for G, such that the iH_i, $(E_\alpha+E_{-\alpha})$, and $i(E_\alpha-E_{-\alpha})$ belong to the compact form of G (Ref. 15, p. 149). Clearly there is no loss in generality in choosing $H_1=iM_{12}$ and $H_2=iM_{34}=M_{30}$. Since L is contained in G we can now write

$$P_1+iP_2=\sum_i a_iH_i+\sum_\alpha b_\alpha E_\alpha, \quad \text{(A2)}$$

where the a_i and b_α are numerical coefficients. But from (1.4) we have

$$[iM_{12},P_1+iP_2]=P_1+iP_2. \quad \text{(A3)}$$

Hence

$$\sum_i a_iH_i+\sum_\alpha b_\alpha E_\alpha = [H_1,\sum_i a_iH_i+\sum_\alpha b_\alpha E_\alpha]=\sum_\alpha r_1(\alpha)b_\alpha E_\alpha. \quad \text{(A4)}$$

Thus

$$P_1+iP_2=\sum_{r_1(\alpha)=1} b_\alpha E_\alpha. \quad \text{(A5)}$$

Similiarly,

$$P_1-iP_2=\sum_{r_1(\alpha)=1} d_\alpha E_{-\alpha}, \quad \text{(A5)}$$

where the d_α are numerical coefficients. Hence

$$P_1=\tfrac{1}{4}\sum_{r_1(\alpha)=1}(b_\alpha+d_\alpha)(E_\alpha+E_{-\alpha}) + \left[\frac{1}{i}(b_\alpha-d_\alpha)\right][i(E_\alpha-E_{-\alpha})]. \quad \text{(A6)}$$

But if P is a subalgebra of the compact form of G, $(b_\alpha+d_\alpha)$ and $1/i(b_\alpha-d_\alpha)$ are real, whence

$$d_\alpha=b_\alpha^*. \quad \text{(A7)}$$

In this case

$$0=[P_1+iP_2,P_1-iP_2]=\sum_{r_1(\alpha)=1;\,r_1(\beta)=1} b_\alpha b_\beta^*[E_\alpha,E_{-\beta}] = \sum_{r_1(\alpha)=1}|b_\alpha|^2H_1+\sum_{i\neq 1}c_iH_i+\sum_\alpha c_\alpha E_\alpha, \quad \text{(A8)}$$

where the c_i and c_α are numerical coefficients, whence

$$b_\alpha=0, \quad r_i(\alpha)=1. \quad \text{(A9)}$$

Since this is impossible for

$$P_1\neq 0, \quad P_2\neq 0, \quad \text{(A10)}$$

we see that P cannot be a subalgebra of the compact form of G.

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π^+ Decay of $_\Lambda$Li^7

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An event unambiguously identified as the π^+ decay of a Li^7 hyperfragment is reported. The event was observed in a stack of L_4 hypersensitized Ilford emulsions exposed to a 1.5-GeV/c K^- beam at CERN. The charge of the hyperfragment was uniquely determined as three, by comparing its measured mean track width with the curves (mean track width versus dip angle) established for $Z=1$, 2, 3, 4, and 5 nuclides found in the same emulsions. The branching ratio R of the π^+/π^- decay modes for $_\Lambda$Li^7, on the basis of the present π^+ decay together with the available world data on π^- decays, is estimated as $R(_\Lambda\text{Li}^7)\sim 1\%$.

I. INTRODUCTION

Although the π^+ emission in free Λ decay is forbidden by the conservation laws, in the presence of a proton a Λ can generate π^+ by virtue of the "stimulation process"

$$\Lambda+p\to n+n+\pi^++35\text{ MeV}. \quad (1)$$

There are several mechanisms which could conceivably contribute to this decay interaction; among them the following have been considered by various authors[1–5]:

(i) The Λ may undergo transition to a virtual Σ^+ state in the presence of a proton inside a hypernucleus and subsequently decay from this state with the emission of a π^+ meson; i.e.,

$$\Lambda+p\to(\Sigma^++n)\to\pi^++n+n. \quad (2)$$

(ii) The Λ may decay through the π^0-mesonic mode and the π^0 so produced may undergo charge exchange with a proton of the hypernucleus; i.e.,

$$\Lambda+p\to(n+\pi^0)+p\to n+n+\pi^+. \quad (3)$$

(iii) The Λ may generate the decay interaction $\Lambda\to n+(\pi^++\pi^-)$, by virtue of the four-fermion weak interaction $(\bar{\Lambda}p)(\bar{p}n)$, and the π^- produced may be subsequently absorbed on a proton inside the hypernucleus; i.e.,

$$\Lambda\to n+(p+\bar{p})\to n+(\pi^++\pi^-), \quad (4)$$
$$\Lambda+p\to n+n+\pi^+.$$

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The experimental determination of the ratio R of the π^+/π^- decay modes for specific hypernuclei is expected to shed considerable light on which of the above-mentioned mechanisms would be of utmost importance.

So far most of the available data on π^+ decays[6–15] belong to the He^4 hypernucleus. Six and two (plus one ambiguous) events of this species have been reported from emulsion[7–13] and helium bubble chamber[14] experiments, respectively. These data yield[16] a mean value of the ratio R for $_\Lambda He^4$ as $\langle R(_\Lambda He^4)\rangle_{av}=2.9\pm1.0\%$. The calculated values of the ratio $R(_\Lambda He^4)$ by von Hippel[5] are 1.1 and 0.4%, corresponding to the decay interactions (2) and (3), respectively, the contribution from (4) being supposed insignificant. The observed value of $\langle R(_\Lambda He^4)\rangle_{av}$ therefore suggests that the dominant mechanism in the π^+ decay interaction is the process (2). A general discussion of the dependence of R on the specific features of hypernuclear species is given in Ref. 5.

The data on the π^+ decay of hypernuclei of higher charge are very meagre and only two probable examples each having the interpretations $_\Lambda Li^7$ or $_\Lambda Be^7$ have so far been reported.[6,15] In this communication we report an unambiguous example of π^+ decay of a Li^7 hypernucleus. The following sections of the paper will be devoted to the details of identification of the event and to an attempt to evaluate R for this species on the basis of the available data.

II. ANALYSIS OF THE EVENT

The event was observed along with a total of 93 π^--mesonic decays of hyperfragments in L_4-hypersensitized Ilford emulsions exposed to a 1.5-GeV/c K^- beam at CERN. The various details of the exposure and scanning of the emulsion stack are described elsewhere.[17]

The primary star containing the hyperfragment (HF) has a configuration $9h+1m+$HF tracks and a short recoil of range $\sim2\ \mu$. The primary interaction thus occurred in a heavy nucleus (Ag or Br) of the emulsion, where the short recoil is presumably due to the heavy spallation residual nucleus. The HF track shows characteristic tapering towards the end of its range, signifying that it carries multiple charge and it has probably come to rest before decay. At the decay point a π^+ meson of 10 882-μ range and a short recoil of range $\lesssim1\ \mu$ were observed. The π^+ meson was identified by its typical decay chain $\pi^+\to\mu^+\to e^+$; the measured range of the muon is 635 μ as against the mean μ^+ range, $(595.5\pm2.1)\ \mu$, $\sigma=(31.6\pm2.1)\ \mu$, calculated for a sample of 104 $\pi^+\to\mu^+\to e^+$ events observed in the present stack. It has been shown[18] that the possibility of a π^- decaying in emulsion and giving rise to a μ^- consistent with the μ^+-range distribution, is 1 in 5000; this possibility has, therefore, not been considered. In Table I we list the relevant data of the HF event.

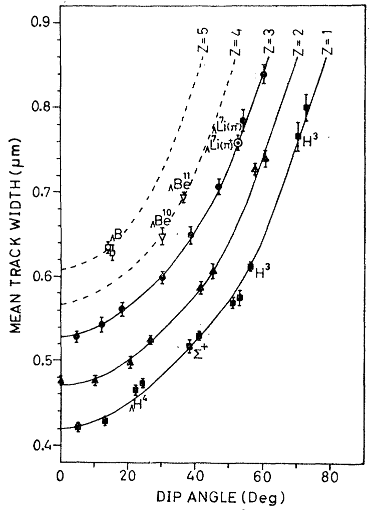

FIG. 1. Plot of the mean track width over a range interval 10–50 μ for $Z=1$, 2, 3 nuclides and over 10–20 μ for $Z=4$, 5 nuclides versus their mean dip angles in the measured range interval. The errors shown on the mean track width are equal to one standard deviation. The errors in the measurement of dip angles (not shown) are typically 2, 1.5, 1, and 0.5° in the intervals 0–20, 20–40, 40–60, and 60–80°, respectively.

Charge of the Hyperfragment

The interpretation of the HF is greatly simplified if its charge can be ascertained unambiguously. Our investigations on the charge measurements in the L_4-hypersensitized emulsions show that with the use of a suitable technique and proper care it is possible to obtain a sufficiently high resolution between various charges. The full details of these investigations will be published shortly. Here we briefly indicate the technique used in the measurements.

The end of the track under investigation is aligned

TABLE I. Experimental data of the event.

Track	Range (μ)	Dip angle (deg)	Azimuthal angle (deg)	Kinetic energy (MeV)
HF	96.6	−45.0	+94	···
π^+	10 882	−27.4	0	24.2
Recoil ($He^{4,6}$)	$\lesssim1$	···	$+(100\pm5)$	$\lesssim0.2$

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[16] See, for example, Refs. 13 and 5.
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TABLE II. The Q_0 values (MeV) in the decay of Li hypernuclei.[a]

${}_{\Lambda}\mathrm{Li}^6$	$\rightarrow$	$\pi^+ + \mathrm{He}^3 + 3n + 16.40$
	$\rightarrow$	$\pi^+ + \mathrm{He}^4 + 2n + 36.98$
	$\rightarrow$	$\pi^+ + \mathrm{He}^6 + 32.28$
${}_{\Lambda}\mathrm{Li}^7$	$\rightarrow$	$\pi^+ + \mathrm{He}^3 + 4n + 10.74$
	$\rightarrow$	$\pi^+ + \mathrm{He}^4 + 3n + 31.31$
	$\rightarrow$	$\pi^+ + \mathrm{He}^6 + n + 32.28$
${}_{\Lambda}\mathrm{Li}^8$	$\rightarrow$	$\pi^+ + \mathrm{He}^3 + 5n + 3.48$
	$\rightarrow$	$\pi^+ + \mathrm{He}^4 + 4n + 24.06$
	$\rightarrow$	$\pi^+ + \mathrm{He}^6 + 2n + 25.03$
${}_{\Lambda}\mathrm{Li}^9$	$\rightarrow$	$\pi^+ + \mathrm{He}^3 + 6n + 1.45$
	$\rightarrow$	$\pi^+ + \mathrm{He}^4 + 5n + 22.03$
	$\rightarrow$	$\pi^+ + \mathrm{He}^6 + 3n + 22.99$
${}_{\Lambda}\mathrm{Li}^{10}$	$\rightarrow$	$\pi^+ + \mathrm{He}^3 + 7n + (-2.10)$
	$\rightarrow$	$\pi^+ + \mathrm{He}^4 + 6n + 18.47$
	$\rightarrow$	$\pi^+ + \mathrm{He}^6 + 4n + 19.43$

[a] We have not considered He^7 as one of the decay products because its existence has not been established.

along the X axis on the scattering stage of a Leitz Orthulux microscope which has an internal eyepiece scale. The two edges of the track can be set against any one of the graticule divisions in succession with the help of an image-shifting device (Clausen micrometer) and the width of the track is obtained from the two readings of the circular scale. In Fig. 1 we display the calibration curves obtained from the measurements of the mean track width in the range interval (10–50) μ of identified particles, such as the Σ^+ and p tracks from $\Sigma^+ \rightarrow p + \pi^0$ decays, H^3 from ${}_{\Lambda}\mathrm{H}^3 \rightarrow \mathrm{H}^3 + \pi^0$, ${}_{\Lambda}\mathrm{H}^4$, ${}_{\Lambda}\mathrm{He}^4$, ${}_{\Lambda}\mathrm{He}^5$, Li^8-hammer, ${}_{\Lambda}\mathrm{Li}^7$ (π^-), ${}_{\Lambda}\mathrm{Be}^{10}$, ${}_{\Lambda}\mathrm{Be}^{11}$, and ${}_{\Lambda}\mathrm{B}$. The high resolution obtained between various charges is evident from the figure. It may be observed that the track width is very sensitive to the steepness of the track. Comparison of the widths of two tracks differing by only a few degrees in the region $\gtrsim 40°$ can alter the results significantly. A proper calibration curve is therefore indispensable for charge identification. The point representing the mean track width of the π^+ HF is in excellent agreement with the curve obtained for Li tracks and is completely resolved both from the $Z=2$ and 4 curves. We therefore conclude that the charge of the HF is three.

Interpretation of the Event

In Table II we list the possible decay schemes of Li hypernuclei along with the energy release Q_0 in their decay, calculated by taking zero Λ binding energies. From the visible energy at the HF decay star, $\simeq 24.2$ MeV, the interpretations ${}_{\Lambda}\mathrm{Li}^{8,9,10}$ are disallowed as they would lead to unacceptable (mostly negative) Λ binding energies. We also rule out the ${}_{\Lambda}\mathrm{Li}^6$ identity, since its core nucleus is highly unstable and the existence of ${}_{\Lambda}\mathrm{Li}^6$ has not been proved (although the decay mode ${}_{\Lambda}\mathrm{Li}^6 \rightarrow \pi^+ + \mathrm{He}^4 + 2n$ for the present event is kinematically possible). We therefore conclude that the hyperfragment is a ${}_{\Lambda}\mathrm{Li}^7$. Kinematically, two decay modes, ${}_{\Lambda}\mathrm{Li}^7 \rightarrow \pi^+ + \mathrm{He}^6 + n$ and $\pi^+ + \mathrm{He}^4 + 3n$ are possible, and in either case, an acceptable Λ binding energy can be obtained.

III. RESULT

We now proceed to make an assessment of the ratio of π^+/π^- decay modes for ${}_{\Lambda}\mathrm{Li}^7$. Since both π^- and π^+ decays are easy to detect without any relative bias, the value of R, to be meaningful, should be deduced on the basis of the world data. We have therefore made every possible effort to collect data from all the individual laboratories. The number of π^- decays of ${}_{\Lambda}\mathrm{Li}^7$ already published in the literature[19] is estimated as 46. The unpublished data[20] contain, in addition, 44 π^- decays. Thus the total number of π^- decays to be considered is 90. On the basis of the present π^+ decay which is the only unambiguous example of ${}_{\Lambda}\mathrm{Li}^7$ identified so far, we thus obtain an order-of-magnitude result on the ratio R as

$$R({}_{\Lambda}\mathrm{Li}^7) \sim 1/90 \sim 1\%.$$

The consideration of any one or both of the two ambiguous π^+ decays,[6,15] however, would raise the value of R accordingly.

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[20] Private communications from Dr. G. T. Goodhead (Oxford), Dr. P. H. Steinberg (Maryland), Dr. D. M. Harmsen (Hamburg), Dr. R. G. Ammar (EFINS-NU), Dr. J. Sacton (Bruxelles, as part of the European K^- collaboration), Dr. J. Schneps, and Dr. Y. W. Kang (Tufts University). We acknowledge the kindness of the above investigators in sending us their unpublished data.