel, Phys. Rev. Letters. 12, 707 (1964).

TABLE III. This table lists the results of these experiments along with the earlier results of Yount and Pine. The incident-electron (and positron) energies and recoil angles are given in the laboratory system. The theory of Lewis was used to calculate the predicted value of R for the exponential and Yukawa models. Also shown for each measurement is the ratio of G_M to G_E scattering calculated from electron-scattering data.

Incident energy (MeV)	Recoil angle (degrees)	Momentum transfer $(\mathrm{BeV}/c)^2$	Momentum transfer F^{-2}	$R\equiv\sigma^+/\sigma^-$ Experiment	R Exponential	R Yukawa	Theory-experiment exponential	Theory-experiment Yukawa	G_M scattering / G_E scattering
205	30	0.01	0.3	0.996 $\pm$0.012	0.992	0.992	-0.004 $\pm$0.012	-0.004 $\pm$0.012	0.007
307	30	0.02	0.6	0.976 $\pm$0.018	0.994	0.994	+0.018 $\pm$0.018	+0.018 $\pm$0.018	0.009
307	30	0.02	0.6	1.006 $\pm$0.018	0.994	0.994	-0.012 $\pm$0.018	-0.012 $\pm$0.018	0.009
307	45	0.05	1.2	1.004 $\pm$0.032	0.994	0.993	-0.010 $\pm$0.032	-0.011 $\pm$0.032	0.136
307	130	0.19	5.0	1.042 $\pm$0.060	1.008	1.028	-0.034 $\pm$0.060	-0.014 $\pm$0.060	4.47
518	45	0.14	3.6	0.998 $\pm$0.023	0.999	0.997	+0.001 $\pm$0.023	-0.001 $\pm$0.023	0.40
850	90	0.76	19.5	1.069 $\pm$0.040	1.032	1.030	-0.039 $\pm$0.040	-0.039 $\pm$0.040	6.12
850	90	0.76	19.5	1.207 $\pm$0.088	1.032	1.030	-0.175 $\pm$0.088	-0.177 $\pm$0.088	6.12
800	80	0.62	15.9	0.991 $\pm$0.084	1.025	1.020	+0.034 $\pm$0.084	+0.029 $\pm$0.084	3.73
850	70	0.60	15.3	1.038 $\pm$0.065	1.021	1.014	-0.017 $\pm$0.065	-0.024 $\pm$0.065	2.91
600	100	0.48	12.4	1.091 $\pm$0.067	1.026	1.030	-0.065 $\pm$0.067	-0.061 $\pm$0.067	4.38
700	50	0.27	7.1	1.010 $\pm$0.031	1.006	1.002	-0.004 $\pm$0.031	-0.008 $\pm$0.031	0.86

high-momentum side of the cut was used in determining the ratio.

Table II gives the results of this experiment. The first column gives the raw data with its statistical errors. The column labeled monitor gives the error due to possible Faraday cup bias. The radiative correction calculated from the formulas of Meister and Yennie[3] is given in the column labeled Rad. Corr. The angles of the incident beams were determined to a relative accuracy of 0.06 deg and the column labeled angle gives the error in the ratio due to this uncertainty. The energy of the incoming beams was determined by knowing the angle and the center position of the peak in the ladder. The error in the energy determination is shown in the next column. Since the mean energies of the incident beams were not always exactly the same due to slight angular changes and shifts within the wide (2%) energy slits used, the mean energies of the recoil particles could differ slighty. The cut, which was taken fairly close to the scattering peak (see Fig. 5), caused a loss or gain of area depending on this shift. The error caused by this correction is shown in the next column. In order to verify that the data analysis had been performed correctly, different cuts were used, different methods of obtaining the ladder parameters tried, and the data was combined by using different methods. These procedures lead to differences in the ratio of order 1.5%. The column labeled analysis is the added error in the ratio arising from these uncertainties. The columns labeled shower counter and subtraction give the added error due to the two methods of analysis, the shower counter method and the subtraction method, respectively. The final column gives the result of the experiment and the final estimated error.

CONCLUSIONS

Table III shows the results of these experiments along with the earlier data of Yount and Pine.[1] All the experimental points have had the radiative corrections subtracted. The columns labeled theory experiment compared the experiments with the theory of Lewis.[9] Two different models of the charge distribution were used. The results obtained from the theory depend

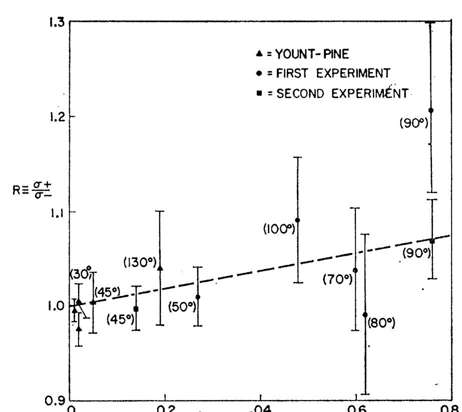

FIG. 6. Final ratios plotted versus momentum transfer.

slightly on the radius chosen, but the variation in the theoretical ratio with radii from 0.7 to 0.9 F is less than 0.5%. The answers shown were obtained with an rms radius of 0.8 F. The column labeled G_M/G_E scattering shows the ratio of magnetic to electric scattering predicted by the experimental fit to the Rosenbluth[21] cross section. Figure 6 shows the data plotted graphically as a function of momentum transfer. The dashed line is the best polynomial fit to the data passing through one at q^2=zero. The data show deviations from one (the first Born prediction) at higher momentum transfer and backward angles. It is probable that there are more two-photon corrections than predicted by Lewis's theory at the larger momentum transfers.

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New Reduction of the Faddeev Equations and Its Application to the Pion as a Three-Particle Bound State*

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A new separation of the angular momentum in the Faddeev equations is given. This separation makes use of the relative angular momentum of two particles, which is combined with the angular momentum of the third particle in the over-all center-of-mass system. With the assumption that the two-body amplitudes factorize in the initial and final momenta, the Faddeev equations are reduced to a coupled set of integral equations in one variable. This set is furthermore simplified in the case of identical particles to only one integral equation. Thereby the statistics is correctly taken into account. The resulting equation is used to investigate possible bound states of three pions with total angular momentum zero, isospin one, and odd parity. The two-body amplitude which determines the kernel is approximated by the isospin-zero, *s*-wave effective-range formula of Chew and Mandelstam. Use is also made of relativistic kinematics. The pion is found as a bound state of three pions in this model. The outcome is, however, strongly dependent on a physical cutoff parameter in the two-body form factor. As a result a detailed investigation of the form factor is desirable.

1. INTRODUCTION

THE Faddeev equations,[1–3] and their validity, for a system of nonrelativistic three particles interacting through two-body potentials between each pair of particles are now well known. These equations are clearly applicable to quantum-mechanical three-particle systems such as the problem of electron-hydrogen atom scattering. They can also be applied to three-body problems in low-energy nuclear physics in which the two-body interactions can be described by some sort of phenomenological potential. Thus in these problems the Faddeev equations are expected to play an important role. The accuracy of the results of such calculations merely depends on how accurately the computations can be carried out.

Our interest in the Faddeev equations is, however, based on their possible application to particle physics. Here, too, very little has been done with the three-particle problems. In nearly all the problems, the three-particle system has been regarded as being two particles, one of which is composed of two particles clumped together. The Faddeev equations, although nonrelativistic, are at least genuine three-particle equations. Furthermore, a remarkable property of the Faddeev equations is that they only require a knowledge of the two-body amplitude (off the energy shell). This is clearly an advantage because at least in the region of resonances and where the effective-range formulas are valid, the two-body amplitude is known fairly well, whereas very little is known about a corresponding po-

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