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Three-Triplet Model with Double $SU(3)$ Symmetry*

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With a view to avoiding some of the kinematical and dynamical difficulties involved in the single-triplet quark model, a model for the low-lying baryons and mesons based on three triplets with integral charges is proposed, somewhat similar to the two-triplet model introduced earlier by one of us (Y. N.). It is shown that in a $U(3)$ scheme of triplets with integral charges, one is naturally led to three triplets located symmetrically about the origin of I_3-Y diagram under the constraint that the Nishijima–Gell-Mann relation remains intact. A double $SU(3)$ symmetry scheme is proposed in which the large mass splittings between different representations are ascribed to one of the $SU(3)$, while the other $SU(3)$ is the usual one for the mass splittings within a representation of the first $SU(3)$.

I. INTRODUCTION

ALTHOUGH the $SU(6)$ symmetry strongly indicates that the baryon is essentially a three-body system built from some basic triplet field or fields, the quark model[1] is not entirely satisfactory from a realistic point of view, because (a) the electric charges are not integral, (b) three quarks in s states do not form the symmetric $SU(6)$ representation assigned to the baryons, and (c) a simple dynamical mechanism is lacking for realizing only zero-triality states as the low-lying levels.

These difficulties may be avoided if we introduce more than one basic triplet. Recently one of us (Y. N.) has attempted a two-triplet model[2] where the members of the triplets t_1 and t_2 had the charge assignment (1,0,0) and $(0,-1,-1)$, as had been proposed earlier by Bacry *et al.*[3] The baryon would be represented by the combination $t_1t_1t_2$, whereas the mesons would correspond to some combination $\sim at_1\bar{t}_1'+bt_2\bar{t}_2'$. The triplets are assumed to have masses large compared to the baryon mass, which would mean that baryons and mesons have very large binding energies. A dynamical mechanism for this is provided by a neutral field coupled strongly to the "charm number"[4] C, which is 1 for t_1 and -2 for t_2, and therefore $C=0$ for baryons and mesons. In analogy with electrostatic energy, we can argue that the potential energy due to the charm field would be lowest when the system is "neutral," namely, $C=0$. Thus all other unwanted configurations with $C\neq 0$, which include among others triplet, sextet, etc. representations, would have high masses, and hence would not be easily observed.

There have been proposed two different ways in which to introduce basic triplet or triplets with integral charges. One approach essentially involves a modification of the Nishijima–Gell-Mann relation by way of introducing an additional quantum number, the triality quantum number,[5] and this has led to considerations of higher symmetry schemes based on rank-three Lie groups.[6] On the other hand, Okubo *et al.*[7] have recently shown that the minimal group required for this purpose is actually the group $U(3)$.[8] It is shown that a triplet scheme may be defined in $U(3)$ such that the triplet always possesses integral values of charge and hypercharge and satisfies the Nishijima–Gell-Mann relation without a modification. The $U(3)$ triplet considered by Okubo *et al.* is of Sakata type; i.e., it consists of an isodoublet and an isosinglet. Actually, the $U(3)$ scheme is much more appealing than those of the rank-three Lie groups on two accounts: firstly, the Nishijima–Gell-Mann relation is satisfied universally by triplets as by octets and decuplets, and secondly as far as the hitherto realized representations are concerned, $U(3)$ is equivalent to $SU(3)$.[9]

In what follows, we show that the $U(3)$ scheme, when fully utilized as described below, naturally and uniquely

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[1] M. Gell-Mann, Phys. Letters **8**, 214 (1964); G. Zweig, CERN (to be published).

[2] Y. Nambu, *Proceedings of the Second Coral Gables Conference on Symmetry Principles at High Energy* (W. H. Freeman and Company, San Francisco, 1965).

[3] H. Bacry, J. Nuyts, and L. van Hove, Phys. Letters **9**, 279 (1964).

[4] This name was originally used in connection with the $SU(4)$ symmetry. B. J. Bjørken and S. L. Glashow, Phys. Letters **11**, 255 (1964); A. Salam, Dubna Conference Report, 1964 (unpublished).

[5] G. E. Baird and L. C. Biedenharn, *Proceedings of the First Coral Gables Conference on Symmetry Principles at High Energy* (W. H. Freeman and Company, San Francisco, 1964); C. R. Hagen and A. J. Macfarlane, Phys. Rev. **135**, B432 (1964) and J. Math. Phys. **5**, 1335 (1964).

[6] For example, see I. S. Gerstein and M. L. Whippmann, Phys. Rev. **137**, B1522 (1965). Earlier references are given in this paper.

[7] S. Okubo, C. Ryan, and R. E. Marshak, Nuovo Cimento **34**, 759 (1964).

[8] The use of $U(3)$ in this connection has also been remarked by I. S. Gerstein and K. T. Mahanthappa, Phys. Rev. Letters **12**, 570, 656(E) (1964).

[9] S. Okubo, Phys. Letters **4**, 14 (1963).

leads to a set of three basic triplets with integral charges, namely an I-triplet (isodoublet and isosinglet), a U-triplet (U-spin doublet and U-spin singlet) and a V-triplet (V-spin doublet and V-spin singlet).[10] These triplets arise from three different ways of defining charge Q, hypercharge Y, and a displaced isospin I_3 in the $U(3)$ group as opposed to the $SU(3)$, in such a way that the charge and hypercharge have integral values, while keeping the Nishijima–Gell-Mann relation intact, and they differ from each other in their quantum-number assignments as well as in their transformation properties under the Weyl reflections.[11] This is described in Sec. II. In Sec. III, a double $SU(3)$ symmetry scheme is proposed based on the three-triplet model in which the large mass splittings between different representations are ascribed to one of the $SU(3)$, and the other $SU(3)$ is, as usual, responsible for the mass splittings within a representation. The low-lying baryon and meson states may be taken as singlets with respect to one of the $SU(3)$. The extended symmetry group with respect to the $SU(6)$ symmetry is briefly discussed.

II. THREE TRIPLETS

We shall denote the infinitesimal generators of $U(3)$ by $A_\nu{}^\mu$ which satisfies the following commutation relations:

$$[A_\beta{}^\alpha, A_\nu{}^\mu] = \delta_\beta{}^\mu A_\nu{}^\alpha - \delta_\nu{}^\alpha A_\beta{}^\mu, \tag{1}$$

where all indices take on the values 1, 2, and 3. The corresponding infinitesimal generators $B_\nu{}^\mu$ of $SU(3)$ are then given by

$$B_\nu{}^\mu = A_\nu{}^\mu - \tfrac{1}{3}\delta_\nu{}^\mu A_\lambda{}^\lambda \tag{2}$$

which satisfy the following equations:

$$[B_\beta{}^\alpha, B_\nu{}^\mu] = \delta_\beta{}^\mu B_\nu{}^\alpha - \delta_\nu{}^\alpha B_\beta{}^\mu \tag{3}$$

and

$$B_\lambda{}^\lambda = 0. \tag{4}$$

Furthermore, the unitary restriction gives

$$(A_\nu{}^\mu)^\dagger = A_\mu{}^\nu, \quad (B_\nu{}^\mu)^\dagger = B_\mu{}^\nu. \tag{5}$$

Let us now briefly summarize the relevant results of Okubo *et al.* In the $SU(3)$ scheme, the charge Q, the hypercharge Y and the third component of isospin I_3 are identified as follows[12]:

$$Q = -B_1{}^1, \tag{6a}$$

$$Y = B_3{}^3 = -B_1{}^1 - B_2{}^2 \quad [\text{by the relation (4)}], \tag{6b}$$

$$I_3 = \tfrac{1}{2}(B_2{}^2 - B_1{}^1). \tag{6c}$$

In the $U(3)$ scheme, the corresponding quantities $\tilde{Q}$, $\tilde{Y}$, and $\tilde{I}_3$ are defined as follows:

$$\tilde{Q} = -A_1{}^1 = Q - \tfrac{1}{3}\tau, \tag{7a}$$

$$\tilde{Y} = -A_1{}^1 - A_2{}^2 = Y - \tfrac{2}{3}\tau, \tag{7b}$$

$$\tilde{I}_3 = \tfrac{1}{2}(A_2{}^2 - A_1{}^1) = I_3, \tag{7c}$$

where

$$\tau = A_1{}^1 + A_2{}^2 + A_3{}^3. \tag{8}$$

With these definitions, the Nishijima–Gell-Mann relation is seen to be equally satisfied by the $U(3)$ and $SU(3)$ theories, i.e.,

$$Q = I_3 + \tfrac{1}{2}Y \tag{9}$$

and

$$\tilde{Q} = \tilde{I}_3 + \tfrac{1}{2}\tilde{Y}, \tag{10}$$

respectively. Since the generators $A_1{}^1$, $A_2{}^2$, and $A_3{}^3$ possess integral eigenvalues in any representation,[13] the identifications of $\tilde{Q}$ and $\tilde{Y}$ to be the charge and the hypercharge, respectively, in $U(3)$ theory shall always lead to integral values for the charge and the hypercharge. In particular, in the three-dimensional representation, the $U(3)$ triplet has the eigenvalues

$$\tilde{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{I}_3 = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{Y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{11}$$

This triplet corresponds to the Sakata triplet which we call an I triplet for short.

We can now generalize the above constructions of the $U(3)$ triplet in the following way. Comparing (6b) and (7b), we see that a particular choice has been made for $\tilde{Y}$. Had we defined $\tilde{Y}$ to be $A_3{}^3$, it would still have integral eigenvalues but the relation (10) would have been violated. This is because $B_\lambda{}^\lambda = 0$ in $SU(3)$ but $A_\lambda{}^\lambda \neq 0$ in general in $U(3)$ and thus some care is needed in defining corresponding quantities in $U(3)$. Making use of (4), the definition in (6) can be written more generally as

$$Q = -B_1{}^1 = B_2{}^2 + B_3{}^3, \tag{12a}$$

$$Y = B_3{}^3 = -B_1{}^1 - B_2{}^2, \tag{12b}$$

$$I_3 = \tfrac{1}{2}(B_2{}^2 - B_1{}^1) = \tfrac{1}{2}(2B_2{}^2 + B_3{}^3) = -\tfrac{1}{2}(2B_1{}^1 + B_3{}^3). \tag{12c}$$

As in (7), replacing $B_\nu{}^\mu$'s in (12) by corresponding $A_\nu{}^\mu$'s, we list all possible candidates for the corresponding quantities in $U(3)$ which are now however not equivalent to each other [they are equivalent, of course, when reduced to $SU(3)$], i.e.,

$$\tilde{Q}: \quad -A_1{}^1, \quad A_2{}^2 + A_3{}^3, \tag{13a}$$

$$\tilde{Y}: \quad A_3{}^3, \quad -A_1{}^1 - A_2{}^2, \tag{13b}$$

$$\tilde{I}_3: \quad \tfrac{1}{2}(A_2{}^2 - A_1{}^1), \quad \tfrac{1}{2}(2A_2{}^2 + A_3{}^3), \quad -\tfrac{1}{2}(2A_1{}^1 + A_3{}^3). \tag{13c}$$

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[10] C. A. Levinson, H. J. Lipkin, and S. Meshkov, Nuovo Cimento **23**, 236 (1961); Phys. Letters **1**, 44 (1962) and Phys. Rev. Letters **10**, 361 (1963).

[11] A. J. Macfarlane, E. C. G. Sudarshan, and C. Dullemond, Nuovo Cimento **30**, 845 (1963).

[12] We use the sign convention of S. P. Rosen, J. Math. Phys. **5**, 289 (1964).

[13] For a derivation of this result, see Eq. (7) of Ref. 7.

To start with, the alternative choices in (13) provide twelve inequivalent ways in which to choose a set of three quantities $\tilde{Q}$, $\tilde{Y}$ and $\tilde{I}_3$ for the $U(3)$ scheme. In every choice $\tilde{Q}$ and $\tilde{Y}$ will have integral eigenvalues, but as can be easily checked the Nishijima–Gell-Mann relation will not be valid for all of them. In fact, there are only three cases for which it is valid and we are thus naturally led to three inequivalent triplets in the $U(3)$ scheme; they are defined by the following three choices:

$$t_I:\quad \tilde{Q}=-A_1{}^1,\qquad \tilde{Y}=-A_1{}^1-A_2{}^2,\qquad \tilde{I}_3=\tfrac{1}{2}(A_2{}^2-A_1{}^1),\tag{14a}$$

$$t_U:\quad \tilde{Q}=A_2{}^2+A_3{}^3,\qquad \tilde{Y}=A_3{}^3,\qquad \tilde{I}_3=\tfrac{1}{2}(2A_2{}^2+A_3{}^3),\tag{14b}$$

$$t_V:\quad \tilde{Q}=-A_1{}^1,\qquad \tilde{Y}=A_3{}^3,\qquad \tilde{I}_3=-\tfrac{1}{2}(2A_1{}^1+A_3{}^3).\tag{14c}$$

Now the first one, t_I, for which

$$\tilde{Y}=-A_1{}^1-A_2{}^2,\tag{15}$$

$$\tilde{I}_3=\tfrac{1}{2}(A_2{}^2-A_1{}^1)=\tfrac{1}{2}(B_2{}^2-B_1{}^1)=I_3\tag{16}$$

corresponds to the I triplet mentioned above.

The structure of the remaining triplets t_U and t_V can be brought to much more transparent and symmetric forms in terms of the U-spin and V-spin subalgebras.[10] As in the case of relations (9) and (10) for $SU(3)$ and $U(3)$, we define the U and V spin of $U(3)$ in exactly the same forms as in $SU(3)$ except that all quantities are tilded quantities. From the $SU(3)$ definitions,[12] we then have

$$\tilde{Y}_U=-\tilde{Q}=-A_2{}^2-A_3{}^3,\tag{17}$$

$$\tilde{U}_3=\tilde{Y}-\tfrac{1}{2}\tilde{Q}=\tfrac{1}{2}(A_3{}^3-A_2{}^2)=\tfrac{1}{2}(B_3{}^3-B_2{}^2)=U_3\tag{18}$$

for (14b), and

$$\tilde{Y}_V=\tilde{Q}-\tilde{Y}=-A_3{}^3-A_1{}^1,\tag{19}$$

$$\tilde{V}_3=-\tfrac{1}{2}(\tilde{Y}+\tilde{Q})=\tfrac{1}{2}(A_1{}^1-A_3{}^3)=\tfrac{1}{2}(B_1{}^1-B_3{}^3)=V_3\tag{20}$$

for (14c). They correspond, therefore, to a U triplet and a V triplet, respectively, and hence the notations t_I, t_U, and t_V. With respect to the $SU(3)$ triplet (quark), these $U(3)$ triplets have their respective "hypercharges" (i.e., Y, Y_U, and Y_V) shifted by the amount of $\frac{2}{3}$ and as such they have quite different transformation properties under the Weyl reflections W_1, W_2, and W_3[11] which are reflections about the axis $I_3=0$, $U_3=0$, and $V_3=0$, respectively. Whereas the $SU(3)$ triplet is invariant under all three Weyl reflections, the $U(3)$ triplets are not. They transform according to

$$W_1:\quad t_I\to t_I,\quad t_U\leftrightarrow t_V;\tag{21a}$$

$$W_2:\quad t_U\to t_U,\quad t_I\leftrightarrow t_V;\tag{21b}$$

$$W_3:\quad t_V\to t_V,\quad t_I\leftrightarrow t_U.\tag{21c}$$

Figure 1 and Table I(a) list the quantum numbers $\tilde{I}_3$ and $\tilde{Y}$ for the single triplet (quark) model; a possible assignment implied by the two-triplet model[2] is shown in Fig. 2 and Table I(b); the corresponding quantum numbers for the three-triplet model are given in Fig. 3 and Table I(c).

TABLE I. Quantum-number assignments for (a) the quark model, (b) the two-triplet model, and (c) the three-triplet model.

(a)

	quark		
$\tilde{I}_3$	$\frac{1}{2}$	$-\frac{1}{2}$	0
$\tilde{Y}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
$\tilde{Q}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$

(b)

	t_1			t_2		
$\tilde{I}_3$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0
$\tilde{Y}$	1	1	0	-1	-1	-2
$\tilde{Q}$	1	0	0	0	-1	-1

(c)

	$t_1(t_I)$			$t_2(t_U)$			$t_3(t_V)$		
$\tilde{I}_3$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$
$\tilde{Y}$	1	1	0	0	0	-1	0	0	-1
$\tilde{Q}$	1	0	0	0	-1	-1	1	0	0

III. DOUBLE $SU(3)$ SYMMETRY

Let us call the three triplets $t_1(=t_I)$, $t_2(=t_U)$, and $t_3(=t_V)$. Each triplet may be characterized in general by the average values, $\bar{I}_3$ and $\bar{Y}$, of $\tilde{I}_3$ and $\tilde{Y}$ for its three members. This specifies the location of the center of the triplet in the $\tilde{I}_3-\tilde{Y}$ diagram. Since $\bar{A}_1{}^1=\bar{A}_2{}^2=\bar{A}_3{}^3=\bar{\tau}/3=\tau/3$, Eq. (14) gives for the three definitions of

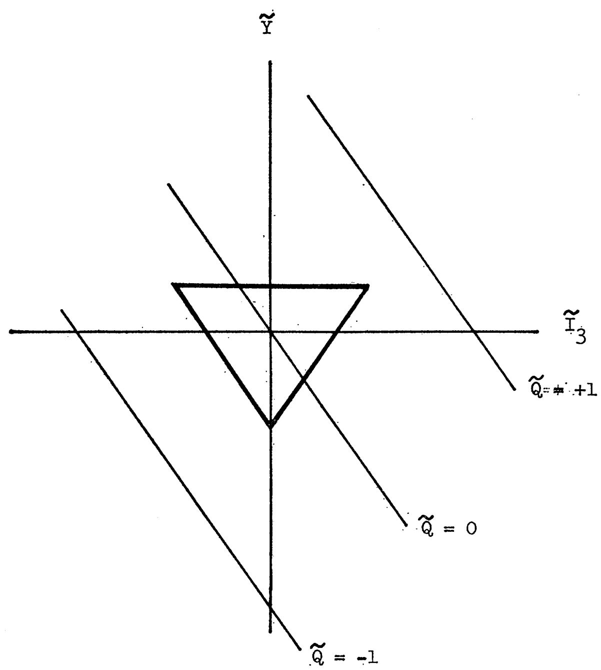

FIG. 1. The single-triplet (quark) model.

$\tilde{I}_3$ and $\tilde{Y}$,

$$\begin{aligned}\bar{I}_3&=0,\ \tfrac{1}{2}\tau,\ -\tfrac{1}{2}\tau,\\ \bar{Y}&=-\tfrac{2}{3}\tau,\ \tfrac{1}{3}\tau,\ \tfrac{1}{3}\tau,\end{aligned}\tag{22}$$

respectively, where $\tau=-1$ for all the triplets. We may define new quantities I_3, Y and $Q=I_3+\frac{1}{2}Y$ by the relations:

$$\begin{aligned}\tilde{I}_3&=\bar{I}_3+I_3,\\ \tilde{Y}&=\bar{Y}+Y,\\ \tilde{Q}&=\bar{I}_3+\tfrac{1}{2}\bar{Y}+I_3+\tfrac{1}{2}Y=\bar{Q}+Q.\end{aligned}\tag{23}$$

It is clear that I_3 and Y play the role of $SU(3)$ generators within each triplet. The charm number C defined in the two-triplet model[2] is then

$$\tfrac{1}{3}C=\bar{Q}=\bar{I}_3+\tfrac{1}{2}\bar{Y}.\tag{24}$$

Now it is interesting to note that according to Eq. (22) and Fig. 3, the centers of the three triplets form an antitriplet, equivalent to an antiquark, symmetrically located around the origin. Let us suppose that the nine members of the three triplets $t_{1\alpha}$, $t_{2\alpha}$, $t_{3\alpha}$, $\alpha=1, 2, 3$ be combined into a single multiplet $T=\{t_{i\alpha}\}$, $i=1, 2, 3$. We can then imagine two distinct sets of $SU(3)$ operations on T. One is the $SU(3)$ acting on the index α for each triplet, while the other $SU(3)$ acts on the index i, which mixes corresponding members of different triplets. T is then a representation $(3,3^*)$ of this group $G\equiv SU(3)'\times SU(3)''$.[14] The quantum numbers of $SU(3)'$ and $SU(3)''$ are identified as $I_3'=I_3$, $Y'=Y$, $I_3''=\bar{I}_3$ and $Y''=\bar{Y}$ in Eq. (22), so that

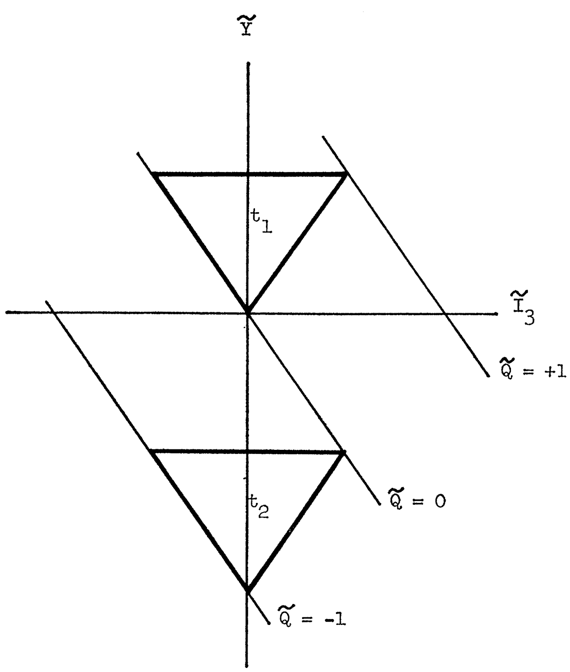

FIG. 2. The two-triplet model.

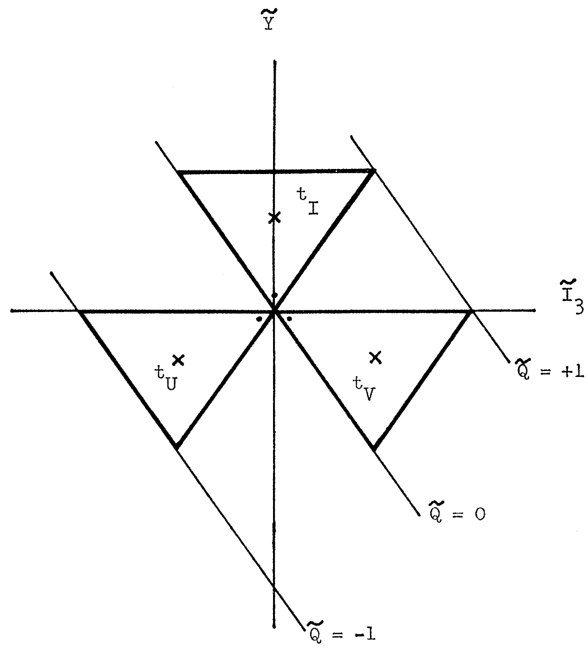

FIG. 3. The three-triplet model.

$$\begin{aligned}&\tilde{I}_3=I_3'+I_3'',\quad \tilde{Y}=Y'+Y'',\\ &\tilde{Q}=I_3'+I_3''+\tfrac{1}{2}Y'+\tfrac{1}{2}Y'',\\ &\tfrac{1}{3}C=I_3''+\tfrac{1}{2}Y''.\end{aligned}\tag{25}$$

A general representation of G may be characterized by four numbers p', q', p'', q'' so that $D(p',q',p'',q'')\sim D(p',q')\times D(p'',q'')$, where $D(p,q)$ is a representation of $SU(3)$. However, in our scheme where the nonet T is the fundamental field, we do not get all the possible representations of G. This can be illustrated by means of the triality numbers[5] $t'=p'-q' \bmod(3)$, $t''=p''-q'' \bmod(3)$. The nonet T has $t'=1$, $t''=-1$. All representations constructed out of T and T^* then satisfy $t'=-t''$.

Let us next consider the meson and baryon states $\sim TT^*$ and $\sim TTT$. The $SU(3)'\times SU(3)''$ contents of these 81- and 729-plets are

$$\begin{aligned}(3,3^*)\times(3^*,3)&=(8,1)+(1,1)+(1,8)+(8,8),\\ (3,3^*)\times(3,3^*)\times(3,3^*)&=(1,1)+2(8,1)+2(1,8)\\ &\quad+(1,10^*)+(10,1)+2(8,10^*)+2(10,8)\\ &\quad+4(8,8)+(10,10^*).\end{aligned}\tag{26}$$

It is an attractive possibility to postulate at this point that the energy levels are classified according to $SU(3)''$. The masses will then depend on the Casimir operators of $SU(3)''$. For example, a simple linear form will be

$$m=m_0+m_2C_2''+m_3C_3'',\tag{27}$$

[14] Such a nonet provides a natural basis for the symmetry of $SU(9)$. However, we will not consider it here.

where C_2'', C_3'' are the eigenvalues of quadratic and cubic Casimir operators of $SU(3)''$. In particular, we may assume that the main mass splitting comes from C_2''. Since this increases with the dimensionality of representation, the lowest mass levels will be $SU(3)''$ singlets. This selects the low-lying meson and baryon states to be (8,1), (1,1) and (8,1), (1,1), (10,1), respectively. In general, all low-lying states will have triality zero, $t'=t''=0$.

As for the baryon number assignment to the triplets, the simplest possibility would be to assign an equal baryon number, i.e., $B=\frac{1}{3}$, to them. In this case the triplets themselves would be essentially stable, and their nine members would behave like an octet plus a singlet of "heavy baryons" as may be seen from Fig. 3. Another simple possibility may be $B=\frac{1}{3}+Y''$, namely $B=(1,0,0)$ for (t_1,t_2,t_3). We expect a mass splitting depending on B or Y'', which may be the origin of the Okubo–Gell-Mann mass formula.

The advantage of the three-triplet model is that the $SU(6)$ symmetry can be easily realized with s-state triplets. The extended symmetry group becomes now $SU(6)'\times SU(3)''$. Since an $SU(3)''$ singlet is antisymmetric, the over-all Pauli principle requires the baryon states to be the symmetric $SU(6)$ 56-plet. Other $SU(6)$ representations such as the 70, will be obtained by bringing in either the orbital angular momentum or the "ρ spin" of the Dirac spinor triplets.

As in the two-triplet model mentioned in the Introduction, the mass formula of the type (27) may be derived dynamically. Instead of the charm number field, we introduce now eight gauge vector fields which behave as (1,8), namely as an octet in $SU(3)''$, but as singlets in $SU(3)'$. Since their coupling to the individual triplets is proportional to λ_i'' [the generators of $SU(3)''$], the interaction energy arising from the exchange of these vector fields will yield the first and second terms of Eq. (27). If these mesons obey again a similar type of mass formula, they will be expected to be massive compared to the ordinary mesons. However, it is not clear whether the resulting short-range character of the interaction can be readily reconciled with the postulated largeness of the interaction energy.

We may characterize the hierarchy of interactions and their symmetries implied by the above model as follows. First, the *superstrong* interactions responsible for forming baryons and mesons have the symmetry $SU(3)''$, and causes large mass splittings between different representations. The scale of mass involved would be comparable or large compared to the baryon mass, namely $\gtrsim 1$ BeV. The lowest states, i.e., $SU(3)''$ singlet states, would split according to $SU(3)'$, which would be the $SU(3)$ group observed among the known baryons and mesons, with their *strong* interactions. The scale of mass splitting would then be $\lesssim 1$ BeV.

When we go to the massive $SU(3)''$ nonsinglet states, there may very well be coupling between the two $SU(3)$ groups similar to the $L\cdot S$ coupling. The levels should be classified in terms of the three sets of Casimir operators formed out of λ_i', λ_i'', and $\lambda_i=\lambda_i'+\lambda_i''$, respectively. The splitting due to the coupling would naturally be intermediate between the above two splittings, namely ~ 1 BeV. Because of this coupling, the separate conservation of the two $SU(3)$ spins, I_3' and Y' on the one hand, and I_3'' and Y'' on the other, would be destroyed, and only the sums $I_3=I_3'+I_3''$ and $Y=Y'+Y''$ would be conserved under *strong* interactions. This in turn would mean that all the massive states are in general highly unstable, and decay strongly to the low-lying states. (In the two-triplet model, we considered only weak decays of $C\neq 0$ states. But strong decays are also a possibility as is contemplated here.)

We have discussed here a possible model of baryons and mesons based on three triplets. How can we distinguish this and other different models mentioned already? Certainly different models predict considerably different structure of massive states. These states are characterized by the triality for the quark model, by the charm number for the two-triplet model and by the $SU(3)''$ representation for the present three-triplet model. If we restrict ourselves to the low-lying states only, however, it seems difficult to distinguish them without making more detailed dynamical assumptions.

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